

Welcome 😊

Agenda: GCD \rightarrow HCF
Properties
GCD code
2 questions.

GCD: Greatest Common Divisor / HCF: Highest Common Factor

$\text{gcd}(a, b) = n \Rightarrow n$ is the largest positive number that divides both a & b completely.

$$\text{gcd}(15, 25) = 5$$

↓	↓
1	5
3	5
5	15
15	

$$\text{gcd}(12, 30) = 6$$

$$\text{gcd}(10, -25) = 5$$

↓	↓
1	-1
2	-5
5	-25
10	1
	5
	25

$$\text{gcd}(0, 8) = 8$$

↓	↓
1	1
2	2
3	4
⋮	8
⋮	
∞	

$$\text{gcd}(0, -10) = \underline{\underline{+10}}$$

$$\text{gcd}(-2, -3) = \underline{1}$$

$$\text{gcd}(0, 0) \Rightarrow \text{NOT DEFINED}$$

↓	↓
1	1
2	2
3	3
4	4
⋮	⋮
∞	∞

$$n \% 0 \Rightarrow \text{N.D.}$$

$$0 \% n \Rightarrow 0$$

Properties of G.C.D

① $\text{G.C.D}(a, b) = \text{G.C.D}(b, a) \Rightarrow$ Commutative.

② $\text{G.C.D}(a, b, c) = \text{gcd}(a, b, c)$
 $= \text{gcd}(b, c, a)$
 $= \text{gcd}(c, a, b)$ } \Rightarrow Associative

Special Property

$A, B > 0$ & $A \geq B$

$$\text{G.C.D}(A, B) = \text{G.C.D}(A-B, B)$$

$$\text{G.C.D}(A, B) = d$$

$$A \% d = 0 \quad B \% d = 0$$

$$A = d \times K_1 \quad B = d \times K_2$$

$$\Rightarrow A-B = d \times (K_1 - K_2)$$

$$(A-B) \% d = 0$$

$\rightarrow \text{gcd}(a, b) = d$ & m is
also a factor of A & B

$$\text{G.C.D}(A-B, B) = m$$

$$(A-B) = m \times K_3 \quad B = m \times K_4$$

$$A = m \times (K_3 + K_4)$$

$$A \% m = 0$$

$\text{G.C.D}(A-B, B) = m$ & d
is also a factor of
 $(A-B, B)$

$$d \geq m$$

$$m \geq d$$

$$\underline{\underline{m = d}}$$

eg: $\gcd(23, 5) \Rightarrow \gcd(23-5, 5) = \gcd(18, 5)$
 \Downarrow
 $\underline{1}$ $= \gcd(13, 5)$
 $= \gcd(8, 5)$
 $= \gcd(3, 5)$
 \Downarrow
 $\underline{1}$

$$\begin{aligned}\gcd(a, b) &= \gcd(a-b, b) \\ &= \gcd(a, -2b, b) \\ &= \gcd(a-3b, b) \\ &\vdots \\ &= \gcd(a-yb, b)\end{aligned}$$

$$\gcd(A, B) = \gcd(A \% B, B)$$

$$\gcd(24, 16) \Rightarrow \gcd(8, 16) \Rightarrow \gcd(8, 16) \dots \infty$$

$$\Rightarrow \gcd(A, B) = \gcd(B, A \% B)$$

$$\gcd(24, 16) \Rightarrow \gcd(16, 8) \Rightarrow \gcd(8, 0) \Rightarrow 8$$

$$\gcd(14, 24) \Rightarrow 2$$

Pseudocode

```
int
{
```

```
    if (b == 0) return a;
```

```
    return gcd(b, a % b)
```

```
}
```

$$T.C \Rightarrow O(\log_2(\max(a, b)))$$

$$\text{gcd}(a, b) = \text{gcd}(a \% b, b)$$

$$\underline{a > b}$$

Case 1

$$b < a/2$$

$$a \% b < b < a/2$$

$$\underbrace{0 \rightarrow b-1}$$

$$a \% b < a/2$$

Case 2

$$b = a/2$$

$$a \% b < b < a/2$$

$$a \% b < a/2$$

Case 3

$$b > a/2$$

$$\Rightarrow 2b > a$$

$$a - 2b < 0$$

$$2a - 2b < a$$

$$a - b < \frac{a}{2}$$

$$a \% b < a/2$$

$$a \% b \Rightarrow a - b$$

$$a - 2b$$

$$a - 3b$$

⋮

Q1 Given an array of N elements, we have to delete 1 element s.t gcd of remaining elements is maximum. Return max. gcd.

eg: $\begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 24 & 16 & 18 & 30 & 15 \end{matrix} \Rightarrow 1$

$\begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 24 & 16 & 18 & 30 & 15 \end{matrix} \Rightarrow 3$

$\begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 24 & 16 & 18 & 30 & 15 \end{matrix} \Rightarrow 1$

$\begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 24 & 16 & 18 & 30 & 15 \end{matrix} \Rightarrow 1$

⁰ 24 ¹ 16 ² 18 ³ 30 ⁴ 15 \Rightarrow 2

Brute force

Delete each element and calculate gcd and find max.

T.C $\Rightarrow N * (N * \log(\max(arr[i])))$

Optimized

⁰ 24 ¹ 16 ² 18 ³ 30 ⁴ 15 \Rightarrow

Pseudo code

int deleteOne (int arr[])

{

PFCMD[N] } TODO $N \log(\max(arr[i]))$
 SFCMD[N] } $N \log(\max(arr[i]))$
 ans = max (SFCMD[1] , PFCMD[N-2])

for (int i=1 ; i<N-1 ; i++)

{

// deleting ith index

left = PFCMD[i-1]

right = SFCMD[i+1]

val = gcd (left, right)

ans = max (val, ans) ;

}

return ans;

T.C $\Rightarrow N \log(\max(arr[i]))$

Q2 PUBG

N players, playing in a game, each has some health $A[i]$

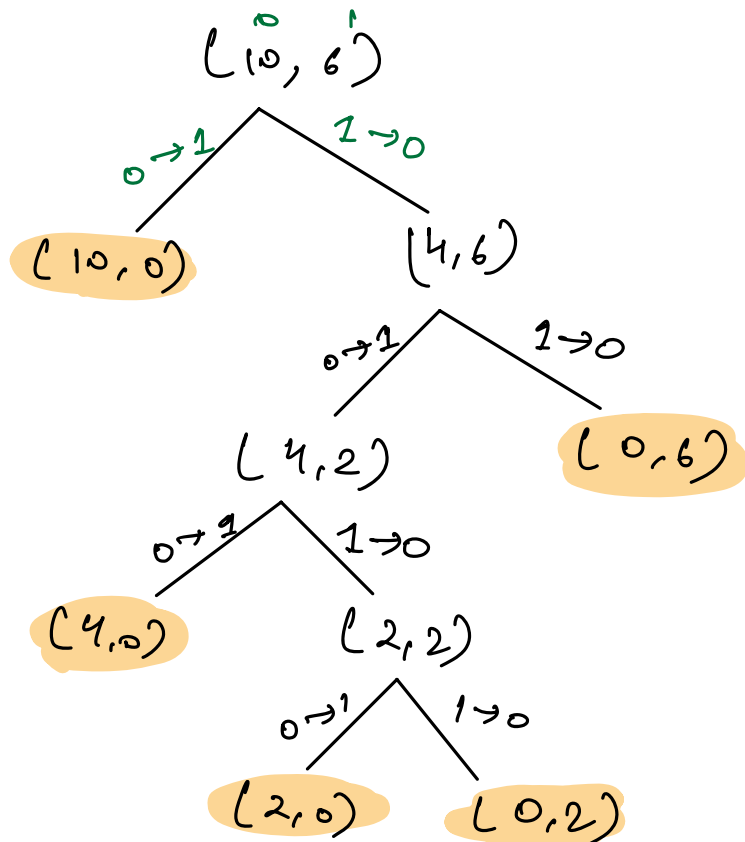
if i^{th} player attacks j^{th} player

1) $A[i] > A[j] \Rightarrow$ player j will die

2) $A[i] < A[j] \Rightarrow A[j] = A[j] - A[i]$

Find the min. health of last surviving player.

eg:



$\begin{smallmatrix} w & s \\ (a, b) \end{smallmatrix}$

$a < b$

$b \rightarrow a$

$\Rightarrow (0, b)$

$a \rightarrow b$

$\Rightarrow (a, b-a)$

1) Weaker player will attack stronger player.

eg:

(48 , 96 , 32 , 36)

(12 , 96 , 32)

(4 , 96) \Rightarrow 4

Pseudocode

```
ans = arr[0];
```

```
for( i = 1; i < N; i++ )
```

```
{
```

```
    ans = gcd ( ans , arr[i] )
```

```
}
```

```
return ans;
```
