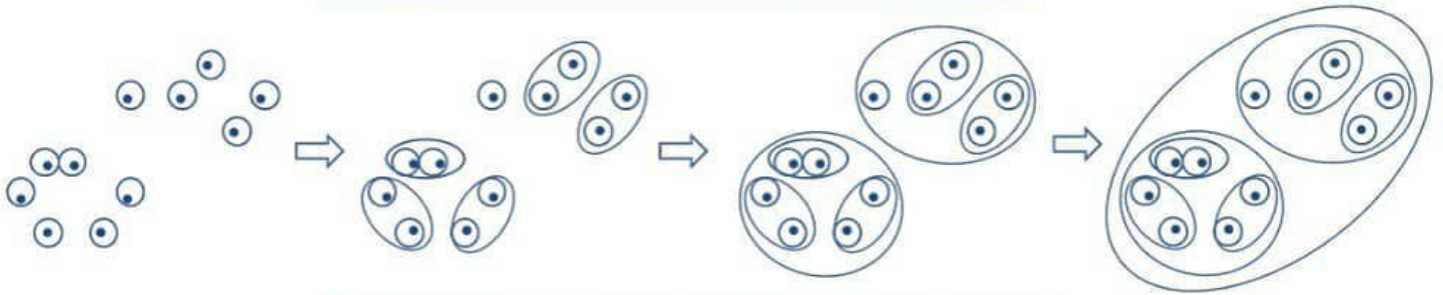


# Hierarchical clustering

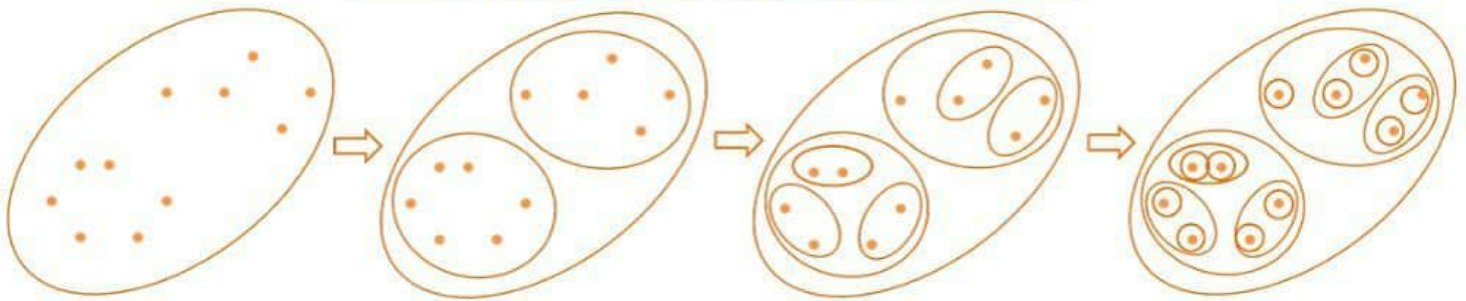
- Two types of hierarchical clustering
- **Agglomerative**: It starts by considering each data point as an individual cluster and in each iteration a pair of similar clusters are merged together until we get a single cluster or K clusters.
- **Divisive**: It is quite opposite to agglomerative, start with one cluster and in each iteration we split the cluster until we get each point as a cluster.
- Traditional hierarchical algorithms use a similarity or distance matrix.
- The main output of Hierarchical Clustering is a dendrogram, which shows the hierarchical relationship between the clusters

# Hierarchical clustering

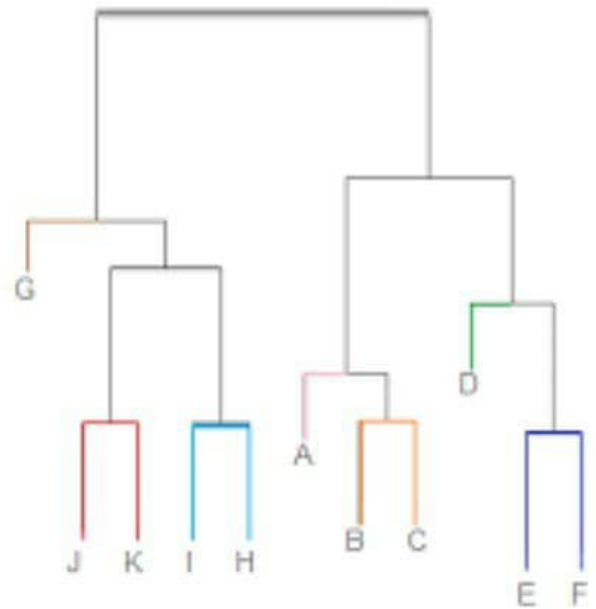
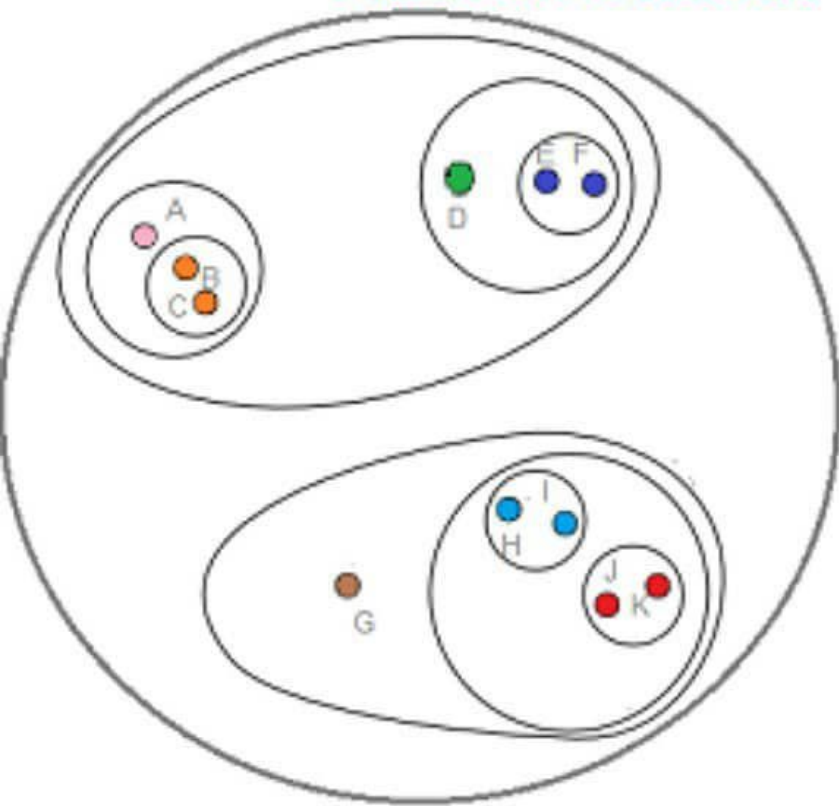
Agglomerative Hierarchical Clustering



Divisive Hierarchical Clustering



# Hierarchical clustering



- **Dendrogram:** A tree like diagram that records the sequences of merges or splits.
- The heights in dendrogram shows how similar they are. You can see E and F are at same height and both are similar compared to other.

# Hierarchical clustering

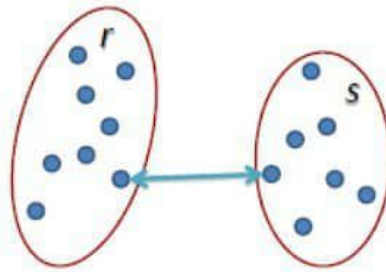
- **Steps**
- Compute the proximity matrix (This stores the distances between each point)
- Let each data point be a cluster
- Repeat: Merge the two closest clusters and update the proximity matrix
- Until only a single cluster remains



# Hierarchical clustering

- How do we calculate the similarity between the points? There are many ways.
- **Complete linkage (Max):** similarity of the farthest pair. One drawback is that outliers can cause merging of close groups later than is optimal.
- **Single-linkage (Min):** similarity of the closest pair. This can cause premature merging of groups with close pairs, even if those groups are quite dissimilar overall.
- **Group average:** similarity between groups.
- **Centroid similarity:** each iteration merges the clusters with the most similar central point.
- **Ward's Method:** This approach of calculating the similarity between two clusters is exactly the same as Group Average except that Ward's method calculates the sum of the square of the distances

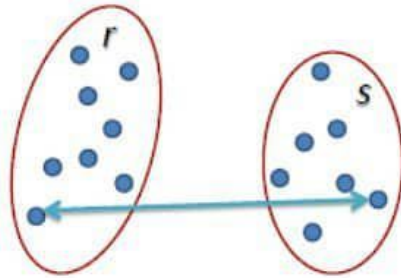
# Hierarchical clustering



$$L(r, s) = \min(D(x_{ri}, x_{sj}))$$

- **Single linkage (min):** The distance between two clusters is defined as the shortest distance between two points in each cluster.
- For example, the distance between clusters “r” and “s” to the left is equal to the length of the arrow between their two closest points.
- **Pros:** This approach can separate non-elliptical shapes if the gap between two clusters is not small.
- **Cons:** MIN approach cannot separate clusters properly if there is noise between clusters.

# Hierarchical clustering

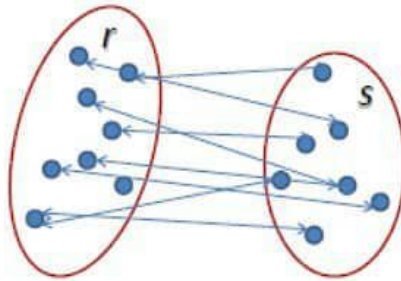


$$L(r, s) = \max(D(x_{ri}, x_{sj}))$$

- **Complete linkage (max):** The distance between two clusters is defined as the longest distance between two points in each cluster.
- For example, the distance between clusters "r" and "s" to the left is equal to the length of the arrow between their two furthest points.
- **Pros:** MAX approach does well in separating clusters if there is noise or outliers.
- **Cons:** Max approach is biased towards globular clusters and tends to break large clusters.



# Hierarchical clustering

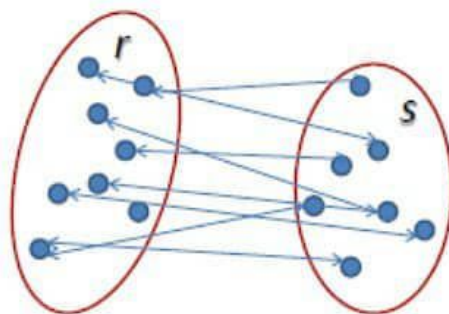


$$L(r, s) = \frac{1}{n_r n_s} \sum_{i=1}^{n_r} \sum_{j=1}^{n_s} D(x_{ri}, x_{sj})$$

- **Group average (AVG):** The distance between two clusters is defined as the average distance between each point in one cluster to every point in the other cluster.
- For example, the distance between clusters “r” and “s” to the left is equal to the average length each arrow between connecting the points of one cluster to the other.
- **Pros:** The group Average approach does well in separating clusters if there is noise between clusters.
- **Cons:** The group Average approach is biased towards globular clusters.



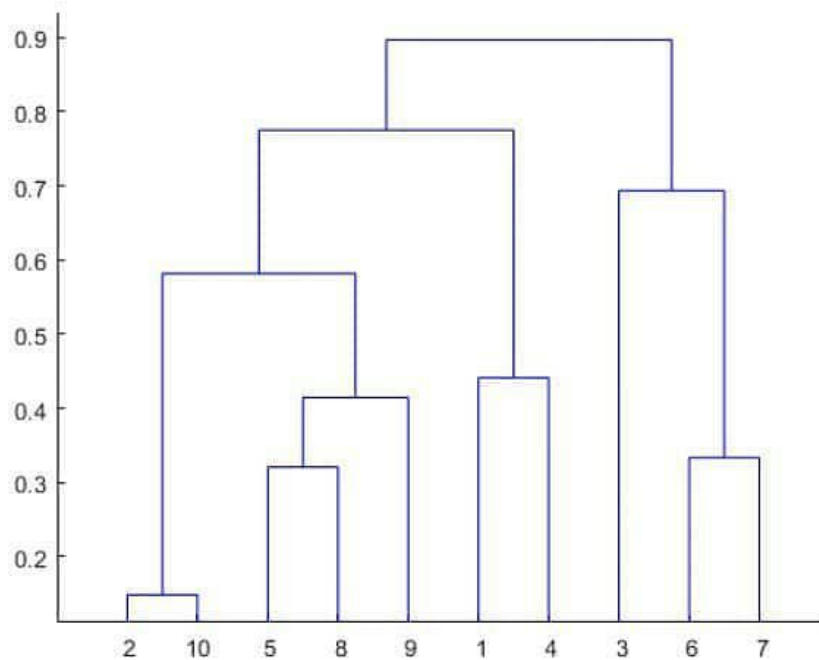
# Hierarchical clustering



$$L(r, s) = \frac{1}{n_r n_s} \sum_{i=1}^{n_r} \sum_{j=1}^{n_s} D(x_{ri}, x_{sj})$$

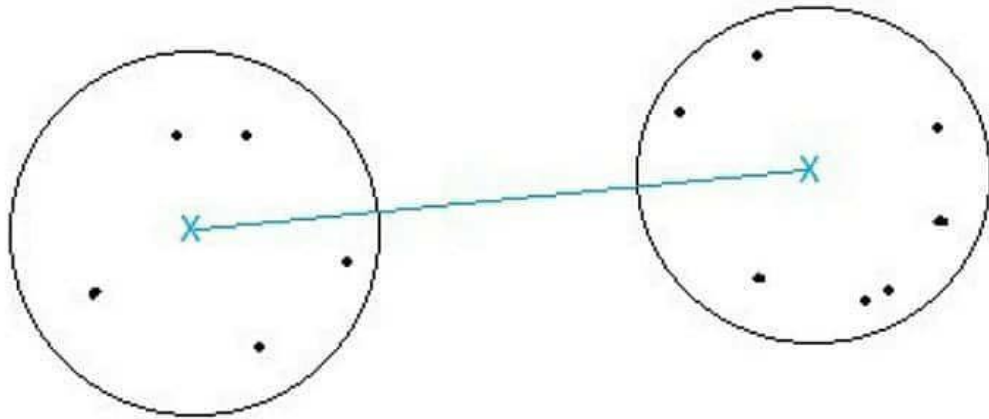
- **Ward's Method:** This approach of calculating the similarity between two clusters is exactly the same as Group Average except that Ward's method calculates the sum of the square of the distances  $X_i$  and  $X_j$ .
- $\text{Similarity}(R, S) = \sum \text{sim}(X_i, X_j) / |R| * |S|$
- **Pros:** Ward's method approach also does well in separating clusters if there is noise between clusters.
- **Cons:** Ward's method approach is also biased towards globular clusters.

# Hierarchical clustering



- **How to choose no of clusters**
- To get the number of clusters for hierarchical clustering, we make use of an awesome concept called a Dendrogram.
- More the distance of the vertical lines in the dendrogram, more the distance between those clusters.

# Hierarchical clustering



- **Centroid similarity:** Compute the centroids of two clusters R & S and take the similarity between the two centroids as the similarity between two clusters. This is a less popular technique in the real world.

# Hierarchical clustering

- **Limitations**
- There is no mathematical objective for Hierarchical clustering.
- All the approaches to calculate the similarity between clusters has its own disadvantages.
- High space and time complexity for Hierarchical clustering. Hence this clustering algorithm cannot be used when we have huge data.