Welcome ☺

Agenda : Bit Manipulation 1

Decimal → binary
binary → decimal
Properties
1 question.

Decimal number system { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

base → 10

$342 \rightarrow 300 + 40 + 2$

$\rightarrow 3\times10^2 + 4\times10^1 + 2\times10^0$

$2563 \rightarrow 2000 + 500 + 60 + 3$

$\rightarrow 2\times10^3 + 5\times10^2 + 6\times10^1 + 3\times10^0$

Binary Number System { 0, 1 } base → 2

$110 \rightarrow 1\times2^2 + 1\times2^1 + 0\times2^0 = 4+2 = 6$

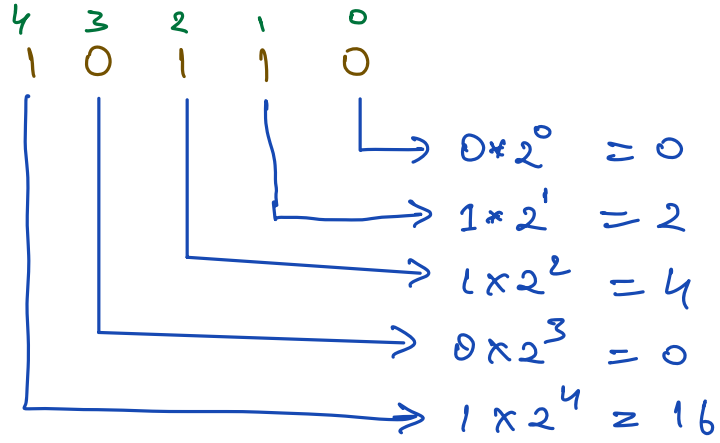
$1011 \rightarrow 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 = 8+2+1 = 11$

0	10	20
1	11	21
2	12	22
⋮	⋮	⋮
9	19	29

$0 \rightarrow 0$ $\quad 10 \rightarrow 2$ $\quad 100 \rightarrow 4$ $\quad 110 \rightarrow 6$

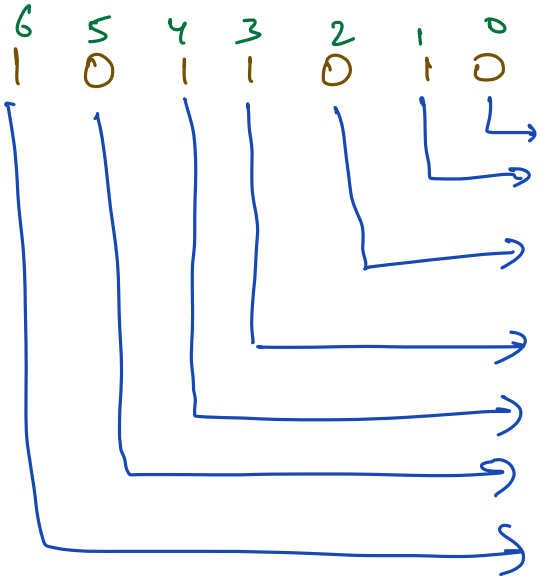
$1 \rightarrow 1$ $\quad 11 \rightarrow 3$ $\quad 101 \rightarrow 5$ $\quad 111 \rightarrow 7$

Binary to decimal



$(10110)_2 = (22)_{10}$

Quiz:



64 + 16 + 8 + 2 = 90

$\Rightarrow 2^6 + 2^4 + 2^3 + 2^1 = 90 \Rightarrow$ ignore 0's and calc. only 1s

Q $(1\ 0\ 2\ 0\ 1\ 0)_2 \Rightarrow$ invalid

Decimal to binary.

2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

reverse order.

$4\ 3\ 2\ 1\ 0$
$1\ 0\ 1\ 0\ 0$

$2^4 + 2^2 = 20$

$(20)_{10} = (10100)_2$

$2^4 + 2^2 = 16 + 4 = 20$

cannot remove zeroes in the right can remove preceding zeroes

$(101)_2$

$2^2 + 2^0 = 5$

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

$5\ 4\ 3\ 2\ 1\ 0$
$1\ 0\ 1\ 1\ 0\ 1$

$2^5 + 2^3 + 2^2 + 2^0 = 32 + 8 + 4 + 1$
$= 45$

$(45)_{10} = (101101)_2$

Addition

$$\begin{array}{ccc} {\scriptstyle 1}\ {\scriptstyle 1} & & \\ 3 & 6 & 8 \\ 4 & 5 & 3 \\ \hline 8 & 2 & 1 \end{array}$$

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 10$

$(2)_{10} = (10)_2$
$(3)_{10} = (11)_2$

Q

$$\begin{array}{cccc} {\scriptstyle 1} & {\scriptstyle 1} & & \\ 1 & 0 & 1 & \rightarrow 5 \\ 0 & 1 & 1 & \rightarrow 3 \\ \hline 1 & 0 & 0 & 0 \rightarrow 8 \end{array}$$

Q

$$\begin{array}{cccc} {\scriptstyle 1} & {\scriptstyle 1} & & \\ 1 & 0 & 1 & \rightarrow 5 \\ 1 & 1 & 1 & \rightarrow 7 \\ \hline 1 & 1 & 0 & 0 \rightarrow 12 \end{array}$$

Quiz

$$\begin{array}{cccccc} & {\scriptstyle 1} & {\scriptstyle 1} & & & \\ 1 & 0 & 1 & 1 & 0 & \rightarrow 22 \\ 0 & 0 & 1 & 1 & 1 & \rightarrow 7 \\ \hline 1 & 1 & 1 & 0 & 1 & \rightarrow 29 \end{array}$$

Bitwise Operators

AND	OR	XOR	NOT	left shift	right shift
&	|	^	!/~	<<	>>

!/~ : 1 → 0, 0 → 1

A	B	A&B	A|B	A^B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

same same puppy shame.

1+1 = 10
addition without carry.

0 → false → unset
1 → true → set

Bitwise Opera's on numbers

① 5 & 6

5 → 1 0 1
6 → 1 1 0
A&B → 1 0 0 → 4

5 & 6 → 4

② 20 | 45

20 → 0 1 0 1 0 0
45 → 1 0 1 1 0 1
A|B → 1 1 1 1 0 1 → 61
5 4 3 2 1 0

20 | 45 → 61

③ 20 ^ 45

20 → 0 1 0 1 0 0
45 → 1 0 1 1 0 1
20^45 → 1 1 1 0 0 1 ⇒ 57
5 4 3 2 1 0

20 ^ 45 → 57

Properties

1) A & 1

A = 10

A → 1 0 1 0
1 → 0 0 0 1
A & 1 → 0 0 0 0
= 0

A = 9

A → 1 0 0 1
1 → 0 0 0 1
A & 1 → 0 0 0 1

A & 1 → 0, if last bit is 0 → A is even
A & 1 → 1, if last bit is 1 → A is odd
→ 0th bit is set

1 0 1 1 0 1
⇒ $2^5 + 2^3 + 2^2 + 2^0$
$2^5 + 2^3 + 2^2$ → even
$+2^0$ → odd
even + odd → odd

1 0 1 1 0 0
$2^5 + 2^3 + 2^2$ → even
$\times 2^0$ → even
even + even = even

2) A & 0 = 0

eg: A → 101
0 → 000
A & 0 → 000

3) A & A = A

eg: A → 101
A → 101
A & A → 101

4) A | 0 = A

eg: A → 101
0 → 000
A | 0 → 101

5) A | A = A

eg: A → 101
A → 101
A | A → 101

6) A ^ 0 = A

eg: A → 101
0 → 000
A ^ 0 → 101

7) A ^ A = 0

eg: A → 101
A → 101
A ^ A → 000

8) Commutative Property

$a \& b = b \& a$

$a \mid b = b \mid a$

$a \wedge b = b \wedge a$

order does not matter

$a \& b \& c = c \& a \& b$

$= c \& b \& a$

$= a \& c \& b$

$= b \& a \& c$

8) Associative Property

$(a \& b) \& c = a \& (b \& c)$

$(a \mid b) \mid c = a \mid (b \mid c)$

$(a \wedge b) \wedge c = a \wedge (b \wedge c)$

Q $a \wedge b \wedge a \wedge d \wedge b \rightarrow a \wedge a \wedge b \wedge b \wedge d$

$0 \wedge 0 \wedge d = d$

Q Given an integer array where every element appears twice except for one element which appears once, find that unique element.

eg: A : [6 9 6 10 9] ans = 10

$d \wedge c \wedge c = d$

$6 \wedge 9 \wedge 6 \wedge 10 \wedge 9 = 6 \wedge 6 \wedge 9 \wedge 9 \wedge 10$

$0 \wedge 0 \wedge 10 = 10$

approach

```
ans = 0
for i -> 0 to (N-1)
    ans = ans ^ A[i]
return ans
```

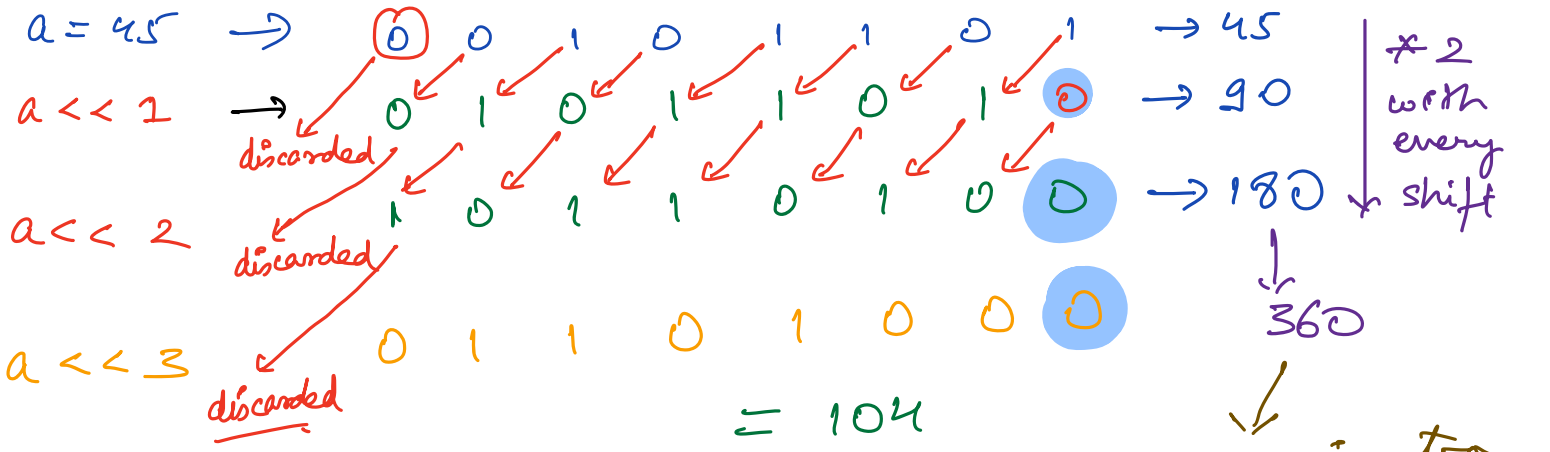
T.C → O(N)

S.C → O(1)

Left Shift (<<)

int → 4 bytes = 32 bits

8 bit number.



8 bit number → 7 6 5 4 3 2 1 0
1 1 1 1 1 1 1 1

= 255

360 is too large to be stored in 8 bits

overflowed

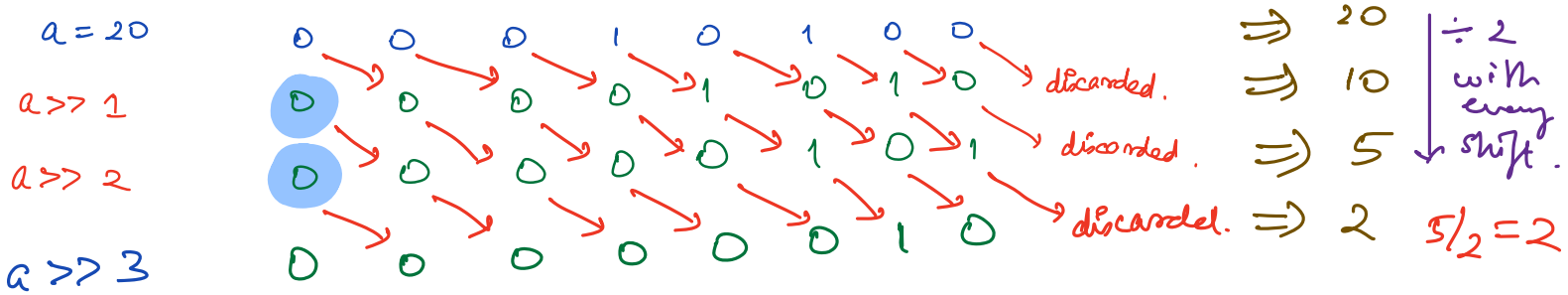
$$a << n = a * 2^n$$

$$1 << n = 2^n$$

$$5 << n = 5^n \times$$

$$5 \times 2^n$$

Right shift.



$$a >> n = a/2^n$$