

Welcome 😊

Agenda: 2D

2-3 problems.

Q Maximum Subsequence Sum / House Robbery Question

Find max. subsequence sum from a given array, where selecting adjacent elements is not allowed.

eg: $[\overset{\checkmark}{9} \quad 1 \quad 8 \quad \overset{\checkmark}{10}]$ Ans = 19

eg: $[\overset{\checkmark}{2} \quad 8 \quad \overset{\checkmark}{10} \quad 3 \quad \overset{\checkmark}{15} \quad 6]$ Ans = 27

Brute force

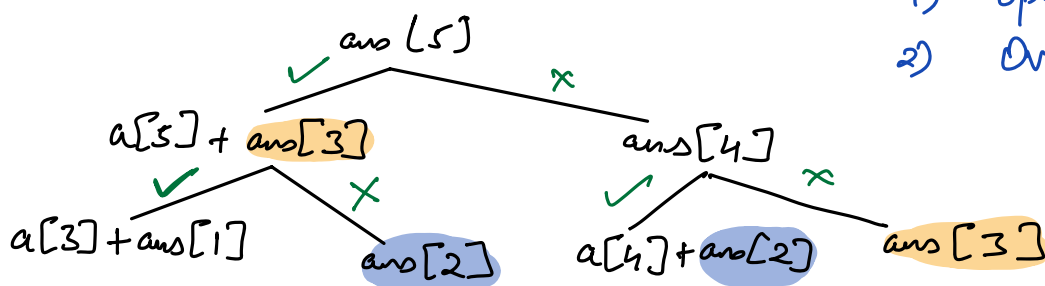
✓ valid subsequence, check and store

select Not select

$A = [\overset{0}{2} \quad \overset{1}{8} \quad \overset{2}{10} \quad \overset{3}{3} \quad \overset{4}{15} \quad \overset{5}{6}]$

Last step
 $A[5] + ans[3]$ $ans[4]$

$ans[i] \Rightarrow$ max. sum from 0 to i



- 1) Optimal sub ✓
- 2) Overlapping subprob. ✓

T.C $\rightarrow O(N)$

S.C $\Rightarrow O(N)$

for ($i \rightarrow 2$ to $N-1$)

$$f \quad ans[i] = \max(\overset{\text{selecting.}}{A[i] + ans[i-2]}, \overset{\text{not selecting.}}{ans[i-1]})$$

3

return ans[N-1]

\Rightarrow You can improve space from $O(N) \rightarrow O(1)$ (4.6)

$$[\overset{\checkmark}{9} \quad 1 \quad 8 \quad \overset{\checkmark}{10}]$$

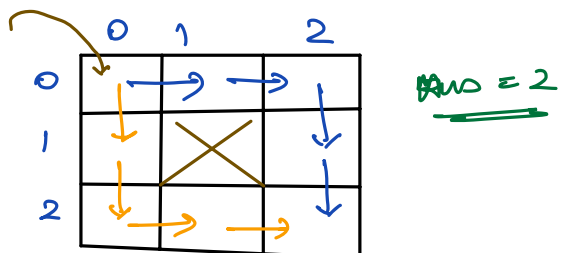
as [9 9 17 19]

Q Given a 2D matrix.

Start \rightarrow from top-left. $(0,0)$

End \rightarrow Bottom - right.

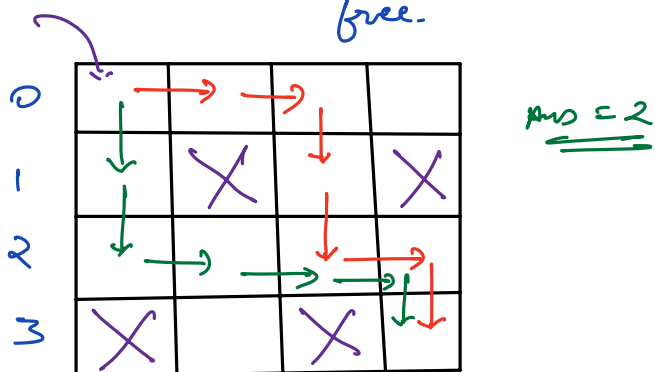
Find # ways to move from top left to bottom right, given there are some blocked cells.



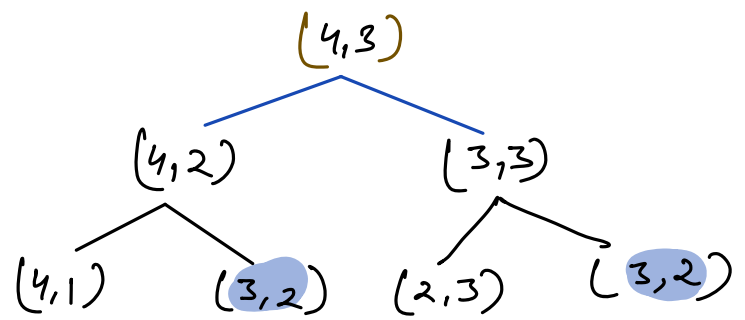
Ans = 2

$$[i, j-1] \xrightarrow{\quad} [i, j]$$
$$A[i][j] = 1$$

0 free.



	0	1	2	3
0		X		
1				X
2	X			
3	X		X	
4				



- 1) Optimal sub ✓
- 2) Overlapping subprob. ✓

$ways[i][j] \rightarrow \# \text{ ways to reach } (i,j)$

$ways[0][0] \rightarrow 1$

Code

for ($i \rightarrow 0$ to $N-1$)

{
 for ($j \rightarrow 0$ to $M-1$)

{
 if ($A[i][j] == 1$) $ways[i][j] = 0$

 else if ($i == 0$ & $j == 0$) $ways[i][j] = 1$

 else if ($i == 0$) // first row.

$ways[i][j] = ways[i][j-1]$

 else if ($j == 0$) $ways[i][j] = ways[i-1][j]$

 else
 $ways[i][j] = ways[i-1][j] + ways[i][j-1]$

}

}
return $ways[N-1][M-1]$

	0	1	2	3
0	1	X	0	0
1	1	1	1	0
2	X	1	2	2
3	0	1	0	2
4	0	1	1	3

Ans = 3

T.C $\rightarrow O(N \times M)$

S.C $\rightarrow O(N \times M)$

only store curr & prev rows
S.C $\rightarrow O(2M) \rightarrow O(M)$

H.W

Q Dungeon & Princess

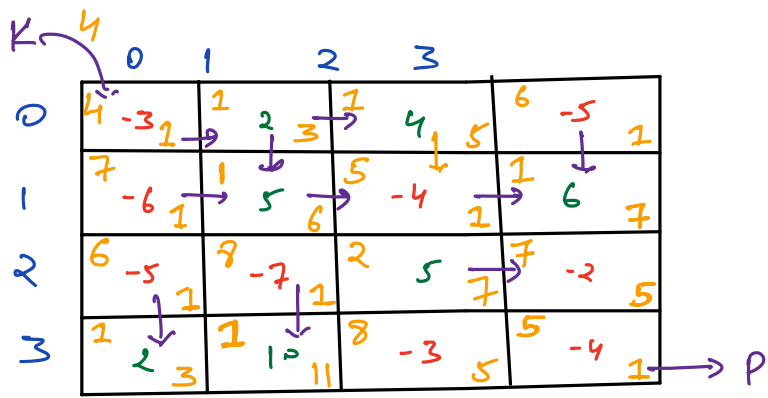
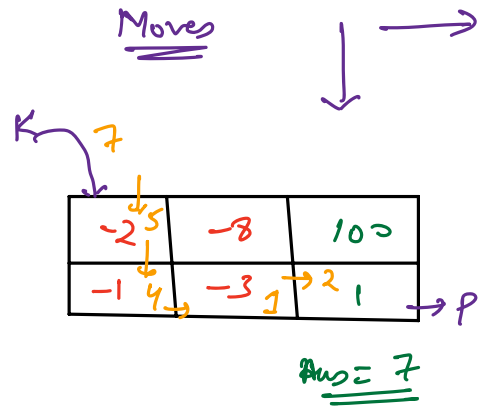
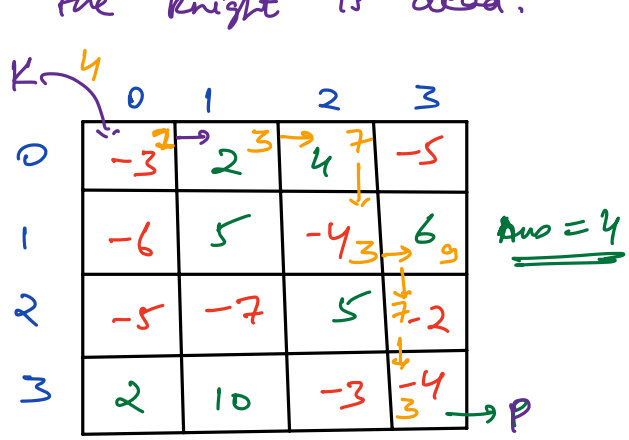
Given a matrix where $A[i][j]$ represents a room s.t

1) $A[i][j] < 0 \rightarrow$ There is a guard in a room that can reduce health by $|A[i][j]|$

2) $A[i][j] = 0 \rightarrow$ Empty room.

3) $A[i][j] > 0 \rightarrow$ There are some magic herbs which can increase the health by $A[i][j]$

Find min. initial health of knight s.t the knight can reach princess alive. If at any point health ≤ 0 ; the knight is dead.



$ans[i][j] \Rightarrow$ min. health req. by knight to enter this cell.

code

```
for (i → (N-1) to 0)
{
    for (j → (M-1) to 0)
    {
        if ((i == (N-1)) && (j == (M-1)))
            ans[i][j] = max(1, 1 - A[i][j])
        else if (i == N-1) // last row
            u = ans[i][j+1]
            ans[i][j] = max(u - A[i][j], 1)
        else if (j == M-1) // last column.
            u = ans[i+1][j]
            ans[i][j] = max(u - A[i][j], 1)
        else // all other cells
            u = min(ans[i+1][j], ans[i][j+1])
            ans[i][j] = max(u - A[i][j], 1)
    }
}

return ans[0][0]
```

T.C $\rightarrow O(N * M)$

S.C $\rightarrow O(N * M)$

optimize. $S.C \rightarrow O(2M)$
 \downarrow
 $O(M)$