Let x = input reserves $\qquad$ Δx = input (sell) amt.

y = output reserves $\qquad$ s = split fraction ≥ 0 & ≤ 1

r = 1 - fee

For a split between 2 pools.

$$\underset{\text{(total output)}}{F} = \frac{y_1 r . \Delta x . s}{x_1 + r \Delta x s} + \frac{y_2 r . \Delta x (1-s)}{x_2 + r \Delta x (1-s)}$$

$$\frac{dF}{ds} = \frac{(x_1 + r \Delta x . s) y_1 r \Delta x - y_1 r \Delta x s (r \Delta x)}{(x_1 + r \Delta x)^2} + \frac{(x_2 + r \Delta x (1-s)) (-y_2 r \Delta x) + y_2 r \Delta x (1-s) r \Delta x}{[x_2 + r \Delta x (1-s)]^2}$$

$$= \frac{x_1 y_1 r \Delta x}{(x_1 + r \Delta x s)^2} - \frac{x_2 y_2 r \Delta x}{(x_2 + r \Delta x (1-s))^2}$$

$\frac{dF}{ds} = 0$ when marginal rate of return is same for both pools.

Similarly,

$$\frac{d^2F}{ds^2} = \frac{-2 x_1 y_1 r^2 \Delta x^2}{(x_1 + r \Delta x)^2} - \frac{2 x_2 y_2 r^2 \Delta x^2}{(x_2 + r \Delta x (1-s))^2}$$

which means $\frac{d^2F}{ds^2}$ is < 0 & optima exists.