Regulating Disclosure: The Value of Discretion

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Abstract

We investigate how to regulate the disclosure of verifiable and contractible information. Within a buyer-seller relationship, we compare generalized mandatory and discretionary (voluntary) disclosure policies. We characterize how the seller's optimal mechanism changes in response to regulation, and study the induced outcomes. Under a regularity condition – concavity of the revenue functions – discretionary policies cannot outperform mandatory disclosure policies regardless of the regulator's objective. Perhaps surprisingly, when this condition fails, discretion may be uniquely optimal. Finally, if the planner can design the verifiable information itself, it is without loss to mandate full disclosure. The results suggest that only in specific instances does discretion expand the set of regulatory possibilities.

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1 Introduction

Verifiable and contractible information determines peoples' opportunities. Smokers pay larger health insurance premiums. High credit scores facilitate favorable mortgage terms. Strong transcripts and recommendation letters advance job applicants through initial screening. Online browsing data is used to steer consumers to web pages, products, and advertisements.

Due to its prominence, contractible information is well-regulated, and existing policies take differing approaches. The majority of observed legislation prohibits the use of particular information outright. A prominent example is the Affordable Care Act (Obamacare), which disallows pre-existing conditions to be used as a determinant of insurance contracts.[1] In contrast, few policies give individuals the option to voluntarily disclose information like the recently-proposed American Data Privacy and Protection Act (ADPPA). This legislation requires companies to allow individuals to opt out of targeted advertising and other forms of personalization. Such discretionary disclosure policies are predicated under the idea that people should "own" their information.

The literature on privacy and price discrimination identifies which information made directly available to a seller may be socially beneficial (Bergemann et al., 2015). Yet, there is comparatively little understanding of how the ownership of that information affects outcomes. Does giving people control over their information, and the option to transact with it, lead to better outcomes than if it was freely available to the seller or non-contractible in the first place? A Coasian argument might suggest that the particular ownership of information, when viewed as a commodity, does not affect outcomes. Alternatively, a classical unraveling intuition hints that even if people own their information, skepticism from a counterparty leads them to fully disclose it nonetheless (Grossman, 1981). However, these arguments fail when an agent has private knowledge of both their preferences and the availability of contractible information.[2]

[1] Other examples include the "Ban the Box" campaign, which seeks to remove criminal history from the job application process. GINA prohibits discrimination according to genetic information. The GDPR provides protections against the misuse of online data in the EU.

[2] For instance, Schmitz (2001) shows the allocation of property rights does affect efficiency in the presence of private information, and efficiency may be maximized when property rights are unassigned. Dye (1985) presents a model in which, with some probability, an agent has no evidence to provide. The equilibrium only involves partial unraveling, and

In this paper, we study how a policymaker should regulate the disclosure and ownership of contractible information. We consider the interaction between a monopoly seller (she) and a buyer (he) in a model of mechanism design with evidence. The buyer has unit-demand and a privately-known valuation for the good. Additionally, with some probability, he may have access to evidence that can be credibly disclosed to the seller. The evidence is useful to the seller because it is statistically informative of the buyer's valuation, so it aids in price discrimination.[3] At the outset, the seller commits to a mechanism which maps the buyer's reported value and disclosed evidence into a price and probability of trade.

Disclosure regulation imposes constraints on how the buyer may misrepresent his evidence. Under a *selective mandate* policy, the buyer does not own his information and has no choice in disclosure. The regulator dictates which information must be disclosed and which information cannot. Alternatively, under a *discretionary* disclosure policy, the buyer owns his information and may choose to either disclose or conceal it. Therefore, the seller must incentivize the buyer to disclose by committing to more trade or better prices.

We begin by studying the seller's optimal mechanism for any given regulatory policy. The analysis is standard when disclosure is mandated: the seller posts the first-best price given her updated beliefs about the buyers type. In contrast, when the buyer has discretion, the optimal mechanism need not take this form. In order to best-incentivize the buyer's disclosure, the seller may find it optimal to randomize over prices and quantities, or to charge prices that are suboptimal conditional on observing certain evidence. We construct the optimal mechanism by characterizing the extreme points of the set of incentive compatible mechanisms, applying similar methods to Kleiner et al. (2021). At these extreme points, the disclosure constraint is related to the kind of mean-preserving-spread constraint found in problems of persuasion (Kamenica and Gentzkow, 2011). We conclude that the seller's value function sums the *monotone concavification* of each posted-price revenue curve across the realizations of evidence. The upshot of this technique is that it reduces the problem to a single-dimensional maximization, identifies the optimal mechanism by inspection, and is well-suited to comparative statics.

With the optimal mechanism in hand, we turn to the question of regulation.

there is concealment on-path. The essential feature that prevents disclosure is that an agent can misreport evidence in a way that is not detectable by the observer, i.e. reporting a different realization of evidence that occurs on-path.

[3]E.g. location data, browsing data, credit history, genetic tests

Given a particular regulation, the seller responds with her choice of an optimal mechanism, and the regulator evaluates the outcome that mechanism induces. Perhaps surprisingly, we show by example that a discretionary disclosure policy can be uniquely optimal if the regulator seeks to maximize consumer surplus or efficiency.[4] The intuition is that a policy of discretionary disclosure provides a downward pressure on the price that the seller charges to buyers with evidence in order to incentivize their disclosure. This downwards force may lead the optimal mechanism under discretion to serve strictly more buyers, and at a lower price, than any selective mandate policy, thus increasing efficiency and consumer surplus.

Our first result (Theorem 1) shows that such examples may be viewed as pathologies, rather than the norm. Under a regularity condition – concavity of the posted-price revenue functions – we show that discretionary disclosure cannot outperform a selective mandate, regardless of the regulator's objective.[5] Specifically, Theorem 1 shows concavity implies that the *outcome* induced by any discretionary disclosure policy must coincide with that of some selective mandate. As an auxiliary result, we also show that the seller has no value to commitment whenever the concavity assumption is satisfied. When the buyer has discretion, the seller's optimal mechanism can be implemented as an equilibrium of a disclosure game in which the seller's price best-responds to the disclosed evidence. Thus, one way to interpret Theorem 1 is that regulation induces a particular equilibrium of the disclosure game.

Our next key result (Theorem 2) extends the basic insight behind Theorem 1 to a setting in which the regulator additionally has the power to determine the information content of the evidence itself. Such flexibility by the regulator may be justified when the evidence resembles an index, such as a credit score, and the regulator may place restrictions on the weights or usage of certain inputs. Alternatively, the regulator may have the ability to coarsen existing information or import new information into the environment.[6] Theorem 2 states that any outcome achievable by some evidence structure and disclosure

[4]Of course, if the regulator wants to maximize the seller's revenue, then fully mandated disclosure will be optimal.

[5]Concavity, which is equivalent to decreasing marginal revenue, is a standard assumption in the reduced-form problem of a monopoly seller. Intuitively, it rules out belief distributions that are sufficiently bimodal. A more thorough discussion of this assumption is found in Section 5 with Theorem 1.

[6]Examples of "coarsening" include replacing individual data with aggregate data, or ZIP-code demographics with city-level demographics.

regulation is also achievable with fully-mandated disclosure (with a possibly-different evidence structure).

The two main results echo the same message: that only in specific instances does discretion expand the set of regulatory possibilities. Moreover, under additional assumptions about the evidence structure, we obtain sharp results showing the sub-optimality of discretionary policies. In particular, we additionally assume a standard Dye (1985) evidence structure in which each type of buyer has some probability of being able to perfectly disclose his valuation to the seller. In this case, we show that any discretionary policy yields lower consumer surplus than prohibiting disclosure altogether and lower total surplus than mandating full disclosure. These observations hinge on a simple comparative statics argument, and in the extensions we show how they apply in more general settings.[7]

Our results rationalize why much existing disclosure regulation takes the form of a mandate or ban, and inform ongoing policy discussions about the benefits of data ownership. A prominent argument in favor of data ownership is that, because online sellers must incentivize disclosure of that information, buyers will be able to reappropriate some of the surplus their information will generate. We demonstrate that this reasoning is incomplete because it ignores the seller's response to regulation. Importantly, the seller does not naively pay the buyer to disclose his evidence, but provides incentives by carefully adjusting the prices he faces upon disclosure. The net effect is that both the total surplus and the division of surplus may change with the information ownership structure.

This paper contributes to a rich literature on price discrimination and privacy. Seminal papers in these fields are attributed to Pigou (1920), Posner (1981), and Riley and Zeckhauser (1983), to name a few. For recent surveys, we direct the reader to Acquisti et al. (2016) and Bergemann and Bonatti (2019). One key innovation relative to this literature is that we study how the *seller* adjusts her optimal mechanism in response to regulation. Other papers which ask related questions, such as Glode et al. (2018), Pram (2021), and Ali et al. (2022), do not afford the seller commitment power: she myopically posts a price given the evidence disclosed by the buyer. The monopolistic power we assume – in addition to being realistic – is important because it allows the

[7]For example, the non-linear preferences over quality, or type-dependent revenue as in the case of an insurer.

seller to trade off direct revenue loss in exchange for incentivizing disclosure.[8] Identifying how the seller optimally manages this tradeoff is unique to this paper and central to our results. Our second contribution stems from our unified framework, which allows us to directly compare the differential impacts of regulatory structures within the same setting. As summarized by Acquisti et al. (2016), other papers in this field primarily focus on "simpler binary models contrasting regulation with its absence." Ichihashi (2020), for example, shows that within a class of mandated disclosure policies, restricting the amount of information the consumer reveals can increase consumer surplus. However, they do not consider giving the consumer the option to withhold evidence after it has been realized. In contrast, our framework allows us to explore a breadth of regulatory regimes, both discretionary and mandatory, rather than focusing on specific instances of regulation.

This paper also advances the literature on mechanism design with evidence through its study of statistical evidence and the analysis techniques involved. Such evidence structures are accommodated in the more abstract branch of this literature, including Green and Laffont (1986), Bull and Watson (2007), and Ben-Porath et al. (2021). Yet, the majority of "applied" papers which seek to solve a particular model assume evidence in which the buyer can perfectly prove his valuation (Dye (1985)). For example, see Pram (2022), Armstrong and Zhou (2022), Hidir and Vellodi (2021). Though convenient, this assumption is both unnecessary and – as we show in the extensions section – leads to incomplete conclusions.[9] The closest paper to ours within this literature is Sher and Vohra (2015), who also develop a model of price discrimination with evidence. They characterize the optimal mechanism by studying the dual and provide an implementation of that mechanism by a sequential bargaining protocol. While the baseline models are quite similar, our fundamental inquiry is different: we study regulation and changes in outcomes *across* evidence structures. For the purpose of answering these questions, their results are not immediately applicable, necessitating the analysis in this paper. We provide a more thorough discussion of the differing

[8]Geddes (2022), for instance, provides evidence that many insurance rating areas are served by a single insurance provider. Monopoly power may also arise due to search costs or other frictions.

[9]Under a Dye model, we show that all consumers are unambiguously hurt by a discretionary disclosure policy, and consumer surplus is maximized when disclosure is banned altogether. This is not true when evidence is statistical in nature, as our leading example shows.

approaches in Section 4 with the characterization of the optimal mechanism.

We now describe the structure for the remainder of the paper. Section 2 develops a tractable model of mechanism design with evidence in which we study the different regulatory regimes. Section 3 provides two numerical examples to illustrate the key tradeoffs in the model, and shows that discretion may be an optimal policy. Section 4 provides a constructive characterization of the optimal mechanism, which is used in Section 5 to present the two main theorems of the paper. Section 6 discusses some extensions of the baseline model to more general evidence structures and preferences. Section 7 concludes with some directions for future work!

2 Model

In this section, we lay out a simple, tractable model of mechanism design with evidence that allows us to study changes in disclosure regulation. The key features of the model that capture the environments we wish to study are

(1) A buyer with privately-known valuation for the good *and* private knowledge of the verifiable information he possesses

(2) A monopolistic seller with commitment power

(3) Regulation, which constrains how information can be disclosed to the seller, is decided **before** the seller designs her optimal mechanism

2.1 Primitives

The seller (the principal, she) designs a mechanism to sell a single good to a unit-demand, risk-neutral buyer (the agent, he) in order to maximize revenue. The buyer's private information is two-dimensional: his valuation and his verifiable evidence. First, he has a private valuation for the good denoted $\theta \in \Theta$ which parametrizes his willingness to pay for the good. We will often refer to θ as the "payoff type" of the buyer. We assume $\Theta = [\underline{\theta}, \bar{\theta}] \subseteq \mathbb{R}_+$, and there is prior belief of the buyer's type given by a CDF F and positive density f. When allocated the object with probability $q \in [0, 1]$ and asked to pay a transfer $t \in \mathbb{R}$, his payoff is $U(q, t; \theta) = \theta \cdot q - t$. The buyer's outside option is normalized to 0.

Additionally, the buyer privately observes his hard evidence. We model evidence by a finite set of realizations E, with a "null" element denoted $e_0 \in E$. E represents all possible of verifiable information the buyer may have, with e_0 meaning that the buyer has no evidence.[10] We relate the valuation of the buyer to the probability distribution over evidence via the function $p : \Theta \to \Delta(E)$. We write $p(e|\theta)$ to denote the conditional probability that the buyer has evidence e given the payoff-type θ. Evidence is thus statistically informative about the buyer's payoff-type, as would be a signal. Additionally, it is *verifiable*. There are technological limitations that prevent the buyer from arbitrarily manufacturing evidence he does not have.

Regulation determines which evidence the buyer is allowed to report to the seller for each realization of $e \in E$. We stress that we do *not* allow regulation to depend on the private valuation of the buyer, only the verifiable evidence that he possesses. Formally, we model *regulation* as a set-valued function $\gamma : E \to 2^E \setminus \emptyset$ where $\gamma(e) \subseteq \{e_0, e\}$. Let Γ denote the set of all such functions. The set $\gamma(e)$ describes the evidence that the buyer is allowed to present to the seller, conditional on his realization of $e \in E$. If $\gamma(e) = \{e_0\}$, that means the buyer cannot present his evidence e and must disclose e_0. The definition also requires $\gamma(e_0) = \{e_0\}$, so if the buyer has no evidence, he cannot disclose anything else. If $\gamma(e) = \{e\}$, the buyer must faithfully disclose his evidence. We can equivalently think of this case as the seller directly observing e whenever it arises. Finally, if $\gamma(e) = \{e, e_0\}$, then the buyer has the *discretion* to provide either e or e_0. That is, he can conceal any evidence he possesses without detection by the seller. In this baseline version of the model, we can think of this as the buyer "opting out" of providing his information.

We are broadly interested in two classes of regulatory policies. A regulatory policy $\gamma \in \Gamma$ is said to be a **selective mandate** if $|\gamma(e)| = 1$ for all $e \in E$. Denote the set of all selective mandate policies by Γ^M. A selective mandate gives the buyer no choice over the evidence he provides: each realization of the evidence either must be revealed ($\gamma(e) = \{e\}$) or cannot be revealed ($\gamma(e) = \{e_0\}$). A regulatory policy γ is said to be **discretionary** if it is not a selective mandate. That is, there exists $e \in E \setminus \{e_0\}$ such that $\gamma(e) = \{e, e_0\}$. We denote the set of all discretionary policies by Γ^D. The sets Γ^M and Γ^D partition the set of regulatory policies Γ.

[10]While we show in the appendix how our results generalize to richer evidence structures, we find that the generality provides minimal insight not captured by this simple framework.

2.2 The Seller's Design Problem

The seller designs a mechanism, taking as given a regulatory policy γ, in order to maximize revenue. In this environment, a revelation principle applies (Bull and Watson, 2007). It is without loss for the seller to consider direct mechanisms that incentivize truthful reporting and full disclosure.

A mechanism can thus be described as a pair of functions $q : \Theta \times E \to [0,1]$, $t : \Theta \times E \to \mathbb{R}$ that satisfy incentive compatibility and individual rationality constraints. Incentive compatibility for a buyer of payoff-type θ and evidence $e \in E$ requires that for all $\theta' \in \Theta$ and $e' \in \gamma(e)$

$$\theta q(\theta, e) - t(\theta, e) \geq \theta q(\theta', e') - t(\theta', e') \tag{IC(θ, e)}$$

Notably, the buyer may report any $\theta' \in \Theta$, yet cannot freely misreport his evidence. He may only report elements of $\gamma(e)$ to the mechanism, which is independent of the buyer's valuation of θ. Additionally, the individual rationality constraint requires the buyer to have non-negative surplus for all (θ, e).

$$\theta q(\theta, e) - t(\theta, e) \geq 0 \tag{IR(θ, e)}$$

Below, we state the maximization problem of the seller. [11]

$$\max_{q,t} \int_\theta \sum_{e \in E} t(\theta, e) p(e|\theta) dF(\theta) \tag{1}$$

$$s.t. \quad \text{IC}(\theta, e) \quad \forall \theta, e \tag{2}$$

$$\text{IR}(\theta, e) \quad \forall \theta, e \tag{3}$$

Finally, an **outcome** describes the mapping between the buyer's payoff-type and the realized quantity and transfer $O : \Theta \to [0,1] \times \mathbb{R}$. Ultimately, we wish evaluate different regulatory regimes based on the outcomes they induce. The outcome is the marginal distribution of the outcome function, integrating across the realized evidence.[12] Formally, a mechanism (q, t) *implements* an outcome $O = (o_q, o_t)$ if $o_q(\theta) = \sum_e q(\theta, e) p(e|\theta)$ and $o_t(\theta) = \sum_e t(\theta, e) p(e|\theta)$. An

[11] Note that it is without loss to require incentive compatibility for off-path realizations of (θ, e)

[12] In principle, one might wish to retain the evidence as part of the definition of an outcome. Our results continue to apply in this case, though an additional randomization by the principal in their implementation of the mechanism may be needed.

outcome is *implementable* if there exists some IC mechanism that implements it.

Finally, the outcome that prevails under a given regulatory regime is one that maximizes the seller's revenue. To that end, let $O(p, \gamma)$ denote the set of outcomes that are implemented by a seller-optimal mechanism when the evidence structure is given by p and the regulation by γ.[13] We emphasize that the seller chooses a mechanism *after* a regulatory policy has been announced.

The regulator has a preference over outcomes, denoted by $\succeq$, and evaluates policies by the outcomes they induce in the seller's optimal mechanism. For two regulatory policies, $\gamma \succeq \gamma'$ iff $O(p, \gamma) \succeq O(p, \gamma')$. In the event that there are multiple optimal outcomes for the seller, the regulator breaks ties according to $\succeq$ from each set. Our task is now to identify, for each regulatory regime, the outcome that is implemented in the seller's *optimal* mechanism.

2.3 Discussion of Assumptions

Before we move on to characterizing the optimal mechanism, we conclude this section with a brief discussion of some of the key assumptions of the model.

Monopolistic Seller: In our leading examples of online marketplaces, insurance providers, and labor markets, it is well-established these markets are not perfectly competitive. In some instances, this is because the provider is truly a monopolist, as we find in some private health insurance markets.[14] Otherwise, they may hold some monopoly power due to search frictions, information costs, or consumer capture. Thus, assuming monopoly power is a reasonable approximation and a useful assumption, in that it allows us to study how the seller responds to legislation in a non-trivial manner. A model of oligopolistic competition may be appealing in this setting, but because we insist on the design approach, we may need to allow the competing principals to engage in more complex behavior, such as eliciting the competing principal's menu from the agent and private communications with the other principal (Attar et al., 2022). While, we find such extensions intriguing and possibly policy-relevant, we believe they obscure the main analysis of this paper, so we leave them to future work.

[13]Though generically unique, in knife-edge cases there the seller may have multiple optimal mechanisms.

[14]Geddes (2022) shows, for example, that a sizeable portion of rating areas in the US have a single private healthcare provider

Commitment Power: In the market settings we describe, it is natural to think of the seller as being able to commit to her response to information provided by the buyer. Though some readers may be disturbed by the seller's ability to commit to a buyer's disclosure, we see no meaningful reason why the seller may be able to commit to respond to a cheap-talk report, but not to disclosure of evidence. We observe such mechanisms frequently in the real world: museums give discounts to seniors and students, car insurance companies give discounts to students with good grades, etc. Moreover, the implementation of pricing by algorithm, as is frequent in online marketplaces, affords this form of commitment power. By definition, such an algorithm is a mapping from provided information, such as browser history, to prices. After concluding the analysis of the baseline model of mechanism design, we illustrate how our results extend to environments without commitment.

Exogenously-Determined Evidence: The main restrictive assumption in our model is that the buyer's evidence exists independently of both the chosen regulation and the seller's optimal mechanism. That is, the buyer cannot generate or acquire evidence in anticipation of disclosing it to the seller. This assumption is justified if either the evidence is controlled by a third-party that the buyer cannot manipulate, such as the result of a genetic test, or the buyer has sufficiently strong incentive to not manipulate it in the first place, either because it is too costly to do so, or because the buyer has other instrumental uses for that evidence. When this assumption is violated, and the buyer does endogenously generate evidence, we return to familiar arguments by Hirshleifer (1971) about the possible inefficiencies generated by information acquisition.[15] Our main theorems need not apply when the evidence generation process is type-dependent, as policies of voluntary and mandatory disclosure provide different incentives to collect evidence in the first place. Although orthogonal to the central analysis, we find this feature to be a promising direction to investigate further.

3 Illustrative Examples

We develop two simple numerical examples that illustrate the key forces in this model, and the ways in which discretion may be a beneficial policy. For

[15] Another complication stems from situations in which the evidence also informs the buyer about his type, as would a statistical test. Then, a change in regulation or mechanism additionally leads to a different distribution of buyer types.

particular distributions over valuations and evidence, we present the seller's optimal mechanism under mandatory and discretionary policies. We also give an indication towards how Theorem 2 applies in these environments, and show that same *outcome* induced by discretion can also be achieved by a mandate with a different distribution over evidence.

In the first example, we show that the seller may find it optimal to randomize over posted prices in order to satisfy the disclosure constraint. It turns out that the seller's optimal mechanism *increases* efficiency relative to any mandate or banning of the evidence. Thus, we demonstrate that discretion may be a strictly valuable regulatory policy. In the second example, we show how a seller might strategically alter her posted prices, even without randomizing, in order to satisfy the disclosure constraint without a significant loss of revenue. In this setting, consumer surplus and efficiency is harmed by discretion.

In both examples, we specialize to the case of $E = \{e_0, e_1\}$, so the buyer either has evidence (e_1), or he does not (e_0). Thus, a selective mandate policy either requires e_1 to be disclosed or bans its use altogether. A discretionary policy allows the buyer to disclose e_1 whenever it is realized. We consider three discrete buyer valuations: $\theta \in \{1, 2, 3\}$. For expositional convenience, we directly describe the joint distribution over the buyer's value and the evidence he holds.[16]

3.1 Optimal Discretion & Randomization

In this example, the prior belief over buyer types is uniform $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. All buyers with valuations of either 1 or 3 are given evidence (e_1), while those with valuation 2 have no evidence (e_0). We describe the optimal mechanisms for the seller in the case where disclosure of e_1 is banned, mandated, and voluntary.

- e_1 banned: the seller posts a price of 2 to all buyers, and earns a revenue of $\frac{4}{3}$

- e_1 mandated: the seller posts a price of 2 to all buyers who present e_0, and a price of 3 to buyers who present e_1. The seller's revenue is $\frac{5}{3}$.

[16]While the examples provided give 0 probability to some pairs of (θ, e), this is not an essential feature. The examples and optimal mechanisms described are not knife-edge, and can be easily modified to the cases when probabilities are strictly positive.

	$\theta = 1$	$\theta = 2$	$\theta = 3$
e_0	0	$\frac{1}{3}$	0
e_1	$\frac{1}{3}$	0	$\frac{1}{3}$

Table 1: Distribution Over (θ, e) for Example 3.1

– e_1 voluntary: the seller posts a price of 2 to all buyers who present e_0. If the buyer presents e_1, then the seller *randomizes* equally between prices of 1 and 3, and posts the realized price. The seller's revenue is $\frac{3}{2}$.

The key takeaway from this example is that the seller may randomize over prices *after* evidence is disclosed in order to satisfy the disclosure constraint when the buyer has discretion. The seller cannot simply post the prices as-if the evidence was mandated, because the buyers with evidence strictly would prefer to conceal it. This randomization over prices can be superior to simply posting the same price to all buyers. Another interpretation, rather than randomization over posted prices, is that the seller gives the buyer a probabilistic allocation of the good, $q(\theta = 1, e_1) = \frac{1}{2}$, and the corresponding expected transfer.

Finally, notice that efficiency is strictly larger under a policy of discretionary disclosure, as opposed to a mandate or ban. Under discretion, all buyers with a valuation of 2 and 3 are served fully, and half of the buyers with a valuation of 1 will purchase the good. Thus, discretion may be strictly beneficial.

Next, we show that the outcome achieved in the seller's optimal mechanism under a policy of discretion can be replicated by an alternative distribution over evidence and mandatory disclosure. Importantly, the prior belief distribution of the buyer's types is fixed, and we only change the likelihood of their obtaining specific evidence.

Table 2 displays the constructed distribution. Half of the buyers with a valuation of $\theta = 1$ are given evidence (e_1), and all the remainder are not. Under a mandated disclosure policy, the seller posts a price of 2 to the buyers with e_0, and a price of 1 to the buyers with e_1. It is easy to verify that the outcome is equivalent to the one under the original distribution under discretionary disclosure. All buyers with valuations of 2 or 3 are served at a

	$\theta = 1$	$\theta = 2$	$\theta = 3$
e_0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
e_1	$\frac{1}{6}$	0	0

Table 2: Mandate-Equivalent Distribution for Example 3.1

price of 2, while only half of the buyers with a valuation of 1 are served at a price of 1.

3.2 Ex-Post Suboptimal Pricing

In this example, the prior belief over the buyer's valuations is $(\frac{3}{7}, \frac{3}{7}, \frac{1}{7})$. Opposite to the previous example, only buyers with a valuation of $\theta = 2$ are given evidence (e_1), while those with valuations 1 and 3 are not.

	$\theta = 1$	$\theta = 2$	$\theta = 3$
e_0	$\frac{3}{7}$	0	$\frac{1}{7}$
e_1	0	$\frac{3}{7}$	0

Table 3: Distribution Over (θ, e) for Example 3.2

The optimal mechanisms for the seller when disclosure of e_1 is banned, mandated, and voluntary are as follows:

- e_1 banned: the seller posts a price of 2 to all buyers, and earns a revenue of $\frac{8}{7}$
- e_1 mandated: the seller posts a price of 1 to all buyers who present e_0, and a price of 2 to buyers who present e_1. The seller's revenue is $\frac{10}{7}$.
- e_1 voluntary: the seller posts a price of 3 to all buyers who present e_0, and a price of 2 to buyers who present e_1. The seller's revenue is $\frac{9}{7}$.

Like the previous example, the seller cannot post the optimal prices as-if disclosure was mandated. However, rather than randomize, the seller chooses to sharply increase the price following no disclosure. While this is not the optimal price *conditional* on e_0, it relaxes the disclosure constraint and allows better price-discrimination of the buyers with evidence e_1. The net result is that efficiency and consumer surplus are both *minimized* under a policy of discretion.

As in the previous example, we see that the seller need not shortsightedly post the optimal price given the evidence she observes. Her commitment power allows her to do strictly better by trading off "direct" revenue loss in exchange for cheaply incentivizing disclosure. In other words, under a discretionary disclosure policy, there is value to the seller's commitment. It is straightforward to verify that there is no equilibrium of a disclosure-and-pricing game in which the seller can attain the revenue of $\frac{9}{7}$. We investigate this observation further after concluding analysis of the baseline model.

We conclude by again presenting a different joint distribution over valuations and evidence that induces the same outcome under a mandate as the original optimal mechanism did under discretionary disclosure.

	$\theta = 1$	$\theta = 2$	$\theta = 3$
e_0	$\frac{1}{7}$	0	$\frac{1}{7}$
e_1	$\frac{2}{7}$	$\frac{3}{7}$	0

Table 4: Mandate-Equivalent Distribution for Example 3.2

Given the distribution described by Table 4, and a policy of mandated disclosure, the seller posts a price of 3 following e_0 and a price of 2 following e_1. The end result is that all buyers with a valuation of 2 and 3 are served, but are left with no surplus. This is the same outcome that was generated by the optimal mechanism under the original distribution and voluntary discretion.

4 The Seller's Optimal Mechanism

Having seen that the seller's revenue-maximizing mechanism need not take the form of posting the myopically optimal price, we seek a characterization of the optimal mechanism. First, we describe the optimal mechanism under the case of mandated disclosure, which is largely standard. Then, we turn our attention to the case of discretionary disclosure and give a constructive characterization of the optimal mechanism.

4.1 Benchmark: Mandated Disclosure

We briefly review the standard analysis in the case of fully-mandated disclosure, as it will highlight the intricacies introduced by the buyer's voluntary disclosure.

We consider the regulatory regime in which $\gamma(e) = \{e\}$ for all $e \in E$. That is, the buyer is required to faithfully disclose her evidence. In this case, the seller has no disclosure constraints to consider, and it is as-if the seller designs a separate mechanism conditional on each observed realization of the evidence. The remaining IC constraint specifies that the buyer should report his type truthfully. The problem is totally separable across realizations of evidence, and we perform the analysis for a single realization of $e \in E$. Let G denote the posterior belief distribution of the buyer's valuation conditional on observing e. The seller maximizes by choice of $q : \Theta \to [0,1]$ and $t : \Theta \to \mathbb{R}$

$$\max_{q,t} \int_\theta t(\theta) dG(\theta) \tag{4}$$

$$s.t. \quad \theta q(\theta) - t(\theta) \geq \theta q(\theta') - t(\theta') \quad \forall \theta, \theta' \tag{5}$$

$$\theta q(\theta) - t(\theta) \geq 0 \quad \forall \theta \tag{6}$$

As usual, the IC constraint is equivalent to the envelope condition and monotonicity. If we denote the indirect utility of the buyer by $U(\theta) := \theta q(\theta) - t(\theta)$, we have

$$U(\theta) = \int_{\underline{\theta}}^{\theta} q(s) ds + U(\underline{\theta}) \tag{7}$$

$$q(\cdot) \text{ non-decreasing} \tag{8}$$

Finally, we identify that at the optimum, $U(\underline{\theta}) = 0$, and the IR constraints are satisfied. After substituting the envelope constraint and performing integration by parts, we simplify the problem to:

$$\max_{q} \int_\theta q(\theta) \left[\theta - \frac{1 - G(\theta)}{g(\theta)}\right] g(\theta) d\theta \tag{9}$$

$$s.t. \quad q(\cdot) \text{ non-decreasing} \tag{10}$$

$$q \in [0,1] \tag{11}$$

The term in brackets, $\theta - \frac{1-G}{g}$ is often referred to as the virtual-value of the buyer. It incorporates the information rents that must be paid to buyers with types higher than θ when type θ is allocated the good. It is known that an optimal mechanism in this case is implemented by a posted-price.[17] All types higher than the posted price of θ^* are fully allocated the object, and all must pay θ^*. This conclusion is immediate by pointwise maximization of q under the integral whenever virtual values are monotone.[18] The conclusion still holds without the monotonicity condition, and the usual argument proceeds by an ironing procedure.

Another way to arrive at this conclusion is to observe that the objective function is linear in the choice object q. Additionally, the choice set (non-decreasing functions to $[0,1]$) is convex and compact in the norm topology. Thus, the optimal mechanism can be identified by the *extreme points* of the constraint set.[19] Choquet's theorem, a celebrated extension of the Krein-Millman theorem, posits that any element of the set can be written as a convex combination of its extreme points. Because the objective is linear, a mechanism is optimal if and only if it is the convex combination of optimal extreme points.

It is easy to see that the extreme points of the set of incentive compatible allocations is the set of step functions that jump from 0 to 1 at some value $\theta_0 \in [\underline{\theta}, \bar{\theta}]$. Such mechanisms correspond exactly to posted-price mechanisms. All types with valuations above θ_0 are served fully, and the envelope condition identifies that they uniformly pay θ_0. The revenue of a posted-price mechanism is simply $R(\theta) = \theta(1 - G(\theta))$. So, we arrive at the following familiar conclusion: whenever there is mandated disclosure – i.e. a market segmentation – the seller posts the optimal price to each segment.

Proposition 1. *Under a selective mandate, there exists an optimal mechanism which posts a price conditional on each realization of the evidence. Every optimal mechanism is a convex combination of optimal posted-price mechanisms.*

[17] For particular distributions, there may be multiple optimal mechanisms, though this is generically not the case.

[18] Single-crossing from below also suffices in this environment.

[19] An extreme point of a convex set S is one that cannot be written as the convex combination of distinct elements. We refer the reader to Kleiner et al. (2021) for a more thorough discussion of the usefulness and applicability of the extreme points of the choice set.

4.2 Voluntary disclosure: Extremal Mechanisms

We now turn to the central case of *fully* discretionary disclosure. That is, $\gamma(e) = \{e_0, e\}$ for all $e \in E$. Given an evidence realization $e \in E$, the buyer may choose to provide either e or e_0 to the mechanism. Of course, if the buyer realizes e_0, he has no choice but to provide it.

Notice that any intermediate case of discretionary disclosure, where the buyer is sometimes given discretion but not for all realizations of e, is easily handled between the previous analysis and this one. Whenever $\gamma(e) = \{e\}$, the seller can freely post her optimal price conditional on observing e without affecting the rest of the mechanism. Whenever $\gamma(e) = \{e_0\}$, an equivalent problem for the seller can be constructed by re-defining the evidence generation function p to map all realizations of e into e_0.

The first step in simplifying the problem is to note that the seller need not consider double-deviations by the buyer. That is, it is sufficient for a buyer of type (θ, e) to not want to misreport either component individually. This is because, conditional on misreporting his evidence, the *best* deviation is to report his type truthfully. Thus, we have two classes of IC constraints: the "within-evidence" constraint and the "across-evidence" or "disclosure" constraint.

Formally, denote the expected utility of a buyer who reports $(\hat{\theta}, \hat{e})$ when his true payoff-type is θ to be

$$U(\hat{\theta}, \hat{e}; \theta) := \theta q(\hat{\theta}, \hat{e}) - t(\hat{\theta}, \hat{e})$$

And the indirect utility of truthfully reporting as

$$U(\theta, e) := \theta q(\theta, e) - t(\theta, e)$$

Incentive compatibility is defined as: for all $\theta, \theta' \in \Theta$, $e \in E$, $e' \in \{e, e_0\}$

$$U(\theta, e; \theta) \geq U(\theta', e'; \theta) \tag{12}$$

Which is equivalent to the two conditions:

$$U(\theta, e; \theta) \geq U(\theta', e; \theta) \quad \forall \theta, \theta' \in \Theta \quad \forall e \in E \tag{within-e}$$

$$U(\theta, e; \theta) \geq U(\theta, e_0, \theta) \quad \forall \theta \in \Theta \quad \forall e \in E \tag{across-e / "disclosure"}$$

The "within-e" constraint has the same characterization as in the classical model. For each $e \in E$, it is equivalent to monotonicity of $q(\cdot, e)$ plus the envelope condition:

$$U(\theta, e) = \int_{\underline{\theta}}^{\theta} q(s, e) ds + U(\underline{\theta}, e) \tag{13}$$

Finally, before re-writing the seller's problem, it is useful to translate to the space of posterior beliefs of the seller *if* she were to directly observe the buyer's evidence. Let $G(\theta|e)$ and $g(\theta|e)$ denote the CDF and corresponding density computed by Bayes rule, and let $p(e)$ denote the ex-ante probability of observing e. Making the usual substitutions, and applying integration by parts, we arrive at a new formulation:

$$\max_{q, U(\underline{\theta}, \cdot)} \sum_{e \in E} p(e) \left(\int_{\theta} q(\theta, e) \left[\theta - \frac{1 - G(\theta|e)}{g(\theta|e)} \right] g(\theta|e) d\theta - U(\underline{\theta}, e) \right) \tag{14}$$

$$s.t. \quad q(\cdot, e) \text{ non-decreasing} \quad \forall e \tag{15}$$

$$\int_{\underline{\theta}}^{\theta} q(s, e) ds + U(\underline{\theta}, e) \geq \int_{\underline{\theta}}^{\theta} q(s, e_0) ds + U(\underline{\theta}, e_0) \quad \forall e, \theta \tag{16}$$

$$U(\underline{\theta}, e) \geq 0 \quad \forall e \tag{17}$$

Besides the disclosure constraint (16), the problem is fairly standard. It is easy to see that $U(\underline{\theta}, e_0) = 0$ at the optimum because reducing it improves the seller's payoff and relaxes the disclosure constraint. However, there is not quite as direct an argument to eliminate $U(\underline{\theta}, e)$ for $e \neq e_0$. Shifting the payoffs down by a constant may violate a binding disclosure constraint.

Indeed, much of the mathematical novelty of the problem lies in the interaction of disclosure constraint, and how we surmount it. It turns out that this constraint may bind in surprising ways, such as only for buyers with low and high valuations, but not for intermediate types. Additionally, it may be that the constraint binds – the solution to the seller's problem violates the constraint if it is ignored – but it does not hold with equality for all (or even the highest) types at the optimum.

One method of attack, as taken in Sher and Vohra (2015), would be to study the dual. This is useful insofar as it provides qualitative features about what might occur in the optimal mechanism, as they discuss in their paper. Ultimately, however, this approach leaves us with a large (infinite) system of equations with endogenous coefficients that characterize the optimum.[20]

[20] Another complication is that it is necessary to carry the monotonicity constraint as well. Imposing a kind of monotone virtual-value assumption is not only unecessary, but it rules out some interesting optimal mechanisms for the seller, such as the one in Example 2.

Notably, it is not straightforward to use this approach to study how the solution changes with the parameters or regulatory policy. This motivates a new perspective of analysis that is more suitable for the purposes of comparative statics.

As in the previous section, we now characterize the set of extremal mechanisms, which will immediately shed light on the shape of the optimal mechanism in this environment. Taking our choice variables to be the function $q : \Theta \times E \to [0,1]$ and values $(U(\underline{\theta}, e))_{e \in E}$, we again observe that the objective function is linear, and the constraint set convex. By imposing a sufficiently large bound on the values of $U(\underline{\theta}, \cdot)$, say $[0, \overline{U}]$ it is also compact. [21] So, we again seek to identify the extreme points of this relation. We present those results in the following lemma, where we exclude the trivial solutions in which $U(\underline{\theta}, \cdot) = \overline{U}$.

Proposition 2. *Suppose $(q, U(\underline{\theta}, \cdot))$ constitute an extreme point of the set of feasible mechanisms. Then:*

(1) There exists $\theta_0 \in [\underline{\theta}, \overline{\theta}]$ such that $q(\theta, e_0) = \mathbf{1}[\theta \geq \underline{\theta}]$

(2) For each $e \in E \setminus \{e_0\}$, either

(i) There exists $\theta_e \leq \theta_0$ s.t. $q(\theta, e_0) = \mathbf{1}[\theta \geq \theta_e]$, and $U(e, \underline{\theta}) = 0$.

(ii) There exist θ_e, θ'_e, with $\theta_e < \theta_0 < \theta'_e$ such that

$$q(\theta, e) = \begin{cases} 0 & \theta < \theta_0 \\ \frac{\theta'_e - \theta_0}{\theta'_e - \theta_e} & \theta \in [\theta_e, \theta'_e) \\ 1 & \theta_e \geq \theta_0 \end{cases}$$

and $U(e, \underline{\theta}) = 0$

(iii) There exists $\theta_e > \theta_0$ s.t. $q(\theta, e_0) = \mathbf{1}[\theta \geq \theta_e]$, and $U(e, \underline{\theta}) = \theta_e - \theta_0$

The proposition is more readily understood by means of a picture. Below, Figure 1 illustrates the indirect utility functions of the buyers in cases (i) - (iii). For the purposes of the illustration, $E = \{e_0, e_1\}$. The slope of the indirect utility is identified with that type's allocation via the envelope formula. The intercept with the y-axis is the payment made to the lowest type. Finally,

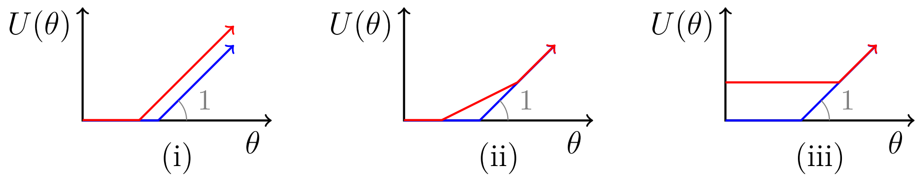


Figure 1: Graphical Illustration of Extremal Mechanisms

the disclosure constraint is captured by the indirect utility following e_1 lying everywhere above that of e_0.

The proof is constructive, and the argument borrows ideas from Kleiner et al. (2021), but it is not implied by their results. This is because our constraints are not quite majorization as they define, and secondly because both the "majorizing" and "majorized" functions are choice variables, not exogenous constraints. We direct the reader to the appendix for details.

Let us interpret the three possibilities for the extreme points. Case (i) corresponds to the seller posting a different price for the good if the buyer shows $e \in E$ or e_0. Case (ii) involves a probabilistic allocation in the intermediate region after the buyer discloses evidence. This can be interpreted as the seller *randomizing* over the posted prices θ_e and θ'_e. The probabilities are pinned down by the indifference of type θ'_e. Case (iii) involves the seller paying the buyer some positive amount when he shows $e \neq e_0$, *regardless* of whether the object is actually allocated, and then posting a price of e_0. From this description, it should be clear that (iii) is suboptimal. The seller benefits from posting the price e_0 to the buyer regardless of what he discloses, without providing a transfer for the evidence itself. This leads to the following observation:

Proposition 3. *The seller never pays the buyer solely for providing evidence. At the optimum,* $U(\underline{\theta}, e) = 0$ *for all* $e \in E$*.*

This result sheds some light on the arguments behind data ownership. Directly paying a buyer for their data is not the cheapest way to incentivize disclosure. Rather than pay for an individual's data directly, the seller finds it more profitable to commit to providing *better deals*. This suggests that only high-valuation buyers with evidence may reap the rewards of a data

[21] Optimality implies that there is no harm in imposing a sufficiently large upper bound - for instance, the total surplus of the efficient allocation.

ownership policy. We investigate such conclusions further in the next section, after we have concluded with a characterization of the optimal mechanism.

We summarize our analysis so far with the following proposition that characterizes optimal mechanisms. That each possibility is actually optimal for some parameters of the model will be apparent once we provide the construction of the seller's value function.

Proposition 4. *Given a discretionary disclosure policy, there exists an optimal mechanism such that*

(1) The seller posts a price θ_0 following the the buyer's disclosure e_0

(2) The seller randomizes over at-most two prices θ_e, θ'_e following the buyer's disclosure $e \neq e_0$

Moreover, all optimal mechanisms can be expressed as a convex combination of such mechanisms.

The optimal mechanism has an appealing property that it does not actually require the buyer's report in order to implement. In a sense, this also alleviates some concerns about the commitment power needed to implement the randomization. If the buyer truthfully reported his type to be some value between the two randomized prices, upon the realization of the higher price, the seller would feel some regret as the object would be unsold. Without strong commitment power, the seller would like to re-draw the price to be the lower one. However, because the optimal mechanism doesn't actually require a report from the buyer, only production of evidence, this mitigates the incentives for the seller to renege after the object is unallocated.

4.3 The seller's Value Function

The final step in our understanding of the seller's optimization problem is to determine exactly which combination of parameters described in Proposition 4 yields the optimal mechanism. We do this by appealing to a concavification argument common in the persuasion literature, which will allow us to easily "read off" the seller's optimal mechanism.

Working backwards, suppose that the seller has already posted a price of θ_0 for any buyer who provides e_0. What is the maximum profit that the seller can make from the segment of buyers who provide evidence $e \in E \setminus \{e_0\}$? Define the posted-price revenue function conditional on evidence e as

$$R_e(\theta) = \theta\,(1 - G(\theta|e))$$

If the seller chooses to post a single price $\theta_e \leq \theta_0$, then her payoff is simply:

$$\max_{\theta_e \leq \theta_0} R_e(\theta_e) \tag{18}$$

On the other hand, suppose that the seller chooses to randomize over prices $\theta_e \leq \theta_0 \leq \theta'_e$. We have already identified that the probabilities of this randomization are pinned down by the binding disclosure constraint of type θ'_e. The probability α of posting θ_e must satisfy

$$\begin{aligned} \alpha(\theta'_e - \theta_e) + (1-\alpha)(\theta'_e - \theta'_e) &= \theta'_e - \theta_0 \\ \implies \alpha &= \frac{\theta'_e - \theta_0}{\theta'_e - \theta_e} \end{aligned}$$

So if the seller wishes to randomize, she offers price $\theta_e \leq \theta_0$ with probability $\frac{\theta'_e - \theta_0}{\theta'_e - \theta_e}$, and price $\theta'_e \geq \theta_0$ with probability $\frac{\theta_0 - \theta'_e}{\theta'_e - \theta_0}$. So, her ensuing revenue would be:

$$\frac{\theta'_e - \theta_0}{\theta'_e - \theta_e} R_e(\theta_e) + \frac{\theta_0 - \theta'_e}{\theta'_e - \theta_0} R_e(\theta'_e) \tag{19}$$

This value is exactly the weighted average of the two posted-price revenues of θ_e and θ'_e. Graphically, this is a point on the line segment between $R_e(\theta_e)$ and $R_e(\theta'_e)$ *evaluated at* θ_0. Indeed, this argument exactly parallels the concavification argument in the persuasion literature. The preceding discussion is summarized by the following result.

Let $\tilde{R}_e(\theta)$ denote the **monotone concavification** of $R_e(\theta)$. $\tilde{R}_e$ is defined as the smallest non-decreasing and concave function that lies above R_e.

Proposition 5. *If the seller posts a price of θ_0 to e_0, the maximum revenue she can earn from the buyers with evidence e is $p(e)\tilde{R}_e(\theta_0)$.*

To make the relationship to persuasion more crisp, consider an additional integration by parts of our objective function. Since $\theta g(\theta) - (1 - G(\theta))$ is the derivative of $\theta(1 - G(\theta))$, the posted-price revenue function, we can write our objective as

$$\max_q \sum_{e\in E} p(e)\left(\int_\theta \theta(1-G(\theta_e))dq(\theta;e)\right) \tag{20}$$

$$s.t.\quad q(\cdot,e) \text{ non-decreasing} \quad \forall e \tag{21}$$

$$\int_{\underline{\theta}}^{\theta} q(s,e)ds \geq \int_{\underline{\theta}}^{\theta} q(s,e_0)ds \quad \forall e,\theta \tag{22}$$

The allocation function $q : \Theta \times E \to [0,1]$ plays the part of the CDF in a persuasion model. Meanwhile, the disclosure constraint acts similarly to a constraint that the "distribution" is a mean-preserving spread of that following e_0. It is not identical to a mean-preserving spread constraint because we do not require equality when evaluated at $\theta = \overline{\theta}$. With this perspective, we can interpret the "concavification" part of the result as applying when the disclosure constraint is *binding*, while the "monotone" qualification applies when the constraint may be slack.

This construction allows us to completely determine the seller's optimal mechanism, which is reported in the following result. An illustration of the monotone concavification, and identifying the optimal prices is presented in Figure 2.

Proposition 6. *The seller's value function is given by*

$$\max_\theta \; p(e_0)R_0(\theta) + \sum_{e\in E\setminus\{e_0\}} p(e)\tilde{R}_e(\theta) \tag{23}$$

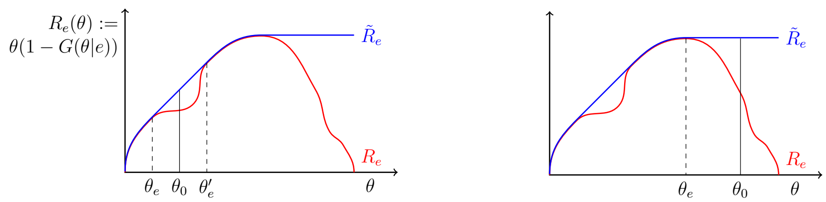
and the maximizer is the optimal posted price to the buyers who provide e_0.

Additionally, given θ_0^*, the monotone concavification identifies the seller's optimal mechanism for each $e \in E \setminus e_0$.[22] Specifically:

- if $R_e(\theta_0^*) = \tilde{R}_e(\theta_0^*)$, it is optimal for the seller to post $\theta_e^* = \theta_0$ to the buyers with e
- if $R_e(\theta_0^*) < \tilde{R}_e(\theta_0^*)$, then $\tilde{R}_e(\theta_0^*)$ is linear. It is optimal to randomize over the endpoints of the linear segment where $\tilde{R} = R$.

[22]There is some awkwardness in stating the result, as it is possible for $R = \tilde{R}$ along a linear segment of R, so the seller *could* still choose to randomize. Similarly, $R < \tilde{R}$ is possible between two peaks of equal height, so the seller could either randomize between the two or post the price corresponding to the lower peak. Thus, the conditions are not mutually exclusive.

– if $R_e(\theta_0^*) < \tilde{R}_e(\theta_0^*)$, then $\tilde{R}_e(\theta_0^*)$ is linear. If the slope is 0, it is optimal to post the revenue-maximizing price $\theta_e < \theta_0^*$



(a) Binding Constraint (b) Non-Binding Constraint

Figure 2: Monotone Concavificiation

The posted-price revenue is plotted against the buyer's valuation. The red curve depicts the posted-price revenue, and the blue is the monotone concavitification. (a) if the seller posts a price of θ_0 to e_0, the optimal randomization over posted prices yields a revenue corresponding to the concavification (b) If the seller posts a price for e_0 that was beyond the revenue-maximizing price, she could still post that optimal price to the buyers who hold e. Hence, the value function lies strictly above the posted-price revenue curve.

5 Results: The (Non)-Value of Discretion

With a characterization of the optimal mechanism in hand, we turn to the question of optimal regulation schemes. We present two results on when discretion *cannot* outperform a selective mandate. The first shows that a regularity condition – concavity of the posted-price revenue functions – is sufficient to guarantee that discretion cannot outperform the best selective mandate policy. The second result shows that, when the regulator additionally has the power to design the evidence structure itself, it is without loss to fully mandate disclosure.

5.1 Exogenous Evidence Structure

First, we study environments where the information content of the evidence is outside of the regulator's control. This is a natural assumption when the evidence represents technological or political constraints. For example, a genetic test cannot be made arbitrarily precise, and it may be politically infeasible to arbitrarily combine different age groups together. Regardless, it still may be in the power of the regulator to designate which of the existing evidence must be disclosed versus what the buyer has a choice over disclosing.

In such an environment, we have the following result. Recall that $O(p, \gamma)$ denotes the outcome(s) induced by the principal's optimal mechanism under probability distribution p and regulation γ.

Theorem 1. *Fix an evidence distribution p, and suppose $R_e(\theta)$ is concave for all $e \in E$.*

$$\bigcup_{\gamma \in \Gamma^M} O(p, \gamma) \supseteq \bigcup_{\gamma \in \Gamma^D} O(p, \gamma)$$

That is, any outcome achievable by some discretionary policy is also achievable by some selective mandate.

Proof. We present the proof for the case when $E = \{e_0, e_1\}$, and relegate the more general proof to the appendix. We show that the outcome that is attained when the buyer has discretion must either coincide with that of mandating e_1 or prohibiting its disclosure.

Letting R_0 and R_1 define the corresponding posted-price revenue curves, define $\theta_0^{FB} := \text{argmax}\, R_0(\theta)$, and $\theta_1^{FB} := \text{argmax}\, R_1(\theta)$. These are the optimal posted prices if the seller could observe the buyer's evidence directly, i.e. under a full mandate.

If $\theta_1^{FB} \leq \theta_0^{FB}$, then the disclosure constraint does not bind, and the outcome that prevails in the optimal mechanism under discretion is equivalent to that under mandated disclosure of e_1.

Instead, suppose $\theta_1^{FB} > \theta_0^{FB}$. The optimal posted price to the set of buyers with e_0 will be denoted by θ_0^*. Because R_0 and $\tilde{R}_1$ are both concave, it must be the case that the $\theta_0^* \in [\theta_0^{FB}, \theta_1^{FB}]$. Whatever the optimal value in this range, we know that $\tilde{R}_1(\theta_0^*) = R_1(\theta_0^*)$ by concavity. Therefore, the seller posts the *same* price of θ_0^* to the buyers with evidence e_1.

Conditional on posting the same price following both e_0 and e_1, the optimal must be the one that maximizes the revenue under the prior belief. Therefore, the outcome in this case corresponds with that of a ban on e_1. □

Corollary 1. *Suppose $R_e(\theta)$ is concave for all $e \in E$. For any regulator preference $\succeq$ over outcomes, the $\succeq$-best discretionary disclosure policy cannot outperform the $\succeq$-best selective mandate.*

Thus, it is without loss of generality for a regulator to restrict attention to selective mandate policies when considering legislation. Allocating "information rights" to the buyer does not perform better than appropriately selecting which evidence should be a priori contractible, and then compelling its disclosure. The concavity assumption required for the argument warrants some discussion. First, concavity of revenue curves is an appealing property because it makes the first-order condition is necessary and sufficient to determine the optimal posted price. This assumption is common within the industrial organization literature precisely because it allows for characterizations via first-order conditions. Roughly, this assumption rules out distributions where the density decays too quickly relative to the buyer's valuation, or is sufficiently bimodal. [23]

Finally, we wish to emphasize that, while we can slightly weaken the concavity condition, such an assumption is needed to deliver the result.[24] It is not an assumption of convenience, as is often the case when non-decreasing virtual value is assumed in the mechanism design literature to sidestep the issue of ironing. While quite similar, concavity of the revenue function and non-decreasing virtual value are not logically related (neither implies the other). An equivalent definition for non-decreasing virtual value is that the *quantile* revenue curve is concave. That is, the function $G(\theta) \mapsto \theta(1 - G(\theta))$ is concave. In contrast, we require $\theta \mapsto \theta(1 - G(\theta))$ concave, which would imply that the quantile revenue curve is single-peaked. We provide a more thorough discussion of the concavity assumption in the appendix.

Consumer Surplus: While Theorem 1 implies that the *best* selective mandate outperforms any discretionary policy, it need not be the case for *every* selective mandate. Notably, existing status-quo policies of *full* disclosure may be inferior to those of discretionary disclosure. For example, when we focus on consumer surplus, we see that the impacts of a discretionary policy are born differently by buyers with and without evidence.

Proposition 7. *The consumer surplus of all buyers with $e \neq e_0$, regardless of valuation θ, is weakly larger under a policy of discretion than under fully-mandated disclosure. The opposite is true for all buyers with no evidence, e_0.*

Notably, the proposition does not say anything about the information content of the evidence that the buyers may have - it may lead the seller to have

[23]If we assume that the density g is differentiable, then the concavity condition reduces to $\theta g' + 2g > 0$. In any region where the density is increasing, concavity holds, but it may fail when the density decreases quickly for large, but unlikely values of θ.

[24]For example, one may weaken concavity to "concavity to the left of the peak" for $e \neq e_0$, and to single-peakedness for e_0

higher or lower beliefs about the buyer's valuation. The result follows because the price (or randomization over prices) that is implemented following any $e \neq e_0$ when the buyer has discretion is necessarily lower than corresponding price when the buyer must disclose the evidence. This conclusion is immediately delivered by the characterization of the seller's value function via the monotone concavification. Intuitively, there is no reason for the seller to raise her prices higher than the first-best upon observing the evidence because it only makes it harder to elicit the evidence but does not increase revenue.

5.2 Designable Evidence Structure

Next, we turn to environments where the regulator may also control the information content of the evidence. This is a reasonable perspective in cases where the evidence is a constructed index, such as a credit score, and the regulator may place restrictions on the weights of different inputs, or otherwise coarsen the evidence. Alternatively, there may be natural ways for the regulator to "coarsen" existing evidence, such as replacing individual-level data with aggregates, or to import new information into the environment. All of these can be conceived as the regulator having some control over the evidence structure itself.

Our goal here is to consider the feasible set that can be achieved by regulation, similarly to Bergemann et al. (2015). Notably, we fix the prior distribution of buyer types, and only consider changes in the evidence those buyers may hold. We look to identify all outcomes that are achievable by joint design of the distribution over evidence and discretion afforded to the buyer. In such an environment, we can arrive at a similar conclusion to Theorem 1 *without* any regularity conditions.

Theorem 2. *Fix any prior distribution F.*

$$\bigcup_{\gamma \in \Gamma^M} \left(\bigcup_{p \in P} O(p, \gamma) \right) = \bigcup_{\gamma \in \Gamma^D} \left(\bigcup_{p \in P} O(p, \gamma) \right)$$

The set of all outcomes that can be achieved by selective mandate policies and discretionary policies under some evidence structure are equivalent.

While the result is stated in terms of selective mandate policies, it applies equally to *fully-mandated* disclosure by appropriate reconstruction of p. Because the set of outcomes that can be achieved under mandated and discretionary policies are equivalent, this simplifies the problem of a regulator, as summarized in the next result.

Corollary 2. *For any regulator preference $\succeq$ over outcomes, when p is a choice object, there is no loss in restricting attention to fully-mandated disclosure policies.*

One direction of Theorem 2 ($\subseteq$) follows immediately. Any outcome that can be achieved under a selective mandate with some evidence distribution can easily be replicated when the buyer has discretion. We can choose the evidence distribution such that e_0 corresponds to the highest posted price from the selective mandate. In this case, the disclosure constraint is non-binding, and the same outcome is achieved.

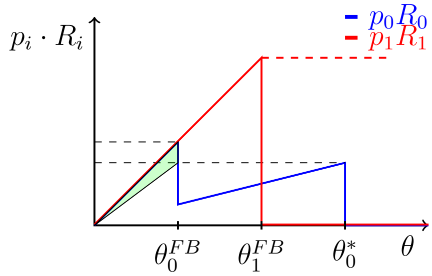
The other direction ($\supseteq$) is more subtle, and the proof proceeds by construction. The idea is that, relative to a mandate, voluntary disclosure provides a downward pressure on the price given to e_0, and an upward pressure on the prices chosen for $e \neq e_0$. This occurs because the seller must satisfy the disclosure constraint. These downwards and upwards forces can be replicated *exactly* by moving probability mass across the evidence realizations and mandating disclosure. We shift low-valuation buyers from e_0 to other realizations of evidence, and conversely for high-valuation buyers.

We provide the full proof in the appendix, but provide an illustration of the fundamental mechanics in a simple example below. The example is not fully general, and does not cover instances where, for example, the seller randomizes over posted prices.

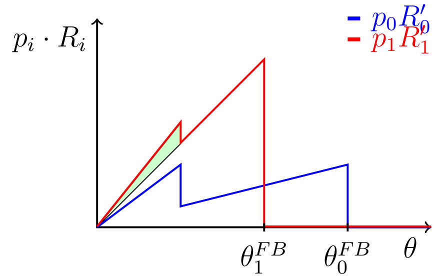
Example 1. *In this example, we again assume $E = \{e_0, e_1\}$. We direct the reader to Figure 3 for a graphical depiction of the revenue curves. These revenue curves can be generated from a joint distribution over evidence and valuations similar to those in Example 3.2. Let θ_0^{FB} and θ_1^{FB} denote the maximizers of R_0 and R_1 respectively, and θ_0^* the optimal posted price to e_0 under a policy of discretion.*

The disclosure constraint binds, since $\theta_0^{FB} < \theta_1^{FB}$ but the revenue curves are not concave, so simply banning or mandating disclosure of e_1 does not replicate the outcome under discretion. In particular, the seller finds it optimal to raise the price following e_0, at a small loss of revenue, in order to satisfy the disclosure constraint.

We construct new revenue curves by moving the valuations of low-type buyers with e_0 to e_1 in such a way that (i) conditional on observing e_0 the



(a) Optimal Mech. with Discretion



(b) Mandated Disclosure Equivalent

Figure 3: Theorem 2 Illustration

The left figure depicts two revenue curves under which the seller's optimal mechanism when the buyer has discretion is to post a price of θ_0^* to e_0 and a price of θ_1^{FB} to e_1. The right figure depicts a different joint distribution over (θ, e), with the same marginal over θ, that achieves the same outcome under a policy of mandated disclosure. Graphically, we move probability mass from e_0 to e_1, which corresponds to the triangular area in the revenue curves.

optimal posted price coincides with the optimal price under discretionary disclosure θ_0^* and (ii) conditional on observing e_1, the optimal posed price remains θ_1^{FB}. The fact that θ_0^* maximizes $p_0 R_0 + p_1 \tilde{R}_1$ guarantees that we are simultaneously able to achieve both of these goals. Specifically, we move probability mass uniformly from the set of buyers with e_0 and $\theta \in [\underline{\theta}, \theta_0^{FB}]$ to instead having e_1 such that the seller is indifferent between charging θ_0^{FB} and θ_0^* upon observing e_0.

Because we did not alter the distribution of buyers with values above θ_1^*, the outcome that is attained under the new distributions with a policy of mandated disclosure is equivalent to the original outcome under voluntary disclosure.

Recall that selective mandate policies effectively induce market segmentations from the perspective of the seller: the segments correspond to the evidence that is observed. Bergemann et al. (2015) demonstrate the richness of market segmentations in terms of the set of welfare pairs for the seller and buyer they can generate. One may view Theorem 2 as qualifying the "richness" of market segmentations in terms of the *outcomes* they produce. Our result is complementary to theirs in that it shows how market segmentations may achieve all outcomes that are generated by discretionary disclosure. Therefore, if the regulator is interested in equality, opportunity, or any other objective that is not a linear combination of surplus, our result shows that the set of market segmentations is a sufficiently rich set of choice objects to consider.

Before we conclude, we discuss briefly what assumptions are critical for driving the result. Perhaps surprisingly, we do not believe that linearity of the buyer's preferences is critical, though we do not have a proof for these cases. In the subsequent extensions section, we provide an instance of how such an argument may apply more generally. Further, in the appendix we provide an example of an environment in which the result is false: there exists some outcome of an optimal mechanism that is sustained by discretion that is not possible under any mandate. The environment is linear insofar as the seller is allowed to randomize over alternatives, which does occur in the optimal mechanism.

Rather, the key feature in this environment which facilitates the result is supermodularity. The buyer's types can be ordered in such a way that the information rents are described by the total mass of higher types. When the buyer has discretion over providing his evidence, "higher" types refer to both those with a higher valuation, but also "higher" evidence. We describe $e \neq e_0$ as "higher" because type (θ, e) may claim to be type (θ, e_0). From this perspective, it is natural to think that there is a way to replicate the mass of "higher" types under a policy of discretion with an appropriately constructed distribution under a policy of mandated disclosure.

To summarize, the two main theorems deliver the central message from this paper: that discretion need not expand the set of regulatory possibilities. When either the concavity condition applies, or the regulator may additionally design the evidence structure itself, allowing for discretion does not magnify the set of possibilities. Under these conditions, any outcome that can be achieved with a discretionary policy can also be achieved with a particular selective mandate.

To be clear, we do not insinuate that discretionary policies are inherently bad. Indeed, they may be superior to any suboptimal status quo policy, regardless of its form. In particular, a discretionary policy may enhance outcomes relative to an environment in which the seller directly observes the buyer's evidence, as is often the case in existing digital markets. Such a result about consumer surplus was illustrated in the previous section: relative to a policy of full disclosure, some buyers strictly benefit from discretion while some are harmed.

Our results indicate when an optimal policy is a selective mandate. This is a convenient set of policies to maximize over because the seller's response is predictable: they post an optimal price, which allows us to use the tools of information design. However, exactly which selective mandate policy is optimal in a given environment necessitates more information: the distribution of buyer values and the regulator's objective. We investigate some implications of a regulator's limited knowledge in the subsequent extensions sections.

6 Non-Committed Seller

The focus of our analysis thus far has been to study how substitution by the seller in response to regulation affects the ultimate market outcome. One may wonder the extent to which our results apply to environments in which the seller has limited (or no) commitment power. In response to cheap talk and evidence disclosure by the agent, the seller posts the price which is a best-response to her current beliefs about the buyer's valuation.

We show that our main results extend readily to this environment – and in some ways more easily. The main intuition is that the regulator can simply mandate disclosure of the evidence being provided along the equilibrium path. Because the seller is already best-responding, she has no reason to adjust the prices. We additionally provide a novel result on the seller's value for commitment. Specifically, we show that when the concave revenue assumption is satisfied, the seller has no value to commitment. That is, her payoff in the optimal mechanism can also be achieved in an equilibrium of the disclosure game.

The description of the model primitives remain unchanged. The buyer's type is drawn from $\theta \in [\underline{\theta}, \bar{\theta}]$ with prior cdf F. The buyer is endowed with evidence $e \in E$, where E has a null-element e_0 corresponding to "no evidence". The mapping $p : \Theta \to \Delta(E)$ describes the relationship between the buyer's value and the evidence he holds. Regulation γ dictates what evidence the buyer may disclose for any realized $e \in E$.

When the seller does not have commitment power over her response to disclosure, we model the interaction between the buyer and seller as a game. The buyer chooses a disclosure strategy σ which maps his valuation and evidence into a (possibly random) disclosure and cheap-talk report. $\sigma : \Theta \times E \to \Delta(\Theta \times E)$. We use $\sigma(\theta', e'|\theta, e)$ to denote the probability the buyer reports (θ, e') when his valuation is θ and evidence is e. Of course, this must be allowed by the chosen regulation, so we require $(\theta', e') \in \text{supp}\sigma(\theta, e)$ implies $e' \in \gamma(e)$. The seller chooses a pricing strategy, which we continue to denote by $t : (\Theta \times E) \to \Delta(\mathbb{R}_+)$. Similarly, we use $t(x|\theta, e)$ to denote the probability the seller posts price x conditional on the report (θ, e). After the seller chooses a price, all buyers with weakly larger valuation purchase the good at the chosen price.[25]

An equilibrium is a disclosure strategy σ, pricing strategy t, and seller beliefs μ such that

(1) (Buyer best-response) For all θ, e, if $(\theta^*, e^*) \in supp\sigma(\theta, e)$ then $\int_{\theta}^{\bar{\theta}}(\theta - x)\,(t(x|\theta^*, e^*) - t(x|\theta', e'))\,dx \geq 0$ for all $\theta' \in \Theta$, $e' \in \gamma(e)$

(2) (Consistent Beliefs) $\mu(\theta|\theta', e') = \frac{\int_E \sigma(\theta', e'|e,\theta)p(e|\theta)f(\theta)de}{\int_\Theta \int_E \sigma(\theta', e'|e,\theta)p(e|\theta)f(\theta)}$ on-path[26]

(3) (Seller best-response) $x^* \in supp(t(\theta, e)) \implies x^* \in \arg\max_x \int_x^{\bar{\theta}} \mu(\theta|\theta, e)d\theta$

We again define an outcome $O : \Theta \to \Delta([0, 1] \times \mathbb{R})$ as the mapping from valuations to the probability of sale and expected price. For a given regulatory policy γ, let $O(\gamma)$ denote the outcome(s) in the equilibria of the disclosure game.

In this formulation of the model, a simple argument delivers the main results from the model with commitment: outcomes that are achievable with discretion are also achievable by a selective mandate policy. Because the seller best-responds to disclosed evidence, the regulator can simply mandate disclosure of the evidence that would be provided on-path by the buyer. The seller's response does not change because the posted price is already optimal given her beliefs.

In the case that $\sigma(\cdot, e)$ is constant and degenerate for each e, this argument tells us that the resulting outcome may be achieved with a selective mandate policy under the *same* evidence structure. Hence, we have a counterpart to Theorem 1. However, if $\sigma(\theta, e) \neq \sigma(\theta', e)$ for some e, then the replication argument does not immediately follow because we require regulation γ to

[25] By the assumption that F has a continuous density f, only a measure-0 of buyers can be indifferent between purchasing and not purchasing conditional on a realized price. Therefore, removing the final stage of the game in which the buyer makes a purchase decision does not change the set of equilibria.

[26] We could also impose an off-path consistency requirement given the evidence disclosed by the buyer. This has no effect on our setting because the seller could always believe the buyer was the highest type consistent with that evidence, and post the corresponding price. In order for off-path disclosure to have bite, we would need another refinement.

depend *only* on the verifiable evidence e and not the buyer's private valuation θ.[27] If instead we interpret the evidence disclosed on path as simply a different underlying evidence structure, then we recover the result Theorem 2.

Proposition 8. *Fix any prior distribution F.*

$$\bigcup_{\gamma \in \Gamma^M} \left(\bigcup_{p \in P} O(p, \gamma) \right) = \bigcup_{\gamma \in \Gamma^D} \left(\bigcup_{p \in P} O(p, \gamma) \right)$$

The set of all outcomes that can be achieved in equilibrium by selective mandate policies and discretionary policies under some evidence structure are equivalent.

Proof. Fix E, the prior distribution F, and the evidence distribution p. Consider an equilibrium of the game (σ, μ, t) in which the buyer has discretion. Now consider an alternative evidence distribution p' defined by $p'(e'|\theta) = \sigma(e'|e, \theta)p(e|\theta)$. That is, p' directly incorporates the buyer's equilibrium strategy into the distribution of evidence. Under p' and fully mandated disclosure, the seller's beliefs unchanged, and hence her best-response is identical. The buyer also has no profitable deviations with respect to his cheap-talk message because of the original equilibrium. $\square$

In some sense, the fact that our results continue to hold in this environment is anticlimactic: they follow almost immediately from our definition of equilibrium. Much more interesting is the role of our regularity assumption – concavity of posted-price revenue curves – in this environment. We show that when concavity is satisfied, the seller does not require commitment power to attain her maximum revenue.

Theorem 3. *Suppose R_e is concave for all e. Then, the seller has no value to commitment. That is, the revenue from the seller's design problem with commitment can be attained in some equilibrium of the disclosure game.*

This result gives us a different interpretation of Theorem 1 as a fixed point. At the optimum, the best price to charge the set of buyers who face a price of x is exactly x. Concavity ensures that the seller cannot strictly benefit from randomization over prices or by moving the price away from x.

[27] It is possible that this happens in equilibrium if the seller is also randomizing prices. For example, high-valuation buyers with evidence may randomize between disclosing – getting the randomization over prices – and concealing to get the expectation.

To our knowledge, this no-value to commitment result does not follow from any existing results in the literature. In particular, the buyer has type-dependent preferences and stochastic evidence, so the results of Glazer and Rubinstein (2004, 2006), Sher (2011), or Hart et al. (2017) do not directly apply. While Ben-Porath et al. (2021) allow for both stochastic evidence and type-dependent preferences, the "partial alignment" condition of their result does not apply.[28]

7 Extensions

7.1 Non-Linear Preferences

Before we conclude, we briefly discuss how these results might extend to models of non-linear preferences. In our baseline model, we assumed that the buyer's utility took the form $U(q,t;\theta) = q\theta - t$. This linearity assumption is clearly critical for our characterization of the optimal mechanism. Identifying the extreme points of the set of incentive compatible mechanisms is only powerful because our objective is linear. When we drop the linearity assumption, the optimal mechanism is more difficult to characterize explicitly, though some general insights may be attained.

We now suppose that the buyer's payoff is given by $U(q,t;\theta) = \theta \cdot v(q) - t$. $v(\cdot)$ is strictly increasing and concave, twice continuously differentiable, and satisfies the Inada conditions $v'(0) = \infty$ and $v' \to 0$. The seller's payoff will be given by $t - c(q)$, where c is twice continuously differentiable, strictly convex and increasing, and satisfies the corresponding Inada conditions.

In order to obtain predictions in this environment, we simplify the relationship between the buyer's payoff-type and his evidence to a Dye (1985) structure. In a Dye structure, each payoff-type of the buyer has a uniquely identifying realization of evidence, which is obtained with some probability. That is, only a buyer of type θ may realize evidence e_θ, otherwise he has no evidence to show. Formally, $E = \Theta \cup \{e_0\}$. We denote the probability that type θ has identifying evidence by $p(\theta) := p(e_\theta|\theta)$. With complementary probability, type θ has no evidence to provide. $1 - p(\theta) = p(e_0|\theta)$. When a buyer holds evidence $e \neq e_0$, he may choose to conceal it and report e_0

[28]Specifically, all types of the buyer are indifferent between receiving the good but being charged their full valuation, and not receiving the good at all. The seller of course prefers the former to the latter.

instead.

Our previous decomposition of the incentive compatibility constraints still applies here. That is, it is sufficient to ensure that each type (θ, e) does not want to misreport in each dimension separately. The assumption of a Dye structure simplifies the problem immensely because, conditional on buyer type θ realizing evidence e_θ, the *only* deviation we need to consider is to reporting (θ, e_0), since only type θ realizes evidence e_θ on-path.

This has two immediate implications for the optimal mechanism: any buyer who presents e_θ is served efficiently, and that type is exactly indifferent between disclosing his evidence and concealing it. Formally, let $q^{FB}(\theta)$ denote the surplus-maximizing quantity. $q^{FB}(\theta) := \arg\max \theta v(q) - c(q)$. We assume for convenience that this quantity is unique. It is also clearly increasing in θ.

Proposition 9. *In the optimal mechanism, $q(\theta, e_\theta) = q^{FB}(\theta)$ for all θ. Additionally, $t(\theta, e_\theta)$ solves $\theta v(q^{FB}(\theta)) - t(\theta, e_\theta) = \theta v(q(\theta, e_0)) - t(\theta, e_0)$.*

The proposition tells us that, once we have determined the allocations of the buyers *without* evidence, the allocations and transfers of the buyers with evidence are immediately pinned down. As usual, an envelope and monotonicity condition characterize incentive compatibility of the buyers without evidence. Ultimately, we are left with determining the *quantities* given to the buyers without evidence, which we succinctly denote by $q(\theta)$.

After the usual substitutions, and dropping the constant terms identified by the previous proposition, the seller's problem can be written:

$$\max_q \int_\theta \left((1 - p(\theta)) \left[\theta v(q(\theta)) - c(q(\theta)) \right] - v(q(\theta)) \frac{1 - F(\theta)}{f(\theta)} \right) f(\theta) d\theta \tag{24}$$

$$s.t. \quad q(\cdot) \text{ non-decreasing} \tag{25}$$

The main observation is that the *direct* value of allocating to the agents without evidence is scaled by $(1 - p(\theta))$, the probability that they realize e_0. In contrast, the information rents are *not* scaled by this constant. Allocating to a buyer with a valuation of θ requires that the seller give a higher transfer to all buyers with higher valuation, *regardless of their evidence*. Because the benefits are scaled by p, but the costs are not, we come to conclude that contractible information actually hurts the buyers in this environment.

Proposition 10. *Consider two evidence distributions p' and p, and denote $q'(\cdot)$ and $q(\cdot)$ the corresponding allocations in the seller's optimal mechanism.*

If $p' \geq p$ pointwise, then $q' \leq q$ pointwise. As a consequence, the consumer surplus of every buyer type and evidence is weakly lower under p' than p.

Proof. The first part follows by application of the Topkis multivariate comparative statics theorem. It is without loss to restrict attention to increasing functions $q : \Theta \to \mathbb{R}_+$ such that $q(\theta) \in [0, q^{FB}(\theta)]$ for all θ. Under this restriction, the objective function is submodular in each pair of $p(\theta), q(\theta)$, linearly separable across $q(\theta), q(\theta')$, and the constraint set forms a lattice.[29]

The second statement follows from the envelope condition, which relates the allocation to the buyer's surplus. Therefore, all buyer types without evidence are worse-off. Buyers with evidence are also harmed because the optimal mechanism makes them indifferent between disclosing and not disclosing. $\square$

Corollary 3. *For any p, discretionary disclosure leads to lower consumer surplus than prohibiting disclosure altogether. Discretionary disclosure also leads to lower total surplus than mandating full disclosure.*

One interpretation of this result is that there is an evidence *externality* when disclosure is discretionary. Any small mass of buyer types having more access to evidence uniformly makes all valuations of consumers worse-off. Intuitively, the seller gains more revenue from those consumers with evidence because she can implement the efficient allocation. Because the outside option of a type-θ buyer with evidence is to simply report that he is type θ without evidence, the seller is incentivized to lower the utility of the θ buyer without evidence. This pressure increases with the probability that the buyer has access to evidence. Lowering the allocation of type θ hurts buyers with higher valuations through the envelope condition, and hurts buyers with lower valuations because the monotonicity constraint may bind.

While this result has a similar message to other results regarding data externalities, the channel is different. Bergemann et al. (2022), for example, study a model in which an intermediary collects and resells data about consumers. In their model, data is social in that data from one consumer is directly informative about another consumer's valuation of a good. Because consumers do not internalize this, the intermediary's cost of acquiring the data is small, and the net effect on consumer surplus is negative. In contrast, in our model there is no intermediary, nor does the seller find it optimal to directly pay the consumer to reveal evidence. Instead, the reduction in

[29]The pointwise maximum and minimum of two increasing functions is increasing.

consumer surplus in our model is driven by a change in the relative value of allocating the good to types with and without evidence. A similar observation arises in Ichihashi (2020), though the timing and features of the models are different. Notably, they allow the buyer to commit to an ex-ante information policy and study the effects of the seller's commitment power. This is different than the "interim" constraint that arises in our model because the buyer's valuation and evidence are realized before he enters the mechanism.

To summarize, when we specialize to a Dye model of evidence, voluntary disclosure of evidence unambiguously hurts the buyer relative to an environment without evidence altogether. This is a robust observation: it relies only on the fact that the information rents must be paid to all types above θ regardless of their evidence, and that the problem can be expressed in a manner that is separable across types of the buyer. While we do not get the same strength of conclusion as Theorem 2, that a selective mandate is superior for *any* objective function, we can provide it for the purposes of consumer surplus, producer surplus, and efficiency.

8 Conclusion

This paper studies a fundamental question of how to regulate the disclosure of information. The key channel we study is the way in which the seller responds to a variety of regulatory policies. Yet, the analysis is not without restrictions. We view this paper as simply the first step in comprehensively understanding the ways in which disclosure should be regulated, and detail several promising directions for future work below.

In our model, the only tool available to the regulator is the design of the evidence structure - both its statistical relation to the buyer's type, and the level of discretion afforded. An open question is how regulating discretion interacts with other policy tools such as price and quantity caps, taxes, and subsidies. In our model, because the seller has no private information, giving the regulator the power to directly control these outcomes trivializes the problem. The regulator can always achieve their first-best by requiring the seller to behave in a specific way. If instead the regulator's tools are limited, or there is uncertainty about the environment – such as a seller's privately-known production cost, or ambiguity about the distribution of consumer values – there is room for rich interaction between the regulator's tools. Because both taxes and incentive constraints ultimately affect the seller's marginal benefit from increasing prices, it would be interesting to identify when the different tools are optimal.

Another promising line of inquiry lies in the question of evidence acquisition, as we briefly discussed while outlining the model. It is clear that different disclosure regulations provide different incentives for the agent to collect evidence in the first place. Specifically, because the agent may always choose to conceal his evidence upon a bad realization, a policy of discretion seems to provide stronger incentives for ex-ante evidence acquisition than one of mandated disclosure. Depending on the exact form of this evidence, this may or may not be beneficial. If the evidence generation is costly, and the seller knows his valuation a prior, then the seller may use evidence generation as an additional way to screen the buyer's type. Whether this is more or less efficient than policies of mandated disclosure is unclear. Further, if the buyer's evidence acquisition provides him information about his valuation, then we relate to the analysis in Roesler and Szentes (2017) on buyer-optimal learning. The additional information generated may be beneficial if the efficient quantities of trade vary with the buyer's valuation, so better targeting is possible, but may also exacerbate the adverse selection problem.

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A Omitted Proofs from Main Text

A.1 Extreme Points

To slightly ease notation, we let the set of buyer types be $[0,1]$. So an IC allocation function is $q:[0,1]\to[0,1]$ non-decreasing. Throughout, we assume that any non-decreasing function is right-continuous and left-continuous at 1. We provide the proof for Proposition 2 in the case where $E=\{e_0,e_1\}$. A more general E introduces more notation, but no conceptual difficulty.

Let $f:[0,1]\to[0,1]$ denote the allocation to the agents who disclose e_0, and $g:[0,1]\to[0,1]$ the allocation to the agents who disclose e_1. A pair (f,g) is IC iff

(1) f,g are non-decreasing

(2) $\int_0^x f(s)ds \leq \int_0^x g(s)ds$ for all $x\in[0,1]$ (Disclosure constraint)

Let $\mathcal{F}$ denote the set of IC (f,g). Our goal is to characterize the set of extreme points of $\mathcal{F}$.

Def: an increasing function $f:[0,1]\to[0,1]$ is **simple** iff there exists a countable set $Y=(y_0,\ldots)$ and associated coefficients $\alpha(y)\in[0,1]$ such that $f=\sum_{y\in Y}\alpha(y)\mathbf{1}[x\geq y]$. In words, a simple function is the countable convex combination of step functions.

Lemma 1. *If (f,g) is an extreme point of $\mathcal{F}$, then both f and g are simple.*

Proof. **Step 1**: First, we show that for any fixed f, whenever the disclosure constraint is non-binding for an interval $[s_0,s_1]$, g must be constant on $[s_0,s_1]$. This argument follows closely Theorem 1 from Kleiner et al. (2021). We reproduce a modified proof here as the construction will be useful later.

Suppose to the contrary $g(s_0)<g(s_1)$, and pick $y\in[g(s_1),g(s_2)]$. Define

$$u(s):=\text{median}\{g(s)-g(s_1),g(s)-g(s_2),y-g(s)\}$$

.

Then, $g\pm u$ is non-decreasing and $g(s_1)\leq(g\pm u)(s)\leq g(s_2)$. Finally, there exists $y\in[g(s_1),g(s_2)]$ such that $\int_{s_1}^{s_2}u(s)ds=0$. If g is non-constant on $[s_1,s_2]$, then there exists a $y\in[g(s_1),g(s_2)]$ such that $u\neq 0$. Therefore, $g=\frac{1}{2}(g+\epsilon u)+\frac{1}{2}(g-\epsilon u)$, where $g\pm\epsilon u$ are both increasing. By appropriate choice of ϵ, we guarantee that $\int_{s_1}^{x}(g\pm\epsilon u)(s)ds\geq\int_{s_1}^{x}f(s)ds$ continues to hold.

Step 2: For any fixed g, whenever the disclosure constraint is non-binding for an interval $[s_0, s_1]$, then f can attain at-most two distinct values on $[s_0, s_1]$. Suppose to the contrary there exists $s^* \in (s_0, s_1)$ such that $f(s_0) < f(s^*) < f(s_1)$. Then, we can apply the previous construction to f on the interval $[s_0, s_1]$, constructing two functions $f \pm u$ that continue to respect the disclosure constraint.

Step 3: On any interval $[s_0, s_1]$ on which the disclosure constraint is binding, both f and g are constant (and $f = g$). The fact that $f = g$ follows immediately from the disclosure constraint and the fundamental theorem of calculus. Now, suppose that f and g are not constant on an interval $[s_0, s_1]$. Then, we can easily construct two increasing functions f', f'' such that $f = \frac{1}{2}f' + \frac{1}{2}f''$. Importantly, we do *not* need to perform this manipulation in a way that preserves the value of $\int_{s_0}^{s_1} f(s)ds$.

One such construction is the following: pick any interval $[x_0, x_1] \subseteq [s_0, s_1]$ where $f(x) \neq f(s_0)$ for some $x \in [x_0, x_1]$. Then, for any $\alpha \in (0, 1)$ define

$$f'(s) = \begin{cases} (1+\alpha)f(s) & s \in [s_0, x_0] \\ f(s) + \alpha f(x_0) & s \in [x_0, x_1] \\ f(x_1) + \alpha f(x_0) + (1-\beta)[f(s) - f(x_1)] & s \in [x_1, s_1] \end{cases}$$

where $\beta = \frac{\alpha f(x_0)}{f(1) - f(x_1)}$. This construction guarantees that $f'(s_0) = f(s_0)$ and $f'(s_1) = f(s_1)$, and that f' and $2f - f'$ are both increasing. Thus, the disclosure constraint continues to hold with equality on the range $[s_0, s_1]$, and is therefore satisfied for all values of $s \geq s_1$. $\square$

Lemma 2. *If (f, g) is an extreme point of $\mathcal{F}$, then f takes at-most two values. Specifically, $f(s) \in \{0, 1\}$ for all $s \in [0, 1]$.*

Proof. Suppose towards the contrary that f takes on 3 values on the interval $[s_0, s_1]$. Then there exist values $s_0 < x_0 < x_1 < s_1$ such that

$$f(x) = \begin{cases} f(s_0) & x \in [s_0, x_0) \\ f(x_0) & x \in [x_0, x_1) \\ f(x_1) & x \in [x_1, s_1] \end{cases}$$

Additionally, by the previous lemma, we know that g also takes the form of step functions in this interval. There exist $y_0 \in [s_0, x_0]$, $y_1, y_2 \in [x_0, x_1]$, $y_3 \in [x_1, s_1]$ on which y is a step function. In the regions $[y_0, y_1]$ and $[y_2, y_3]$, the disclosure constraint is non-binding. Importantly, it must be the case that $\int_{y_0}^{y_1} g(s)ds = \int_{y_0}^{y_1} f(s)ds$, and similarly $\int_{y_2}^{y_3} g(s)ds = \int_{y_2}^{y_3} f(s)ds$. Substituting in the corresponding values, we obtain two conditions for the values of $g(y_0)$ and $g(y_2)$:

$$(g(y_0) - g(s_0))[x_0 - y_0] = (f(y_1) - g(y_0))(y_1 - y_0)$$
$$(g(y_2) - f(y_1))(x_1 - y_2) = (f(y_3) - g(y_2))(y_3 - x_1)$$

The key feature is that these are all linear relationships. Thus, increasing $f(y_1)$ by ϵ would have a *linear* effect on the values of $g(y_0)$ and $g(y_2)$ required to maintain the disclosure constraints. Thus, by *simultaneously* increasing $f(y_1)$ by $\pm\epsilon$, and $g(y_0), g(y_2)$ by $\epsilon\frac{y_1-x_0}{x_0-y_0}$ and $\epsilon\frac{y_3-x_1}{x_1-y_2}$ accordingly, we can maintain *all* disclosure constraints. To summarize, starting from (f, g) in which f took on at-least 3 values, we construct a pair $(f \pm \epsilon, g \pm \delta)$ that satisfy the IC constraints. Hence, (f, g) could not have been extremal.

To conclude, it is easy to see that if f takes on two values, they must be in $\{0, 1\}$. The same construction as the previous argument applies here - if $f \in (0, 1)$, we can add and subtract an ϵ to the value and correspondingly modify g to preserve the disclosure constraint. □

Finally, we-reincorporate the possible transfer to the lowest-type agent with evidence, which we denote by u_0. Thus, the disclosure constraint becomes $\int_0^x f(s)ds \leq \int_0^x g(s)ds + u_0$ for all $x \in [0, 1]$. We now seek extremal triplets of (f, g, u).

Lemma 3. *If (f, g, u) is extremal, there exists $s \in [0, 1]$ such that the disclosure constraint is binding. Let $s^* = inf\{s : \int_0^s f(z)dz = \int_0^s g(z)dz + u_0\}$ denote the first binding constraint. Then, $g = 0$ on $[0, s^*]$, and the previous characterization applies to f, g in $[s^*, 1]$.*

Proof. The first claim is immediate. If it is non-binding, then simply consider $u_0 \pm \epsilon$ which continues to satisfy IC. Second, we must have $g = 0$ on $[0, s^*]$ by the construction used in step 3. If $g \in (0, 1)$, then modifying g by $\pm\epsilon$ on $[0, s^*]$, and modifying $u_0 \pm \epsilon \cdot s^*$ yields two IC mechanisms that average to (f, g, u_0). Finally, the previous characterization applies without modification to $[s^*, 1]$, as the lower bound can simply be normalized to 0. □

A.2 Theorem 1

Proof. Consider evidence structure E and concave revenue curves (R_e). Under a policy of discretion, let θ_0^* denote the optimal posted-price for the agents without evidence, and let θ_e^* the corresponding prices to those with evidence. We know that the seller need not randomize over posted prices because the revenue function is strictly concave. Let E_0 denote the set of $e \in E$ for which $\theta_0^* = \theta_e^*$.

We first show that for all $e \in E \setminus E_0$, it must be the case that $\theta_e^* = \arg\max R_e(\theta)$. Suppose not, and there existed θ' such that $R_e(\theta') > R_e(\theta_e^*)$. Since $e \in E \setminus E_0$, we know $\theta_e^* < \theta_0^*$. By optimality, if such a superior θ' existed, it must be $\theta' \geq \theta_0^*$. However, by concavity, for any $\alpha \in (0,1)$, $R(\alpha\theta' + (1-\alpha)\theta_e^*) > R(\theta_e^*)$. Thus, by choice of sufficiently small α, the seller could have strictly improved her payoff without violating any IC constraints. Thus, it must be the case that $\theta_e^* = \arg\max R_e(\theta)$.

We now translate this outcome to a selective mandate. Consider a policy of banning disclosure of E_0, and mandating all $E \setminus E_0$. Note that this is equivalent to the seller being *constrained* to post the same price to all buyers in E_0, and unconstrained in the pricing of $E \setminus E_0$. By the previous observation, the seller cannot strictly improve by charging a different price to the agents with $e \in E \setminus E_0$.

A similar argument rules out that the seller may improve by charging a different price to the set of buyers in E_0. If there were an improvement, since each revenue curve is concave, a small movement in the direction of that improvement would again strictly improve the seller's revenue. Thus, the same posted prices are optimal.

□

A.3 Theorem 2

We provide the proof for when $E = \{e_0, e_1\}$, as the more general proof follows the same ideas with some additional "accounting". If the optimal mechanism under a policy of discretion can be implemented by either banning or mandating disclosure of e_1, we are finished. So let us suppose towards the contrary that it cannot. There are two fundamental cases: either the seller randomizes over prices following e_1, or the seller raises the price of e_0 above its first-best level in order to satisfy the discretion constraint, as in the diagram.

Case 1: Randomization. By the characterization of the seller's value function, we know θ_0^* solves $\max_\theta p_0 R_0(\theta) + p_1 \tilde{R}_1(\theta)$. Additionally, we know that at θ_0*, because the seller randomizes, $R(\theta_0^*) < \tilde{R}(\theta_0^*)$. Let θ_1, θ_1' denote the posted prices over which the seller randomizes after observing e_1. We know that we must have $\theta_1 < \theta_0^* < \theta_1' < \theta_1^{FB}$, where θ_1^{FB} maximizes R_1.

Rather than describe the changes in the underlying distributions, we perform manipulations on the revenue curves themselves. For this purpose, we absorb the constant terms p_0 and p_1 into the revenues R_0 and R_1. We provide a brief overview of the technique before providing the details. We modify R_1 in two steps: first, we remove probability mass around the peak such that θ_1' becomes the optimal posted price. Second, we proportionally move buyers with valuations in $[\theta_1', \bar{\theta}]$ from e_1 to e_1. The net result of these two changes is that the seller is *indifferent* between posting the prices of θ_1 and θ_1' when e_1 is observed, as they are both optimal. Thus, she can randomize between the two in order to replicate the policy under discretion. Finally, we must check that the seller finds posting θ_0^* optimal after observing e_0.

A convenient lemma that allows us to work directly with revenue functions is stated below.

Lemma 4. *A function $R : [0, \bar{\theta}] \to \mathbb{R}$ is an (improper) revenue curve for some (improper) CDF iff (i) $\alpha R(x) \leq R(\alpha x)$ for all x,$\alpha \in [0,1]$ (ii) $R(\theta)/\theta \leq 1$ for all θ, and (iii) $R(0) = R(\bar{\theta}) = 0$*

Define $\delta = \frac{R_1(\theta_1') - R_1(\theta_1)}{\theta_1' - \theta_1}$. This is the slope of the line segment connecting the values at θ_1 and θ_1'. Define $R_1'(\theta) = \min\{R_1(\theta), R_1(\theta_1')\}$, and $S_1 = R_1 - R_1'$. Notice that R_1' is indeed generated by some CDF based on the geometric characterization of revenue curves provided earlier. Additionally, that CDF must have probability at-least δ above the value of θ_1'. Now we uniformly delete mass from $[\theta_1', \bar{\theta}]$ such that the total probability deleted is exactly δ. In terms of revenue curves, we are subtracting the following:

$$S_1' = \begin{cases} \delta x & x \in [0, \theta_1'] \\ \delta R_1' & x \in [\theta_1', \bar{\theta}] \end{cases}$$

Denote the resulting revenue curve by R_1^*. That is, $R_1^* = R_1 - S_1 - S_1'$. It is easy to verify that, by construction, the maximizers of R_1^* are both θ_1 and θ_1'. This follows from the fact that the line segment between the two points had a slope of δ, making them optimal out of all prices less than θ_1', and that by subtracting S_1, θ_1' became optimal out of all prices $[\theta_1', \bar{\theta}]$. Notice that during this construction we did *not* modify the probabilities of any types less than θ_1'. This is essential, as for these types, a randomization over the two prices is *not* equivalent to facing the average price. On the other hand, it is equivalent for all types larger than θ'_1, because they will purchase the good regardless.

Finally, we re-assemble the pieces that were removed from R_1 onto R_0. Define $R_0^* = R_0 + S_1 + S'_1$. It is routine to verify that R_0^* is a valid revenue curve. To conclude, we note that θ_0^* is now the optimal posted price, which follows from optimality of θ_0^* in the original problem with discretion.

B An Environment in which Theorem 2 Fails

Consider a setting in which the agent has one of two types, $\theta \in \{\theta_1, \theta_2\}$, equally likely. The principal has three possible actions $\{x, y, z\}$. Table 5 below describes the payoff-pairs (v, u) for the principal and the agent, respectively. The payoffs are constructed in a way that the principal and agent preferences are very misaligned.

	x	y	z
θ_1	(1,1)	(5,0)	(2,2)
θ_2	(5,0)	(1,1)	(2,2)

Table 5: Payoffs under which Theorem 2 fails

Consider an evidence structure $E = \{e_0, e_1\}$ under which type θ_1 always gets e_1, while type θ_2 always gets e_0. Thus, θ_1 can claim to be θ_2, but not vice-a-versa.

The principal's optimal mechanism under a policy of discretionary disclosure involves giving type θ_1 a 50-50 lottery over actions y and z. Type θ_2 is given outcome x for certain. The lottery is chosen so that type θ_1 is indifferent between the lottery over y and z versus getting x for sure. Notice that type θ_2 would strictly prefer the lottery to her allocation of x, which yields a payoff of 0.

However, under any mandated disclosure policy and *any* beliefs of the principal, the optimal mechanism does not involve giving z with positive probability. Thus, regardless of the distribution of types and evidence, any mandated disclosure policy will only generate outcomes where x and/or y occur with positive probability.

The pathology within this example is that z is the agent's most-preferred choice, but the principal would rather randomize over x and y. Thus, using z to provide incentives to *both* agents is ineffective, as it does not change the relative proportions of x and y required to induce truth-telling. So it is useless whenever the principal faces "reciprocal" IC constraints. However, if only one direction of IC constraint is required, as is the case with discretionary disclosure, z is effective at incentivizing exactly *one* of the two types (θ_1) to report truthfully.

C Arbitrary Evidence Structures

Our baseline model considers a very simple style of evidence structure. The only way the buyer is allowed to misreport his evidence is to conceal it altogether. That is, the buyer can only opt-out of providing his evidence. We briefly develop a more general model of evidence in this environment, and show that our main results extend.

We model the evidence environment as a finite partially-ordered set $(E, \rhd)$. The partial order captures the primitive ways in which the buyer may misreport his evidence. Specifically, for any $e \in E$, under a policy of full discretion, the buyer is allowed to disclose any $e' \in E$ such that $e \rhd e'$. In this setting, regulation can be described by a function $\gamma : E \to 2^E \setminus \emptyset$ such that $e' \in \gamma(e) \implies e \rhd e'$ *and* if $e'' \in \gamma(e')$ and $e' \in \gamma(e)$, then $e'' \in \gamma(e)$. In this setting, we need to explicitly impose transitivity of the regulation, otherwise the revelation principle may fail.

The formalism of the problem is otherwise unchanged. The seller's mechanism is a price and quantity schedule as a function of the buyer's reported valuation and evidence. $q : \Theta \times E \to [0, 1]$ and $t : \Theta \times E \to \mathbb{R}$. Incentive compatibility for type (θ, e) requires that for all $\theta' \in \Theta$ and $e' \in \gamma(e)$

$$\theta q(\theta, e) - t(\theta, e) \geq \theta q(\theta', e') - t(\theta', e') \qquad (\text{IC}(\theta, e))$$

And the problem of the seller is

$$\max_{q,t} \int_\theta \sum_{e\in E} t(\theta,e)p(e|\theta)dF(\theta) \tag{26}$$

$$s.t. \quad \text{IC}(\theta,e) \quad \forall\theta,e \tag{27}$$

$$\text{IR}(\theta,e) \quad \forall\theta,e \tag{28}$$

When $\triangleright$ is a tree, there is an easy generalization of the characterization of the optimal mechanism that can be described inductively. For evidence e, the seller randomizes over 2^{k-1} prices, where k is the number of $\triangleright$-lower evidence: $k = |\{e' \in e : e \triangleright e'\}|$. The value function can be found by recursively taking the monotone concavificiation.

Unfortunately, when $\triangleright$ is not a tree, there is not a convenient description of the extreme points of the set of IC mechanisms. For example, suppose $E = \{e_0, e_1, e_2\}$ and $e_1 \triangleright e_0$, and $e_2 \triangleright e_0$. In this case the seller may find it optimal to randomize over an arbitrary number of prices for all of e_0, e_1, e_2. It turns out that, regardless, our two main results extend to these settings.

C.1 Theorem 1

Let $G(\theta|e)$ be the posterior belief CDF, with associated density g, after observing evidence e. Using Bayes rule, this is just

$$g(\theta|e) = \frac{p(e|\theta)f(\theta)}{\int_\Theta p(e|\theta')f(\theta')} \tag{29}$$

Recall that the posted-price revenue function for evidence e is given by

$$R_e(x) = x(1 - G(x|e))$$

Theorem 4. *Suppose R_e is concave for all $e \in E$.*

$$\bigcup_{\gamma\in\Gamma^M} O(p,\gamma) \supseteq \bigcup_{\gamma\in\Gamma^D} O(p,\gamma)$$

That is, any outcome achievable by some discretionary policy is also achievable by some selective mandate.

By construction. (i) We show first that the seller need not randomize over prices due to the concavity assumption. Randomization lowers the seller's payoff and simultaneously increases the information rents paid to the buyers. (ii) Then, we show that the seller's price is actually ex-post optimal conditional on the set of agents that disclose particular evidence. This implies that if the corresponding disclosures were mandated, the seller has no incentive to change her mechanism.

Lemma 5. *If R_e is concave for all $e \in E$, then an optimal mechanism posts a price for each $e \in E$.*

Proof. By (strong) induction. The inductive variable $k \in \mathbb{N}$ is the number of lower-ranked evidence in E. That is each $e \in E$ will be identified with $k = |\{e' \in E : e \rhd e'\}|$.

Base case, $k = 1$: First, consider any $\rhd$-minimal element of E, denoted e_0. To ease notation, denote the allocation $q_0(\theta) := q(\theta, e_0)$ and similarly for t_0. By the same logic as in the baseline model, at an optimal mechanism $t_0(0) = 0$. Notice that the allocation function q_0 can be interpreted as a randomization over posted prices, and can be extended to a CDF $q_0 : \Theta \to [0, 1]$. In the case that $q_0(\bar{\theta}) < 1$, we can let the corresponding CDF jump to 1 at $\bar{\theta}$.

Denote the expected allocation as $\bar{q}_0 = \int s dq_0(s)$. Because the buyer's utility is convex in the distribution of prices, the seller posting a single price $\bar{q}_0$ (weakly) lowers the utility of *all* buyer valuations relative to the original distribution by Jensen's inequality. Any buyer with a valuation of $\theta \geq supp(q_0)$ is indifferent between the randomization and $\bar{q}_0$ because they will always purchase the good.

Therefore, replacing q_0 with $\bar{q}_0$ creates no new incentives for a buyer with evidence $e \rhd e_0$ to deviate. By the concavity assumption, this is also an improvement for the seller.

Inductive Step, $k \in \mathbb{N}$:

Suppose the optimal mechanism posts a price for all $e' \in E$ such that $|\{e'' \in E : e' \rhd e''\}| < k$. Denote each price by $x(e')$.

Now fix any $e \in E$ that is $\rhd$-higher than k elements of E. Let $t_k \in \mathbb{R}$ denote the transfer to the lowest type, and $q_k : \Theta \to [0, 1]$ denote the quantity functions in the optimal mechanism. We essentially repeat the same argument as the base case, replacing q_k with its expectation $\bar{q}_k$. But, there are two extra considerations (i) we show that $t_k = 0$ (ii) this creates no disincentives for buyers with e to misreport to a lower e'.

(i) **At an optimal mechanism** $t_k = 0$.[30] This is fairly intuitive, but note that we cannot just reduce t_k to 0 without possibly violating an IC constraint. This may cause a buyer with evidence e to find it optimal to misreport. Rather, we modify *both* t_k and q_k. Let $U_k(\theta) = t_k + \int_0^\theta q_k(s)ds$ denote the indirect utility of the agent in the mechanism.

First, we identify the "outside option" of type (θ, e) as the maximum utility they could attain by reporting (θ, e') for some e' lower than e, which we denote as $U_0(\theta)$.

$$U_0(\theta) := \theta - \max_{e' \in E : e \rhd e'} x(e') \tag{30}$$

U_0 is convex and increasing in θ because it is the maximum over convex and increasing functions. All the IC constraints can be summarized as $U_k(\theta) \geq U_0(\theta)$ for all θ. Notice that, by optimality, there must be some value of θ for which $U_k(\theta) = U_0(\theta)$, otherwise the seller could strictly benefit by reducing t_k by small-enough ϵ without violating any constraints.

Consider the first value of for which the IC constraint binds, denoted $\theta_0 := \inf \theta : U_k(\theta) = U_0(\theta)$. Since $t_k' > 0$ and $U_0(0) = 0$ it must be that $\theta_0 > 0$. Since θ_0 is the *first* point of intersection, it must be that the (left-handed) derivatives are ordered $U_k'(\theta_0) \leq U_0'(\theta_0)$.[31] Consider a new indirect utility function $V^\epsilon : \Theta \to [0, 1]$ defined by:

$$V^\epsilon(\theta) = \begin{cases} U_k(\theta) - \epsilon U_0'(\theta_0)(\theta_0 - \theta) & \theta \in [0, \theta_0] \\ U_k(\theta) & \theta \in [\theta_0, \bar{\theta}] \end{cases} \tag{31}$$

Notice that by choice of ϵ sufficiently small, we have a convex function V such that (a) $V' > U_k'$ on $[0, \theta_0]$ (b) $V < U_k$ on $[0, \theta_0)$ (c) $V = U_k$ on $[\theta_0, \bar{\theta}]$ (d) $V \geq U_0$ for all θ. We interpret this V as an alternative indirect utility function, implementable by some quantity and transfer schedule. The alternative mechanism which implements V gives the seller strictly larger payoff because the both the allocation is pointwise higher (a) and the associated transfer is lower (b). Additionally, no disclosure IC constraints are violated (d).

[30] This result is also true without the concavity assumption, and the argument is written in a way to make this clear. In particular, all we need to do is consider the best deviations for each buyer type, which form a convex and increasing "outside option" for the buyer.

[31] If U_k and U_0 are not differentiable at θ_0, then there exist subgradients with the same property since both functions are convex.

(ii)**An optimal mechanism posts a price**. Now that we've seen that the transfer to the lowest type is necessarily 0, all that remains is the allocation function $q_k : \Theta \to [0,1]$. Again, we view q_k as a CDF which describes the randomization over posted prices. As in the base case, replacing q_k with its expected price $\bar{q}_k$ is an improvement for the seller by the concavity assumption. It makes all types with evidence e weakly worse-off, so it creates no new incentives for other types with evidence $e' \rhd e$ to misreport as having e.

Finally, it does not create any new incentives for any buyer (θ, e) to misreport his evidence as e'. This follows directly from incentive compatibility of the highest type $\bar{\theta}$. Because this type always buys the good at any price $[0, \bar{\theta}]$, he is risk-neutral and indifferent between a lottery over prices and its expectation. Hence, $\bar{q}_k$ must be weakly smaller than the posted price for any evidence $e' \in E$ such that $e \rhd e'$.

□

Next, we show that under the concavity assumption, the prices that the seller charges are actually best replies to the beliefs induced by subsets of the evidence.

To be precise, consider a seller-optimal mechanism which posts a price $x(e)$ for each $e \in E$. We can use the prices to partition the evidence into equivalence classes. For each $e \in E$, consider the set of evidence which posts the same price.

$$S(e) = \{e' \in E : x(e) = x(e')\} \tag{32}$$

The revenue curve conditional on the set S is similarly defined

$$R_S(\theta) = \theta(1 - G(\theta|S)) \tag{33}$$

Where $G(\theta|S)$ is again computed using Bayes rule. Importantly, $R_S = \sum_{e \in S} R_e p(e)$. That is, the revenue curve conditional on set S is a weighted average of the revenue curves of its elements. Hence, R_S is also concave. Finally, we conclude that conditional on the seller only knowing $e \in S(e)$, then $x(e)$ is the optimal posted price.

Lemma 6. *Suppose R_e is concave for all $e \in E$. In an optimal mechanism, $x(e) \in \arg\max_x R_{S(e)}(x)$ for all $e \in E$*

Proof. Since E is finite, for any sufficiently small ϵ, posting a price of $x(e) \pm \epsilon$ for all realizations in $S(e)$ maintains IC. Suppose towards the contrary that $x(e) \notin \arg\max R_{S(e)}(x)$, and let $y \in \mathbb{R}$ be a strict improvement, such that $R_{S(e)}(x) < R_{S(e)}(y)$. But then we have for any $\alpha \in (0, 1)$.

$$R_{S(e)}(x) < R_{S(e)}(\alpha x + (1-\alpha)y) < R_{S(e)}(y)$$

Sufficiently small but positive α will continue to satisfy the IC constraints, hence there is a strict improvement available to the seller. □

Finally, the regulator can induce the same outcome under mandatory disclosure by requiring that the buyer provide a particular element of each equivalence class $S(e)$. Given this disclosure, the seller must find it optimal to post $x(e)$, as in the optimal mechanism with discretion.

C.2 Theorem 2 Preliminary: The Dual

In order to prove Theorem 2 in the more general setting, we assume that the set of buyer valuations is finite, so $\Theta = \{\theta_0 = 0, \theta_1, \ldots, \theta_n = \bar{\theta}\}$. It does not seem that finiteness is essential for this result, but the current method uses a iterative construction. We study the dual of the problem, and use an interpretation provided by Sher and Vohra (2015) to inform the construction. First, we describe the dual optimum for the problem.

Let $\mu(\theta, e)$ denote the constraint corresponding to $q(\theta, e) \leq 1$. Let $\lambda(\theta, e, \theta', e')$ denote the constraint that type (θ, e) can report to be (θ', e') for all $\theta' \in \Theta$ and $e' \lhd e$. Though it is sufficient to only consider adjacent binding constraints, it turns out to be useful to include all possible deviations. To slightly simplify notation, for a given (θ, e) let U and L denote the upper-than and lower-than sets of $\Theta \times E$, defined by

$$U(\theta, e) = \{(\theta', e') : \theta' \in \Theta, e' \in E, e' \rhd e\} \tag{34}$$
$$L(\theta, e) = \{(\theta', e') : \theta' \in \Theta, e' \in E, e \rhd e'\} \tag{35}$$

That is, $U(\theta, e)$ describes all types that could claim to be (θ, e) could make, and $L(\theta, e)$ is the set of all reports that (θ, e) could make.

The dual is:

$$\min_{\mu,\lambda} \sum_{\theta} \sum_{e} \mu(\theta, e) \tag{36}$$

$$s.t. \quad \sum_{(\theta',e')\in L(\theta,e)} \lambda(\theta, e, \theta', e') - \sum_{(\theta',e')\in U(\theta,e)} \lambda(\theta', e', \theta, e) = f(\theta)p(e|\theta) \quad \forall \theta, e \tag{37}$$

$$f(\theta)p(e|\theta)\left(\theta - \frac{\sum_{(\theta',e')\in U(\theta,e)} \lambda(\theta, e, \theta', e')(\theta' - \theta)}{f(\theta)p(e|\theta)}\right) \leq \mu_t \tag{38}$$

$$\lambda(\theta, e, \theta', e') \geq 0 \tag{39}$$

$$\mu(\theta, e) \geq 0 \tag{40}$$

As noted in Sher and Vohra (2015), the key observation is that the expression in parentheses in equation 37 can be interpreted as an endogenous "virtual value".

$$\psi(\theta, e) = \theta - \frac{\sum_{(\theta',e')\in U(\theta,e)} \lambda(\theta, e, \theta', e')(\theta' - \theta)}{f(\theta)p(e|\theta)} \tag{41}$$

and , at the optimum

$$\mu(\theta, e) = \max\{\psi(\theta, e), 0\}$$

To summarize, there are two main observations. First, there is an analogue of the virtual value in this setting. Types are allocated with probability 1 when their virtual value is positive, 0 if negative, and possibly-interior probability when the virtual value is 0. Importantly, including all the IC constraints, including the non-adjacent ones, means we do not have to additionally assume that the virtual valuations are monotone. They will necessarily be that way at the optimum.

Second, equation (37) can be interpreted as a flow of cumulative probability. Imagine that the second term was 0. Then, this says that the total weight of the dual variables associated with (θ, e) wanting to mimic other types (θ', e') is just the probability that (θ, e) is realized. In other words, $\lambda(\theta, e, \theta', e)$ describes how the information rents given to (θ, e) are born by lower type and evidence pairs. The second summation describes the cumulative "inflow" into type (θ, e) from higher types and evidence.