

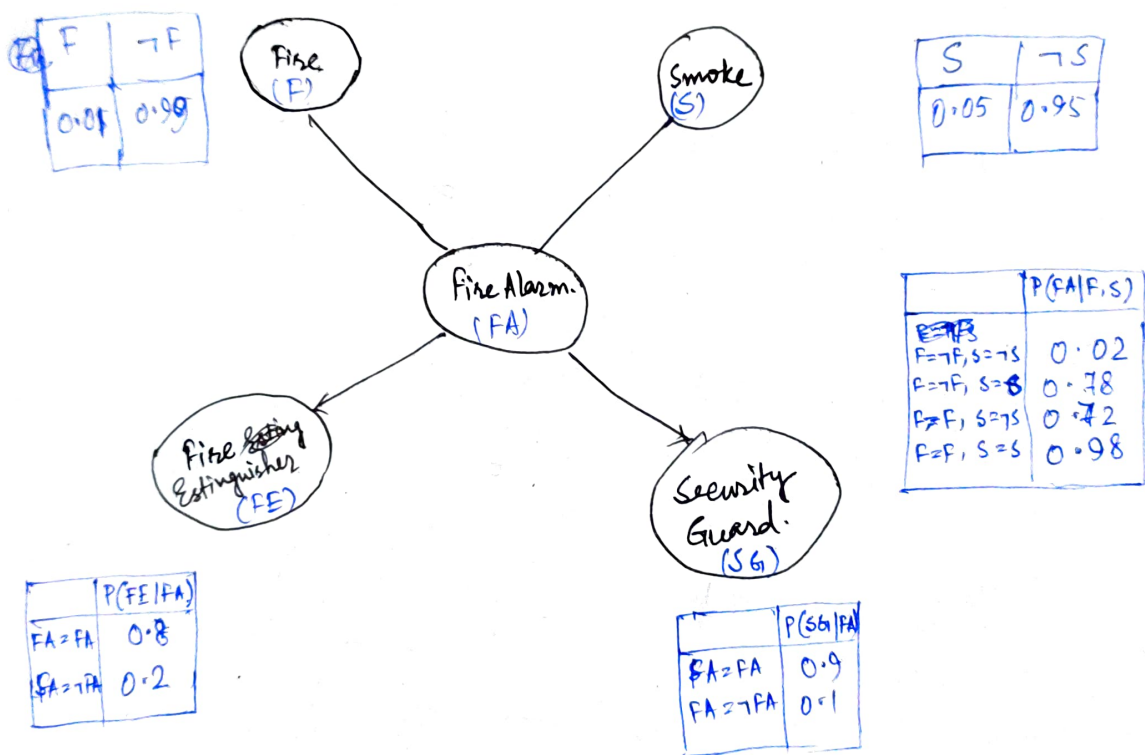
1. Independent events:

In the context of probability an independent event is defined as the event whose occurrence is not dependent upon any other event.

Ex: If we throw a dice and getting a no. as 2 is not dependent upon any other event.

Conditional Independence:

Conditional independence are called as the situations where a observation is irrelevant when evaluating the certainty of a hypothesis.



$$\text{Here } P(FE, SG, FA, F, S) = P(FE|FA) \times P(SG|FA) \times P(FA|F, S) \times P(F) \times P(S)$$

where  $P(FE|FA)$  is probability of charging fire extinguisher given fire alarm has been initiated.

$P(SG|FA)$  is probability of coming security guard given fire alarm has been initiated.

$P(FA|F, S)$  is the probability of initiating fire alarm given Fire and or Smoke found.

$P(F)$  is the probability of finding fire

$P(S)$  is the probability of finding smoke.

For the previous example. the Markov Blanket for node FA is

$$MB(FA) = F, S, FE \text{ and } SG$$

$$MB(FE) = FA$$

$$MB(SG) = FA$$

$$MB(F) = FA$$

$$MB(S) = FA$$

2.

Lets assume.

(C)  $P(A) \rightarrow a_1 = 0.183; a_2 = 0.462; a_3 = 0.355$

$P(B) \rightarrow b_1 = 0.407; b_2 = 0.593$

$P(C) \rightarrow c_1 = 0.697; c_2 = 0.303$

~~$P(M|X)$~~

	$x_1$	$x_2$
$m_1$	0.562	0.436
$m_2$	0.278	0.722
$m_3$	0.807	0.193

~~$P(X|Y)$~~

$P(M|Y)$

	$y_1$	$y_2$	$y_3$
$m_1$	0.523	0.138	0.339
$m_2$	0.327	0.397	0.276
$m_3$	0.260	0.474	0.266

from here we can calculate.

$$\begin{aligned}
 P(M|x_2 y_3) &= P(m_1|x_2 y_3) P(m_2|x_2 y_3) P(m_3|x_2 y_3) \\
 &= 0.436 \times 0.339 \quad 0.722 \times 0.276 \quad 0.193 \times 0.266 \\
 &= 0.148 \quad 0.199 \quad 0.051
 \end{aligned}$$

$$P(a_2 | x_2, y_3) \propto P(a_2, x_2, y_3)$$

$$= \alpha \sum_{B, C, M} P(a_2, B, C, M, x_2, y_3)$$

$$P(a_2, B, C, M, x_2, y_3)$$

$$= P(M | a_2, B, C) P(M | x_2) P(M | y_3) P(a_2) P(B) P(C)$$

note denoting  $p(a_2)$  as  $a_2$  similarly for others.

$$P(a_2, B, C, M, x_2, y_3)$$

$$= \alpha P(a_2) \times P(B) \times P(C) \times P(M | a_2, B, C) (P(m_1 | x_2, y_3) + P(m_2 | x_2, y_3) + P(m_3 | x_2, y_3))$$

$$+ P(a_2) \times P(B) \times P(C) \times P(M | a_2, B, C) (P(m_1 | x_2, y_3) + P(m_2 | x_2, y_3) + P(m_3 | x_2, y_3))$$

$$+ P(a_2) \times P(B) \times P(C) \times P(M | a_2, B, C) (P(m_1 | x_2, y_3) + P(m_2 | x_2, y_3) + P(m_3 | x_2, y_3))$$

$$+ P(a_2) \times P(B) \times P(C) \times P(M | a_2, B, C) (P(m_1 | x_2, y_3) + P(m_2 | x_2, y_3) + P(m_3 | x_2, y_3))$$

$$= 0.462 \times 0.407 \times 0.697 \times$$

$$P(a_2, B, C, M, x_2, y_3)$$

$$= P(a_2) P(B) \times P(C) (P(m_1 | a_2, B, C) P(m_1 | x_2, y_3) + P(m_2 | a_2, B, C) P(m_2 | x_2, y_3) + P(m_3 | a_2, B, C) P(m_3 | x_2, y_3))$$

$$+ P(a_2) P(B) \times P(C) (P(m_1 | a_2, B, C) P(m_1 | x_2, y_3) + P(m_2 | a_2, B, C) P(m_2 | x_2, y_3) + P(m_3 | a_2, B, C) P(m_3 | x_2, y_3))$$

$$+ P(a_2) P(B) \times P(C) (P(m_1 | a_2, B, C) P(m_1 | x_2, y_3) + P(m_2 | a_2, B, C) P(m_2 | x_2, y_3) + P(m_3 | a_2, B, C) P(m_3 | x_2, y_3))$$

$$+ P(a_2) P(B) \times P(C) (P(m_1 | a_2, B, C) P(m_1 | x_2, y_3) + P(m_2 | a_2, B, C) P(m_2 | x_2, y_3) + P(m_3 | a_2, B, C) P(m_3 | x_2, y_3))$$

$$= 0.462 \times 0.407 \times 0.697 (0.109 \times 0.148 + 0.316 \times 0.199 + 0.573 \times 0.051)$$

$$+ 0.462 \times 0.407 \times 0.303 (0.457 \times 0.148 + 0.320 \times 0.199 + 0.221 \times 0.051)$$

$$+ 0.462 \times 0.693 \times 0.697 (0.370 \times 0.148 + 0.330 \times 0.199 + 0.299 \times 0.051)$$

$$+ 0.462 \times 0.593 \times 0.303 (0.888 \times 0.148 + 0.079 \times 0.199 + 0.2032 \times 0.051)$$

$$= 0.0593 + 0.0556 + 0.0163$$

$$= 0.0132$$

(M | ABC) table is given in the next page



	$m_1$	$m_2$	$m_3$
$a_1 b_1 c_1$	0.426	0.263	0.310
$a_2 b_1 c_1$	0.718	0.179	0.109
$a_1 b_2 c_1$	0.485	0.272	0.242
$a_1 b_2 c_2$	0.452	0.453	0.094
$a_2 b_1 c_2$	0.109	0.316	0.573
$a_2 b_1 c_2$	0.557	0.320	0.221
$a_2 b_2 c_1$	0.370	0.330	0.299
$a_2 b_2 c_2$	0.888	0.079	0.032
$a_3 b_1 c_1$	0.334	0.261	0.404
$a_3 b_1 c_2$	0.482	0.246	0.277
$a_3 b_2 c_1$	0.363	0.266	0.369
$a_3 b_2 c_2$	0.382	0.116	0.500

2.

(a)

Pseudocode:

$a[3]$

for  $i = 1$  to 3.

$a[i] = \text{random value between } 0 \text{ to } 1.$

~~$a[i]$~~  / ~~sum~~

for  $i = 1$  to 3

$a[i] / (a[1] + a[2] + a[3])$

$b[2]$

for  $i = 1$  to 2

$b[i] = \text{random value between } 0 \text{ to } 1.$

for  $i = 1$  to 2

$b[i] / (b[1] + b[2])$

$c[2]$

for  $i = 1$  to 2.

$c[i] = \text{random between } 0 \text{ to } 1.$

divide each element by sum of 2.

$m[12][3]$ .

for  $i = 1$  to  $12$ .

for  $j = 1$  to  $3$ .

$m[i][j] = \text{random value between } 0 \text{ to } 5$ .

$\text{sum}[12]$ .

for  $i = 1$  to  $12$ .

$\text{sum } i = \text{sum of element of } i\text{th row of } m$ .

~~$m[i][j] = m[i]$~~

for  $i = 1$  to  $12$

for  $j = 1$  to  $3$ .

$m[i][j] = m[i][j] / \text{sum}[i]$ .

~~$x = y[3]$~~

~~for  $i = 1$  to  $3$ .~~

~~$x[i] = x - y[i]$~~

$x[3][2]$ .

for  $i = 1$  to  $3$ .

for  $j = 1$  to  $2$ .

$x[i][j] = \text{random value between } 0 \text{ to } 5$ .

~~$x$~~  divide each element of each row by the sum of the elements of the row.

$y[3][3]$

for  $i = 1$  to  $3$ .

for  $j = 1$  to  $3$ .

$y[i][j] = \text{random value between } 0 \text{ to } 5$

divide each element of each row by the sum of the elements of the row.

$$x - y[3].$$

for i = 1 to 3

$$x - y[i] = x[i][3] \times y[i][3].$$

$$a_1[4][3], a_2[4][3], a[4][3].$$

for i = 1 to 4

for j = 1 to 3

$$q[i][j] = m[i][j]$$

for i = 5 to 8

for j = 1 to 3

$$a_1[i][j] = m[i][j]$$

$$a_2[i-4][j] = m[i][j]$$

for i = 9 to 12

for j = 1 to 3

$$a_3[i-8][j] = m[i][j]$$

$$a_1 - x_2 - y_3[3], a_2 - x_2 - y_3[3], a_3 - x_2 - y_3[3].$$

for i = 1 to 4

for j = 1 to 3

$$a_1 - x_2 - y_3[i] = q[i][j] \times x - y[i] \times b[i/2] \times c[i/2]$$

$$a_2 - x_2 - y_3[i] = a_2[i][j] \times x - y[i] \times b[i/2] \times c[i/2]$$

$$a_3 - x_2 - y_3[i] = a_3[i][j] \times x - y[i] \times b[i/2] \times c[i/2]$$

~~Q10~~ ~~Q10~~

for  $i = 1$  to 3

$$a_1 x_2 y_3 = a_1 x_2 y_3 + a_1 - x_2 - y_3 [i]$$

$$a_2 x_2 y_3 = a_2 x_2 y_3 + a_2 - x_1 - y_3 [i]$$

$$a_3 x_2 y_3 = a_3 x_2 y_3 + a_3 - x_2 - y_3 [i].$$