Welcome ☺

Agenda : Prime Numbers

Q1 Find the count of factors of N

$N \% n == 0 \Rightarrow n$ is a factor of N

eg: 32 ⇒ 1 2 4 8 16 32 ⇒ 6

$N = i * j \Rightarrow j = N/i$

$i^2 \leq N$

$i \leq \sqrt{N}$

```
ans = 0
for ( n → 1 to √N )
{
    if ( N % n == 0 )
    {
        if ( n != N/n )
            ans += 2
        else
            ans += 1
    }
}
return ans
```

36

n	N/n
1	36
2	18
3	12
4	9
6	6
9	4
12	3
18	2
36	1

T.C ⇒ $O(\sqrt{N})$

Prime Numbers

a +ve no: which has only 2 factors. 1 & number itself

⇒ Smallest prime no: ⇒ 2

Q Given an integer N, find all prime numbers from 1 to N

eg: N = 10 ⇒ {2, 3, 5, 7}

N = 4 ⇒ {2, 3}

Brute force $\sum_{1}^{N} i$ check if it is prime or not

check ⇒ $\sqrt{N}$

T.C ⇒ $N\sqrt{N}$

Sieve of Eratosthenes.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
~~T~~	T	T	~~T~~	T	~~T~~	T	~~T~~	~~T~~	~~T~~	T	~~T~~	T	~~T~~	~~T~~
F	✓	✓	F		F		F	F	F		F		F	F

2 ⇒ 2×2 2×3 2*4 2*5 . . . .

Obs ⇒ All the multiples of prime no. are not prime.

2*5 ≡ 5*2 = 10

5*3 = 15

5*4 = 20

5*5 = 25 ✓✓ ⇒ directly check i^2 multiple

5*6 = 30

drawback ⇒ → You can only find out prime no.s using Sieve for range 1 to N.

Not work for range (N_1 to N_2)

→ till $N \leq 10^6$

Pseudocode

```
∀i  isP[] = T
isP[0] = F      isP[1] = F
for( i=2 ; i*i ≤ N ; i++ )
{
    if( isP[i] ) // Prime no.
    {
        for( j= i*i ; j≤ N ; j= j+(i)
        {
            isP[j] = F
        }
    }
}

for( i→ 2 to N)
{
    if( isP[i] )    print(i)
}
```

N = 17

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
~~T~~	T	T	~~T~~	T	~~T~~	T	~~T~~	~~T~~	~~T~~	T	~~T~~	T	~~T~~	~~T~~	~~T~~	T
F			F		F		F	F	F		F		F	F	F	

T.C

i ⇒	# iterations
2 ⇒ 4, 6, 8...	N/2
3 ⇒ 9, 12, 15....	N/3
5 ⇒ 25.....	N/5

$$\left(\frac{N}{2}+\frac{N}{3}+\frac{N}{5}+\frac{N}{7}\cdots\right) < \left(\frac{N}{2}+\frac{N}{3}+\frac{N}{4}+\frac{N}{5}+\frac{N}{6}\cdots\right)$$

$$N*\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots\right)$$

$$N*\sum_{i=2}^{N}\frac{1}{i} \simeq N*\int_{2}^{N}\frac{1}{n}\,dn$$

$$\simeq N\log N$$

T.C of Sieve of Eratosthenes $< O(N\log N)$

Q) Given an integer array N, find the count of factors of N for all numbers from 1 to N

eg: N=8

1	2	3	4	5	6	7	8
1	2	2	3	2	4	2	4

Brute force $\Rightarrow$ $N\sqrt{N}$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2	2	3	2	4	2	4	3	4	2	6	2	4	4

```
for ( i=1 ; i<=N ; i++)
{
    for ( j=i ; j<=N ; j=j+i)
    {
        cnt[j]++
    }
}
return cnt
```

$\Rightarrow N+\frac{N}{2}+\frac{N}{3}+\frac{N}{4}\cdots\frac{N}{N}$

$\Rightarrow N\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\cdots\frac{1}{N}\right) \Rightarrow N\log N$

Q4 Given an integer N, find the smallest prime factor for all numbers from 2 to N.

eg: N=10

2	3	4	5	6	7	8	9	10
2	3	2	5	2	7	2	3	2

$\forall i$ spf[ ] = i

isP[0] = F isP[1] = F

```
for( i=2 ; i*i <= N ; i++ )
{
    if( spf[i]==i ) // Prime no.
    {
        for( j= i*i ; j<= N ; j=j+(i) )
        {
            if( spf[j]==j)   spf[j]=i ⇒ only update once.
        }
    }
}
return spf.
```

N=10

	0	1	2	3	4	5	6	7	8	9	10
SPF	0	1	2	3	~~4~~ 2	5	~~6~~ 2	7	~~8~~ 2	~~9~~ 3	~~10~~ 2

TC ⇒ N log N

SC ⇒ O(1/N)

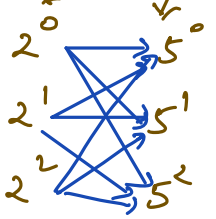
Q5 Given an integer array, # A[i] find the count of factors.

eg: 10 ⇒ 4

8 ⇒ 4

Brute force ⇒ $O(Q * \sqrt{N})$

⇒ 100 ⇒ $2^2 * 5^2$

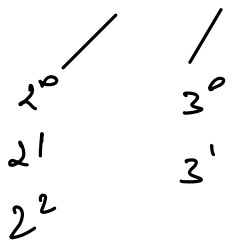


$2^0 * 5^0 = 1$ $2 * 1 = 2$ $4 * 1 = 4$

$1 * 5 = 5$ $2 * 5 = 10$ $4 * 5 = 20$

$1 * 25 = 25$ $2 * 25 = 50$ $4 * 25 = 100$

12 ⇒ $2^2 * 3$



⇒ 1, 2, 3, 4, 6, 12

$N = P_1^X P_2^Y P_3^Z$

factors ⇒ $(X+1) * (Y+1) * (Z+1)$

N = 60 ⟶ spf[60] = 2

$\frac{60}{2} = \frac{30}{2} = 15$ spf[15]

$\frac{15}{3} = 5$ spf[5] = 5

$\frac{5}{5} = 1$

60 ⇒ $2^2 * 3 * 5$

1. Find spf till max input ⇒ $N \log N$

```
factors (N)
{
    ans = 1
    v = spf[N]
    while (v > 1)  N>1
    {
        cnt = 0
        while (N % v == 0)
        {
            cnt++
            N = N/v
        }
        ans *= (cnt+1)
        v = spf[N]
    }
    return ans
}
```

⇒ T.C $O(\log N)$

$\frac{60}{2} \Rightarrow \frac{30}{2} \Rightarrow \boxed{\frac{15}{3}}$

$\frac{5}{5} \Rightarrow \boxed{1}$

T.C ⇒ $N \log N + Q * \log N$