

Introduction to Electrical & Electronics Circuits

**Course Code: EE
101**

Department: Electrical Engineering

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Sub-Topics:

- Maximum power transfer theorem



Review

- Instantaneous power in 1- ϕ circuit pulsates at $2F$
- Single phase motors require special resilient mountings
- Complex power, $S = VI^* = P + jQ$

$$P \rightarrow \text{active power} = VI \cos\theta \text{ W} =$$

$$I^2 R = \angle_V^I$$

$$Q \rightarrow VI \sin\theta \text{ VAR} = I^2 X$$

$$S = VI^* \text{ VA}$$

- In DC, if P & V are known, I can be determined
- However In AC, P, V & $\cos\theta$ or V & S should be known

to determine I



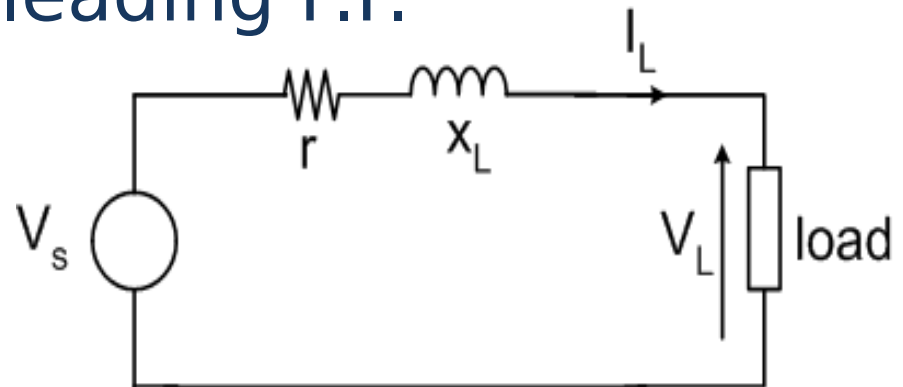
For a given P

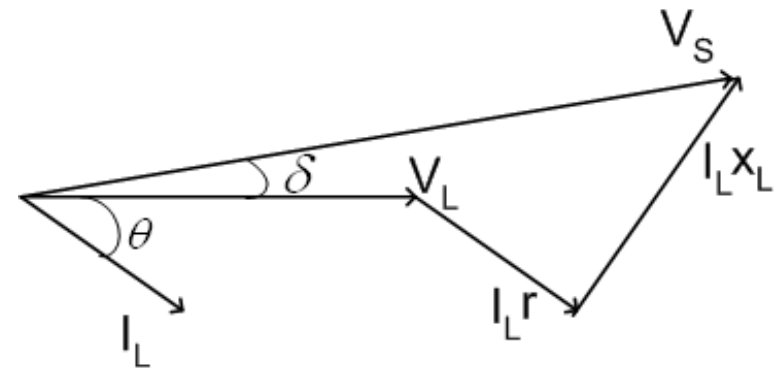
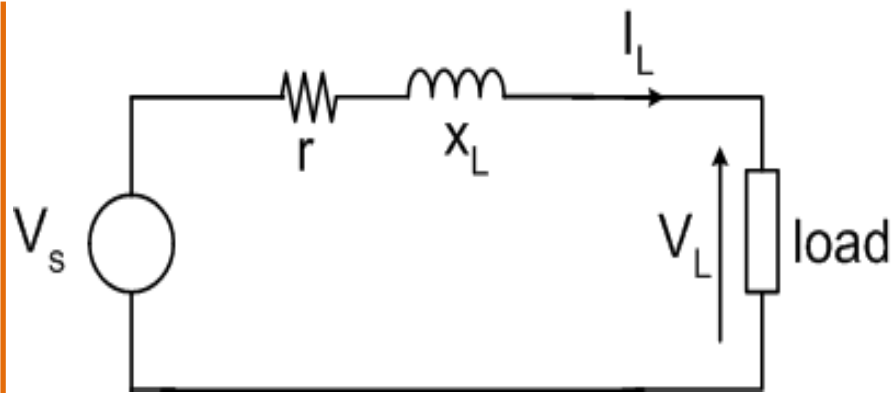
- I drawn by load as $\cos \theta$ (P.F.)

⇒ Drop in the line

⇒ $I^2 R$, $I^2 X$ in the line and therefore 'S' ↑
of source
⇒ $I^2 R$ loss in source also increases

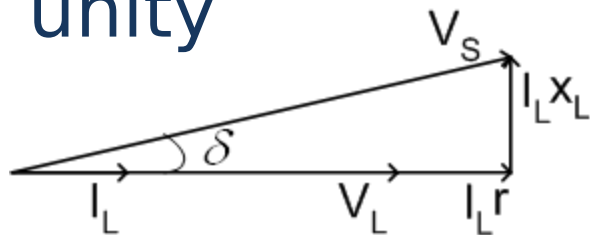
- $V_L < V_s$ for lagging & unity P.F.
- $V_L \leq V_s$ or $V_L > V_s$ for leading P.F.





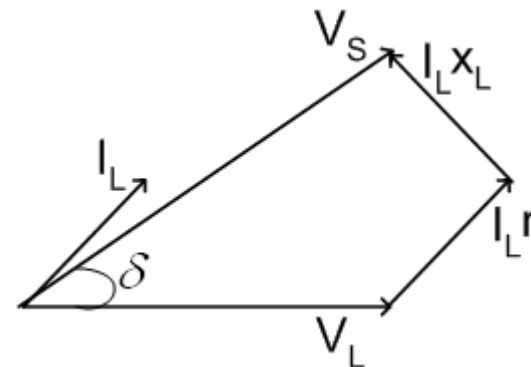
$$|V_s| > |V_L| \quad \text{Source P.F.} = \cos \angle_{V_s}^{I_s = I_L}$$

If the P.F. is unity



$$|V_s| > |V_L|$$

If P.F. is leading



$$|V_s| \geq |V_L| \quad \text{or} \quad |V_s| \leq |V_L|$$



Maximum Power Transfer Theorem:

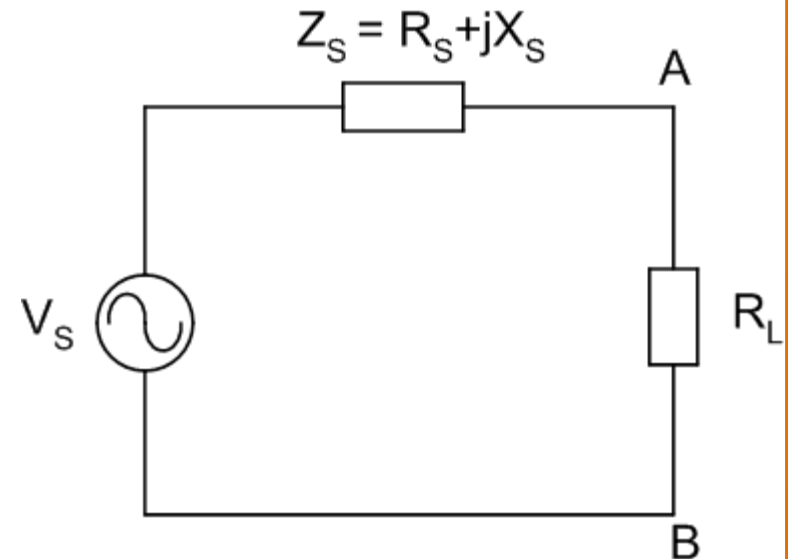
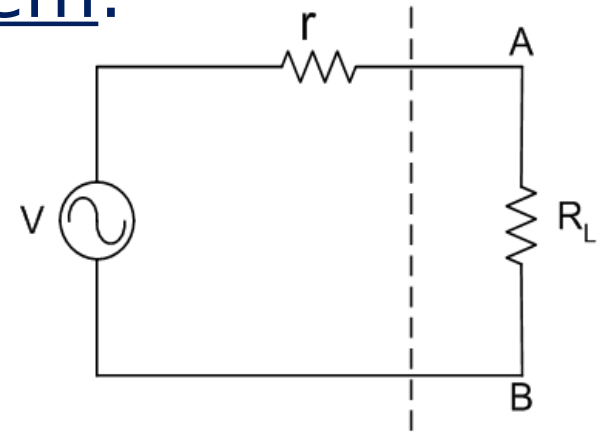
'P' transferred to the load is maximum when ' $r = 0$ '

Generally load is ~~variable~~ the value of load which results in maximum power transfer?

Case I: Load

$$I = \frac{V_s}{\sqrt{(R_s + R_L)^2 + X_s^2}}$$

$$P = I^2 R_L = \frac{V_s^2 R_L}{(R_s + R_L)^2 + X_s^2}$$



$$P = P_{max} \text{ when } \frac{dP}{dR_L} = 0$$

$$R_L^2 = R_S^2 + X_S^2 \text{ or } R_L = |Z_S|$$

$$X_S = 0, P = P_{max} \text{ when } R_L = R_S$$

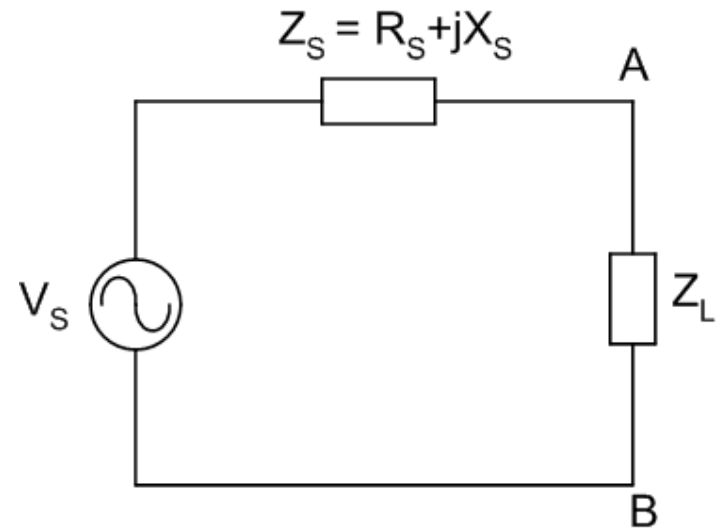
If

Case $Z_L = R_L + jX_L$

ii:

$$I = \frac{V_S}{(R_S + R_L) + j(X_S + X_L)}$$

$$\therefore P = I^2 R_L = \frac{V_S^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$



If R_L is held constant $P = P_{\max}$ when $X_S =$

X_L
If R_L is variable $P = P_{\max}$ when $R_L = R_S$

\therefore If $R_L = R_S$ & $X_L = -X_S$ $Z_L = Z_S^*$



Frequency response of R-C and R-L circuits:

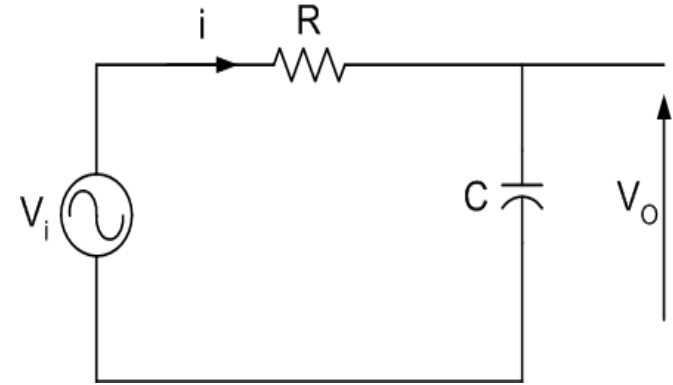
$$i = \frac{V_i}{R + \frac{1}{j\omega C}}$$

$$V_o = \frac{1}{j\omega C} \left(\frac{V_i}{R + \frac{1}{j\omega C}} \right) = \frac{V_i}{1 + j\omega RC}$$

$$\therefore \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC} = A \Rightarrow \text{Gain} \Rightarrow \text{Transfer function}$$

$$\therefore |A| = \frac{1}{\sqrt{1 + (\omega RC)^2}} \quad \& \quad \theta = -\tan^{-1} \omega RC$$

$$\omega = 0; \quad \frac{V_o}{V_i} = 1 \quad \& \quad \theta = 0$$



For low values of ω , $\frac{V_0}{V_i} \approx 1$ & $\theta \approx 0^\circ$

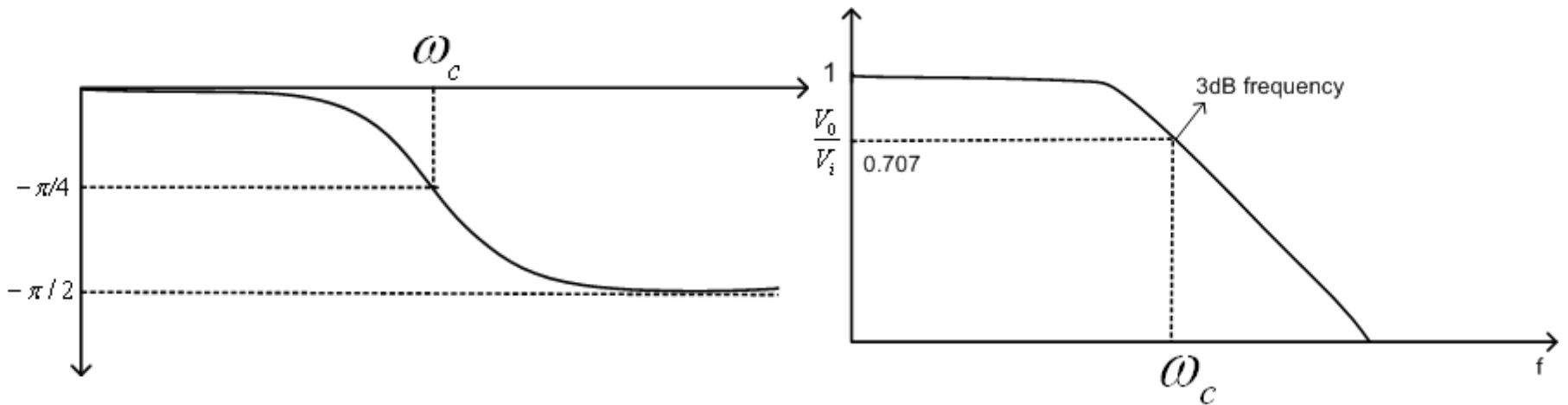
As $\omega \uparrow$, $\frac{V_0}{V_i} \downarrow$

At $\omega = \omega_c = \frac{1}{RC}$, $\frac{V_0}{V_i} = 0.707$, $\theta = -45^\circ$

As $\omega \uparrow$ above ω_c , $\frac{V_0}{V_i} \downarrow$ rapidly & $\theta \uparrow$ towards $-\frac{\pi}{2}$

(Q for $\omega > \omega_c$, $\frac{V_0}{V_i} = \frac{1}{j\omega RC}$ if $\omega \uparrow$ by 10 times, $\frac{V_0}{V_i} \downarrow$ by 10 times)





⇒ o/p always lags the i/p

⇒ Lag Network

$$\frac{V_o}{V_i} = \text{Voltage gain} = A$$



Gain in dB = $20 \log A$

At $\omega = \omega_c$ Gain in dB = $-20 \log(\sqrt{2}) = -3 \text{ dB}$

At $\omega = 0$, $V_0 = V_i$

\Rightarrow Power transfer to the output terminal \propto

V^2 At $\omega = \omega_c$ $|V_0| \downarrow$ to $\frac{1}{\sqrt{2}}$

\therefore Power reduces to $\frac{1}{2}$

$\Rightarrow \omega_c \rightarrow$ Half power frequency



Observations:

⇒ The signal does not get attenuated if $f \ll f_c$

⇒ The signal gets attenuated if it's $f \gg f_c$

$$v_i = A \sin \omega_1 t + B \sin(7\omega t + \theta_1) + C \sin(9\omega t + \theta_2)$$

$$\omega_c = 3\omega_1$$

$$v_0 ; A \sin \omega_1 t$$

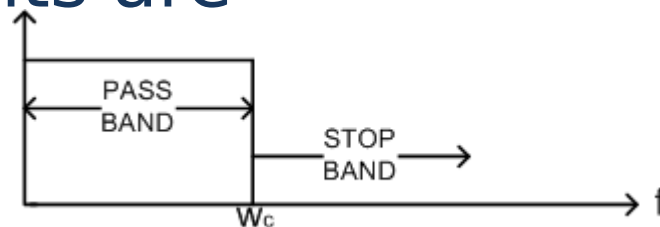
⇒ Only low f components are available

at o/p
⇒ Allows low frequency components

⇒ High frequency components are

filtered out

⇒ **LOW PASS FILTER**

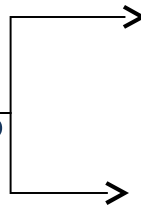


⇒ FILTER: frequency selector circuit used to pass a certain band of frequencies & to stop or attenuate remaining band of frequencies

Active(Gain

could be >1)

⇒ There are two types



Passive(Max gain <1 , only
passive elements)



⇒ FILTER: frequency selector circuit used to pass certain band of frequencies & to stop or attenuate remaining

⇒ There are two types of filter

→ Active (Gain could be > 1)

→ Passive (Max gain 1, only)

passive elements)

