Conic section Assignment

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Problem Statement **- If x=9 is the chord of contact of the hyperbola $x^2 - y^2 = 9$ then the equation of the corresponding pair of tangents is**

Solution

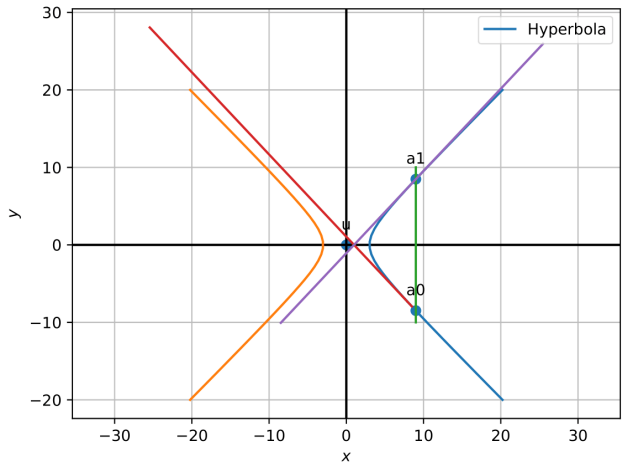

Figure 1:

The given equation of hyperbola $x^2 - y^2 = 9$ can be written in the general quadratic form as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{3}$$

$$f = -9 \tag{4}$$

The point of intersection of the lines x=9 and x=4 to the parabola is given by

The points of intersection of the line

$$L: \quad \mathbf{x} = \mathbf{q} + \mu\mathbf{m} \quad \mu \in \mathbf{R} \tag{5}$$

with the conic section are given by

$$\mathbf{x}_i = \mathbf{q} + \mu_i \mathbf{m} \tag{6}$$

where

$$\mu_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}} \left(-\mathbf{m}^T(\mathbf{V}\mathbf{q} + \mathbf{u}) \pm \sqrt{\left[\mathbf{m}^T(\mathbf{V}\mathbf{q} + \mathbf{u})\right]^2 - \left(\mathbf{q}^T\mathbf{V}\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)} \right) \tag{7}$$

From the line x-9=0 the vectors q,m are taken,

$$\mathbf{q} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} \tag{8}$$

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{9}$$

by substituting eq(2),(3),(4),(8),(9) in eq(7)

$$\mu_1 = -8.48528137 \tag{10}$$

$$\mu_2 = 8.48528137 \tag{11}$$

substituting eq(8),(9),(10) in eq(6) the intersection points on the parabola are

$$\mathbf{a}_0 = \mathbf{q} + \mu_1\mathbf{m} \tag{12}$$

$$\mathbf{a}_1 = \mathbf{q} + \mu_2\mathbf{m} \tag{13}$$

Equation of a tangent at a point is

$$(\mathbf{V}\mathbf{q} + \mathbf{u})^\top \mathbf{x} + \mathbf{u}^\top \mathbf{q} + f = 0 \tag{14}$$

Equation at a_0 is

$$t_1 = (\mathbf{V}\mathbf{a_0} + \mathbf{u})^\top \mathbf{x} + \mathbf{u}^\top \mathbf{a_0} + f = 0 \tag{15}$$

Equation at a_0 is

$$t_2 = (\mathbf{V}\mathbf{a_1} + \mathbf{u})^\top \mathbf{x} + \mathbf{u}^\top \mathbf{a_1} + f = 0 \tag{16}$$

The equation of the corresponding pair of tangents is

$$t = t_1 * t_2 \tag{17}$$

That is Equation of pair of tangents is

$$t = ((\mathbf{V}\mathbf{a_0} + \mathbf{u})^\top \mathbf{x} + \mathbf{u}^\top \mathbf{a_0} + f)((\mathbf{V}\mathbf{a_1} + \mathbf{u})^\top \mathbf{x} + \mathbf{u}^\top \mathbf{a_1} + f) \tag{18}$$

Construction

Points	**intersection points**
a_0	$\begin{pmatrix} 9 \\ -8.48528137 \end{pmatrix}$
a_1	$\begin{pmatrix} 9 \\ 8.48528137 \end{pmatrix}$