Introduction to Electrical & Electronics Circuits

Course Code: EE 101

Department: Electrical Engineering

Instructor Name: B.G. Fernandes

E-mail id: bgf @ee.iitb.ac.in

Sub Topics:

- Norton's theorem
- Source conversion
- Superposition theorem
- Time domain response of RL and RC circuit
  - Step response
  - Pulse response

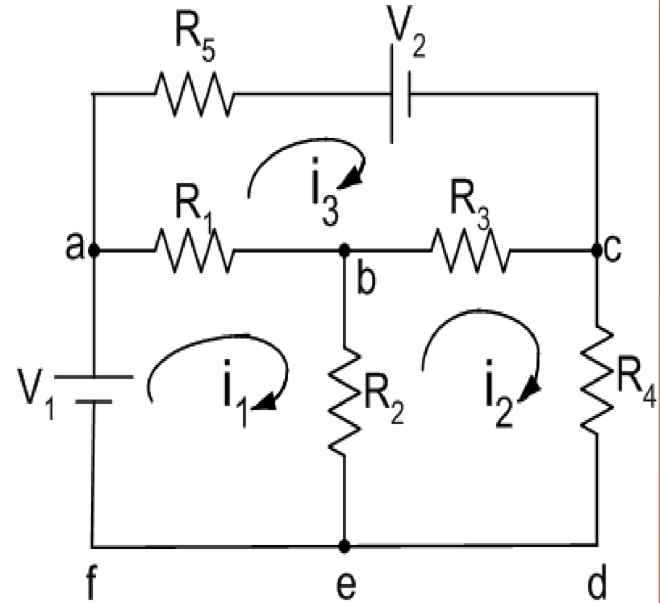
Review

- Mesh current method

$$[R][I]=[V]$$

R – Known, V- Forcing function-known

I – to be determined

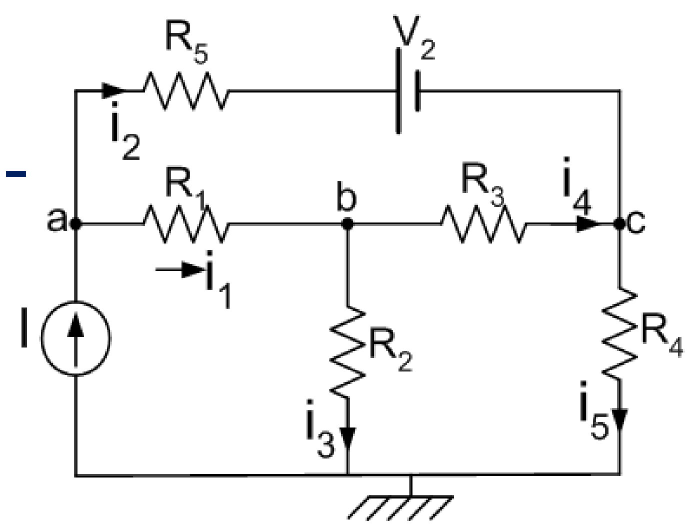


- Nodal Voltage method

$$[G][V]=[I]$$

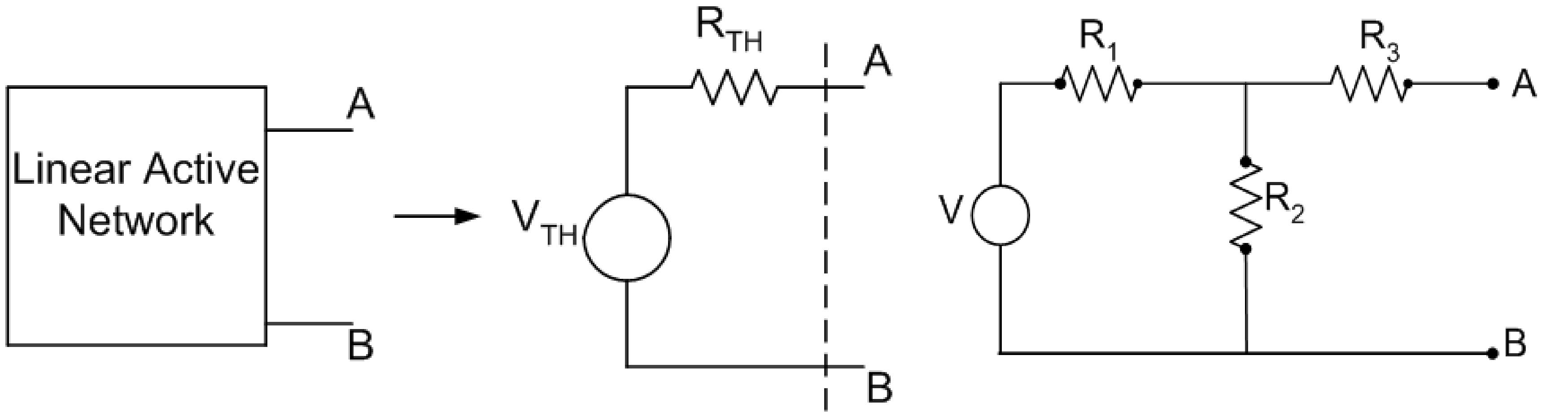
G – Known, I- Forcing function-known

V– to be determined



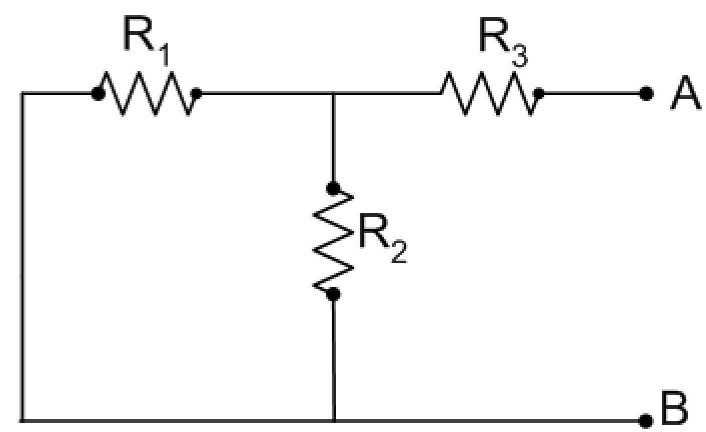
$\Longrightarrow$ Having determined [V], branch currents can be calculated

Thevenin's Theorem

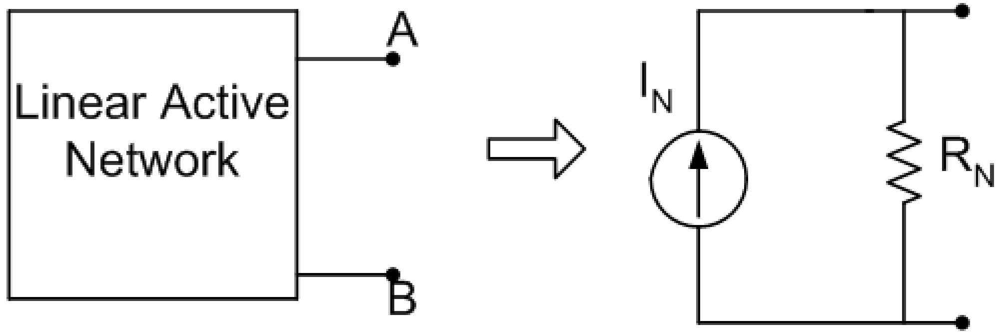


V_{TH} = 'V' across $R_2 = \left(\frac{V}{R_1 + R_2}\right) R_2$

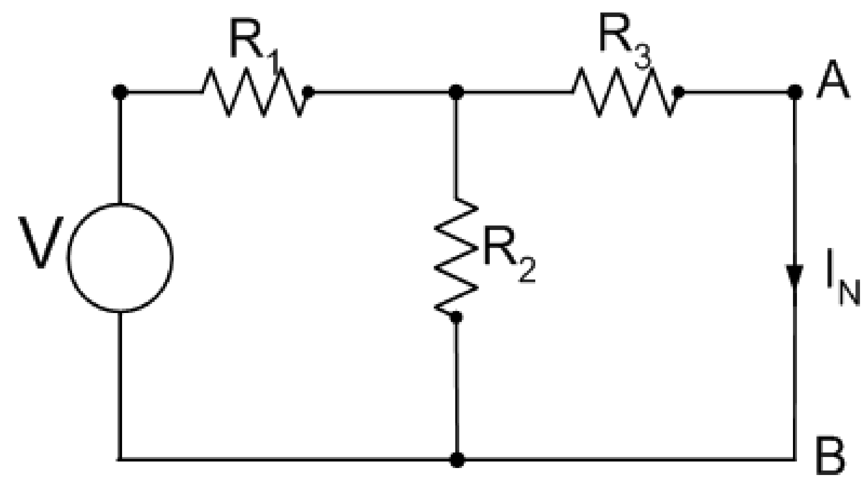
$$R_{TH} = R_3 + \frac{R_1 R_2}{R_1 + R_2} \qquad i_L = \frac{V_{TH}}{R_{TH} + R}$$



Norton’s Theorem:



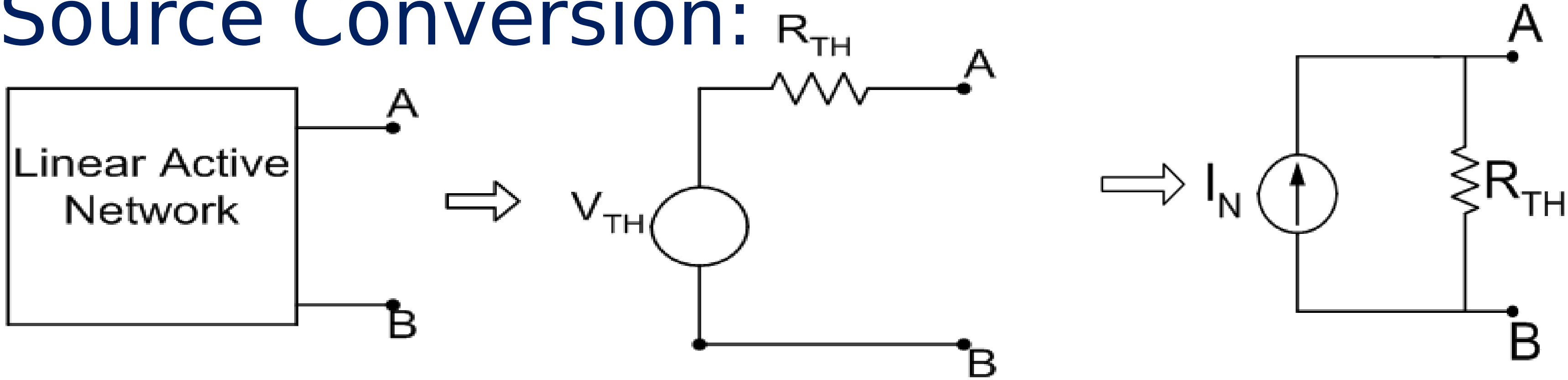
$I_N \longrightarrow$ Current through the short circuit ap
the terminals



$R_N = R_{TH}$

$$I_N = \frac{V}{\left(R_1 + \dfrac{R_2 R_3}{R_2 + R_3}\right)} \frac{R_2}{R_3 + R_2}$$

Source Conversion:



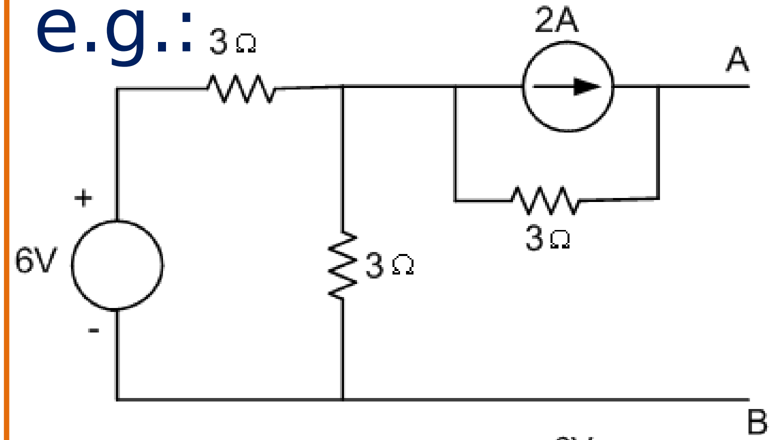
Short circuit the terminals AB

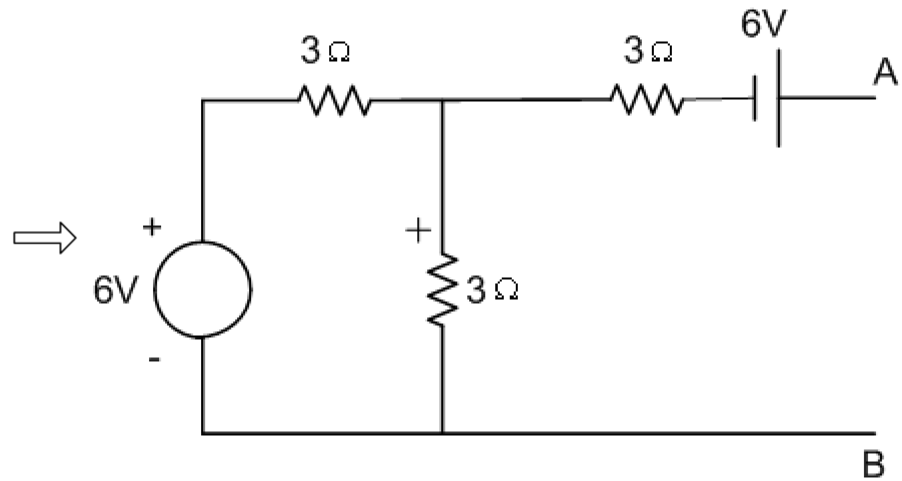
Using Thevenin's theorem, $I_{AB} = \dfrac{V_{TH}}{R_{TH}}$

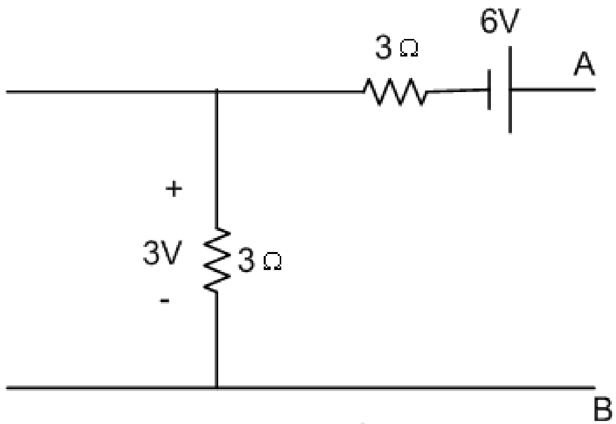
From Norton's theorem, $I_{AB} = I_N$

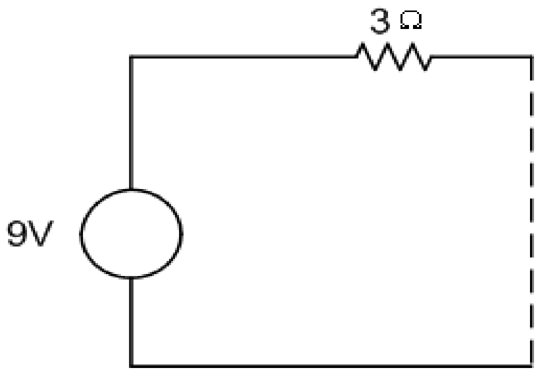
$$\Rightarrow \frac{V_{TH}}{R_{TH}} = I_N \qquad \therefore V_{TH} = I_N R_{TH}$$

e.g.:









$$R_{TH} = 4.5\Omega$$

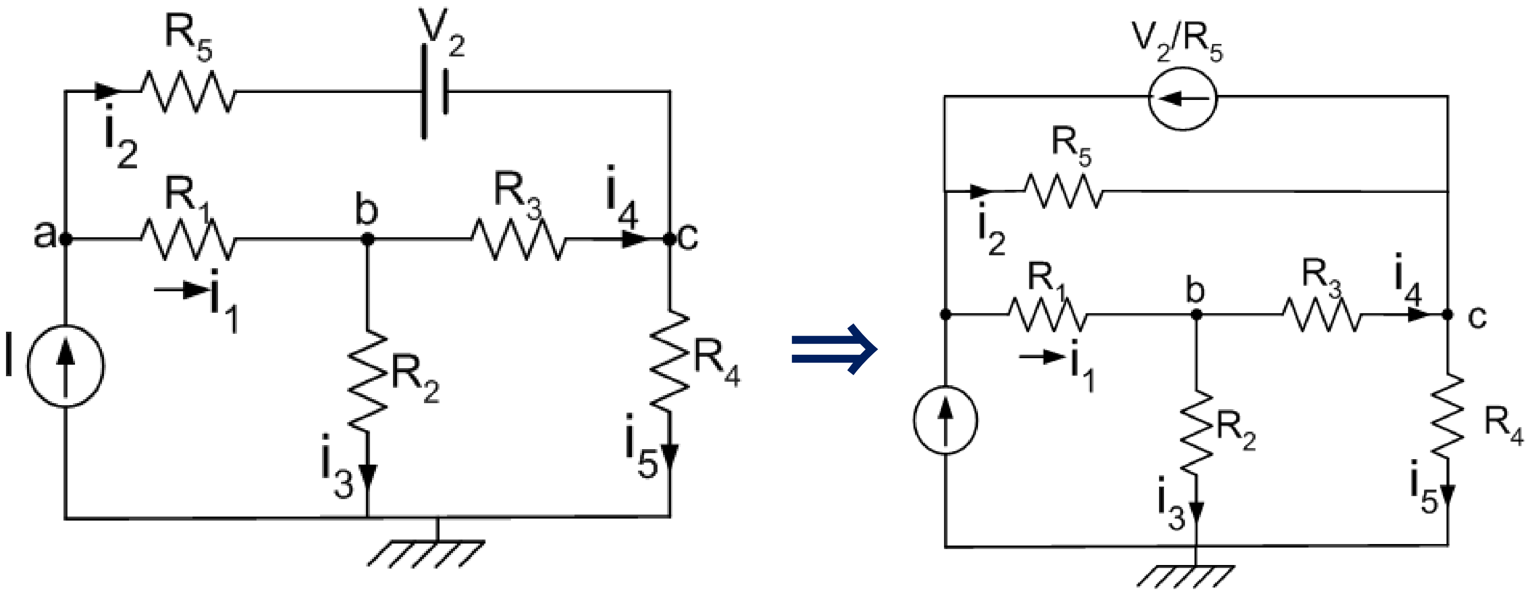
$$V_{TH} = \left(\frac{6}{6} \times 3\right) + 6$$

$$= 9V$$

$$I_{SC} = I_N = 2A$$

Circuit used for nodal analysis

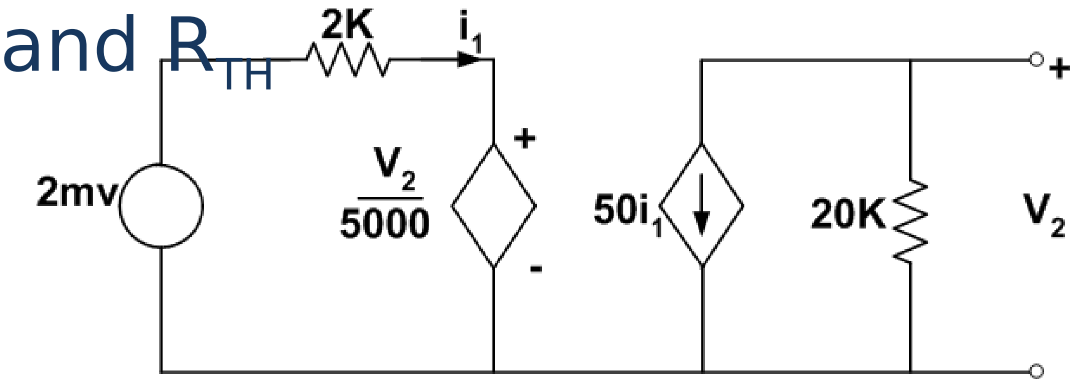
e.g.



$$\begin{pmatrix} (G_1+G_5) & -G_1 & -G_5 \\ -G_1 & (G_2+G_3+G_1) & -G_3 \\ -G_5 & -G_3 & (G_3+G_4+G_5) \end{pmatrix} \begin{pmatrix} v_a \\ v_b \\ v_c \end{pmatrix} = \begin{pmatrix} I+\dfrac{V_2}{R_5} \\ 0 \\ -\dfrac{V_2}{R_5} \end{pmatrix}$$

Equivalent circuit of a transistor amplifier:

Determine V_2 (V_{TH}) and R_{TH}



$$V_2 = -20\times10^3 \times 50 i_1$$

$$2\times10^{-3} = 2\times10^3 i_1 + \frac{V_2}{5000}$$

$$\therefore i_1 = 1.11\times10^{-6}\ \text{A}$$

$$\&\ V_2 = -1.11\ \text{V}$$

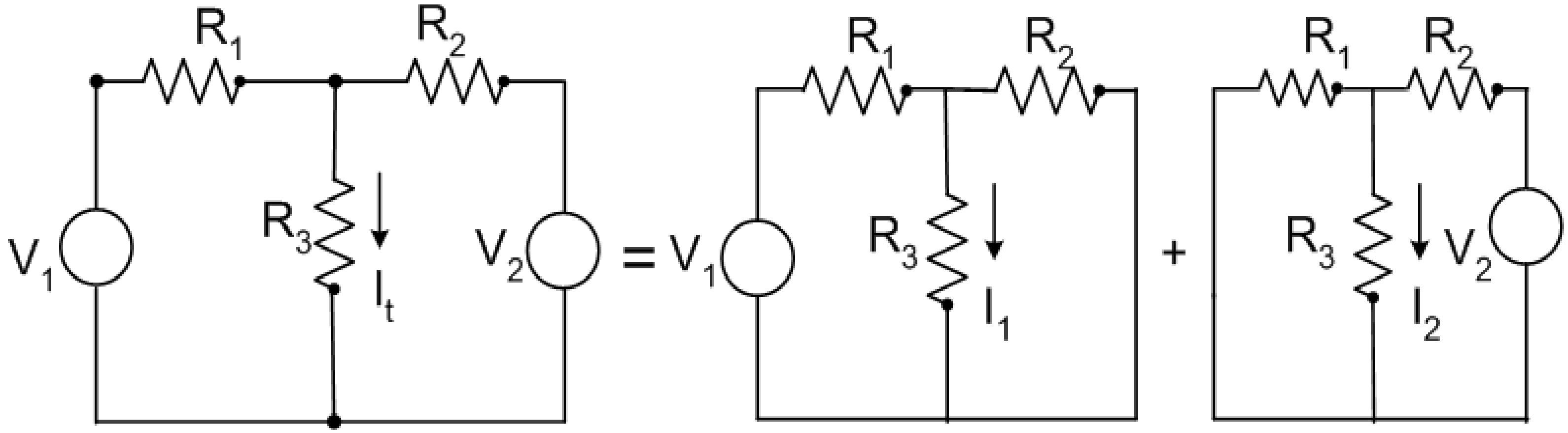
$$R_{TH} = \frac{V_{TH}}{i_{sc}}, i_{sc}\,|_{V_2=0} = -50 i_1$$

$$2\times10^{-3} = 2\times10^3 \left(\frac{-i_{sc}}{50}\right)$$

$$\therefore i_{sc} = -50\times10^{-6}, \therefore R_{TH} = 22.2k\Omega$$

Superposition Theorem:

e response in any element of linear network having
o or more sources is the sum of responses obtained
ch source acting separately, with all other sources s
ual to zero.



$$I_t = I_1 + I_2$$

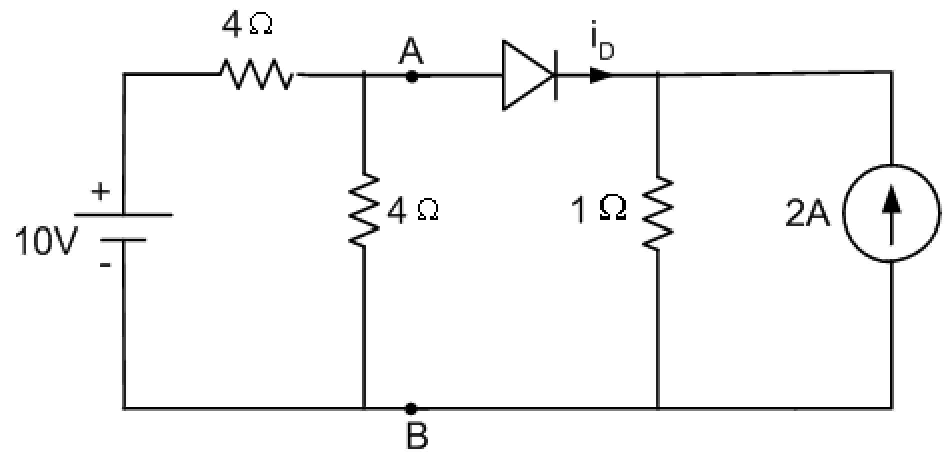
If $x_1 \longrightarrow y_1$

& $x_2 \longrightarrow y_2$

For a linear system, if $(x_1 + x_2) \Longrightarrow (y_1+y_2)$

Determine i_D

(Assume diode is ideal)



Circuit is non-linear

'V' across 1Ω = $(2+i_D)$ = V_{AB}

∴ 'i' supplied by battery

$$= \frac{(2+i_D)}{4} + i_D = \frac{5i_D + 2}{4}$$

Applying KVL

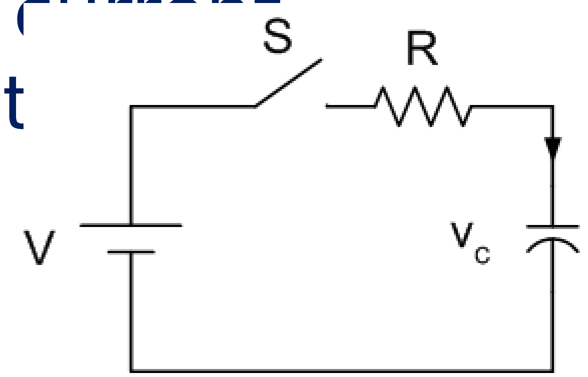
$$10 = \frac{5i_D + 2}{4} * 4 + (2 + i_D)$$

$$i_D = 1A$$

Time domain response of RC and RL circ

- No transients in purely resistive circuit
- I cannot change instantaneously in an inductor
- V across a capacitor cannot change instantaneo

Step response: DC voltage or current suddenly applied to the circuit



RC circuit: $V_c = V_f + (V_{ci} - V_f)e^{-\frac{t}{\tau}}$

where, τ = Time constant

Circuit is assumed to attain steady state at t = 5

V_{ci} = capacitor voltage at t = 0, V_f is the final val

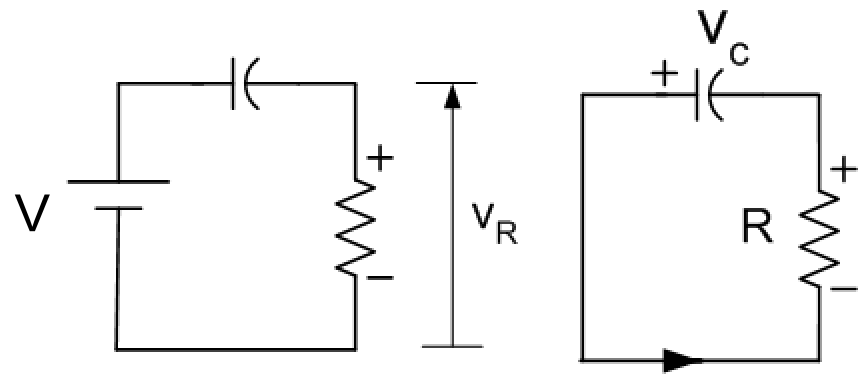
Step response of R-C circuit

Case i)

$$v_c = 0 \text{ at } t = 0$$

$$v_c = 0 \text{ at } t = 0^+$$

$$V_R = V \;\; \& \;\; i = \frac{V}{R}$$

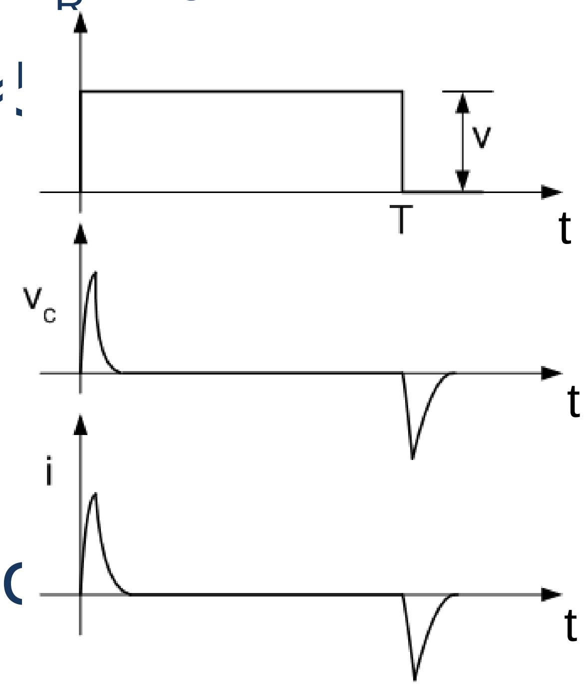


At steady state v_c= V, I = 0 ∴ V_R = 0

Steady state is attained at t ≈!

At t = T⁺

$$v_R = -v_c \;\; \& \;\; i = -\frac{v_c}{R} = \frac{-v_R}{R}$$



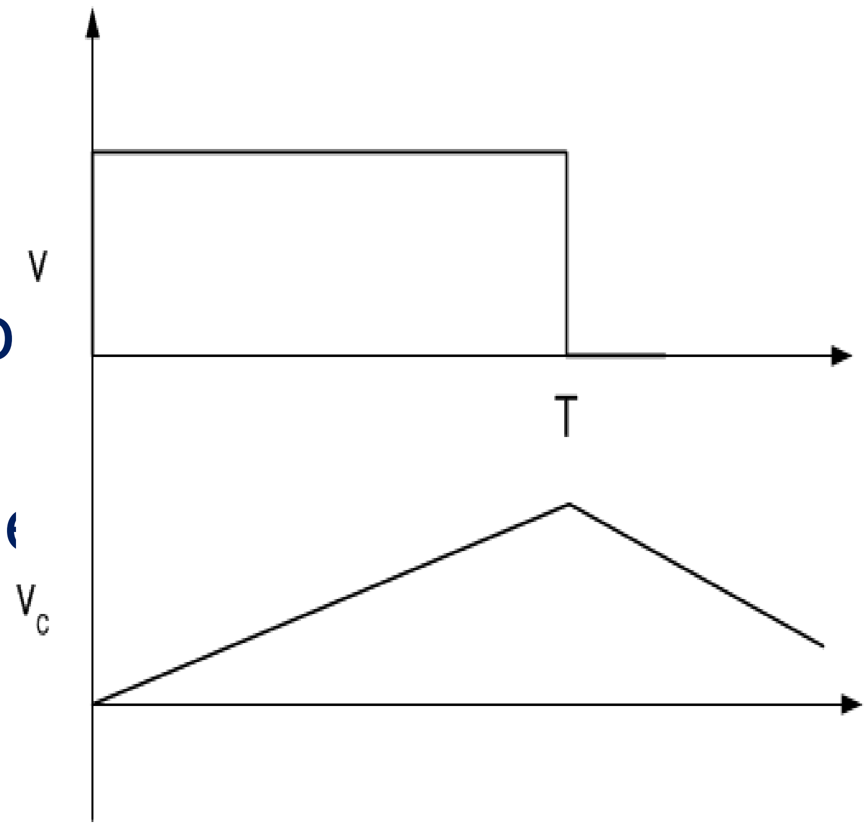
At steady state, v_c and i = 0.

Observation: ‘i’ through ‘C’ c
change instantaneously

Case 2: τ ? T

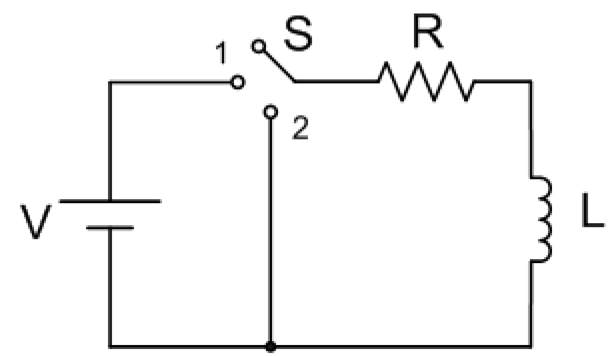
Vc increases gradually
So, $|v_c|$ is a small fractio
V at t = T.
Initial portion of v_c is line
$v_c \Rightarrow$ integral of V

$\Rightarrow$ Integrator



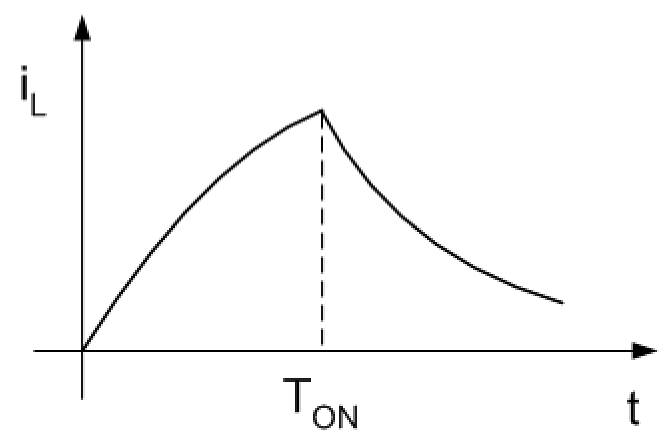
Step response of R-L circuit

‘S’ at 1:



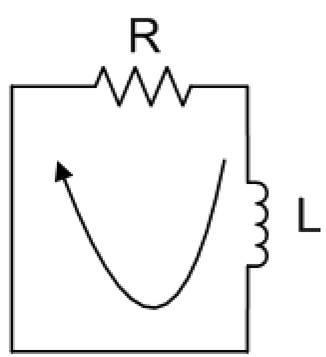
$$V = iR + L\frac{di}{dt}$$

$$i_L = \frac{V}{R}\left(1 - e^{\frac{-t}{\tau}}\right)$$



‘S’ at 2: ‘i’ will decay

$$i_L = \frac{V}{R}e^{\frac{-t}{\tau}}$$

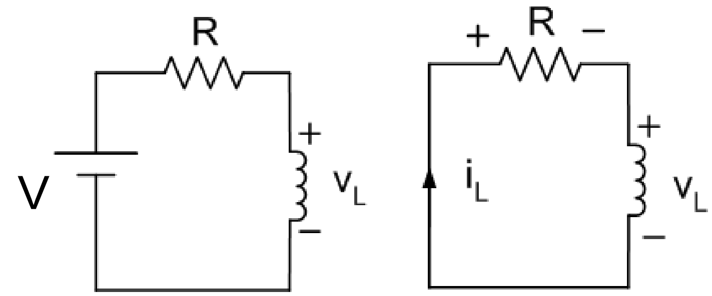


Step response of R-L circuit

$T ? \tau$

$i = 0$ a $t = 0$

$i = 0$ at $t = 0^+$ $\therefore v_L = V$



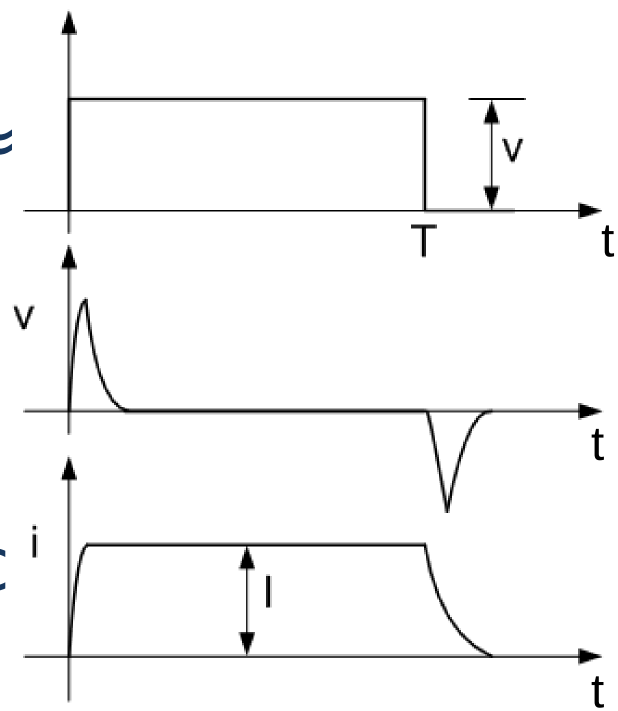
At steady state $\Rightarrow v_L$ 0

Steady state is attained at t ≈

At t = T⁺ $V = 0,\ i = I,\ v_L = -IR$

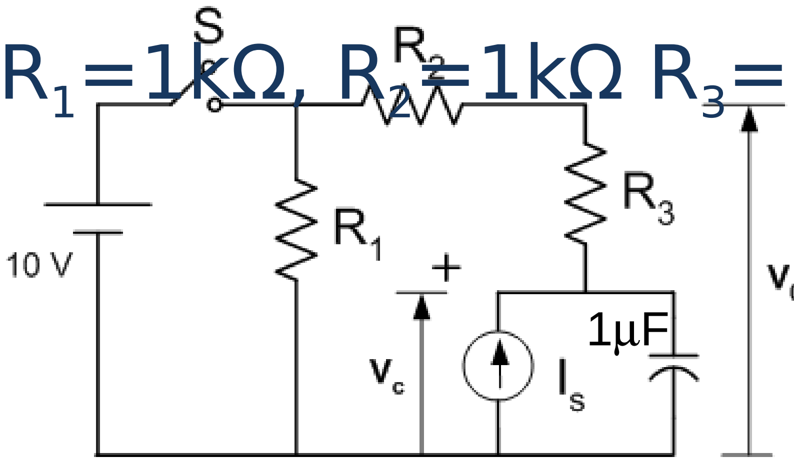
$\Rightarrow$ Differentiator

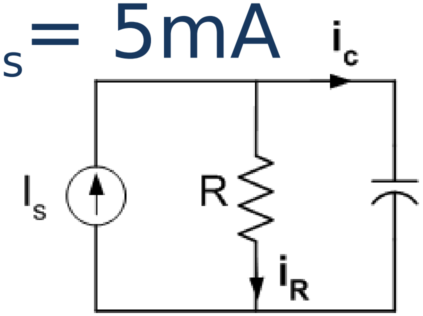
Observation: ‘V’ across ‘L’ c change instantaneously



Problem:

a) 'S' is kept open for long time. 'S' is then closed at an instant of time considered to be t = 0 msec. Determine V_c & V_o at $t = 0^+$ if $R_1 = 1k\Omega$, $R_2 = 1k\Omega$ $R_3 = 2k\Omega$ & $I_s = 5mA$





Eq circuit when S is open:

I_s charges 'C'

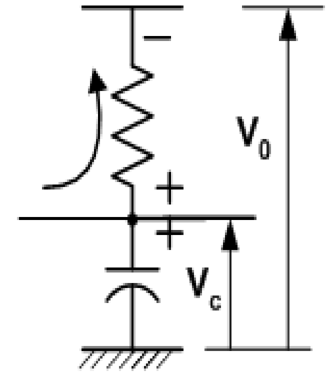
$$\therefore \text{As } v_c \uparrow, \; i_R \uparrow \& \;\; \therefore \; i_c \downarrow \qquad \because i_s = i_c + i_R$$

$\Rightarrow$ At Steady state when $i_c = 0$ & $i_R = 5mA$

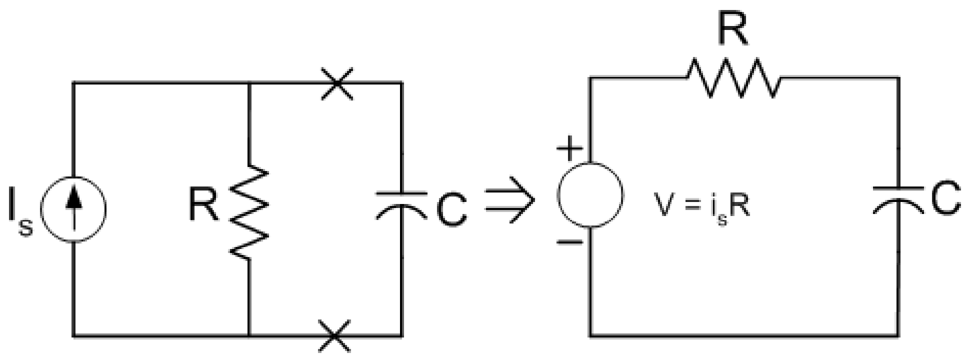
$$\therefore v_c \text{ when } i_c = 0 \text{ is } 5 \times 4 = 20V$$

$$\therefore v_{c(t=0^-)} = 20V \ \&$$

$$v_{o(t=0^-)} = 20 - 2\times 5 = 10V$$

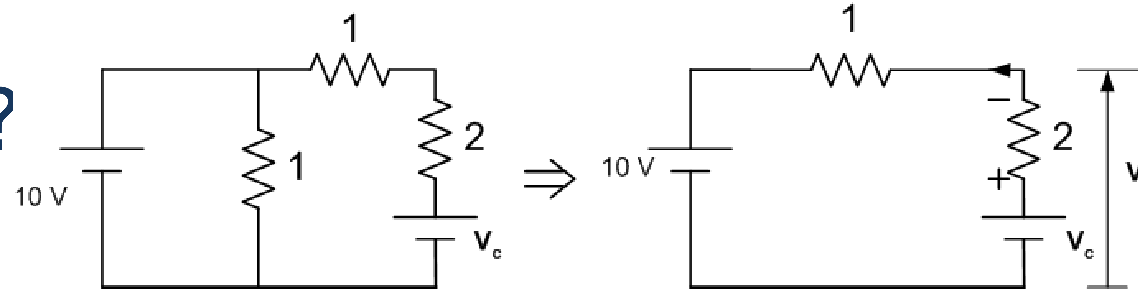


OR



Steady state $V_c = V = i_sR$ and $\tau = RC$

V_o at $t = 0_+$?

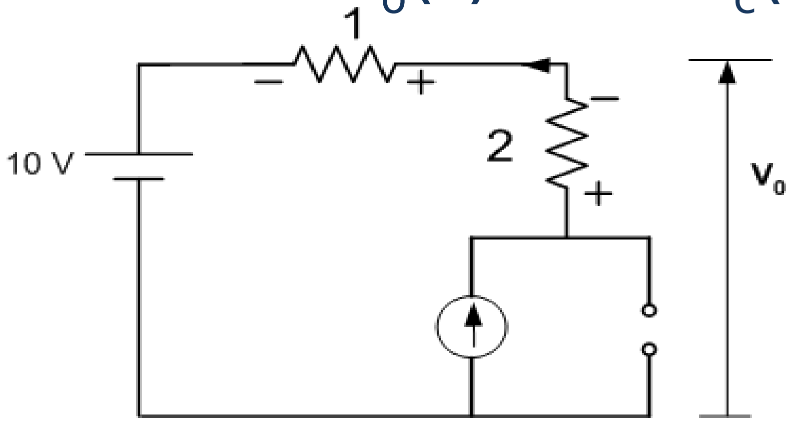


$$v_{c(0^-)} = v_{c(0^+)} = 20\text{V}$$

$$\therefore V_{o_{(0^+)}} = 20 - \frac{(20-10)\times 2}{3} = 13.33V$$

b) For t ≥0, find the expressions for $V_o(t)$ and $V_c(t)$

V_c attains a new value governed by circuit parameters.



Equivalent circuit at steady s

At steady state

$$i_c = 0$$

$$v_o = 10+5\times1=15V\&$$

$$v_c = 10+5\times3=25V$$

$$\tau = 3m\sec$$

$$v_c = v_f + (v_{in} - v_f)e^{\frac{-t}{\tau}}$$

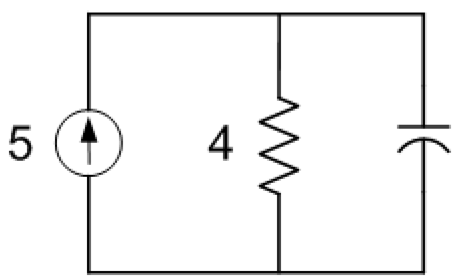
$$v_f = 25V, v_{in} = 20$$

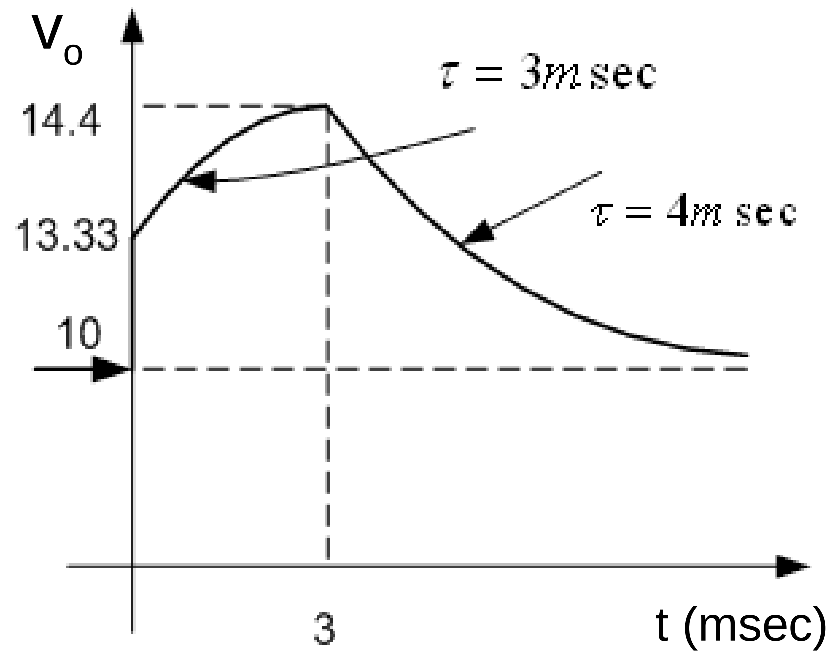
$$\therefore v_c = 25 - 5e^{\frac{-t}{\tau}}$$

$$V_o = v_c - \frac{(v_c - 10)\times 2}{3} = 15 - \frac{5}{3}e^{\frac{-t}{\tau}}$$

c) ‘S’ is again opened at t = 3msec sketch V_o for $0 \le t \le 10$msec

At t = 3msec, V_o=14.4V. When S is opened equiva circuit is





$V_o \rightarrow 10$ with τ= 4msec