Umbrella Mesh - Actuation Experiments

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June 2023

Contents

1 Introduction

This document is the report for my semester project, spring 2023, supervised by KUSUPATI Uday and YINGYING Ren, two Ph.D. students at EPFL's Geometric Computing Laboratory. They've developed an inverse design framework for *Umbrella Mesh* (UM) optimization. UM, a new class of volumetric deployable structures, is composed of units made out of rigid plates, hinge joints, and elastic beams (arms) as shown in Figure 1-(a).
The power of UM holds in the fact that, at undeployed state, the structure has zero energy and is super compact, but especially that it can be designed to deploy into a targeted 3D surface (from 0D to 3D: point-like to 3D-surface). As shown in Figure 2., UM's compactness beholds to the designer's choice who can juggle between arms' length and the number of units to reach her desire.

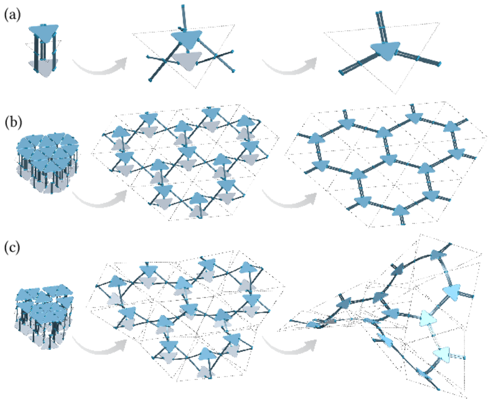


Figure 1: UM's concept [1]

Actuating units by joining the plates together, thanks to external actuators, deploys the compact structure into its target 3D surface. Once deployed, bistable UM structures do not need those external actuation forces anymore to prevent them to retract back into the compact form. Switching from a stable state to the other requires a bistable structure to pass through an unstable equilibrium point, which has higher internal energy. The deployment path highly influences intermediates states, thus the unstable equilibrium point the structure needs to overcome.
As the Figure 1-(c), shows, complex meshes with varying undeployed heights reach incompatibilities at some expansion point, which leads the structure to buckle out of the plan, invading 3D space.

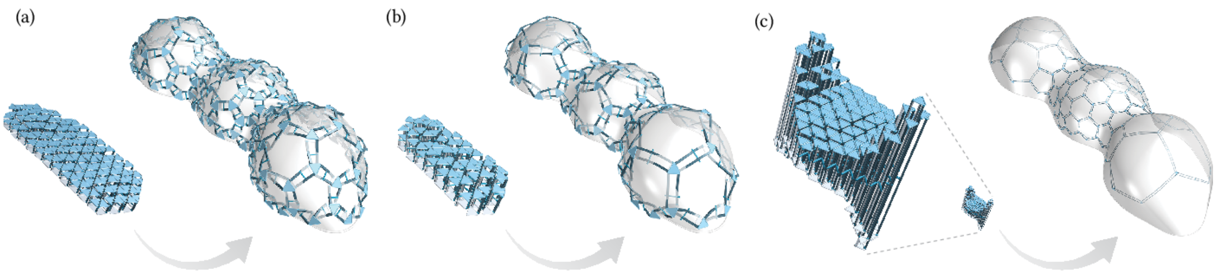


Figure 2: UM design choices [1]

2 Goal and Design

The main goal of this project is to analyze different strategies to deploy bistable UM structures, in order to reduce the maximal energy required to deploy from a stable compact state to a stable deployed state.

To familiarize with the new playground, simple rectangular with constant height meshes have been manipulated. The idea here is to implement tools allowing the user to choose units' target heights, and analyze the deployment steps internal stresses, units' height, and elastic energy. Choosing units' height automatically set them active, which is opposed to the other units that are free to deploy depending on the structure global equilibrium state undergoing such actuation constraints. Active units' actuators ensure plates' normal vectors to be aligned (plates are required to be parallel and overlapping), while free units' plates reach a position satisfying the equilibrium of the entire structure. The deployment is processed in *step* here, meaning that the set of active unit do not update while deployment. Active units reach their target height through the 'steps' number linear interpolation point between initial and target heights.

Once simple rectangular grid tools have been implemented, optimized meshed provided by the supervisors became the new center of interest. The first thing was to adapt the implementation allowing the user to deploy meshes in terms of *phase*. During one phase, the set of active units is constant, but as soon as new units are set active/free, the deployment enters a new phase. Let's mention here that phases are composed of steps. The user is now allowed to specify the target height for each unit, for each phase, and the number of steps for each phase.

3 Rectangular Grid

As an initiation to the new deployable structures, rectangular grid with constant height have been manipulated. To deploy into three-dimensional space, UM should reach some incompatibility at the plane level, leading the structure to buckle out. The framework to optimize the mesh grid to fit the target surface once deployed incorporates instabilities through units' height variation. Here the incompatibilities are resulting from varying target heights.
The two main observations deduced from the different tests done with regular rectangular grids are the following. Firstly, uni-dimentional structure never buckles out of their axis, because no incompatibilities could be induced at deployment. The other main observation is that to be bistable, UM must have variation in their units' height. Constant height grid can buckle out of the plane, allowing deployment into 3D space, but the structure would always shrink back to the compact state once units are freed.

4 Mesh Grid

We will now analyze mode advances meshes. Those meshes are the result of the optimization pipeline aiming to fit an arbitrary 3D-surface once deployed. The provided meshes are bi-stable, and all of them are degree 3 (equilateral triangular plates with three arms). As opposed to rectangular grids, the target height is 100%, meaning that the active units should be fully deployed.

4.1 Optimized meshes

This section aims to present the different advanced grid and their respective characteristics. The meshes are ordered in terms of dome number, from no dome to a total of four. As demonstrated later, each dome has local bistability behavior, meaning that a partially deployed structure can stay partially deployed after freeing the units.

4.1.1 Saddle

The *Saddle* mesh deployed state models a unique saddle point. This mesh is composed of 32 units, and its minimal and maximal heights measure respectively 0.287 and 0.396. As the name let suggest, this mesh has no complete dome.

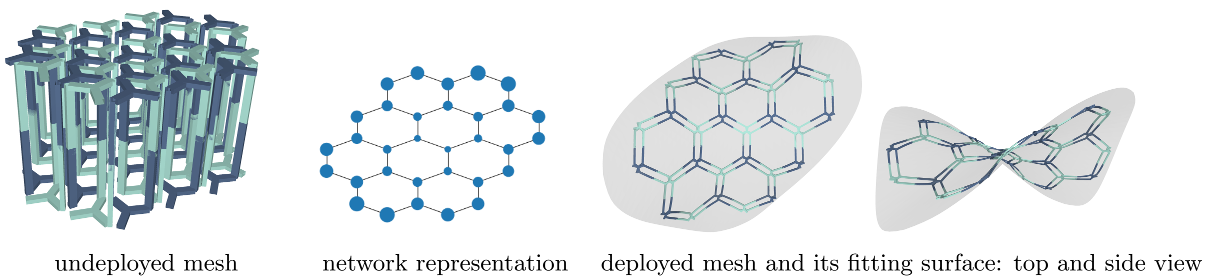
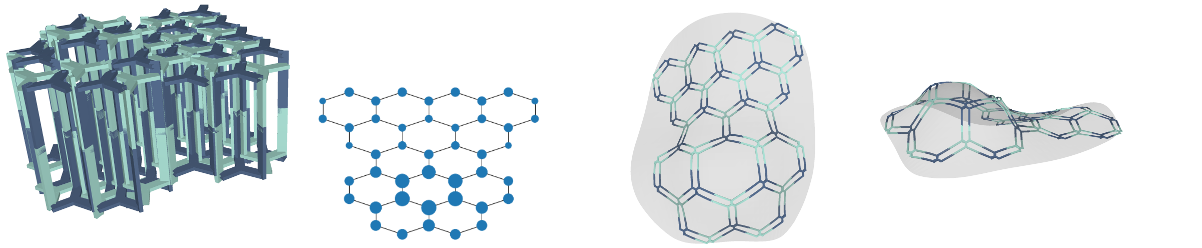


Figure 3: Descriptive image of *Saddle* mesh

Let's note here that the network representation node's size is proportional to the corresponding unit's undeployed height.

4.1.2 Igloo

The *Igloo* mesh deployed state models a the igloo and its entrance. This mesh is composed of 57 units, and its minimal and maximal heights measure respectively 0.221 and 0.443. This advanced mesh has one unique dome.

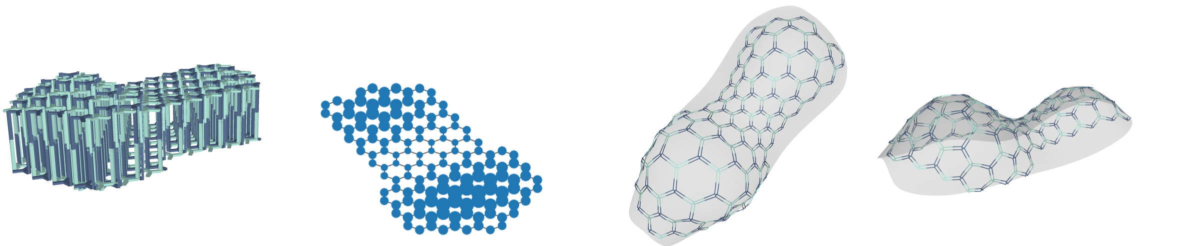


undeployed mesh | network representation | deployed mesh and its fitting surface: top and side view

Figure 4: Descriptive images of *igloo coarse* mesh

4.1.3 Squid

The *Squid* mesh deployed state models a double dome squid-like shape. This mesh is the most fine grained one, with 144 units. Its minimal and maximal heights are respectively 0.128 and 0.294.

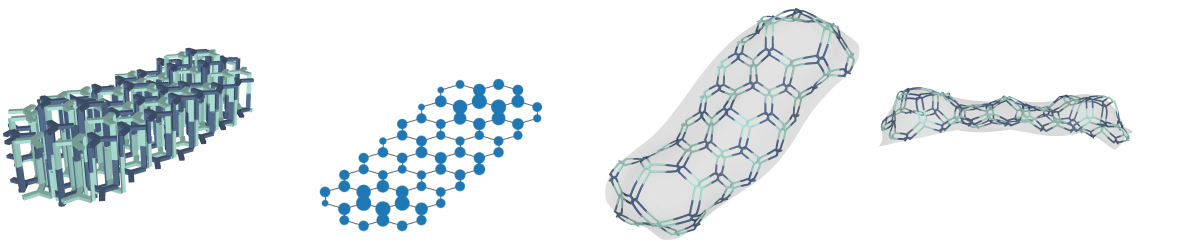


undeployed mesh | network representation | deployed mesh and its fitting surface: top and side view

Figure 5: Descriptive images of *Squid* mesh

4.1.4 Peanut

This mesh, called the *Peanut*, shapes into three domes once deployed. This mesh is somehow coarse compared to the complexity of the target surface. It has 64 units and its respective minimal and maximal undeployed units heights are 0.082 and 0.238.

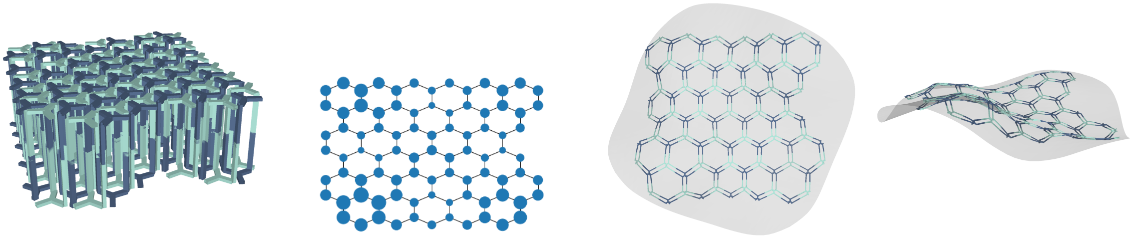


undeployed mesh | network representation | deployed mesh and its fitting surface: top and side view

Figure 6: Descriptive images of *Peanut* mesh

4.1.5 Lilium

This last mesh is called the *Lilium*, which is a genus name for herbaceous flowering plants growing from bulbs. Its deployed shape form four dome, with one of them in the opposed direction to the others. This mesh has 87 units and its respective minimal and maximal undeployed units heights are 0.201 and 0.279.



undeployed mesh network representation deployed mesh and its fitting surface: top and side view

Figure 7: Descriptive images of *Lilium* mesh

4.2 Distinctive Units

Meshes are composed of units, some of which are unique in terms of their absolute or relative position and height. It is thus possible to group them by class and to treat them accordingly. This section help itself with the *Lilium* mesh to depict five classes, which will later be used to deploy meshes.

4.2.1 Boundary

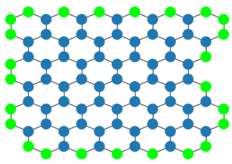


Figure 8: boundary units

Boundary units are such that the degree of the node in the network representation is lower than the degree of the unit composing the structure. As all advanced meshes are D3, a unit is part of the boundary if and only is the number of connected arm is lower than 3.
The Figure 8 highlight boundary units: green-colored node indicate the boundary.

4.2.2 Neighbors

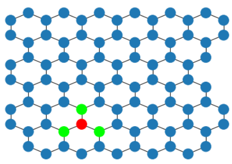


Figure 9: neighbors units

For arbitrary unit u_0, its *neighbors* are the units directly linked to it. The Figure 9 highlight u_0 in red and its neighbors in green.

4.2.3 Surroundings

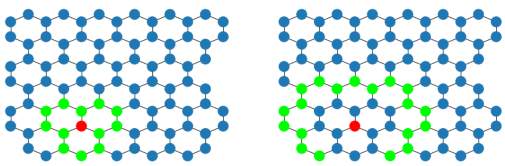


Figure 10: surroundings level 1 and level 2

The *surroundings* of arbitrary unit u_0 are defined such that they would form a closed boundary centered around $u0$ in an infinite grid-like mesh; let's note that the advanced meshes are finite, thus the surroundings could be interrupted by boundary units. Surroundings are characterized by their *level* attribute, which quantifies the distance the wave has spread out from its starting unit u_0. In other words, the level 2 surroundings are the surroundings of level 1, and level 3 surroundings are the surroundings of level 2, and so on.
Figure 10 emphase starting unit in red and its surroundings in green.
Here are some notes about surroundings: let's observe that if u_a surround u_b at level X, then u_b too surround u_a at level X. Let's also note that surroundings only evolve in the forward direction from the starting point. In other words, surroundings only contain presumably unvisited units (they cut the space into a inside and outside sub-region). This imply that when the surroundings from unit u_a should enter the inside from the surroundings u_b, the two surroundings are merged together to surrounds the combined inside regions.

4.2.4 Bumps and Depressions

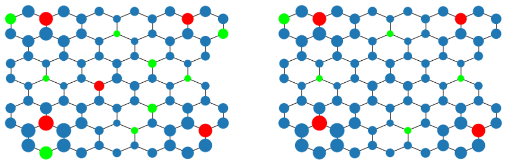


Figure 11: extremes height units, compared to its neighbors and surroundings level 1

A *bump* is a unit that has the tallest undeployed height when juxtaposed with its comparison units.
A *depression* is the opposite: a unit that has the smallest undeployed height when juxtaposed with its comparison units.
On Figure 11, green-colored nodes are depressions and bumps are red-colored nodes.

The comparison points can be chosen in many manners: Figure 11 shows bumps and depressions when compared to the neighbors (left) or the surroundings level 1 (right). From now on, if not specify, extremes units (depressions and bumps) are classified in comparison to surroundings level 1. Neighbors-compared extremes point are respectively named Nbump and Ndepression, and when compared to all units forming the mesh, they are named Abump and Adepression.
Let's note that the Figure 11 situation considers only local units to compare with. Such a situation can lead to unintuitive results such as bump with lower undeployed height than depression. The left image shows it: the bottom left depression has a bigger undeployed height than the central bump (the node size shows it).
Let's note that, if the comparison points are the entire structure (Abump and Adepression), there will be exactly one bump and one depression, or none if all units have equal undeployed height.

5 Deployment

Deploying UM structure can be done following different strategies, each of which has some pros and cons. This section first presents global approaches comparing all meshes together, then focuses on several strategies, that depend on the structure.

To compare the deployment process, the following metrics are observed:

- *Elastic Energy*: mechanical potential energy induced by linear elastic deformation.
- *Von-Mises (VM) stress*: scalar value significant for ductile material under complex loading. It is used to predict yielding by comparing VM stress to the yield strength. Lower values could be assumed as linear elastic behavior.
- *Height of Unit*: distance between plates. The Target surface is defined such that each unit's deployed height is null.

Such criteria are useful to quantify how close to the target surface the deployed structure is, the amount of energy required to reach this state, and under what stress the structure undergoes. An important remark is plates are considered rigid compared to arms, thus their stresses are not considered for this analyze. As their role is to connect arms together, their hub nature implies complex stress interaction that would deserve care for physical model building.

5.1 Naive Deployment

This section presents some naive approaches for UM deployment process. The goal here is to observe if occurring, any similar comportment of the analyzed advanced meshed undergoing such processes.
Those deployment processes are called naive because they deploy all active units at the same time. In other words, the deployment is composed of only one phase, which is composed of 15 steps.
The naive deployment focuses on some of the distinctive units presented in Section 4.2: the four approaches presented here respectively actuate all the bumps, all the depressions, the boundary units, and all-units.

5.1.1 Metrics

Let's recall that the comparison units for bumps and depressions are surroundings level 1 (the default approach).

Blue curves are the all-units approach, and the orange one represents the deployment of boundary units. Red and green curves are respectively depicting the deployment process of bumps and depressions.
At first, the three metrics are shown for each mesh, then a global discussion highlighting observations is done.

- *Saddle*

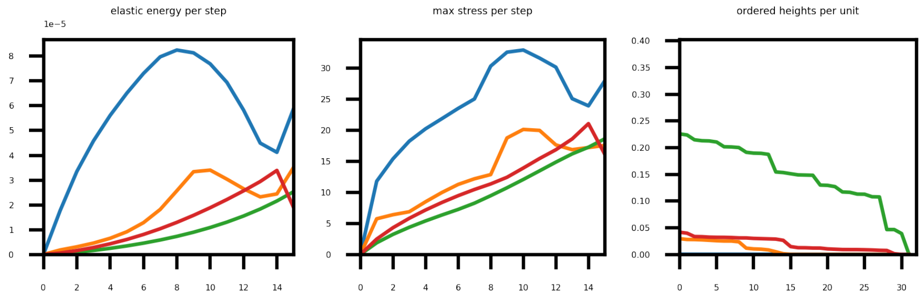


Figure 12: elastic energy, maximal VM stress and deployed height of *Saddle*

- *Igloo*

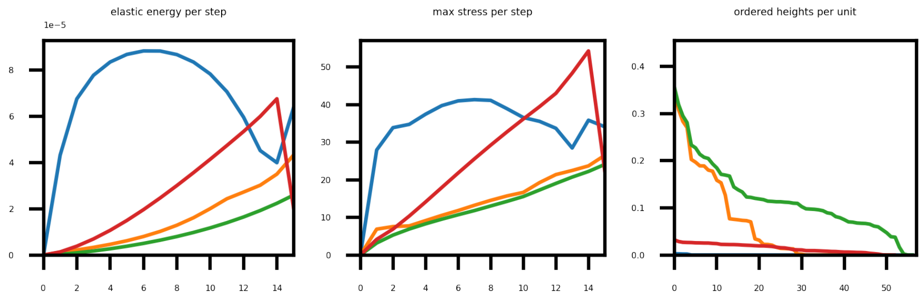


Figure 13: elastic energy, maximal VM stress and deployed height of *Igloo*

- *Squid*

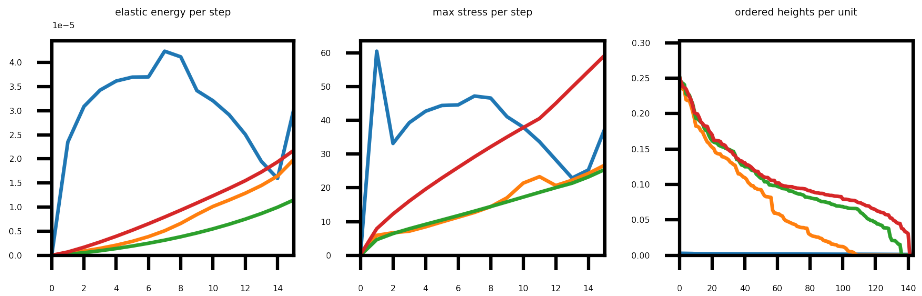


Figure 14: elastic energy, maximal VM stress and deployed height of *Squid*

- *Peanut*

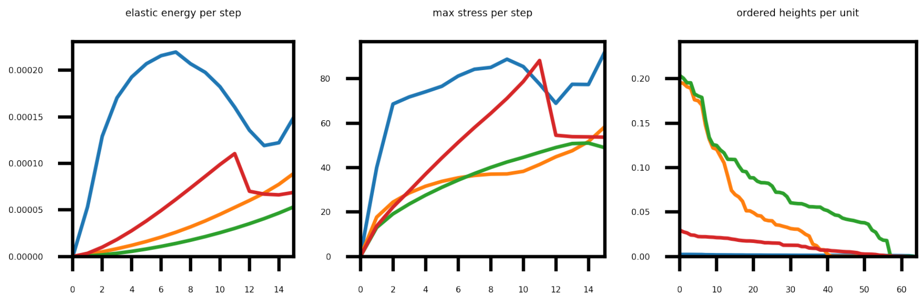


Figure 15: elastic energy, maximal VM stress and deployed height of *Peanut*

- *Lilium*

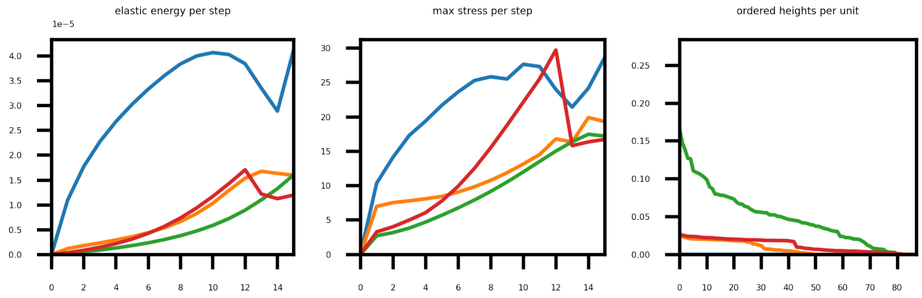


Figure 16: elastic energy, maximal VM stress and deployed height of *Lilium*

5.1.2 Discussion

The all-units strategy is considered as baseline because it is the most simple strategy ensuring the deployed structure reaches the target surface (by definition, the target surface is deployed state with all units having zero height).

First of all, let's notice that the all-units approach (blue curve) always reaches the highest energy value. This process constrains each unit to deploy at the same rate by preventing them from either naturally deploying or shrinking back to the compact state. Intermediates states would require high external actuation energy to maintain the structure partially deployed, which translates to high elastic energy.

This strategy also fits the target surface the best, but all meshes have elastic energy increasing at the last step. This means that the target surface is not reached at the second equilibrium rest state, but would require external actuators to be fully fitted.

Only depressions activated strategy (green curve) never overcome the energy barrier. This can be deduced by the monotonic energy curve and the high y-intercept value of the ordered height curve.

Actuating only bumps (red curve) seems promising because four out of five meshes overstep the energy barrier required to reach the second equilibrium rest state. But this is not always true: *Squid* mesh does not reach the deployed stable equilibrium by deploying only bumps. The final state 3D model and its VM stresses are shown in Figure 17. Clearly, bumps are fully deployed, but the shape is still planar.

Boundary actuation (orange curve) is an in-between situation, not truly meaningful as it might contain bumps and/or depressions or neither. Boundary units still can be useful for specific mesh deployment strategies.

The *Igloo* mesh shows that higher elastic energy is not correlated with higher stress value: the highest value of stress is the bumps actuated strategy, as opposed to the highest elastic energy which is reached by the all-units strategy. Let's also note that for the last step, the blue energy did increase, but the stress dose decreased. This increase in elastic energy implies more overall strain deformation. But as the stress decrease, those strains' deformations should be spread widely across units rather than concentrate onto some only.

Bumps only actuating have big and brutal energy drops (releasing). Figure 17 is a hint about it: this mesh design does not allow the structure to buckle out of plan with such a scenario, but other meshes do, such as the *Igloo*. The strain energy is, at first, concentrated around the bumps, which is then brutally released into the entire structure once its buckles out of plan. Such deployment processes follow paths that have steep energy curves nearby the unstable equilibrium point. The energy curve is not sufficiently fine-grained to ensure this, and no analytical solution has been developed for this specific case, but this curve let suggest it could be part of the differentiability class C^0 but not C^1.

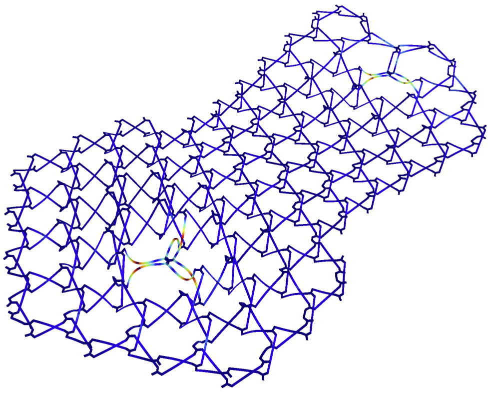


Figure 17: VM stresses of bumps only deployed *Squid*

5.2 Customized Deployment

This section presents four deployment process families per optimized meshes, which are *depressions and bumps*, *shortest path*, *surroundings*, and *increasing height*.

Here are the main goals behind the presented strategy: reduce maximal strain energy, and reduce maximal VM stress while fitting the target surface as close as possible. The different approaches are guided by the number of active units, the units category, and the actuation ordering. As written previously, non-regular meshes buckle out of plan only after some expansion threshold, thus it should be that deploying first meshes with small heights could allow the structure to expand widely before reaching the buckling limit. This approach should reduce internal stress by spreading the strain energy among more units.

Below the graphs, the legends shortly describe the strategy followed for that specific deployment process (color-based). Let's specify that 'then' means a new phase. But here are some clarifications:
* As observed, bumps are really interesting units to fully deploy. Thus, the *shortest path* strategies are aimed at actuating all the bumps. Paths connect depressions to bumps: for each bump, the shortest path is computed from all depressions, and the shortest of the shortest path is kept for that bump. The network representation used to compute such shortest path has weight on the edges, which is the absolute difference between the two linked units' heights.
It is possible to force at least one of the shortest paths to account for the smallest depression. This is done by firstly computing the shortest path from this depression to each bump, keeping the shortest of those paths, and proceeding as usual for the remaining bumps.
* 'Two actuators' means that two actuators progress along the same path, in such a way that the two actuators move one unit closer to the targeted bump per phase. The idea here is the first actuator would deploy further the structure, while the second actuator keeps the already deployed units deployed, so the active units must be linked.
* A strategy approach is to actuate two units per path. If paths have different lengths, two different approaches are analyzed: the classic and the 'end together'. For the classic approach, all the actuator pairs evolve at the same rate and as soon as one reaches its bump, it stays there, active, waiting for the others to end. The 'end together' approach is sort of the opposed approach: at first, all pairs deploy their two starting units, then only the pair of the longest path progress. Once the remaining phases are the same, the pair of such a path joins the race. Thus, all bumps are deployed at the same phase, which is the last one.
* The colors for the shortest path do not indicate the height of the units, but rather how the path evolves from depression (green) to bump (red).
* *surroundings* approach deploys units in decreasing surrounding level value, and each unit belonging to the same level is actuated at the same phase. In other words, waves converge to bumps, and for each phase, the active units are closer to the central unit. As for the previous remark, the idea here is that surroundings closer to bumps should contain higher units, thus the next phase should keep the previously

deployed units deployed, plus deploy new units.
* After examining the wave actuation deployment, it is possible to point out the *belt* of the mesh. The belt is characterized as the surrounding units with the highest level value for which the overstepping of the unstable equilibrium point occurs.
* *increasing height* deploys one new unit per phase, in a cumulative approach. This strategy ensures the structure is fully deployed, but requires a lot of actuators. It should be done in a cumulative approach to ensure the structure not to locally shrink back to its compact state after freeing some units.

5.2.1 Saddle

- *depressions and bumps*

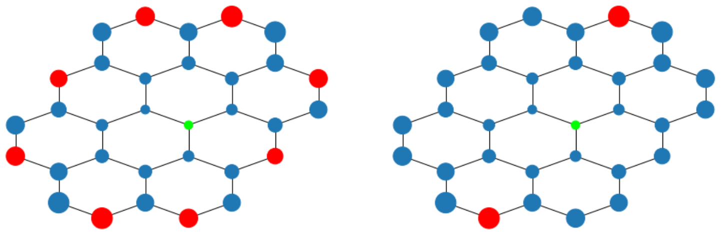
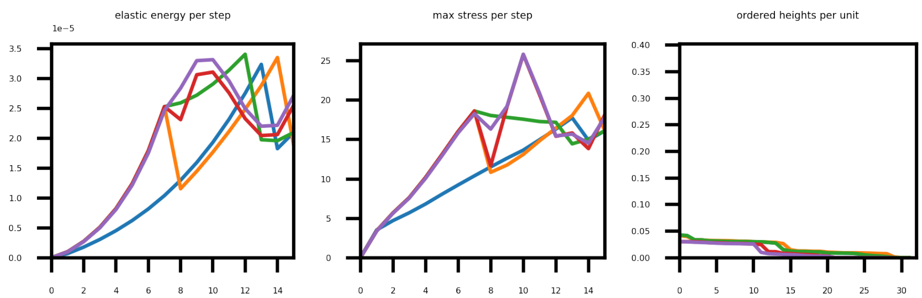


Figure 18: *Saddle*'s depressions and bumps compared to neighbors (left) and surroundings level 1 (right)



blue: depression and bumps
orange: depression then bumps
green: depression then depressions and bumps
red: Ndepression then Nbumps
purple: Ndepression then Ndepression and Nbumps

Figure 19: depressions and bumps actuation for *Saddle*

Clearly, all strategies reach approximately the same maximal energy value, but the blue curve has the lowest maximal stress. All strategies reach descent and comparable deployed state, described by the right graph. Neighbors' comparison to select extreme units seems to soften the energy release post unstable equilibrium point, this is mostly due to the fact that more units are controlled by actuators.

- *shortest paths*

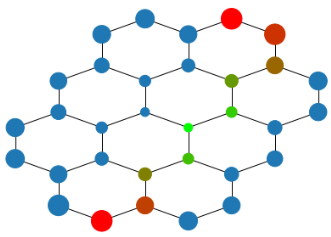
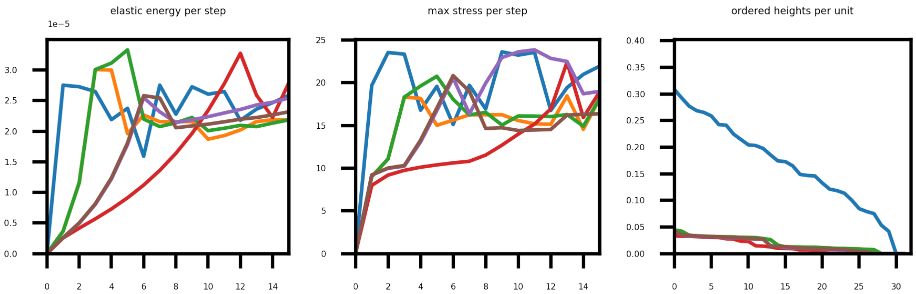


Figure 20: *Saddle*'s shortest paths



blue: two actuators, one path after the other
orange: two actuators per path
green: two actuators per path, end together
red: actuate all paths' units at the same time
purple: first actuate all units of path a, then all units of path b
brown: first actuate all units of path b, then all units of path a

Figure 21: shortest path actuation for *Saddle*

Path a is the one linked to the bottom bump, and path b is the other one.
Let's observe, on the height graph, that the deployed structure resulting from the blue strategy is far from the target surface. This is due to the complete release of opposed bump, while a saddle point only has one global bistable behavior (no dome).

- *surroundings*

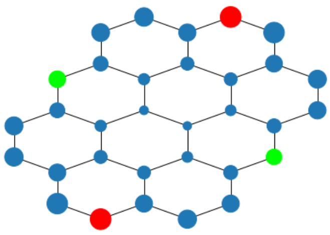
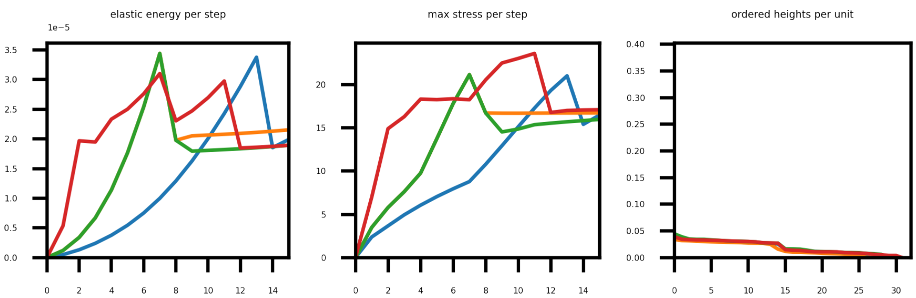


Figure 22: *Saddle*'s belt and bumps



blue: belt only
orange: belt then belt and bumps
green: belt then bumps
red: wave converging to bumps

Figure 23: surrounding actuation for *Saddle*

First of all, let's observe that all the strategies reach a descent deployment state with heights graph. The saddle's belt is a bit strange (only two units), which means that the structure quickly oversteps the unstable equilibrium point.

- *increasing height*

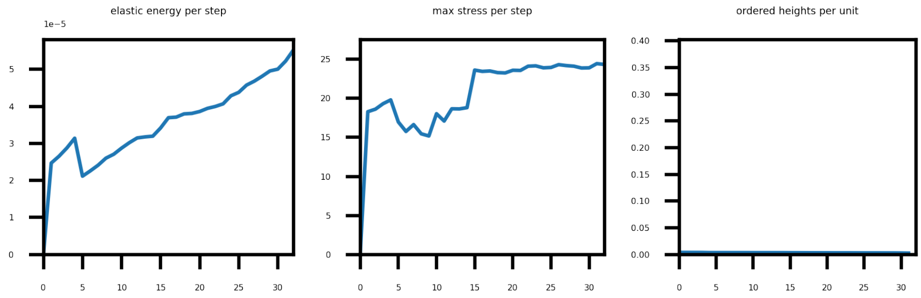


Figure 24: increasing height actuation for *Saddle*

This strategy reaches the same final energy as the ‘all-units‘ shown in Figure 12, but the maximal value is lower.

5.2.2 Igloo

- *depressions and bumps*

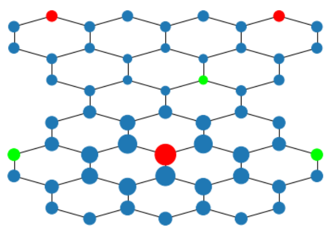
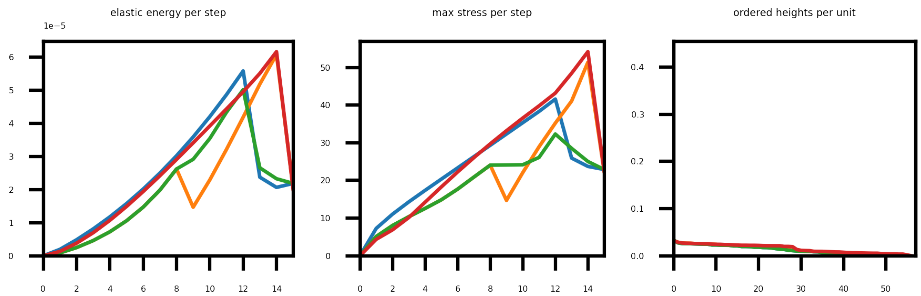


Figure 25: *Igloo*’s depressions and bumps



blue: depressions and bumps
orange: depressions then bumps
green: depressions then depressions and bumps
red: only main bump

Figure 26: depressions and bumps actuation for *Igloo*

The main bump is the one related to the dome.
The presented results show that those strategies demand overcoming a high energy barrier before reaching the second stable equilibrium state. And that once overcame, the strain energy release is brutal. Those strategies do seem not that promising for this mesh. But let’s first discover the other results before letting them aside.

- *shortest paths*

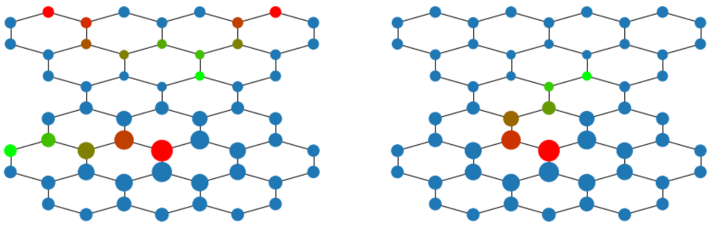
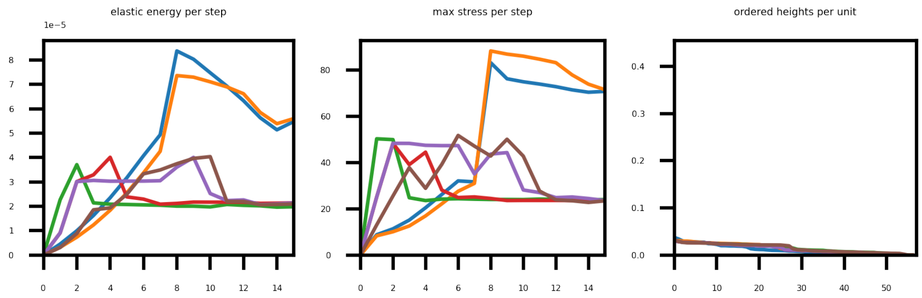


Figure 27: *Igloo*'s shortest paths, bump and depression compared to surroundings level 1 (left) and all units (right)



blue: all paths' units at the same time; level 1
orange: all paths' units at the same time; all units
green: two actuators: actuate the main bump path, then the others; level 1
red: two actuators per path; level 1
purple: two actuators per path, end together; level 1
brown: two actuators per path; all units

Figure 28: shortest path actuation for *Igloo*

Let's observe that the blue and orange strategies demand to overcome a bigger strain energy barrier, which is governed by the main bump.
The other strategies have similar maximal stress and energy values, but close to half than actuate all paths' units at the same time. This is a great observation because, as all strategies reach a descent deployed state, only using two actuators is a good plus.

- *surroundings*

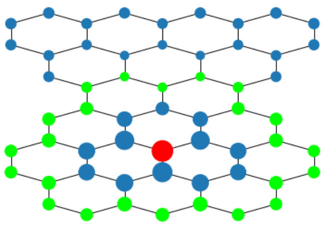
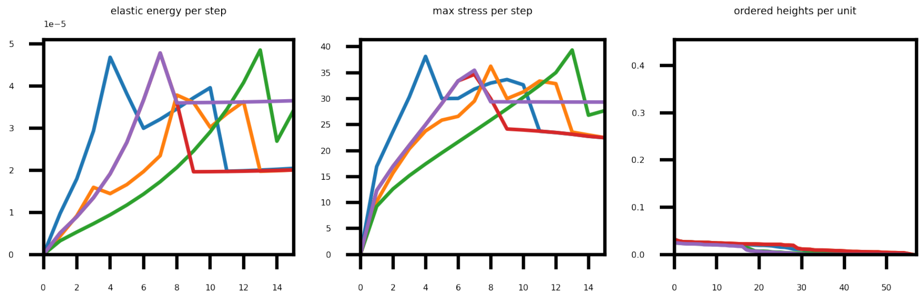


Figure 29: *Igloo*'s belt and main bump



blue: wave converging to bumps
orange: wave converging to main bump
green: belt around main bump
red: main bump's belt, then only main bump
purple: main bump's belt, then also main bump

Figure 30: surrounding actuation for *Igloo*

Let's observe that belt activation is somehow similar to actuating the main bump only. The brutal energy release seems too violent for such strategies to be used with physical large-scale models.

- *increasing height*

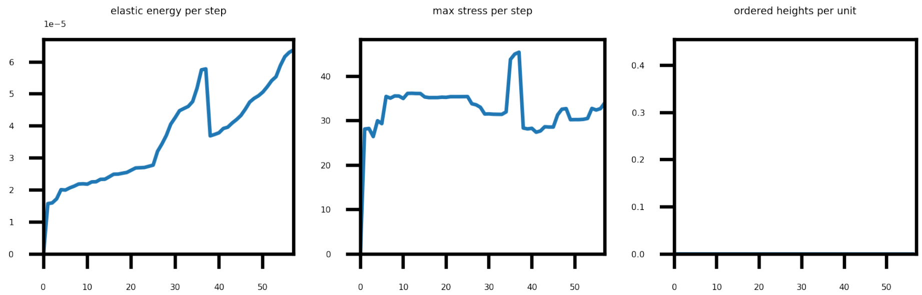


Figure 31: increasing height actuation for *Igloo*

5.2.3 Squid

- *depressions and bumps*

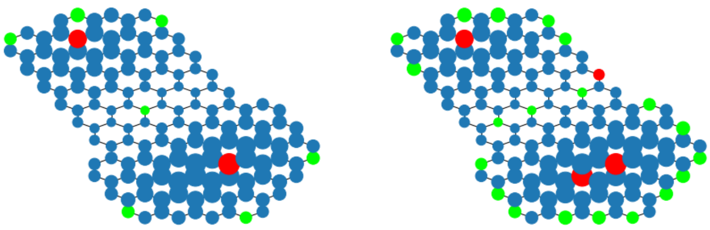
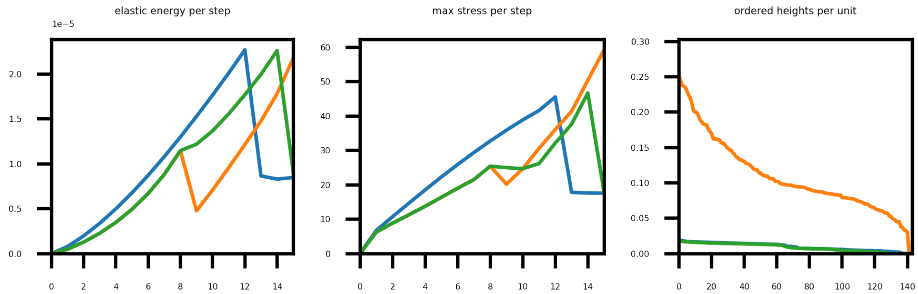


Figure 32: *Squid*'s depressions and bumps surr and neigh



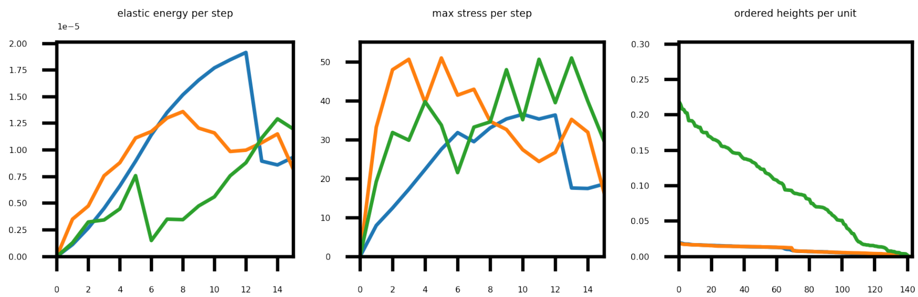
blue: Nbumps only
orange: depressions then bumps
green: depressions then depressions and bumps

Figure 33: depressions and bumps actuation for *Squid*

As for bumps only, Nbumps approach did not overcome the unstable equilibrium. And the two other strategies store huge amounts of strain energy that is brutally released.

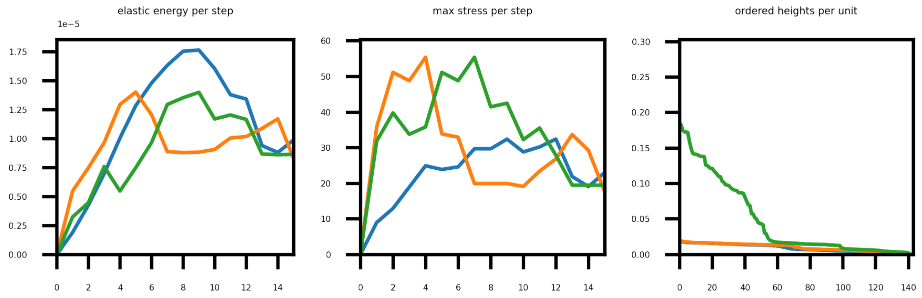
- *shortest paths*

Figure 34: *Squid*'s shortest paths
left(a): default configuration, mid(b): force smallest depressions, right(c): do not consider extremes that are boundary units



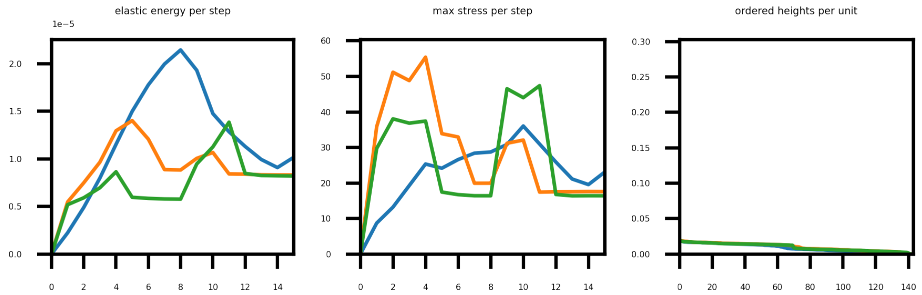
blue: all paths units at once
orange: two actuators: path to main dome, then path to secondary one
green: two actuators: path to secondary dome, then path to main one

Figure 35: shortest paths (a) actuation for *Squid*



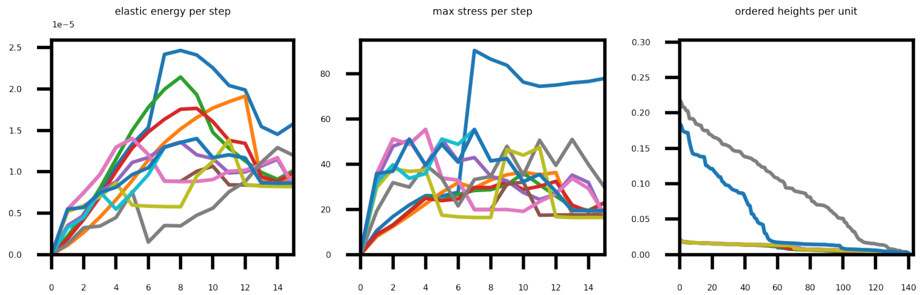
blue: all paths' units at once
orange: two actuators: path to main dome, then path to secondary one
green: two actuators: path to secondary dome, then path to main one

Figure 36: shortest paths (b) actuation for *Squid*



blue: all paths units at once
orange: two actuators: path to main dome, then path to secondary one
green: two actuators: path to secondary dome, then path to main one

Figure 37: shortest paths (c) actuation for *Squid*



all curves in one graph

Figure 38: shortest paths actuation for *Squid*

Let's note that not all strategies reach a descent deployed state.
Let's also observe, from configuration (b), that

- *surroundings*

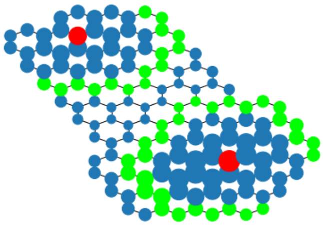
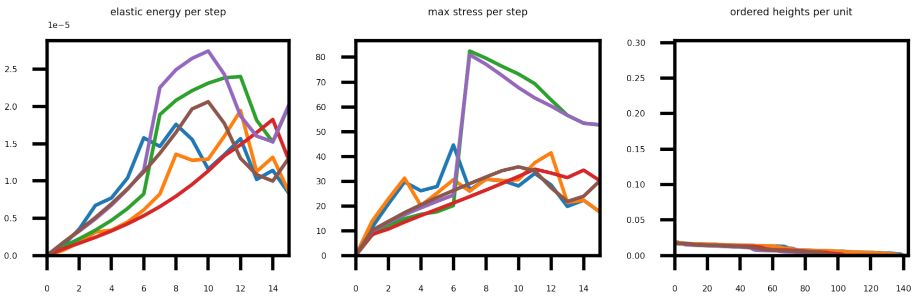


Figure 39: *Squid*'s belts



blue: wave converging to bumps
orange: wave converging to bumps, neighbors
green: belt only
red: belt only, neighbors
purple: belt and bump
brown: belt and bump, neighbor

Figure 40: shortest paths actuation for *Squid*

- *increasing height*

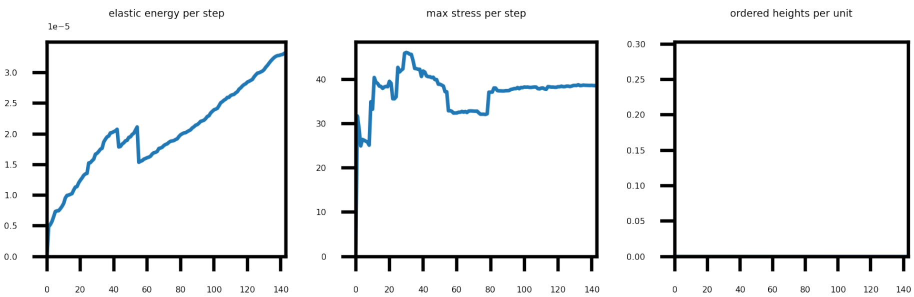


Figure 41: increasing height actuation for *Squid*

5.2.4 Peanut

- *depressions and bumps*

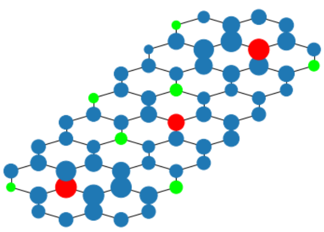
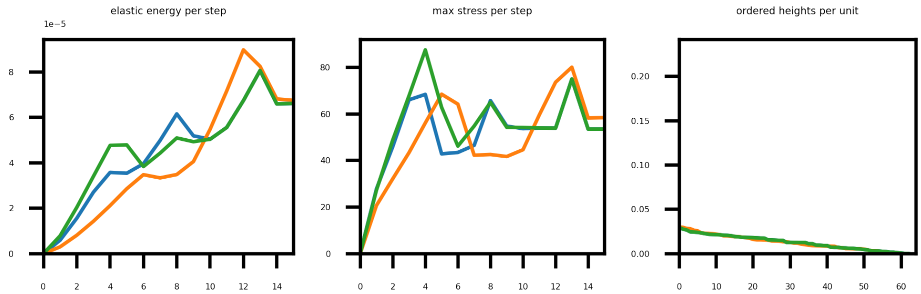
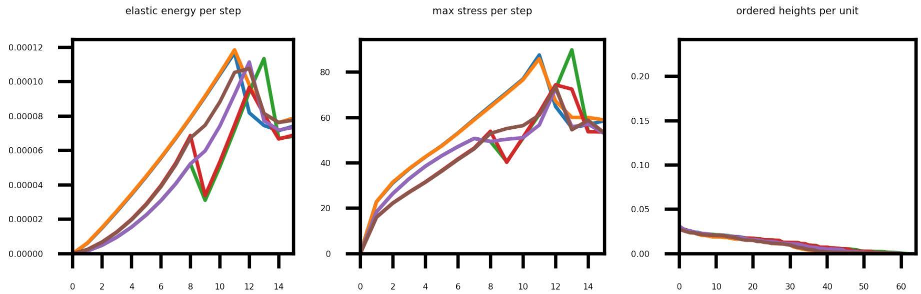


Figure 42: *Peanut*'s depressions and bumps



blue: first actuate central bump, then bottom one, and finally top one
orange: bottom, mid, top, one after the other
green: central bump first, then the two others

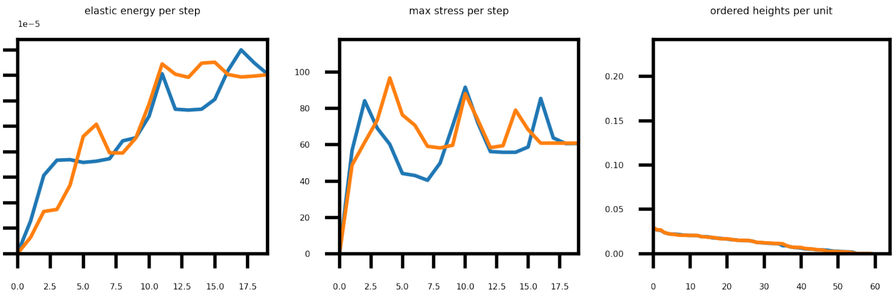
Figure 43: depressions and bumps actuation for *Peanut*



blue: depressions and bumps
orange: depressions and bumps, neighbors
green: depressions first, then bumps
red: depressions first, then bumps, neighbors
purple: depressions first, then also bumps
brown: depressions first, then also bumps, neighbors

Figure 44: depressions and bumps actuation for *Peanut*

- *shortest paths*



blue: central dome first
orange: central dome last

Figure 45: depressions and bumps actuation for *Peanut*

- *surroundings*

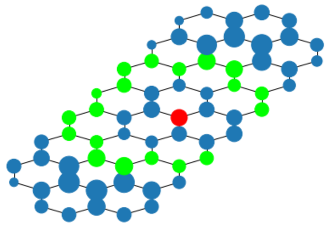
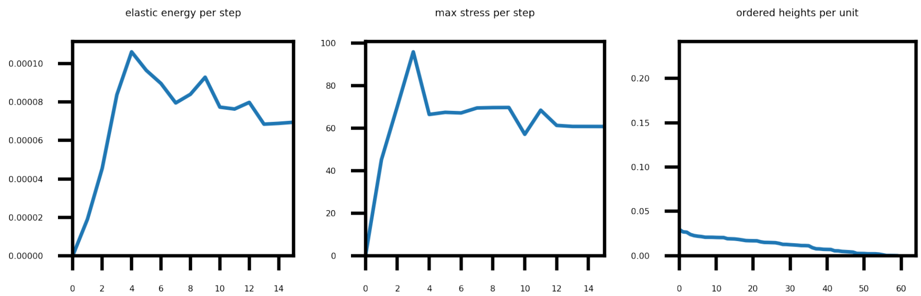


Figure 46: *Peanut*'s wave at intermediate stage



blue: wave converging to central bump

Figure 47: depressions and bumps actuation for *Peanut*

- *increasing height*

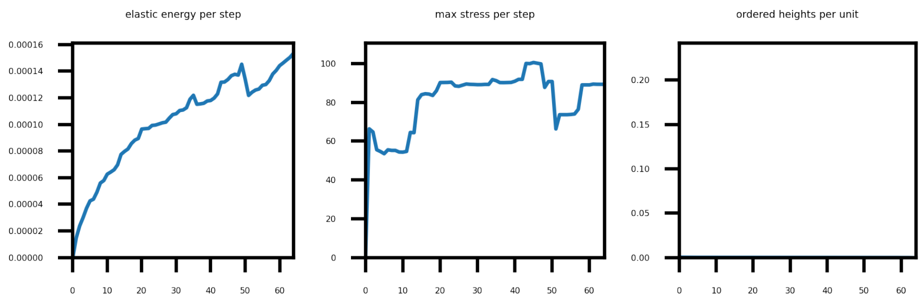


Figure 48: increasing height actuation for *Peanut*

5.2.5 Lilium

6 Conclusion

Bistable *Umbrella Mesh* deployable structure deployment path has huge potential for stress and strain energy optimization. Baseline deployment, shown at Section 5.1.1, could be highly improved for all the analyzed meshes. Metrics could be divided by three for some structures such as the *Saddle* mesh.
The deployment processes followed in this study are fat from being exhaustive, but already shows that considerable gain of energy can happens with some deployment optimisation. This optimisation is structure dependent, different meshes could demand less energy for a specific strategy, but another strategy would better suit other mesh.

Other than deployment, UM structure also shrink back.

Finding a good deployment path, that reduces the maximal strain energy the structure should overstep to reach the second stable equilibrium state is somehow equivalent to finding a good catalysts.
Other than deploment path, playing with scale let's imagine structures as big as building, like stadium shelters [1], to children toy. Deployment % of units highly influences stiffness of deployed structure, due to higher beam's static head. Such research which would also be a topic to explored.

References

[1] Yingying Ren et al. “Umbrella meshes”. In: *ACM Transactions on Graphics (TOG)* 41 (2022), pp. 1–15.