OPTIMISATION

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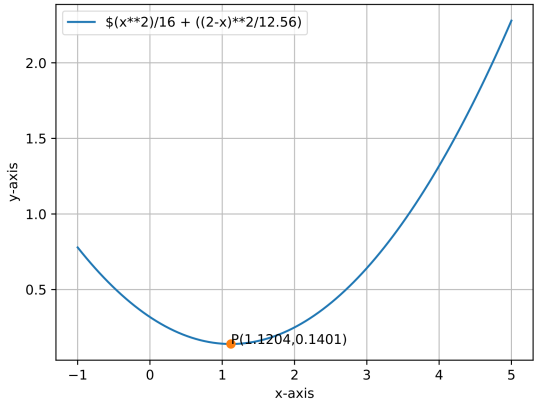
Problem statement

A wire of length 2 units is cut into two parts which are bent respectively to form a square of side 'a' units and a circle of radius ;r; units .If the sum of areas of the square and the circle so formed is minimum then : To Find: The relation between the radius of circle and side length

Given:
Length of the wire is 2 units

1 Construction



2 solution

perimeter of the square is $\mathbf{x}$ units. (1)

circumfernce of circle is $(2-x)$ units. (2)

side length of square a $= x/4$ (3)

radius of the circle $r = \dfrac{2-x}{2\pi}$ (4)

So, the total length is

$$4x + 2\pi r = 2 \tag{5}$$

Now by using the formula for the area of the circle and square is:

Area of square$= a^2$ (6)

Area of the circle$= \pi r^2$ (7)

Now, the combined area(A)

$$A = a^2 + \pi r^2 \tag{8}$$

$$A = \frac{x^2}{16} + \frac{(2-x)^2}{4\pi} \tag{9}$$

Objective function:

$$A = \min_x \frac{x^2}{16} + \frac{(2-x)^2}{4\pi} \tag{10}$$

(11)

constraints:

x>0

3 Calculation of Minima using Gradient Descent algorithm

minima using Gradient Descent method The minimum value is caluculated by using gradient descent method.

$$x_{n+1} = x_n - \alpha \nabla A(x_n) \tag{12}$$

$$\implies x_{n+1} = x_n - \alpha\left(\frac{x}{8} - \frac{2-x}{6.28}\right) \tag{13}$$

where

1. $\alpha = 0.001$
2. x_{n+1} is current value
3. x_n is previous value
4. precession = 0.00000001
5. maximum iterations = 100000000

The combined area has minimum value at 1.1204 i.e

$$\frac{8}{4+\pi} \tag{14}$$

The combined area is minimum with side length a is

$$a = \frac{2}{4+\pi} \tag{15}$$

The area of the circle is minimum with radius

$$r = \frac{1}{4+\pi} \tag{16}$$

Hence, the

$$\boxed{\text{a = 2r}} \tag{17}$$