Welcome ☺

Agenda : Permutations
Combinations
Properties
Addition & multiplicⁿ rule
Pascal triangle
$N_{C_R}$ %p
Find $N^{th}$ column of excel.

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Q 3 T/F questions in a test
How many ways can a person answer them?

⇒ T/F ⇒ 2 2 2 = 2*2*2 = 8

⇒ T or F or — ⇒ 3 3 3 ⇒ 3*3*3 = 27

⇒ Always make sure to have proper understanding of choices

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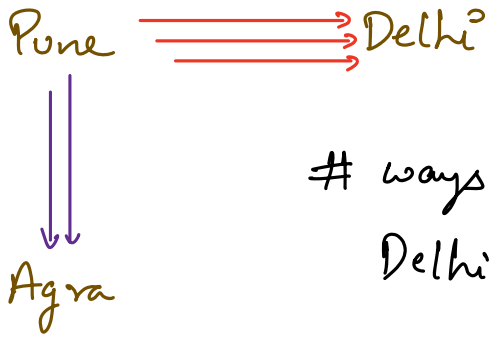
Multiplicⁿ & Addition Rule ⇒ AND & OR RULE

1) Pune ⟹ Delhi ⟹ Agra

# ways to travel from Pune to Agra via Delhi

#ways to travel — Pune to Delhi ⇒ 3 (AND) Delhi to Agra ⇒ 2 → * → 6

AND ⟶ *

2) 


# ways to travel from Pune to Delhi OR Agra.

# ways to travel to Delhi + # of ways to travel to Agra.

OR ⟶ +

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Permutations ⟹ Arrangement ⇒ order matters

Q # ways to arrange N distinct characters.

eg: ABC ⟹ $\underset{\downarrow}{3} * \underset{\downarrow}{2} * 1 = 6$ ways

3 contenders. 2 " 1

⇒ ABC ACB BAC

BCA CAB CBA

→ N distinct char ⇒ $N * N-1 * N-2 * N-3 * \ldots * 1$

# ways = $N * (N-1) * (N-2) * \ldots \Rightarrow N!$

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Q # ways to arrange R out of N distinct characters.

eg: d a t e

r=2 N=4

4 * 3 ✗ ✗

4 * 3 = 12

r=3 N=5 $\quad 5 * 4 * 3 \Rightarrow 60$

$$\underbrace{N \quad N-1 \quad N-2 \quad N-3 \quad N-4 \quad N-5 \ldots N-(R-1)}_{R}$$

$$\#\text{ways} = {}^{N}P_{R} \Rightarrow N*(N-1)*(N-2)\cdots*(N-R+1)*\frac{(N-R)(N-R-1)\cdots*(1)}{(N-R)(N-R-1)*\cdots(1)}$$

$$\boxed{{}^{N}P_{R} \Rightarrow \frac{N!}{(N-R)!}}$$

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Combinations $\Rightarrow$ Selection → order does not matter.

Q # ways to select R out of N distinct characters ?

eg: d a t e

r=2 N=4

$\Rightarrow$ da, dt, de, at, ae, te $\Rightarrow 6$

(da → ad)

same selection: d, a / a, d → different arrangements

# ways to arrange R out of N distinct char $\rightarrow$ ${}^{N}P_{R}$ $\Rightarrow$ 1) select R 2) arrange R

# ways to arrange R characters. $\Rightarrow$ R!

> # ways to select R out of N distinct chars $\Rightarrow$ $\frac{{}^{N}P_{R}}{R!}$

(selecting R out of N char) * (arrange R chars) $\Rightarrow$ arranging R out of N distinct chars.

# ways of Selection AND Arrangement $\Rightarrow$ ${}^{N}P_{R}$

# ways to ARRANGE R char $\Rightarrow$ R!

# ways to Selection $\Rightarrow$ $\frac{{}^{N}P_{R}}{R!} = \frac{N!}{(N-R)! * (R!)} \Rightarrow {}^{N}C_{R}$

> ${}^{N}C_{R} \Rightarrow \frac{N!}{R! * (N-R)!}$

$0 \leq R \leq N$

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## Properties of Combinations

1) ${}^{N}C_{0} = 1$
   - $\downarrow$
   - don't select anything.

2) ${}^{N}C_{N} = 1$
   - $\downarrow$
   - select everything.

3) $^{N}C_{R} = {}^{N}C_{N-R}$

$$\frac{N!}{(N-R)!\,R!} \qquad \frac{N!}{(N-(N-R))! * (N-R)!}$$

$$= \frac{N!}{R! * (N-R)!}$$

4) $\boxed{{}^{N}C_{R} + {}^{N}C_{R+1} = {}^{N+1}C_{R+1}}$

$n! \Rightarrow n * (n-1)!$

$$\frac{N!}{R!\,(N-R)!} + \frac{N!}{(R+1)!\,(N-R-1)!}$$

$$\frac{N!}{R!\,(N-R)*(N-R-1)!} + \frac{N!}{(R+1)*R!\,(N-R-1)!}$$

$$\frac{N!}{R!\,(N-R-1)!}\left(\frac{1}{N-R} + \frac{1}{R+1}\right)$$

$$\frac{N!}{R!\,(N-R-1)!}\left(\frac{R+1+N-R}{(N-R)(R+1)}\right)$$

$$\Rightarrow \frac{N!}{R!\,(N-R-1)!}\left(\frac{N+1}{(N-R)(R+1)}\right)$$

$$\Rightarrow \frac{(N+1)!}{(R+1)!(N-R)!} = {}^{N+1}C_{R+1}$$

Q Calculate $^{N}C_{R}$

1) small values of N & R

eg: $N = 10 \quad R = 3$

$$\frac{N!}{(N-R)!\ R!} = \frac{10!}{7!\ 3!}$$ ✓

1) Calculate factorial from 1 to N

2) store $R!$ , $(N-R)!$ , $N!$

3) Ans $= \dfrac{N!}{R!\ (N-R)!}$

2) $0 \le R \le N \le 10^5$ ⟹ overflow

$^{N}C_{R}$ % p → prime

$$\left(\frac{N!}{R!\ (N-R)!}\right) \% p$$

$$\Rightarrow \left(N! * (R!)^{-1} * (N-R)!)^{-1}\right) \% p$$

$$\Rightarrow \left((N!\%p) * ((R!)^{-1}\%p) * (((N-R)!^{-1}\%p)\right) \% p$$

$(R!)^{-1}\%p$ → fermats → $(R!)^{P-2}\%p$

$(N-R)!^{-1}\%p$ → $(N-R)!^{P-2}\%p$ → calculate using fast power.

$a^b$ fastPower ( a , b , p)

## Pseudocode

```
nf = 1      nrf = 1      rf = 1
f = 1
for ( i → 2 to N)
{
   f = (f * i) % p
   if ( i == n)      nf = f
   if ( i == (n-r))  nrf = f
   if ( i == r)      rf = f
}
return ( (nf * fastPower(nrf, p-2, p)) % p * fastPower( rf, p-2, p)) % p
```

fastPower → $O(\log(P-2)) = O(\log(P))$

T.C ⇒ $O(N + \log(P))$

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Q Print $N^{th}$ row of Pascal triangle (mod)

$^1C_0$ $^1C_1$

$^2C_0$ $^2C_1$ $^2C_2$

$^3C_0$ $^3C_1$ $^3C_2$ $^3C_3$

⋮

⇒

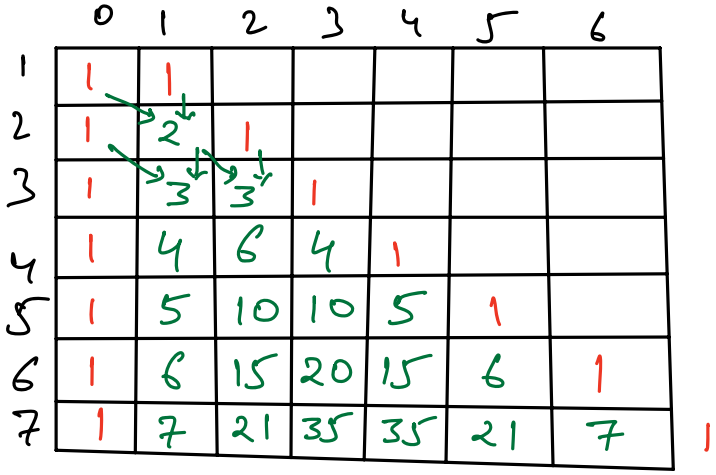
1 1

1 2 1

1 3 3 1

⋮

o/p ⇒ $^NC_0$ $^NC_1$ $^NC_2$ . . . . $^NC_N$




$$^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$$

$$^{1}C_{0} + {}^{1}C_{1} = {}^{2}C_{1}$$

Code

```
NCR[][]
for(i -> 1 to N)
{
  ncr[i][0] = ncr[i][i] = 1
  for(j -> 1 to i-1)
  {
    ncr[i][j] = (ncr[i-1][j] + ncr[i-1][j-1]) % m
  }
}
return ncr[N] => print Nth row
```

T.C

$O(N^2)$

S.C

$O(N^2)$ reduce $\rightarrow$ $O(2N) \simeq O(N)$

Hint

Using 2 1D arrays

prev array

curr array.

## Q Excel Column Title.

Given a +ve no. N, find the column title of $N^{th}$ column in excel sheet

A B C . . . . . Z AA AB AC . . . . BA BB...

N ⇒ 1 ⇒ A

26 ⇒ Z

27 ⇒ AA

30 ⇒ AD

⇒ 26 base number system

⇒ N = 26 → Z

↳ N %. 26 = 26 %. 26 == 0

A ⇒ 1 ⇒ 0

B ⇒ 2 ⇒ 1

C ⇒ 3 ⇒ 2

⋮

Z ⇒ 26 ⇒ 25

$(N-1)\ \%.\ 26$

### Decimal to 26 base

N = 30

| | |
|---|---|
| 26 | (30-1) |
| 26 | (1-1) |
| | 0 |

3 → D

0 → A

AD

Code

```
ans = ""
for ( N>0 )
{
    ans = ((char) (N-1)%26 + 'A') + ans
                  (0→25)
    N = (N-1)/26
}
return ans
```

T.C $O(\log_{26} N)$