CIRCLE ASSIGNMENT

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Problem Statement – **Circles with radii 3,4,5 touch each other externally if P is the point of intersection of tangents to these circles at their point of contact.Find the distance of P from the point of contact.**

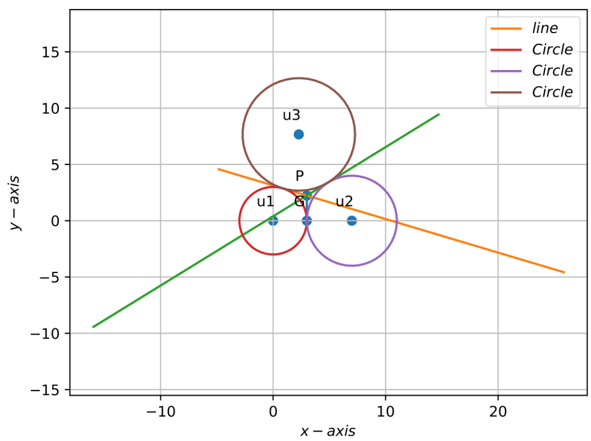

Figure 1: Perpendcular line

Construction

The input parameters are as follows

Symbol	Value	Description
r_1	3	radius
r_2	4	radius
r_3	5	radius
u_1	$\begin{pmatrix}0\\0\end{pmatrix}$	
u_2	$\begin{pmatrix}7\\0\end{pmatrix}$	
u_3	$\begin{pmatrix}2.28571428751\\7.66651878\end{pmatrix}$	$e_1{}^T(\mathbf{a})$
m	$\begin{pmatrix}7\\0\end{pmatrix}$	$e_2{}^T(\mathbf{b})$

solution

step 1

The general equation of the circle is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where V is the identity matrix
Let the equation of the circles with radii 3,4and 5 are

$$\mathbf{x}^\top \mathbf{x} + 2\mathbf{u_1}^\top \mathbf{x} + f_1 = 0 \tag{2}$$

$$\mathbf{x}^\top \mathbf{x} + 2\mathbf{u_2}^\top \mathbf{x} + f_2 = 0 \tag{3}$$

$$\mathbf{x}^\top \mathbf{x} + 2\mathbf{u_3}^\top \mathbf{x} + f_3 = 0 \tag{4}$$

Common tangent between the circles with $\mathbf{u_1}$ and $\mathbf{u_2}$

$$2(\mathbf{u_1}^\top - \mathbf{u_2}^\top)\mathbf{x} + f_1 - f_2 = 0 \tag{5}$$

$$2(\mathbf{u_2}^\top - \mathbf{u_3}^\top)\mathbf{x} + f_2 - f_3 = 0 \tag{6}$$

$$2(\mathbf{u_3}^\top - \mathbf{u_1}^\top)\mathbf{x} + f_3 - f_1 = 0 \tag{7}$$

Solving the above tangent equations we get the point

$$\mathbf{P} = \begin{pmatrix}3\\2.236\end{pmatrix} \tag{8}$$

To find the point of contact ,Foot of the point P to the line formed by the points $\mathbf{u_1}$ and $\mathbf{u_2}$

$$\mathbf{G} = \mathbf{u_1} + \mathbf{m}^\top \frac{(P - u_1)}{\|m\|^2}\mathbf{m} \tag{9}$$

where

$$\mathbf{m} = \mathbf{u_1} - \mathbf{u_2} \tag{10}$$

we get the point

$$\mathbf{G} = \begin{pmatrix}3\\0\end{pmatrix} \tag{11}$$

The distance between the two points G and P is

$$\mathbf{D_1} = \|\mathbf{P} - \mathbf{G}\| \tag{12}$$