Optics

Coherent Sources: The two light sources that generate light waves of (i) same amplitude (ii) same wave length (iii) constant phase difference (or) path difference are known as coherent sources. Some times it is possible to get the interference phenomenan even if the first two conditions are not satis-fied, but the third condition must be ful filled.

The interference is a result of superposition of two light waves generated from the two coherent sources.

Interference: The variation of Intensity in the region of superposition of two (or) more waves of same frepuency whose phase relationship does not change with time is known as Interference of light. Inter-ference of waves occurs according to the principle of superposition.

Superposition Principle : It states that the resultant displacement at a point in a medium is the algebraic sum of the displacements due to individual waves passing through that point simultaneously.

Consider two light waves (S.H. waves) having same amplitude that have got a phase difference of 'δ' i.e $y_1 = a \sin wt$ and $y_2 = a \sin(wt+\delta)$

When these two waves superpose, a new wave is formed. It is represented by $y = A\sin(wt+\phi)$

$$\therefore \quad y = y_1 + y_2 = A\sin(wt+\phi)$$

$$\therefore \quad A\sin(wt+\phi) = a\sin wt + a\sin(wt+\delta)$$

$$A\left[\sin wt.\cos\phi + \cos wt.\sin\phi\right] = a\sin wt + a\sin wt.\cos\delta + a\cos wt.\sin\delta$$

Comparing the coefficients of sin wt and cos wt on both sides, we get

$$A\cos\phi = a(1+\cos\delta) \quad \text{--- (1)}$$

$$A\sin\phi = a\sin\delta \quad \text{--- (2)}$$

Squaring and adding (1) and (2), we get

$$A^2[\cos^2\phi + \sin^2\phi] = a^2(1+\cos\delta)^2 + a^2\sin^2\delta$$

$$= a^2[1+\cos^2\delta + 2\cos\delta + \sin^2\delta]$$

$$= a^2[2+2\cos\delta]$$

$$\therefore \quad A^2 = 2a^2(1+\cos\delta)$$

$$\therefore \quad A^2 = 2a^2 \times 2\cos^2\frac{\delta}{2}$$

$$A^2 = 4a^2\cos^2\frac{\delta}{2}$$

$$\therefore \quad I = I_0\cos^2\frac{\delta}{2}, \quad \text{wher } I_0 = 4a^2$$

$$(\because I \propto A^2)$$

if $\delta = 0, 2\pi, 4\pi, \ldots$ we get maximum intensity

if $\delta = \pi, 3\pi, 5\pi, \ldots$ we get minimum intensity

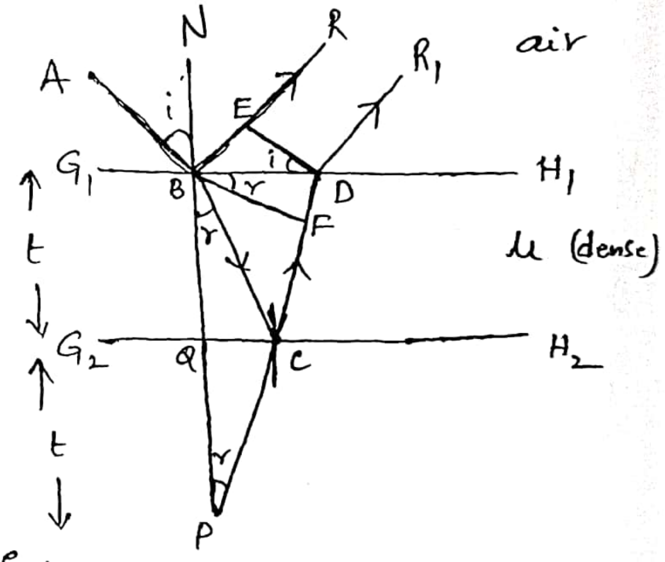
where δ = phase difference between two light waves

When the path difference between the two beam is even multiples of $\lambda/2$ (i.e. $2n\cdot\lambda/2$) (or) even multiples of 'π' phase difference ie $2n\cdot\pi$, where $n = 0, 1, 2, 3 \ldots$ then the superposition of such two light waves gives maximum intensity (or) it is also known as the condition for getting maximum intensity.

Similarly when the path difference is equal to odd multiples of $\lambda/2$ i.e $(2n \pm 1)\lambda/2$ (or) when the phase difference is equal to odd multiples of 'π' i.e $(2n \pm 1)\pi$, then the super position of such two light waves gives minimum intensity. i.e we get darkness at the point of superposition.

Interference in thin films due to reflected light
(COSINE LAW)

Let G_1H_1 and G_2H_2 be the two surfaces of a transparent film of uniform thickness 't' and refractive index 'μ' as shown.



Suppose a ray 'AB' of monochromatic light be incident on its upper surface.

This ray is partly reflected along BR and refracted along BC. After one internal reflection at 'c', we obtain the ray CD. After refraction at D, the ray finally emerges out along DR_1 in air. obviously DR_1 is parallel to BR our aim is to find out the effective path difference between the rays BR and DR_1. For this purpose we draw a normal DE on BR and normal BF on DC. We also produce DC in the backward direction which meets BQ at P. In this figure,

$\angle ABN = i$, $\angle QBC = r$

From the geometry of the figure, $\angle BDE = i$, $\angle QPC = r$

The optical path difference between the rays BR & DR_1 is given by Δ = path in film (BC+CD) − path in air (BE)

$$\Delta = \mu(BC+CD) - 1 \times BE$$

We know that $\mu = \dfrac{\sin i}{\sin r} = \dfrac{BE/BD}{FD/BD} = \dfrac{BE}{FD}$

$$\Rightarrow BE = \mu FD.$$

$$\therefore \quad \Delta = \mu(BC + CD - FD)$$

$$\Delta = \mu(BC + CF + \cancel{FD} - \cancel{FD}) = \mu(BC + CF)$$

$$\therefore \quad \Delta = \mu(PC + CF) = \mu PF$$

From $\triangle$le BPF, $\cos r = PF/BP \Rightarrow PF = BP\cos r$
$= 2t\cos r$

$$\therefore \quad PF = 2t\cos r$$

Then $\boxed{\Delta = 2\mu t \cos r}$

The optical path difference is usually called as cosine law.

It should be remembered that a ray reflected at a surface backed by denser medium suffers an abrupt phase change of π (or) a path difference of $\lambda/2$.

Thus the path difference becomes $2\mu t \cos r \pm \lambda/2$

If $\Delta = 2\mu t \cos r \pm \lambda/2$ is equal to $n\lambda$, then maximum occurs. i.e. $2\mu t \cos r \pm \lambda/2 = n\lambda$

$$2\mu t \cos r = (2n \pm 1)\lambda/2$$

If this condition is fulfilled, the film will appear bright in the reflected light.

If $\Delta = 2\mu t \cos r \pm \lambda/2$ is equal to $(2n \pm 1)\lambda/2$, then minima occurs i.e. $2\mu t \cos r \pm \lambda/2 = (2n \pm 1)\lambda/2$

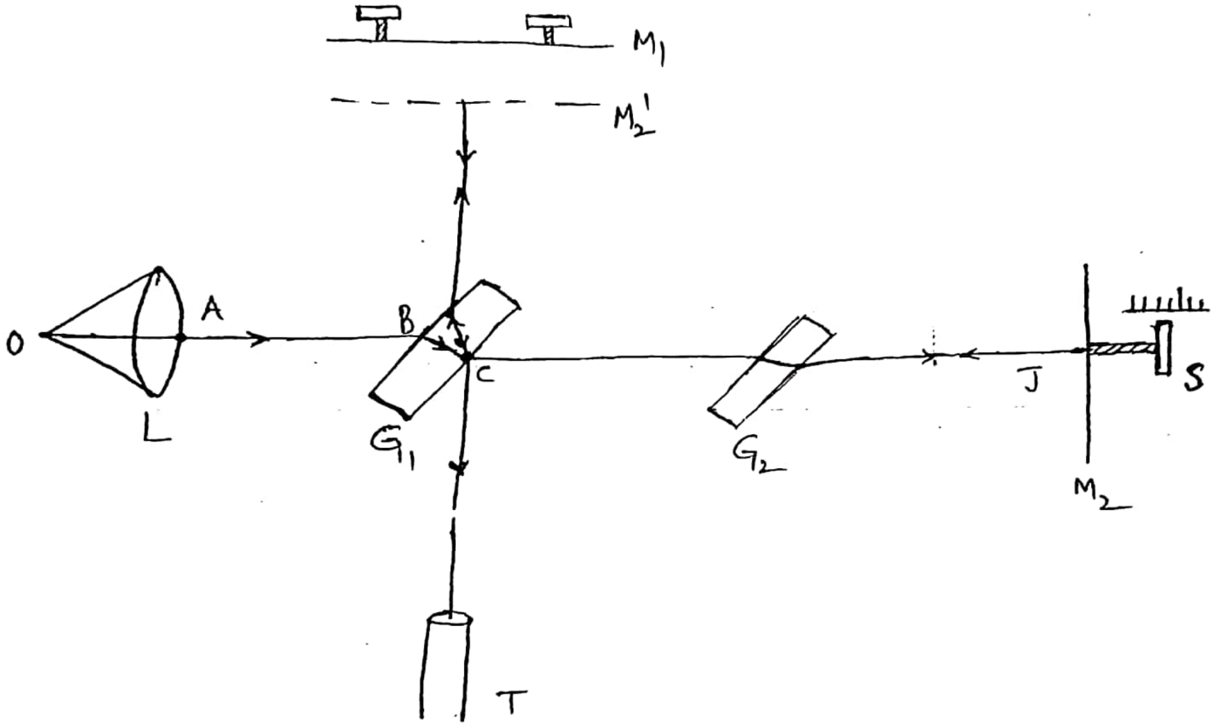
$$2\mu t \cos r = n\lambda$$

If this condition is fulfilled, the film will appear dark in the reflected light.

Ex: When white light is reflected by the films like soap bubbles, oil layers on water a variety of colours could be seen. This is due to interference between light waves reflected by the front and back surfaces of these films.

Michelson Interferometer:

It consists of two optically plane mirrors M_1 and M_2 which are at right angles to each other.



There are two optically flat glass plates G_1 and G_2, which are placed parallel to each other. The two plates are inclined at an angle of 45° with the horizontal. The face of the glass G_1 towards G_2 is semi silvered so that it is capable of reflecting as well as refracting.

The two mirrors M_1 and M_2 are placed at an equal distance from 'G_1'.

First of all the light rays from a source are rendered parallel with the help of a Convex lens. The parallel light rays are incident on a glass plate G_1. Then a part of light beam is reflected from the bottom of the plate G_1 and moves towards M_1. Another part of light beam is refracted and moves towards M_2 through 'G_2'

<u>Beam – I</u> :- It is the reflected light beam from G_1. It moves towards M_1 and is reflected at M_1 and comes back to G_1 and enters into a Telescope T. In this process this light beam travells thrice through 'G_1' before it reaches the telescope.

<u>Beam – 2</u> :- It is the refracted light beam from G_1. It moves towards M_2 through G_2 and is reflected at M_2 and comes back to 'C' through G_2 and reaches the telescope.

This beam travells once through G_1 and twice through G_2. The two light beams have travelled equal distance in air and glass plates. Then these two light beams act as coherent beams. The virtual image of M_2 in G_1 is formed at M_2'. The two interfering beams come by reflection from M_1 and the other which is reflected from M_2 functions as if it had been reflected from M_2'. i.e the two interfering light beams may be taken as reflected rays from M_1 & M_2'. Then an air film is formed between M_1 & M_2' and interference occurs through this film, which can viewed through telescope.

Depending on the thickness of the air film and also the inclination between them, the shape of the frinzes are determined.

When M_1 and M_2' are <u>parallel</u> to each other, the thickness of air film is constant and circular frinzes are formed.

When M_1 and M_2' are inclined to each other the air film enclses wedge shaped. Then curved fringes are formed. When M_1 and M_2' intersect at Middle parallel straight fringes are formed.

The cross wires of the telescope is made to coinicide with one of the fringe. By bringing M_2 towards G_2 slightly, the fringes cross over the cross wire of the telescope. Number of fringes that have crossed can be counted. Let it be 'n'.

The distance of the mirror moved towards G_2 can be measured with the help of a scale attached to the micrometer screw's.

Then $d = \frac{n\lambda}{2} \Rightarrow \boxed{\lambda = \frac{2d}{n}}$

With this equation we can determine the wave-length of light used very accurately.

Determination of difference in wave lengths:

Let the source has two wavelengths, λ_1 and λ_2 $(\lambda_1 > \lambda_2)$ which are very close to each other.

The two wave lengths form their seperate fringe patterns. But as λ_1 and λ_2 are very close to each other and thickness of air film is small, the two patterns coincide practically. If the mirror M_1 is moved slowly, the two patterns seperate slowly and when the thickness of air film is such that the dark fringe of λ_1 falls on the bright fringe of λ_2, the result is maximum indistinctness. Now, the mirror M_1 is further moved say through a distance 'd' so that the next indistinct position is reached. In this position, if n fringes of λ_1 cross over the cross-wires of the telescope, then $(n+1)$ fringes of λ_2 should cross over the cross wires of telescope T.

Hence $d = \frac{n\lambda_1}{2}$ and $d = \frac{(n+1)\lambda_2}{2}$

then $n = \frac{2d}{\lambda_1}$ and $n+1 = \frac{2d}{\lambda_2}$

Consider $(n+1) - n = \frac{2d}{\lambda_2} - \frac{2d}{\lambda_1} = 2d\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2}\right)$

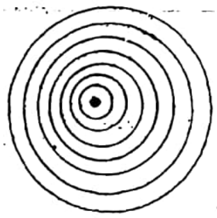
$\Rightarrow 1 = 2d\left(\frac{\lambda_1 - \lambda_2}{\lambda^2}\right)$ $(\because \lambda_1 \approx \lambda_2)$

$$\therefore \boxed{\lambda_1 - \lambda_2 = \frac{\lambda^2}{2d}}$$

NEWTON'S RINGS

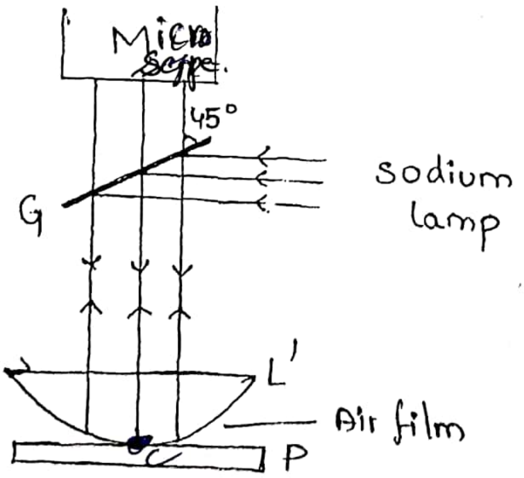
→ When a plano convex lens with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between the lower surface of the lens and the upper surface of the glass plate. The thickness of the air film at the point of contact is 0.

→ If monochromatic light is allowed to fall normally and the film is viewed in reflected light, alternate bright and dark concentric circular rings are seen. ~~these circular rings are seen.~~ These circular rings were first observed by Newton and hence are called Newton's rings.



Experimental Arrangement –

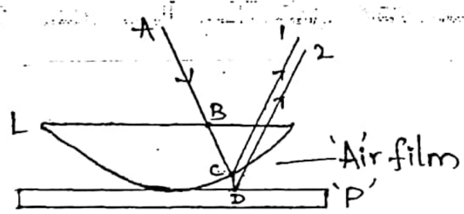
the experimental arrangement for Newton's rings is as shown.



L is a plano convex lens of larger radius of curvature placed on a plane glass plate 'p'. The lens touches the glass plate at c. The light from monochromatic source such as sodium lamp falls on a glass plate 'G' which is inclined at an angle of 45°.

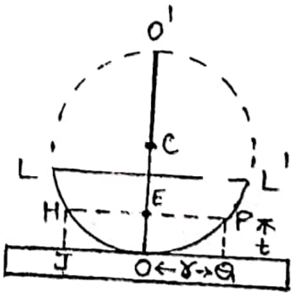
→ the glass plate 'G' reflects normally a part of the incident light towards the air film enclosed by the lens L and the glass plate 'P'. A part of the light incident on the lens 'L' is reflected by the curved surface of the lens 'L' and the other part transmitted is reflected back by the upper surface of the glass plate 'P'.

→ these two reflected rays interfere and give rise to interference pattern in the form of circular rings. these rings can be viewed through a travelling microscope.



Newton's Rings By Reflected Light –

Let 'R' be the radius of curvature of the lens 'L' and let a Newton ring (either dark or bright) be located at the point 'Q'.



The thickness of the air film at Q is PQ = t. the radius of newton's ring at Q is 'OQ' say 'r' (i.e OQ = r)

From the property of the circles,

$$EP \times EH = OE \times O'E$$

$$r \times r = t \times O'E$$

$$r^2 = t \times (OO' - OE)$$

$$r^2 = t \times (2R - t)$$

$$r^2 = 2Rt - t^2$$

$$r^2 = 2Rt \quad [\because t^2 \text{ can be neglected as t is small Compared to R.}]$$

Hence $$t = \frac{r^2}{2R} \quad \text{--- (1)}$$

For bright ring -

we have for the bright ring, $2t + \lambda/2 = n\lambda$ (or)

$$2t = (2n-1)\lambda/2 \quad \text{--- (2)}$$

Substitute (1) in (2), we get

$$2 \times \frac{r^2}{2R} = (2n-1)\lambda/2$$

$$\therefore r^2 = \frac{(2n-1)\lambda R}{2} \Rightarrow \boxed{r = \sqrt{\frac{(2n-1)\lambda R}{2}}}$$

For Dark Ring -

we have $$2\mu t \cos r + \lambda/2 = (2n+1)\lambda/2$$

$$\therefore 2t + \lambda/2 = 2n\lambda/2 + \lambda/2$$

$$2t = 2n\lambda/2 + \lambda/2 - \lambda/2$$

$$\therefore 2t = n\lambda \quad \text{--- (3)}$$

substitute (1) in (3), we get

$$2 \times \frac{r^2}{2R} = n\lambda \rightarrow r^2 = n\lambda R$$

$$\boxed{r = \sqrt{Rn\lambda}}$$

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Determination Of Wavelength Of Monochromatic Light –

Let 'R' be the radius of curvature of the curved surface in contact with the glass plate.

We know that the radius of n^{th} dark ring is given by $r_n = \sqrt{nR\lambda}$ $\Rightarrow D_n = 2\sqrt{nR\lambda}$

i.e $D_n = \sqrt{4nR\lambda}$ $\Rightarrow$ $\boxed{D_n^2 = 4nR\lambda}$ — (4)

Similiarly, the diameter of $(n+m)^{th}$ dark ring is 'D_{n+m}'

$$\therefore \quad \boxed{D_{(n+m)}^2 = 4(n+m)R\lambda} \text{ — (5)}$$

$$D_{n+m}^2 - D_n^2 = 4(n+m)R\lambda - 4nR\lambda$$

$$= 4R\lambda[\cancel{n}+m-\cancel{n}]$$

$$= 4Rm\lambda$$

$$\therefore \quad \boxed{\lambda = \frac{D_{n+m}^2 - D_n^2}{4Rm}}$$

First of all the centre of the cross wires of the Travelling Microscope is made to coincide with any one of the dark fringe on left side, say n^{th} ring i.e 10^{th} ring the reading of the micrometer screw (attatched with eye piece) is noted. Now, the Travelling Microscope is moved to the right side and the readings of micrometer are noted successively at $(n-2)^{th}$, say 8^{th} ring, $(n-4)^{th}$, say 6^{th} ring etc till we are very near to the central dark spot. Again move the Travelling Microscope to right side in the same direction and coincide the centre

Determination Of Radius Of Curvature Of the Lens -

Similiarly, the radius of curvature can be measured using the formula

$$R = \frac{D_{n+m}^2 - D_n^2}{4\lambda m} \quad \text{--- (1)}$$

Here,

λ = wavelength of monochromatic light is a standard value i.e $5893\,A^\circ$ (or) 5893×10^{-8} cms.

the value of $\frac{D_{n+m}^2 - D_n^2}{m}$ is calculated from the graph

Substituting these values in equation (1), we can find the radius of the plano convex lens.

NOTE -

the Central Spot in "Newton's rings" is dark due to the following reason -

- we know that the path difference between the 2 light rays is $\boxed{2\mu t \cos r + \lambda/2}$
- At the point of contact, the value $t = 0$, for air film the refractive index $\mu = 1$ and for normal incidence $r = 0$
- By Substituting these values in the above equation, the path difference $= 2(1)(0)\cos 0^\circ + \lambda/2 = \lambda/2$, Hence which is the condition of minimum intensity.

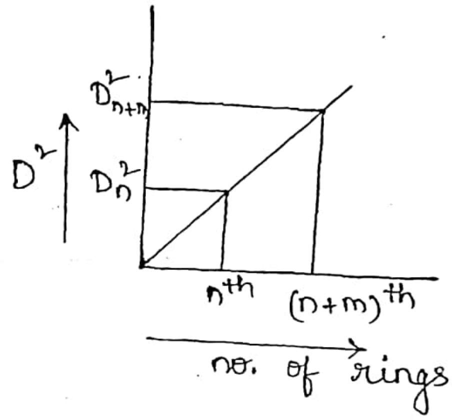
Thus, the Central Spot is dark.

of the cross wire of the T. Microscope with the dark rings successively.

crossing the central dark spot in the same direction the corresponding readings are noted on the other side for the successive rings say 2^{nd} ring, 4^{th} ring, 6^{th} ring, 8^{th} ring etc.

A graph is drawn between the no. of rings on x-axis and the square of the corresponding diameter on y-axis.

the graph is as shown.



From graph -

the value $\dfrac{D^2_{n+m} - D^2_n}{m}$ can be calculated.

The radius of curvature of plano convex lens can be obtained with the help of spherometer, using the formula

$$R = \frac{l^2}{6h} + \frac{h}{2} \quad \text{—} ①$$

Here l → distance between the two legs of spherometer

h → difference of the readings of the spherometer when it is placed on the lens as well as when placed on the plane surface.

Substituting these values in ①, R can be calculated.

→ thus, By knowing the values of R and $\dfrac{D^2_{n+m} - D^2_n}{m}$, we can calculate the wavelength (λ) of the monochromatic light.

Diffraction of Light

when a light falls on obstacles (or) apertures whose size is comparable with the wavelength of light, there is a departure from straight line propagation, the light bends around the corners (or) edges of the obstacles and enters into the geometrical shadow. out side the shadow several bright and comparitively dark bands are observed. These bright and dark bands are known as Diffraction frinzes. The bending of light beam at the edge of an obstacle is called Diffraction.

If the wavelength of light is very small, such bending is not pronaunced and hence light appears to travel in straight line.

for diffraction to be more effective, the size of the obstacle must be comparable with wave length. Since the wavelength of light is (in the visible range) in the range of 4000Å ~~-~~ 6500Å

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the size of the obstacle must be comparable with this to cause Diffraction.

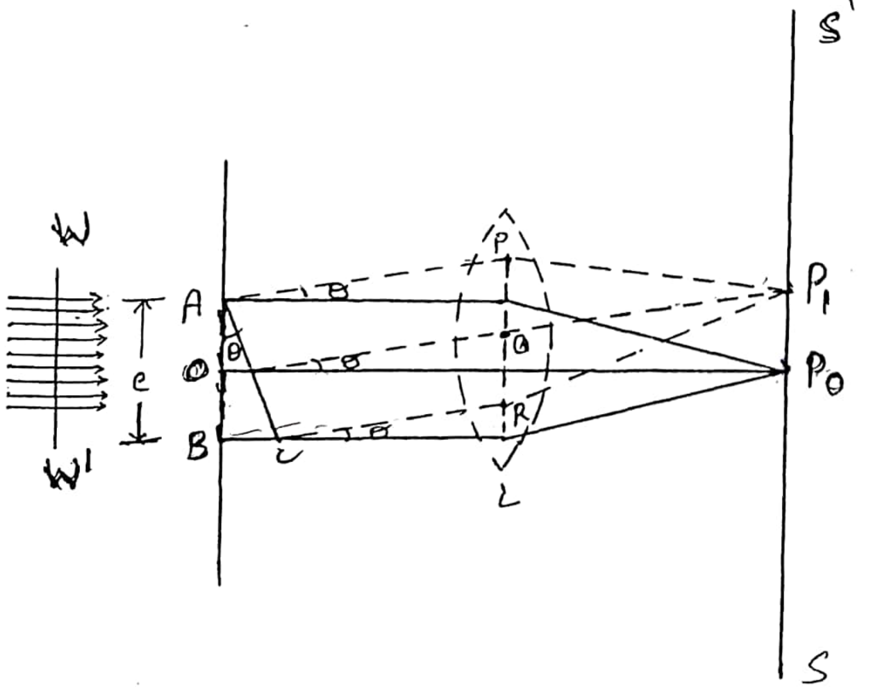
There are two types of diffraction, as shown

1. <u>Fresnel's Diffraction</u>:- In this diffraction, the source & the screen are at finite distances from the obstacle causing diffraction. No lenses are required to observe this type of diffraction on the screen. The centre of the diffraction zone (or) pattern may be bright (or) dark.

2. <u>Fraunhofer diffraction</u>:- In this diffraction the source and the screen are at infinite distances from the obstacle causing diffraction. The incoming light from a source is rendered parallel with a lens and diffracted beam is focussed on the screen with another lens. The centre of diffraction pattern is always bright.

Fraunhofer diffraction due to a single slit:

Consider a slit AB of width 'e'. The plane of the slit is perpendicular to the plane of the paper. A plane wave front WW' of mono-chromatic light of wave length 'λ' is incident on the slit AB. Every point on the wave front in the slit will act as a source of second-ary waves. The secondary waves travelling in the direction of OP_0 are brought to focus at P_0, on the screen. The secondary waves from AB which are brought to focus at 'P_0' have no path difference. Hence the intensity at P_0 is high and it is known as central maximum.



The secondary waves in the slit AB which make an angle 'θ' with OP_0 direction are brought to focus at P_1 on the screen. The intensity at 'P_1' depends on path difference between the waves at A and B reaching to the point P_1. To find the path difference, a perpendicular AC is drawn to BR from A. Now the path difference between the secondary waves from A and B in the direction of OP_1 is BC.

$\therefore$ $BC = AB \sin\theta = e \sin\theta$.

The corresponding phase difference is '$\frac{2\pi}{\lambda} e \sin\theta$'.

Let us consider the width of the slit is divided into n-equal parts. Then the phase difference between two successive parts is $\frac{1}{n} \times \frac{2\pi}{\lambda} e \sin\theta$

Let $\frac{2\pi}{n\lambda} e \sin\theta = d$ and let the amplitude of the wave in each part be '$\underline{a}$'.

The Resultant amplitude 'R' using Vector-addition method is $R = a \dfrac{\sin nd/2}{\sin d/2}$

i.e $R = a \dfrac{\sin n\left(\frac{2\pi}{dn}\frac{e\sin\theta}{2}\right)}{\sin\left(\frac{2\pi}{dn}\frac{e\sin\theta}{2}\right)} = a \dfrac{\sin\left(\frac{\pi}{d} e\sin\theta\right)}{\sin\left(\frac{\pi e\sin\theta}{nd}\right)}$

Let $\alpha = \dfrac{\pi e\sin\theta}{d}$, then $R = a\dfrac{\sin\alpha}{\sin\alpha/n}$.

$\therefore R = a\dfrac{\sin\alpha}{\sin\frac{\alpha}{n}} \approx a\dfrac{\sin\alpha}{\frac{\alpha}{n}}$ ($\because \frac{\alpha}{n}$ is very small)

$\therefore R = na\dfrac{\sin\alpha}{\alpha} \Rightarrow R = A\dfrac{\sin\alpha}{\alpha}$ where $na = A$

$\therefore \boxed{R = A\dfrac{\sin\alpha}{\alpha}}$ — (1)

The intensity of light $I = R^2 = A^2\left[\dfrac{\sin\alpha}{\alpha}\right]^2$ — (2)

Principal Maximum:

When $\alpha \to 0$, the value of $\frac{\sin\alpha}{\alpha}$ in equation becomes 1. i.e $\dfrac{\sin\alpha}{\alpha} = 1$

Hence $\dfrac{\pi e\sin\theta}{d} = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$

i.e Intensity is maximum at P_0 corresponding to $\theta = 0$.

Hence the secondary waves travelling normal to the slit can produce maximum intensity called Principal maximum.

Minimum intensity positions :-

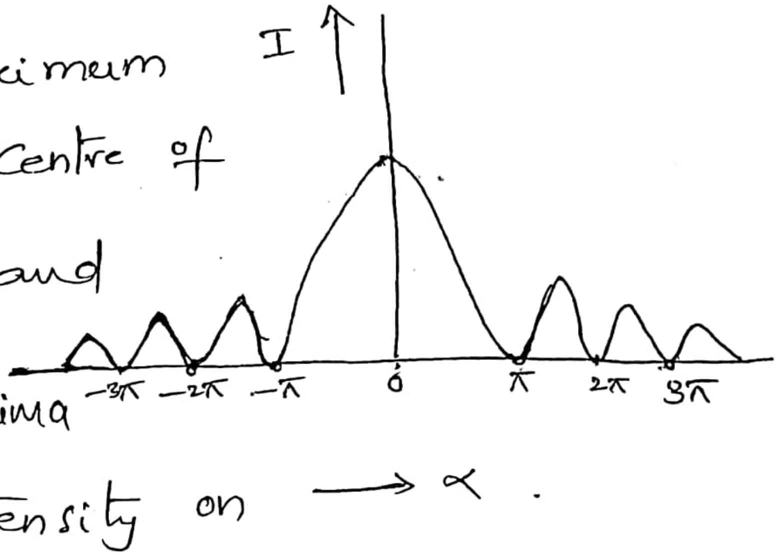
The intensity will be minimum when 'sin α' in equation ② is zero.

i.e $\sin\alpha = 0 \Rightarrow \alpha = \pm m\pi$, for $m = 1, 2, 3, \ldots$

ie $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

∴ We know that minimum intensity positions are found on both sides of principal maximum.

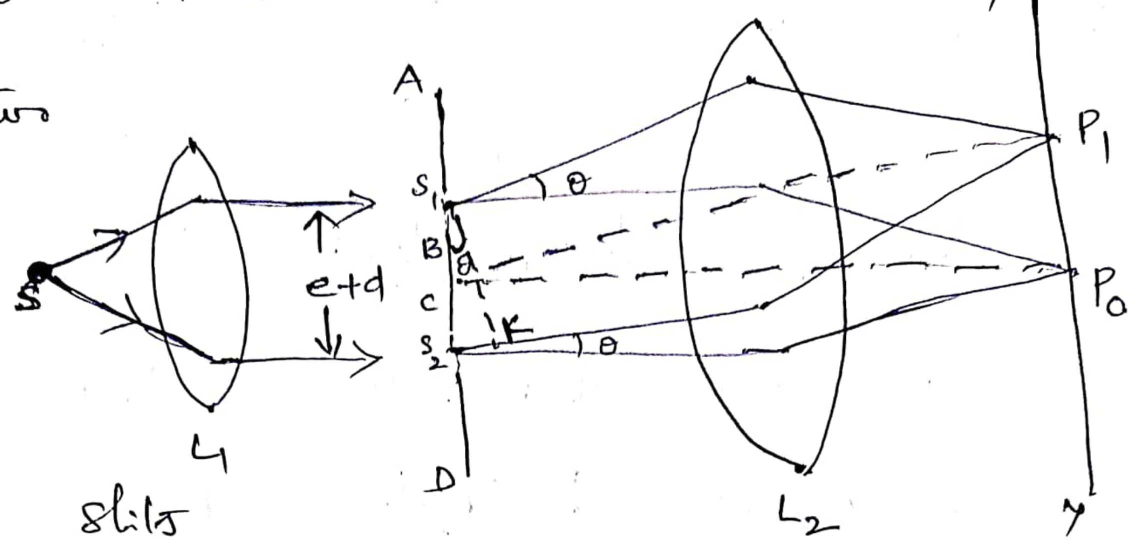
A graph is drawn between Intensity of light versus 'α'. The diffraction pattern is as follow



The principal maximum occurs at the centre of diffraction pattern and the subsidiary maxima of decreasing intensity on both sides of principal maximum.

Fraunhofer diffraction due to double slit:

Consider two slits S_1, S_2 of equal width 'e'.



These two slits are seperated by a distance of 'd'. The distance between the corresponding middle points of two slits is 'e+d'. Let a monochromatic parallel beam of light of wave length 'λ' be incident normally on the two slits. The diffracted light from these two slits S_1, S_2 is focussed on a screen XY. The diffraction at two slits is the combination of diffraction as well as interference

When a plane wave-front incident on both slits normally, every point within the slits become the sources of secondary waves which travels in all possible directions. The secon-dary waves travelling along the direction of

-incident light are brought to focus at P_0 while the secondary waves travelling in a direction making an angle of 'θ' with the direction of incident light come to focus P_1.

We know that, the resultant amplitude of all waves diffracted at each slit is given by $R = \frac{A \sin\alpha}{\alpha}$, $A \rightarrow$ constant

$\alpha \rightarrow \frac{\pi e \sin\theta}{\lambda}$.

For simplicity consider the two slits as equivalent to two coherent sources S_1, S_2. Let each source be sending a wavelet of amplitude $(A \sin\alpha/\alpha)$. ∴, the resultant amplitude at a point 'P_1' on the screen will be a result of interference between two waves of amplitude $A \sin\alpha/\alpha$ with a phase difference of 'δ' (say). To find 'δ', draw a perpendicular S_1K on S_2K.

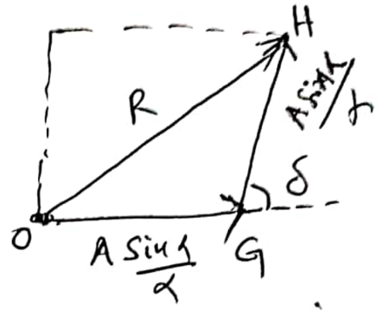
The path difference between the waves from S_1 and S_2 = S_2K.

$= (e+d) \sin\theta$

phase difference $\boxed{(\delta) = \frac{2\pi}{\lambda}(e+d)\sin\theta}$

The Resultant amplitude 'R' at 'P' can be obtained using parallelogram law of Vectors.
[By using the formula, $R^2 = a^2 + b^2 + 2ab\cos\theta$]

Now, $R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2 + 2\,\frac{A\sin\alpha}{\alpha}\cdot\frac{A\sin\alpha}{\alpha}\cdot\cos\delta$



$\therefore R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A\sin\alpha}{\alpha}\right)^2\cos\delta,$

where $\delta \rightarrow$ phase diff. between them.

$\therefore R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2\left[1+1+2\cos\delta\right]$

$\therefore I = R^2 = 2\left(\frac{A\sin\alpha}{\alpha}\right)^2\left[1+\cos\delta\right]$

$\therefore$ Intensity at P is $I = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\delta/2$

but $\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta$ $\left(\because 2\cos^2\frac{\delta}{2} = 1+\cos\delta\right)$

Hence, $I = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cdot\cos^2\frac{\frac{2\pi}{\lambda}(e+d)\sin\theta}{2}$

$\therefore I = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cdot\cos^2\frac{\pi}{\lambda}(e+d)\sin\theta$

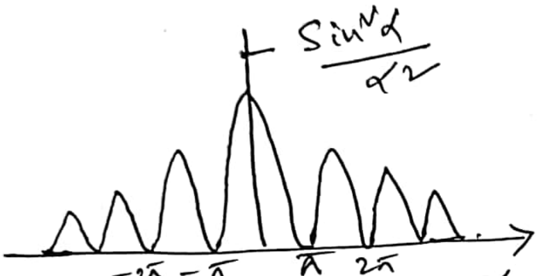
let $\frac{\pi}{\lambda}(e+d)\sin\theta = \beta$.

$$\therefore \boxed{I = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cdot\cos^2\beta.}$$

$\therefore$ The Resultant intensity $I = 4A^2 \frac{\sin^2\alpha}{\alpha^2} \cdot \cos^2\beta$

Here $A^2 \frac{\sin^2\alpha}{\alpha^2}$ is the diffraction due to individual slit and $\cos^2\beta$ is due to the interference of secondary waves on the double slits.

$\therefore$ The diagrams are as shown.



(a) diffracti pattern due to each slit (single slit)

Interference pattern due to the superposition of waves from two slits.

<u>Conditions for maximum intensity & minimum intensity</u>

For the intensity I to be maximum, $\cos^2\beta = 1$

$\because \cos^2\beta = 1 \Rightarrow \beta = \pm m\pi$,

Hence, $\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi \Rightarrow \boxed{(e+d)\sin\theta = \pm m\lambda}$

This is the condition maximum intensity.

If $m=0$, hence $\sin\theta = 0 \Rightarrow \boxed{\theta = 0}$

ie when $\theta = 0$ between the secondary wavelets we will get principal maximum. This is also called central (or) zero order maximum.

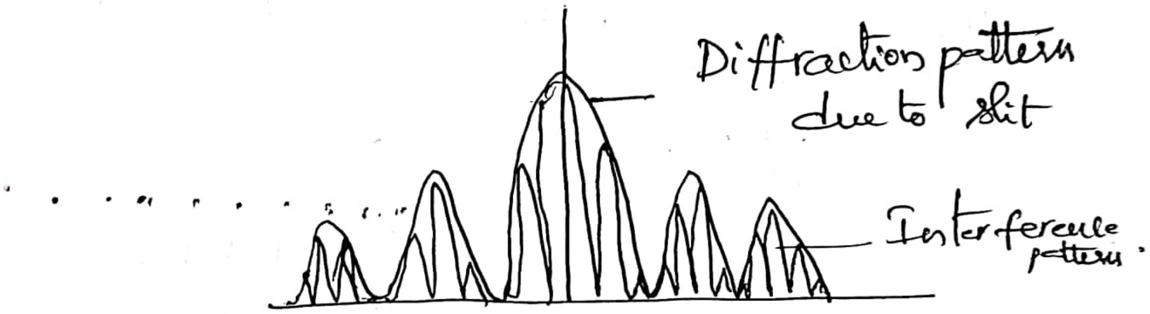
For minimum intensities, $\cos^2\beta = 0$

then $\beta = \pm (2m \pm 1)\pi/2$

$\therefore \quad \frac{\pi (e+d)}{\lambda} \sin\theta = (2m \pm 1)\pi/2$

$\therefore \quad \boxed{(e+d)\sin\theta = (2m \pm 1)\lambda/2}$

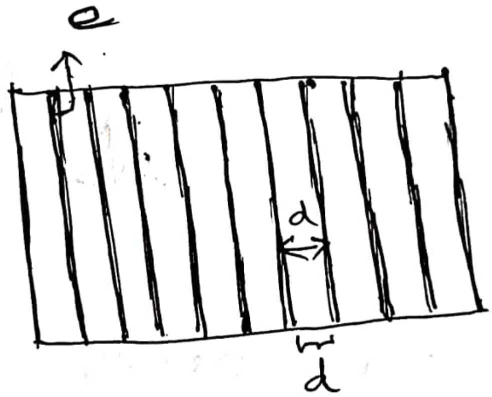
$\therefore$ The final intensity profile due to double slits.



Diffraction Grating:

An arrangement which consists of a large number of parallel slits of same width and seperated by equal opaque spaces is known as Diffraction Grating.

According to Fraunhofer, a grating consist of large no. of parallel wires placed very close side by side at regular intervals. Now, gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass with a fine diamond point. The ruled lines are opaque to light and the space between them is transparent to light.



Let 'e' be the width of opaque region (line) and 'd' be the width of slit (transparent). Here 'e+d' is called Grating element.

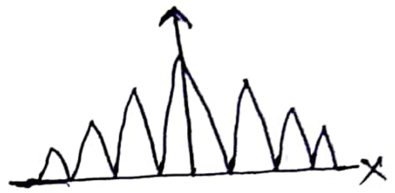
If 'N' be the no. of lines/inch on grating

Then $N(e+d) = 1 \text{ inch} = 2.54$

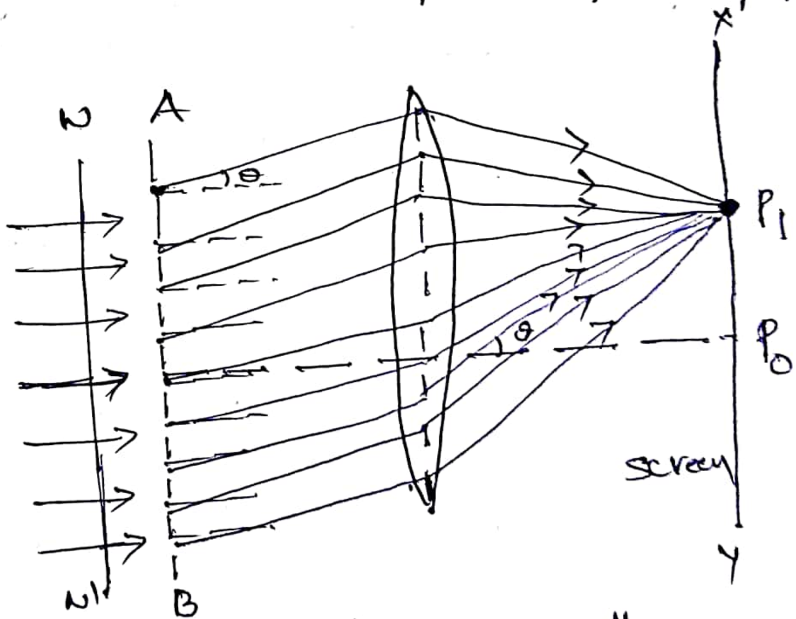
$$\therefore \boxed{(e+d) = \frac{2.54}{N}}$$

Scanned with CamScanner

When light falls on grating, the light gets diffracted through each slit and forms a diffraction pattern, known as diffraction spectrum. The diffraction pattern consists of principal maxima flanked by 1st order maxima, second order maxima etc followed by minimas.



The following figure shows a plane transmi--ssion grating placed ⊥ar to the plane of the paper



Let 'e' be the width of each slit and 'd' be the width of each opaque region. Suppose a parallel beam of monochromatic light of wave length 'λ' be incident normally on the grating. By Huygen's principle each slit (source) sends secondary waves in all directions. The secondary waves travelling in the direction of incident light will focussed at P_0 on the screen. The point 'P_0' will be a central maximum. The secondary waves travelling in a direction inclined an angle 'θ' with the direction of incident light will reach at P_1 in different phases. As a result bright and dark bands on both sides of central maxima are formed.

The intensity at 'P_1' can be obtained using the theory of Fraunhofer diffraction due to single slit. The waves coming from all points in a slit along the direction 'θ' are equivalent to a single wave of amplitude

$A \sin\alpha / \alpha$, where $\alpha = \pi e \sin\theta / \lambda$

If there are 'N' slits, then we have N-diffracted waves. The path difference between two consecutive slits is $(e+d)\sin\theta$

$\therefore$ The corresponding phase difference is

$\frac{2\pi}{\lambda} \times (e+d)\sin\theta$ between two consecutive waves.

Let $\frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta$, which is constant.

By the method of vector addition of amplitudes, the resultant amplitude $R = \dfrac{a \sin nd/2}{\sin d/2}$

Here $a = \dfrac{A \sin\alpha}{\alpha}$, $n = N$, $d = 2\beta$.

$$\therefore \quad R = \frac{A \sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}$$

$$\therefore \quad I = R^2 = \left(\frac{A \sin\alpha}{\alpha}\right)^2 \cdot \left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

$$\boxed{I = R^2 = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 \cdot \left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)}$$

where $I_0 = A^2$.

Here the factor $\left(\frac{A \sin\alpha}{\alpha}\right)^2$ is the distribution of intensity due to single slit. The factor $\frac{\sin^2 N\beta}{\sin^2\beta}$ is the distribution of intensity as a combined effect of all slits.

Condition for Principal maxima

For principal maxima, the intensity must be maximum. ie $I \propto \frac{\sin^2 N\beta}{\sin^2\beta}$

I = maximum, when the denominator is minimum

$$\therefore \quad \sin\beta = 0 \Rightarrow \beta = \pm m\pi$$

$$\Rightarrow \quad \frac{\pi}{\lambda}(e+d) \sin\theta = \pm m\pi$$

$$\therefore \quad \boxed{(e+d) \sin\theta = \pm m\lambda}, \quad \text{where } m = 0, 1, 2, \ldots$$

This is the condition for principal maximum. This is called Grating equation.

if $m=0 \Rightarrow \theta=0 \Rightarrow$ 0th order principal maximum

if $m=1 \Rightarrow$ 1st order principal maximum

if $m=2 \Rightarrow$ 2nd order principal maximum

The expression for intensity of principal maximum can be obtained as,

We have $I = I_0 \dfrac{\sin^2\alpha}{\alpha^2} \times \dfrac{\sin^2 N\beta}{\sin^2\beta}$

Here, since $\beta \to \pm m\pi$, then the above equation can be written as. $I = I_0 \dfrac{\sin^2\alpha}{\alpha^2} \lim\limits_{\beta\to\pm m\pi} \dfrac{\sin^2 N\beta}{\sin^2\beta}$

ie $I = I_0 \dfrac{\sin^2\alpha}{\alpha^2} \cdot N^2$

$\left(\because \lim\limits_{\beta\to\pm m\pi} \dfrac{\sin N\beta}{\sin\beta} = N \right.$ from L-Hospital rule. $\left. \right)$

$\therefore \boxed{I = I_0 N^2 \dfrac{\sin^2\alpha}{\alpha^2}}$

$\therefore \boxed{I \propto N^2}$

Condition for minimum intensity:

Intensity = minimum $\Rightarrow \sin^2 N\beta = 0$

$\sin N\beta = 0 \Rightarrow N\beta = \pm m\pi \Rightarrow \boxed{\beta = \pm \dfrac{m\pi}{N}}$

$\therefore \dfrac{\pi}{d}(e+d)\sin\theta = \pm \dfrac{m\pi}{N}$

$\Rightarrow \boxed{(e+d)\sin\theta = \pm \dfrac{md}{N}}$

→ if $m=0$, then $(e+d)\sin\theta = 0$, i.e this corresp-
-onds to zeroth order principal maximum.

→ If $m = N$, then $(e+d)\sin\theta = \pm d$, it corresponds to first order maxima.

→ If $m = 2N$, then $(e+d)\sin\theta = \pm 2d$, it corres-
-ponds to 2nd order maxima.

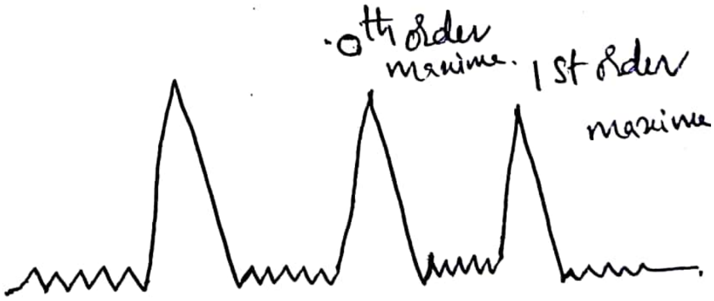
So, m can't take the values of $0, N, 2N, \ldots$

Hence, m can take the possible values from 0 to $N-1$. ie $m = 1, 2, 3, \ldots (N-1)$

Then only the intensities will be minimum.

The intensity distribution is as shown.

(a) diffraction due to individual slit.



(b) Interference of waves coming from N-slits (Grating).

∴ It can be concluded that between zeroth order maxima and 1st order principal maxima, there will be $(N-1)$ minima and between any two minimas, there will be secondary maximas.

Rayleigh's Criterion:

The image of two point objects which are close to each other are said to be just resolved if the central maxima of one diffraction pattern falls on the first minimum of the other diffraction pattern. This phenomenon was discovered by Rayleigh's and hence this phenomenon is known as Rayleigh's criterion.

Resolving power: The ability of an optical instrument to form seperate diffraction principal maxima of two wavelengths which are very close to each other,

$$\text{Resolving power (R.P)} = \frac{\text{wavelength of one spectral line}}{\text{diff. between these spectral lines}}$$

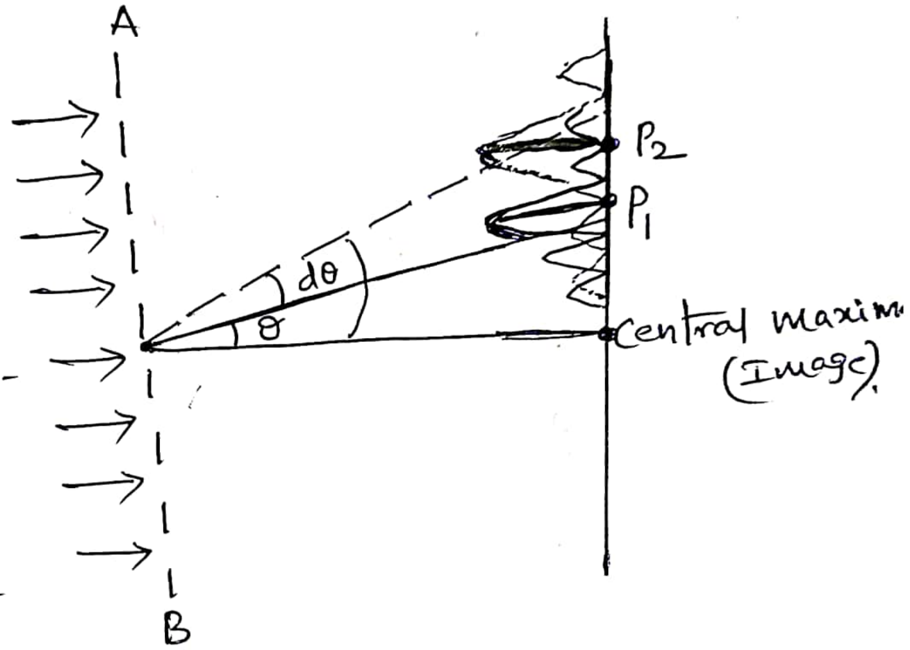
Let λ, $\lambda+d\lambda$ be the two wave lengths of two spectral lines respectively, then

$$R.P = \frac{\lambda}{(\lambda+d\lambda)-\lambda} = \frac{\lambda}{d\lambda}.$$

$$\therefore \boxed{R \cdot P = \frac{\lambda}{d\lambda}}$$

To find R.P of a grating, Consider a plane optical grating with grating element $(e+d)$. Let 'N' be the total no. of slits of grating. xy be the Screen.

Let the light be consists of wave-lengths λ and $\lambda+d\lambda$ and grating element $(e+d)$.



When a beam of light having two wavelengths λ and $\lambda+d\lambda$ be incident normally on grating. Here P_1 is the maxima of the wavelength 'λ' with an angle of diffraction θ. P_2 is the maxima of wavelength $\lambda+d\lambda$ with an angle of diffraction $\theta+d\theta$.

According to Rayleigh's Criterion the two wavelengths of λ and $\lambda+d\lambda$ will be resolved if the maxima of $\lambda + d\lambda$ in the direction of $\theta + d\theta$ falls over the 1st minima of the wave length 'λ' in the direction of $\theta + d\theta$.

The Principal maximum of 'λ' in the direction of 'θ' is given by $(e+d)\sin\theta = n\lambda$ — ①

Illy the minima equation of 'λ' in the direction of 'θ' is $N\cdot(e+d)\sin\theta = m\lambda$, where $m = Nn+1$

∴ The minima equation of 'λ' in the direction of $(\theta + d\theta)$ is given by $N(e+d)\sin(\theta+d\theta) = (Nn+1)\lambda$

$$\therefore N(e+d)\sin(\theta+d\theta) = (Nn+1)\lambda \quad \text{— ②}$$

Illy The Principal maxima of $\lambda + d\lambda$ in the direction of $\theta + d\theta$ is $(e+d)\sin(\theta+d\theta) = n(\lambda+d\lambda)$ — ③

Multiply with 'N' on both sides,

$$N(e+d)\sin(\theta+d\theta) = Nn(\lambda+d\lambda) \quad \text{— ④}$$

From ② and ④, LHS of ② & ④ are equal and hence, we have

$$(Nn+1)\lambda = Nn(\lambda+d\lambda)$$

$$\Rightarrow \cancel{Nn\lambda} + \lambda = \cancel{Nn\lambda} + Nn\,d\lambda$$

$$\lambda = nN\, d\lambda \Rightarrow \boxed{\frac{\lambda}{d\lambda} = Nn} \quad — (5)$$

Here $N \rightarrow$ no. of lines on grating

$n \rightarrow$ order of diffraction

$$\therefore \boxed{R.P = \frac{\lambda}{d\lambda} = N.n}$$

$\therefore$ It can be concluded R.P is directly proportional to no. of lines on grating and proportional to order of spectrum.

From (1), we have $n\lambda = (e+d)\sin\theta$

ie $n = \frac{(e+d)\sin\theta}{\lambda}$ — (6)

put (6) in (5), we get

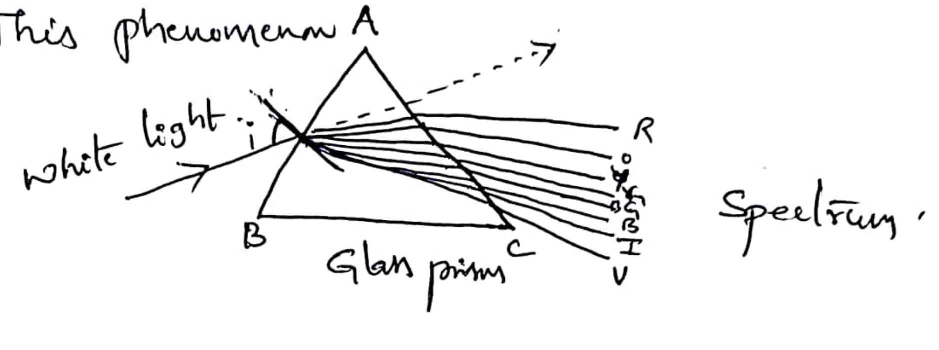
$$\boxed{\frac{\lambda}{d\lambda} = N\frac{(e+d)\sin\theta}{\lambda}}$$

Resolving power of a grating is given by

$$\boxed{\frac{\lambda}{d\lambda} = N\frac{(e+d)\sin\theta}{\lambda}}$$

Dispersion of light:

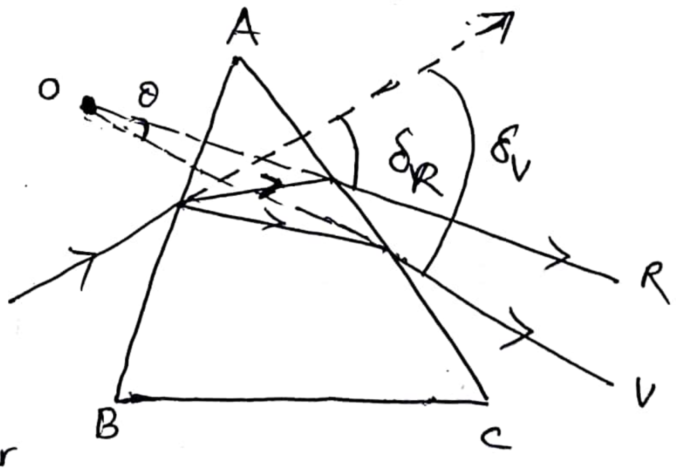
When a white light is allowed to fall on a prism, then the prism split that light into 7 constituent colours. The pattern of these colours is known as Spectrum. This phenomenon is called Dispersion of light.



Angular dispersion:

The difference between the angles of deviation produced by any two colours of light while passing through a prism is called Angular dispersion. It is denoted by θ.

Let δ_V, δ_R be the angular deviations of violet, red colours light respectively.



Then angular dispersion of red and violet is $\boxed{\theta = \delta_V - \delta_R}$

Let $\mu_R \rightarrow$ R.I of prism for red light

$\mu_V \rightarrow$ R.I of prism for violet light

For a thin prism of angle 'A'.

then, $\delta = (\mu - 1) A$ is the expression for angle of deviation.

$\therefore \quad \delta_R = (\mu_R - 1) A, \quad \delta_V = (\mu_V - 1) A$

$\therefore$ The angular dispersion 'θ' due to Red & Violet Colours is given by $\theta = (\mu_V - 1) A - (\mu_R - 1) A$

$$\therefore \quad \boxed{\theta = (\mu_V - \mu_R) A}$$

<u>Dispersive Power</u>: It is defined as the ratio of angular dispersion to the mean deviation produced by a prism. It is denoted by ω.

$$\text{Dispersive power } (\omega) = \frac{\text{Angular dispersion}}{\text{Mean deviation}}$$

W.K.T, angular dispersion 'θ' $= (\mu_V - \mu_R) A$

Mean deviation $\delta_y = (\mu_y - 1) A$

$$\therefore \quad \omega = \frac{(\mu_V - \mu_R) A}{(\mu_y - 1) A} \Rightarrow \boxed{\omega = \frac{\mu_V - \mu_R}{\mu_y - 1}}$$

$$\therefore \quad \boxed{\omega = \frac{\mu_V - \mu_R}{\mu_y - 1}}$$

Dispersive power depends on the nature of material of the prism.