

L : no. of layers $L = 4$

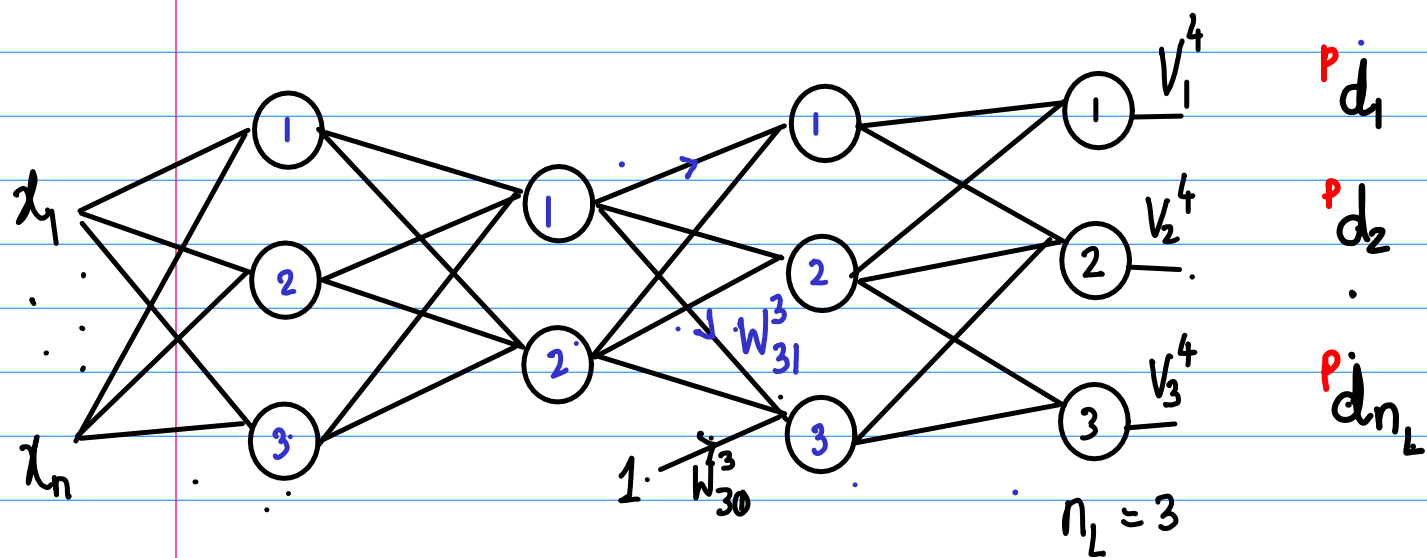
n_i : no. of neurons in layer 'i'

v_g^i : output of neuron g in layer i
 when x^p is presented at the input

x^p : p th pattern

x_i^p : i th feature/attribute/co-ordinate
 of the p -th pattern

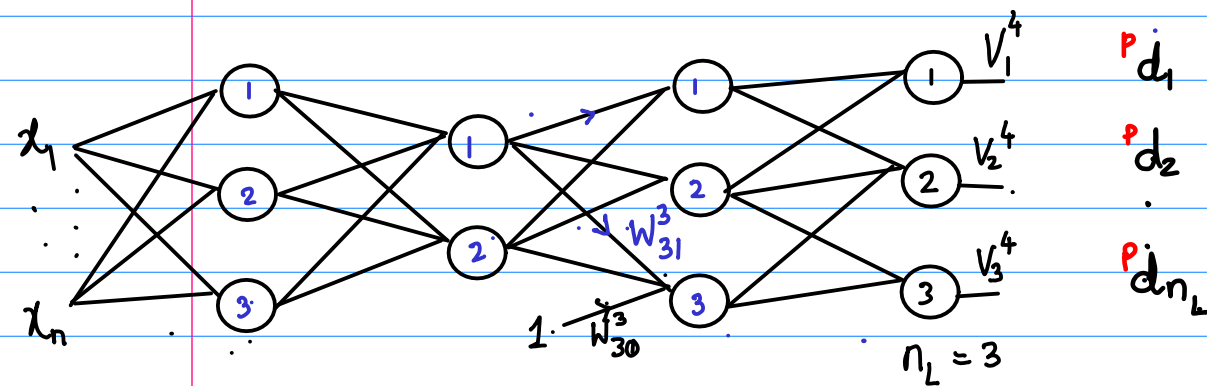
1 1 / \ 9 9
 6 6 6 - ..



$$\text{Min } \frac{1}{2} \frac{1}{P} \frac{1}{n_L} \sum_{p=1}^P \sum_{i=1}^{n_L} (d_i^p - V_i^L)^2$$

loss function
MSE
squared loss

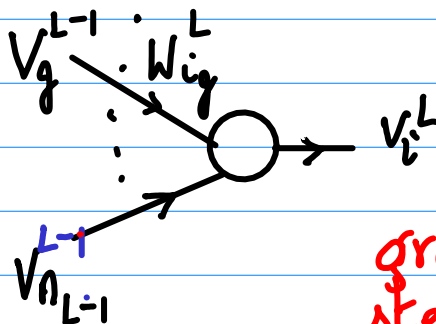
cross entropy



$$\text{Min } \frac{1}{2} \frac{1}{P} \frac{1}{n_L} \sum_{p=1}^P \sum_{i=1}^{n_L} (d_i^p - V_i^L)^2$$

loss function
MSE
squared loss

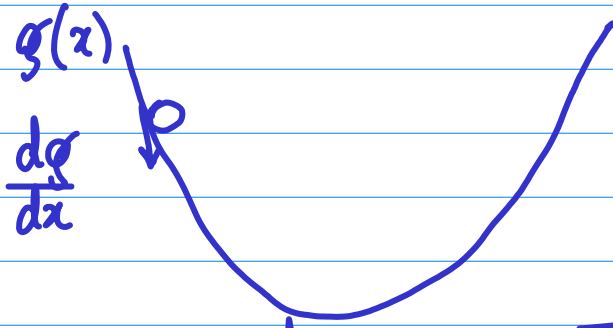
L
 w_{ig}^L



gradient descent
steepest descent

$${}^p V_i^L = f({}^p \text{net}_i^L)$$

$$E = \frac{1}{2} \sum_{p=1}^P \sum_{i=1}^{n_L} ({}^p d_i - {}^p V_i^L)^2$$



$$\dot{x} = \frac{dx}{dt} = - \frac{dg}{dx} \quad \rightarrow x$$

$$x = (x_1, x_2)^T \quad g(x_1, x_2)$$

$$\begin{aligned} \frac{dx_1}{dt} &= - \frac{\partial g}{\partial x_1} \\ \frac{dx_2}{dt} &= - \frac{\partial g}{\partial x_2} \end{aligned}$$

$$\frac{dx}{dt} \equiv \underline{\dot{x}} = -\nabla g(x)$$

$${}^p V_i^L = f({}^p \text{net}_i^L)$$

$$E = \frac{1}{2} \sum_{p=1}^P \sum_{i=1}^{n_L} ({}^p d_i - {}^p V_i^L)^2$$

$${}^p \text{net}_i^L = \sum_{g=1}^{n_{L-1}} W_{ig}^L {}^p V_g^{L-1} + W_{i0}^L$$

$${}^p \text{net}_i^L = \sum_{g=0}^{n_{L-1}} W_{ig}^L {}^p V_g^{L-1}$$

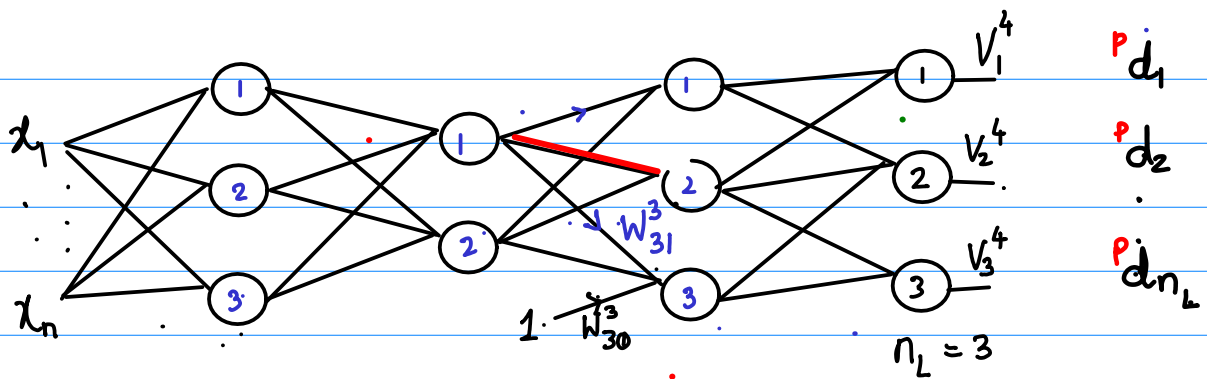
$$^p v_i^L = f(^p net_i^L)$$

$$E = \frac{1}{2} \sum_{p=1}^P \sum_{i=1}^{n_L} (^p d_i - ^p v_i^L)^2$$

$$^p net_i^L = \sum_{g=0}^{n_{L-1}} w_{ig}^L \cdot ^p v_g^{L-1}$$

$$\frac{dw_{kg}^L}{dt} = - \frac{\partial E}{\partial w_{kg}^L}$$

$$= \frac{1}{2} \cdot 2 \cdot \sum_{p=1}^P (^p d_k - ^p v_k^L) \cdot f'(^p net_k^L) \cdot ^p v_g^{L-1}$$



$$E = \frac{1}{2} \sum_{p=1}^P \sum_{i=1}^{n_L} (d_i - V_i^L)^2$$

$$\frac{dW_{21}^3}{dt} = - \frac{\partial E}{\partial W_{21}^3}$$

$$V_i^L = f(\text{net}_i^L)$$

$$= \sum_{p=1}^P \sum_{i=1}^{n_L} (d_i - V_i^L) \cdot \frac{dV_i^L}{dW_{21}^3}$$

$$= \sum_{p=1}^P \sum_{i=1}^{n_L} (d_i - V_i^L) \cdot f'(\text{net}_i^L) \cdot \boxed{\frac{d \text{net}_i^L}{dW_{21}^3}}$$

$$\text{net}_i^L = W_{i1}^L \cdot V_1^{L-1} + W_{i2}^L \cdot V_2^{L-1} + W_{i3}^L \cdot V_3^{L-1}$$

$$\text{net}_i^4 = W_{i1}^4 \cdot V_1^3 + \boxed{W_{i2}^4 \cdot V_2^3} + W_{i3}^4 \cdot V_3^3 + W_{i0}^4$$

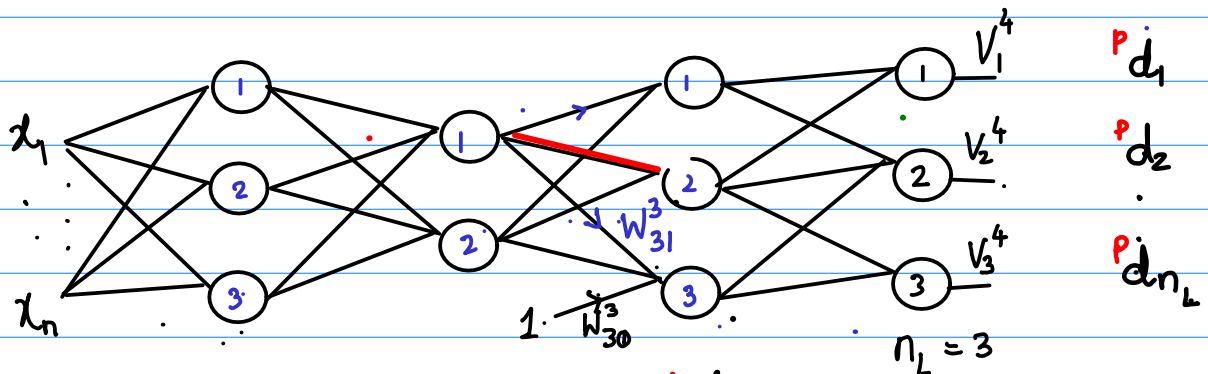
$$\frac{d \text{net}_i^L}{dW_{21}^3} = W_{i2}^4 \cdot \frac{dV_2^3}{dW_{21}^3} = W_{i2}^4 \cdot f'(\text{net}_2^3) \cdot \frac{d \text{net}_2^3}{dW_{21}^3}$$

$$V_2^3 = f(\text{net}_2^3)$$

$$\text{net}_2^3 = W_{21}^3 \cdot V_1^2 + W_{22}^3 \cdot V_2^2 + W_{20}^3$$

$$net_2^3 = W_{21}^3 P V_1^2 + W_{22}^3 P V_2^2 + W_{20}^3$$

$$\frac{d net_2^3}{d W_{21}^3} = P V_1^2$$



$$\delta_2^3 = (d_1 - V_1^4) f'(net_1^4) \cdot W_{12}^4$$

$$+ (d_2 - V_2^4) f'(net_2^4) \cdot W_{22}^4$$

$$+ (d_3 - V_3^4) f'(net_3^4) \cdot W_{32}^4$$

$$\frac{d W_{21}^3}{dt} = \delta_2^3 \cdot f'(net_2^3) \cdot V_1^2$$

$$\frac{d W_{21}^3}{dt} \approx \frac{W_{21}^3(t + \Delta t) - W_{21}^3(t)}{\Delta t} = -\frac{\partial E}{\partial W_{21}^3}$$

$$\frac{d W_{ij}}{dt} \approx \frac{W_{ij}(t + \Delta t) - W_{ij}(t)}{\Delta t} = -\frac{\partial E}{\partial W_{ij}}$$

$$\Rightarrow W_{ij}(t + \Delta t) = W_{ij}(t) + \Delta t \left[-\frac{\partial E}{\partial W_{ij}} \right]$$

$$W_{ij}(k+1) = W_{ij}(k) + \underset{\substack{\downarrow \\ \text{learning rate}}}{\eta} \left[\frac{-\partial E}{\partial W_{ij}} \right]$$