

LINE ASSIGNMENT

Uday Kumar

immadisettyudaykumar15@gmail.com

FWC22086

IITH Future Wireless Communication (FWC)

MATRICES

Problem Statement – A straight line L is perpendicular to the line $5x-y=1$. The area of the triangle formed by the line and the coordinate axis is 5. Find the equation of line L.

solution

step 1

let the given equation is

$$\mathbf{n}_1^T(x) = c \quad (1)$$

Direction vectors of the perpendicular line is

$$\mathbf{n}_1^T \mathbf{n}_2 = 0 \quad (2)$$

$$\mathbf{n}_2 = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \quad (3)$$

The perpendicular line meets the coordinate axis at two points and forms a triangle with area $Q=5$

Let the two points be M,N and O is the origin

The two points lies on the line with direction vector \mathbf{m} , we get the condition

$$\mathbf{n}_2^T(\mathbf{M}) = \mathbf{n}_2^T(\mathbf{N}) \quad (4)$$

Solving the above two equations we get

$$a = 5b$$

substitute $a=5b$ then the points become

$$\mathbf{M} = \begin{pmatrix} 5b \\ 0 \end{pmatrix} \quad \mathbf{N} = \begin{pmatrix} 0 \\ b \end{pmatrix} \quad (5)$$

To find the value of b

Given the area of the triangle formed by the points OMN is Q

$$\mathbf{V}_1 = \mathbf{O} - \mathbf{M}, \mathbf{V}_2 = \mathbf{O} - \mathbf{N} \quad (6)$$

$$\frac{1}{2} \|(\mathbf{V}_1) \times (\mathbf{V}_2)\| = Q$$

by solving we get

$$b = \sqrt{2}$$

To get the value of c_2

$$\mathbf{n}_2^T(\mathbf{M}) = c_2 \quad (7)$$

we get

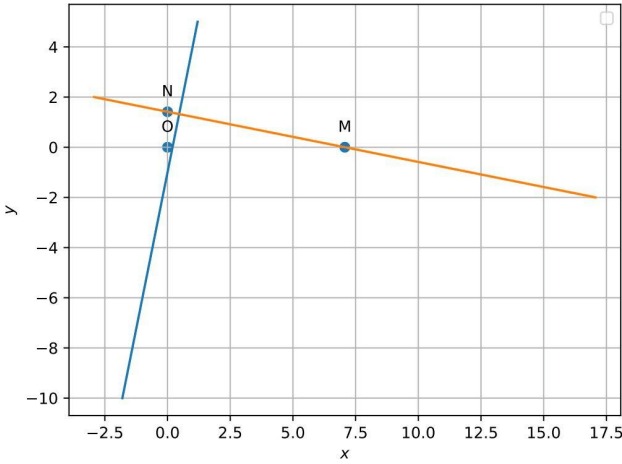


Figure 1: Perpendicular line

Construction

The input parameters are as follows

Symbol	Value	Description
\mathbf{n}_1	$\begin{pmatrix} 5 \\ -1 \end{pmatrix}$	normal vector
c_1	$\begin{pmatrix} 1 \end{pmatrix}$	constant
\mathbf{n}_2	$\begin{pmatrix} 1 \\ 5 \end{pmatrix}$	direction vector
e_1	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	
e_1	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	
M	$\begin{pmatrix} a \\ 0 \end{pmatrix}$	$e_1^T(\mathbf{a})$
N	$\begin{pmatrix} 0 \\ b \end{pmatrix}$	$e_2^T(\mathbf{b})$

$$c_2=5\sqrt{2}$$

The final equation is

$$\mathbf{n}_2^T(x) = c_2 \quad (8)$$