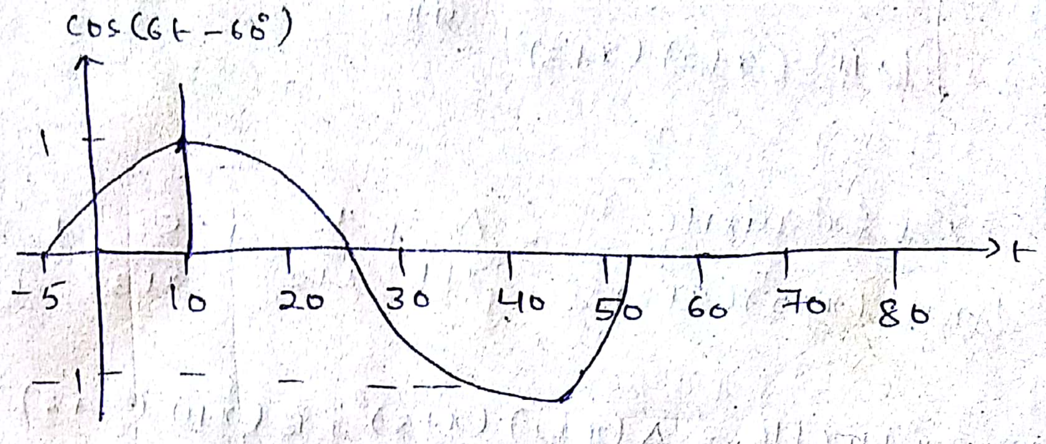
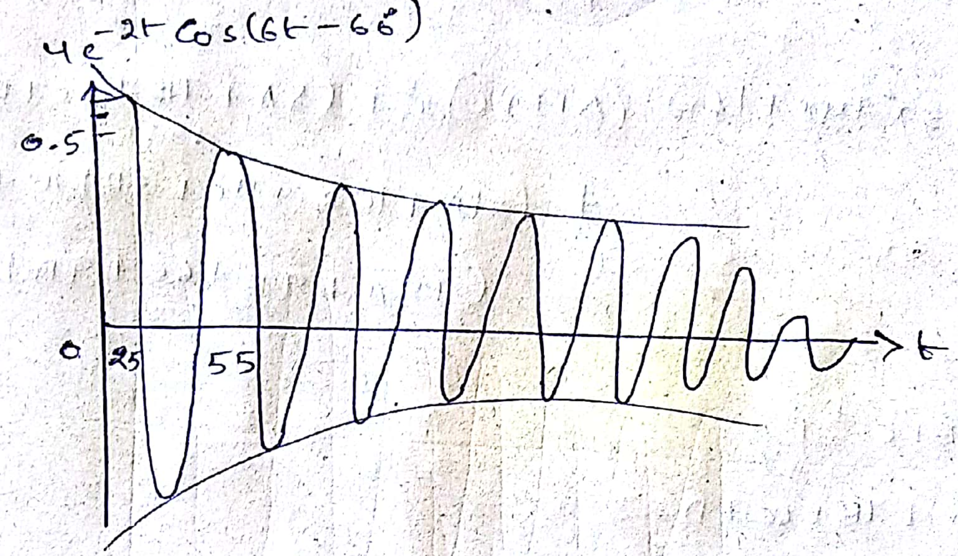
1. Sketch signal $f(t) = 4e^{-2t}\cos(6t - 60^\circ)$





2)

i) $\int \frac{x^3+3x^2+4x+6}{(x+1)(x+2)(x+3)^2}\,dx.$

Sol.

$$\frac{x^3+3x^2+4x+16}{(x+1)(x+2)(x+3)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3} + \frac{D}{(x+3)^2}$$

$$x^3+3x^2+4x+16 = A(x+2)(x+3)^2 + B(x+1)(x+3)^2 + C(x+1)(x+2)(x+3) + D(x+1)(x+2).$$

$$x^3+3x^2+4x+16 = (A+B+C)x^3 + (8A+7B+6C+D)x^2 + (21A+15B+11C+3D)x + (18A+9B+6C+9D).$$

$A+B+C = 1$;

$8A+7B+6C+D = 3$

$21A+15B+11C+3D = 4$

$18A+9B+6C+9D = 6.$

By solving these we get.

$A = \frac{13}{6}$

$B = \frac{-26}{3}$

$C = \frac{11}{2}.$

$D = \frac{-2}{3}$

$$= \int \frac{13}{6(x+1)}\,dx + \int \frac{-26}{3(x+2)}\,dx + \int \frac{11}{2(x+3)}\,dx + \int \frac{-2}{3(x+3)^2}\,dx.$$

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$= \frac{13}{6}\ln|x+1| - \frac{20}{3}\ln|x+2| + \frac{11}{2}\ln|x+3| + \frac{2}{3(x+3)}$

ii) $\frac{4x^2+2x+18}{(x+1)(x^2+4x+13)}$

Sol. $\frac{4x^2+2x+18}{(x+1)(x^2+4x+13)} = \frac{A}{(x+1)} + \frac{Bx+C}{x^2+4x+13}$

$4x^2+2x+18 = A(x^2+4x+13) + (Bx+C)(x+1)$

$= (A+B)x^2 + (4A+B+C)x + (13A+C).$

$A+B=4$

$4A+B+C=2$

$13A+C=18$

By solving these we get

$A=2;\ B=2,\ C=-8.$

$= \frac{2}{x-1} + \frac{2x-8}{x^2+4x+13}$

iii) $\frac{4x^3+16x^2+23x+13}{(x+3)^3(x+2)}$

Sol. $\frac{4x^3+16x^2+23x+13}{(x+3)^3(x+2)} = \frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$

$4x^3+16x+23x+13 = A(x^3+3x^2+3x+1) +$

$B(x^3+4x^2+5x+2) + C(x^2+3x+2) + D(x+2).$

$= (A+B)x^3 + (3A+4B+3C)x^2 + (3A+5B+3C+D)x +$

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$(A+2B+2C+2D)$

$A+B=4$;

$3A+4B+3C=16$

$3A+5B+3C+D=23$

$A+2B+2C+2D=13$

By solving these we get.

$A=\frac{-3}{2}$, $B=\frac{23}{5}$, $C=\frac{-1}{5}$, $D=\frac{12}{5}$

$$=\frac{\frac{-3}{2}}{x+2}+\frac{\frac{23}{5}}{x+1}+\frac{\frac{-1}{5}}{(x+1)^2}+\frac{\frac{12}{5}}{(x+1)^3}$$

iv) $\dfrac{3x^2+9x-20}{x^2+x-6}$.

Sol $\cancel{\dfrac{3x^2+9x-20}{x^2+x-6}} = \cancel{\dfrac{A}{x+3}+\dfrac{B}{x-2}}$.

$\cancel{= Ax-2A+Bx+3B}$

$\cancel{=(A+B)x+3B-2A}$

$\cancel{A+B=9}$

$\cancel{3B-2A=20}$

$\cancel{A=2}$

Sol $\dfrac{3x^2+9x-20}{x^2+x-6} = \dfrac{Ax+b}{x+3} + \dfrac{c}{x-2}$.

$3x^2+9x-20 = (Ax+b)(x-2)+c(x+3)$.

put $x=2$ for c.

we get

$c=2$.

$3x^2+9x-20 = Ax^2 - Ax + bx - b + 2x + 6$.

$3x^2+9x-20 = Ax^2 + x(b-A+2) - (b-6)$.

$A=3$

$b-A+2=9$.

$b=16$.

$= \dfrac{3x+16}{x+3} + \dfrac{2}{x-1}$.

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2.

i) $\int x^2 \cos ax \, dx = \frac{1}{a^3}\left(2ax\cos ax - 2\sin ax + a^2x^2\sin ax\right)$

$f(x) = x^2$

$g(x) = \cos ax$

$f(x)$	$g(x)$
x^2	$\cos ax$
$2x$	$\frac{\sin ax}{a}$
2	$-\frac{\cos ax}{a^2}$
0	$\frac{\sin ax}{a^3}$

$= \frac{x^2 \sin ax}{a} + \frac{2x}{a^2}\cos ax - \frac{2\sin ax}{a^3}$

$= \frac{1}{a^3}\left(2ax\cos ax - 2\sin ax + a^2x^2\sin ax\right)$

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ii) $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$.

$\int u\,dv = uv + \int v\,du$.

$u = \cos bx$, $dv = e^{ax}$

$du = -b\sin bx\,dx$ $v = \frac{e^{ax}}{a}$.

$= \cos bx\,\frac{e^{ax}}{a} - \int \frac{e^{ax}}{a} - b\sin bx\,dx$.

$= \cos bx\,\frac{e^{ax}}{a} + \frac{b}{a}\int e^{ax}\sin bx\,dx$.

again $u = \sin bx$, $dv = e^{ax}$

$du = b\cos bx\,dx$, $v = \frac{e^{ax}}{a}$.

$= \cos bx\,\frac{e^{ax}}{a} + \frac{b}{a}\left[\sin bx\,\frac{e^{ax}}{a} - \int \frac{e^{ax}}{a}\,b\cos bx\,dx\right]$

$= \cos bx\,\frac{e^{ax}}{a} + \frac{b}{a}\sin bx\,\frac{e^{ax}}{a} - \frac{b^2}{a^2}\int e^{ax}\cos bx\,dx$.

$\int e^{ax}\cos bx\,dx + \frac{b^2}{a^2}\int e^{ax}\cos bx\,dx = \cos bx\,\frac{e^{ax}}{a} + \frac{b}{a^2}\sin bx\,e^{ax}$.

$\int e^{ax}\cos bx\,dx = \cos bx\,\frac{e^{ax}}{a} + \frac{b}{a^2}\sin bx\,e^{ax}$.

$= \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$.

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iii) $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$.

$= \int \frac{1}{a^2\left(\frac{x^2}{a^2}+1\right)} dx$.

$= \frac{1}{a^2} \int \frac{1}{\left(\left(\frac{x}{a}\right)^2+1\right)} dx$. let $t = \frac{x}{a}$

$dt = \frac{dx}{a}$.

$= \frac{1}{a^{\cancel{2}}} \int \frac{\cancel{a}}{t^2+1} dt$

$= \frac{1}{a} \int \frac{1}{t^2+1} dt$

$= \frac{1}{a} \tan^{-1} t$ $\because t = \frac{x}{a}$.

$= \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$

4\. ii) $y'' - 3y' + 2y = 0$.

sol. $\left(\frac{dy}{dx}\right)^2 - 3\left(\frac{dy}{dx}\right) + 2y = 0$.

$m(D)y = (D^2 - 3D + 2)y$.

Auxilary equation of the above equation is

$m^2 - 3m + 2 = 0$.

$(m-1)(m-2) = 0$.

$m = 1, 2$.

$y(x) = ae^{x} + be^{-2x}$.

i) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 - 3x + 2y = 8$.

sol. $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 - 3x + 2y = 8$.

Differentiate with x

$\frac{d^3y}{dx^3} + 3\left(\frac{dy}{dx}\right)^2 - 3 + 2\frac{dy}{dx} = 0$.

Differentiate with x.

$\frac{d^4y}{dx^4} + 6\left(\frac{dy}{dx}\right) + 2\frac{d^2y}{dx^2} = 0$. let $\frac{d}{dx} = D$;

$D^4y + 6Dy + D^2y = 0$.

$y(D^4 + 6D + D^2) = 0 \Rightarrow D(D^3 + 2D + 6) = 0$.

$D^3 + 2D + 6 = 0$.

$D_1 = -1.456$

$D_2 = 0.7280 + 1.8i$

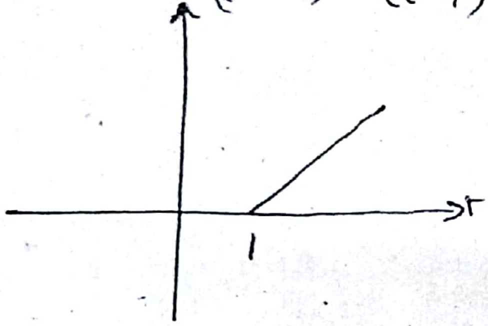
$D_3 = 0.72 - 1.8i$.

$\therefore y = A_1 e^{-1.456t} + e^{0.72t}(A_2 \cos(1.8)t + A_3 \sin(1.8)t)$.

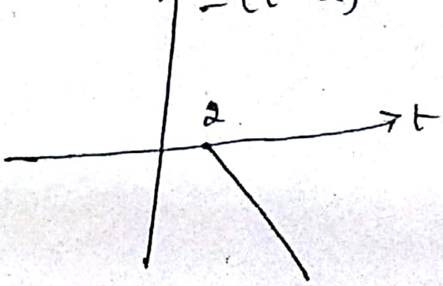
5. Plot $f(t)$ & $f(-3t+4)$ where $f(t)$ is.

$$f(t) = (t-1)u(t-1) - (t-2)u(t-2) - u(t-4).$$

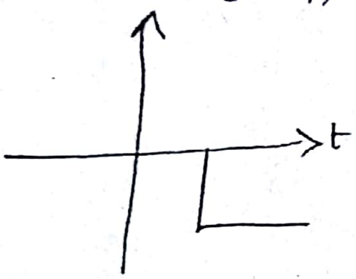
$(t-1)u(t-1)$



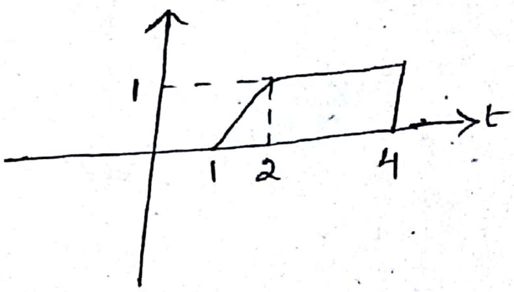
$-(t-2)u(t-2)$



$-u(t-4)$



$f(t)$



$f(-3t-4)$