Lab Challenge 05 – Distributions of Random Variables

Due Date: 11:59 pm, 4 days after class

Each challenge is graded out of 2 points:

- 0 points – no attempt or no progress to a solution
- 1 point – challenge not fully completed or completed with major errors
- 2 points – challenge fully completed with at most a small error

Deliverables

1. A single pdf document containing your solutions to the challenges you completed.
2. An RStudio file (.R extension) containing a *complete* script used to generate your results.

Challenges

1. As of Jan 29, 2022, 79.7% of BC residents are fully vaccinated against COVID-19. Suppose BCIT officials wish to determine whether the rate of vaccination among BCIT students differs from this. They select a random sample of $n = 200$ students and define $X =$ the number of vaccinated students in the sample.

   Assume that:
   - BCIT student *do* have a 79.7% vaccination rate.
   - X can be modelled as a binomial variable with $n = 200$ and $p = 0.797$.

   a. Generate a probability histogram for X. Label the axes and give it a title. On the same graph plot a normal distribution with the same μ and σ as X.
   b. Calculate $P(155 \leq X \leq 165)$.
   c. The vaccination rate for students in the sample is equal to the sample proportion $\hat{p} = \frac{X}{n}$. Calculate the probability $P(0.740 \leq \hat{p} \leq 0.855)$.

2. Suppose that a data center detects 1.56 hard drive errors during each hour of operation, on average. Assuming that hard drive errors are independent and random in time (which may not be realistic), we can model this situation using a *Poisson random variable*

   $$X = \text{the number of hard drive errors during one hour of operation}$$

   a. Find $P(X = 0)$
   b. Find the number of errors, m, such that $P(X > m)$ is less than 5%.
   c. Plot a probability histogram of X. Label the axes and give it a title. On the same graph, plot a normal distribution with the same mean and standard deviation as X. Does the normal distribution provide a good approximation of X?

3. Here you will investigate the effect of *replacement* when choosing a five-card "hand". Suppose you randomly select five cards from a standard deck of cards. Let $X =$ the number of aces selected.
   a. Assuming that the cards were select *with replacement*, use the binomial distribution to generate a probability distribution of X. What is the probability of getting 4 aces?
   b. Assuming that the cards were select *without replacement*, use probability laws (or use simulation) to generate a probability distribution of X. What is the probability of getting 4 aces?