Lab Report: Lab5 - Computational Statistics

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Introduction

Implementation of 2 Assignment questions of Computational Statistics Lab 5 .

Contributions

Member: Dhanush Kumar Reddy Narayana Reddy, Liu Id: dhana004, Contribution: Report writing and coding of question 1.

Member: Udaya Shanker Mohanan Nair, Liu Id: udamo524, Contribution: Report writing and coding of question 2.

Question 1

Question 2

Given a Gumbel distribution with scale parameter 1 and location parameter $\mu + c$, where $c = \log(\log(2))$. And this distribution has the following distribution function. For this distribution, median of a random variable is .

Now we are gone to generate random variables(Gumbel) using Inverse Transformation Method.

The CDF of the Gumbel distribution is given by

$$F(x) = \exp(-\exp(-(x - \mu - c)))$$

where

$$c = \log(\log(2))$$

Inverse Transformation of this function for a random variable is given by $Y \sim Y(0, 1)$.

$$X = F^{-1}(Y)$$

$$F(x) = \exp(-\exp(-(x - \mu - c)))$$

Set $Y = F(x)$, where $Y \sim Y(0, 1)$

Take the natural logarithm on both sides:

$$\log(Y) = -\exp(-(x - \mu - c))$$

Take the logarithm again:

$$\log(-\log(Y)) = -(x - \mu - c)$$

Solve for x:

$$x = -\log(-\log(Y)) + \mu + c$$

```
## Loading required package: lattice
```

```
##
## Attaching package: 'BSDA'
```

```
## The following object is masked from 'package:datasets':
##
##     Orange
```

Now we need to find out power of the test for median values ranging from 0 to 2, where for each value we will using number of observations, n as 13 and then used sign test(used SIGN.test in this case).

I have tested for 50 different values of median.

Jotting Power of few median values.

```
## Median Value:  0   Power: 0.023
```

```
## Median Value:  0.5306122   Power: 0.185
```

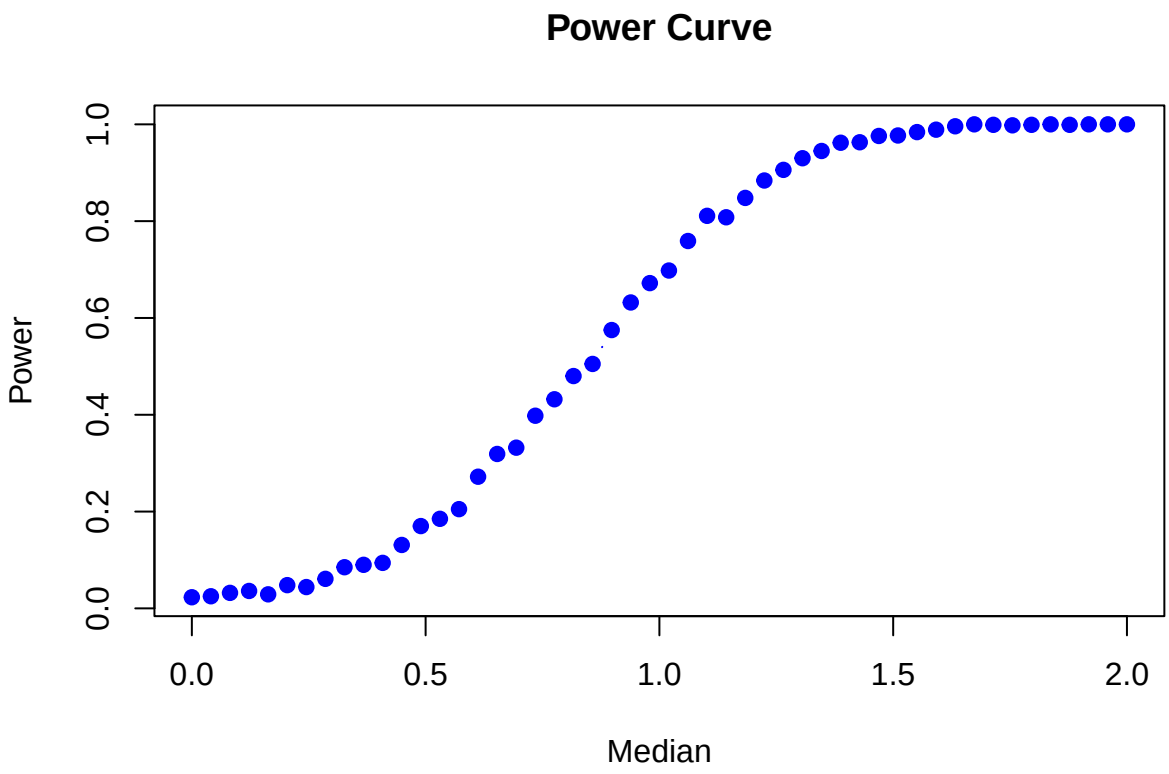
```
## Median Value:  1.020408   Power: 0.698
```

```
## Median Value:  1.510204   Power: 0.977
```

```
## Median Value:  2   Power: 1
```

from this we understand as the value of median increases the power also increases.

Now plotting the power curve



Appendix

Question 1

Question 2

```
library(BSDA)
c <- log(log(2))
gumbel_fn <- function(median,n = 13) {
  Y <- runif(n)
  X <- -log(-log(Y)) + median + c
  return(X)
}
median_values <- seq(0, 2, length.out = 50)
len <- length(median_values)
power <- numeric()
for (j in seq_along(median_values)) {
  median <- median_values[j]
  count <- 0
  for (i in 1:1000) {
    data <- gumbel_fn(median)
    test <- SIGN.test(data)
    if(test$p.value < 0.05){
```

```
      count <- count + 1
    }
  }
  power[j] <- count / 1000
}
cat("Median Value: ", median_values[1],"  Power:",power[1])
cat("Median Value: ", median_values[14],"  Power:",power[14])
cat("Median Value: ", median_values[26],"  Power:",power[26])
cat("Median Value: ", median_values[38],"  Power:",power[38])
cat("Median Value: ", median_values[50],"  Power:",power[50])

plot(median_values, power, type = "b", pch = 19, col = "blue",
     xlab = "Median", ylab = "Power", main = "Power Curve")
```