Find out the Θ-notation for the function:

$f(n) = 27n^2 + 16n$

Sol Let $f(n) = 27n^2 + 16n$

To obtain big Theta notation, we have to find out,

$$\boxed{c_1 * g(n) \le f(n) \le c_2 * g(n)}$$

If we assume

$c_1 = 27$ and $c_2 = 43$

For $n \geq 1$ for

$g(n) = n^2$ we get

$$\boxed{27n^2 \le 27n^2 + 16n \le 43n^2}$$

for $n \geq 1$

$\therefore f(n) = \Theta(n^2)$ for $c_1 = 27$, $c_2 = 43$

Answer the following,

(i) Find big theta (Θ) and big omega (Ω) notation

1) $f(n) = 14 * 7 + 83$

2) $f(n) = 83n^2 + 84n$.

(ii) Is $2^{n+1} = O(2^n)$? Explain.

Sol To obtain big Omega -

$f(n) = 14 * 7 + 83$ we can write it in polynomial form as.

$f(n) = 2n^2 + 11n + 6$ where $n = 7$

Now if $g(n) = 7(n)$ then

we get $f(n) > 7(n)$ Hence.

$$2n^2 + 11n + 6 = \Omega(n)$$

To obtain big Theta notation.

we have to find out

$$\boxed{c_1 * g(n) \leq f(n) \leq c_2 * g(n)}$$

If we assume $c_1 = 7$ and $c_2 = 26$ Then with $n = 7$,

$$7n \leq 2n^2 + 11n + 6 \leq 26n. \text{ is True.}$$

$\therefore f(n) = \Theta(n)$ where $n = 7$, $c_1 = 7$ and $c_2 = 26$

2. $f(n) = 83n^2 + 84n \in \Omega(n^2)$

If $\boxed{c_1 * g_1(n) \leq f(n) \leq c_2 * g_2(n)}$ Then $f(n) \in \Theta(g(n))$

$\therefore f(n) = \Theta(n^3)$ where $c_1 = 83$ and $c_2 = 167$ with $n \geq 1$

(iii) $2^{n+1} = O(2^n)$ is True because

True

$$2^{n+1} = 2 \cdot 2^n \leq c \cdot 2^n, \text{ where } c \geq 2$$

Check equalities (True / False):

(i) $5n^2 - 6n \in \Theta(n^2)$

(ii) $n! \in O(n^n)$

(iii) $2n^2 2^n + n\log n \in \Theta(n^2 2^n)$

(iv) $\sum_{i=0}^{n} i^2 \in \Theta(n^3)$

(v) $n^2 \in \Theta(n^3)$

(vi) $2^n \in \Theta(2^{n+1})$

(vii) $n! \in \Theta((n+1)!)$

Sol

(i) $\boxed{5n^2 - 6n \in O(n^2)}$ True because

$f(n) = 5n^2 - 6n$

$g(n) = n^2$

and $f(n) \le c * g(n)$ is true.

(ii) $f(n) = n! = 1 * 2 * 3 * 4 * \cdots n$

$g(n) = n^n = n * n * n * \cdots * n$

$f(n) \le c * g(n)$ is true.

Hence $n! \in O(n^n)$ is false.

(iii) $f(n) = 2n^2 2^n + n\log n$

$g(n) = n^2 2^n$

(iii) $f(n) = 2n^2 2^n + n\log n \in \Theta(n^2 2^n)$ ✗ False.

$g(n) = n^2 2^n$

If $n = 16$, then

$f(n) = 2(16)^2 (2)^{16} + 16 \log 16$

$g(n) = (16)^2 (2)^{16}$

This shows that,

$$\boxed{f(n) \geq c * g(n)}$$

Hence $\underline{2n^2 2^n + n\log n \in \Theta(n^2 2^n)}$ is False

(iv) $\sum_{i=0}^{n} i^2 \in \Theta(n^3)$ ✓ True.

$f(n) = \sum_{i=0}^{n} i^k = 0 + 1 + 2^k + 3^k + \dots + n^k = \frac{n^{k+1}}{k+1}$

$g(n) = \Theta(n^{k+1})$

$\sum_{i=0}^{n} i^k \in \Theta(n^{k+1})$ is $\underline{\text{true}}$

$n^2 \in \Theta(n^3)$ ✓

(v) $f(n) = n^2$ True

$g(n) = n^3$

As $c_1 * g(n) \leq f(n) \leq c_2 * g(n)$ is true.

Hence $\underline{n^2 \in \Theta(n^3)}$ is true.

(vi) $2^n \in \Theta(2^{n+1}) \longrightarrow$ True.

Let $2^n = \frac{1}{2} \cdot 2^{n+1}$

$2^n \leq 2^{n+1}$

Hence $2^n \in \Theta(2^{n+1})$ is true.

(vii) $n! \in \Theta((n+1)!) \longrightarrow$ False

$$n! = 1 * 2 * 3 \dots * n$$

$$(n+1)! = (n+1) * n * \dots 2 * 1.$$

$\therefore\ n! \in \Theta(n+1)!$ is False

Prove or disprove that $f(n) = 1+2+3+\dots+n \in \Theta(n$

Sol let $f(n) = 1+2+3+\dots n = \frac{n(n+1)}{2} = \frac{1}{2}n^2.$

$g(n) = n^2$

By definition $f(n) \in \Theta(g(n))$ if

$$\boxed{c_1 * g(n) \le f(n) \le c_2 * g(n)}$$

If $c_1 = \frac{1}{4}$ and $n = 2$ Then

$\boxed{c_1 * g(n) \le f(n)} \Rightarrow \frac{1}{4}(2^2) \le (2^2)$ is TRUE

If $c_2 = 1$ and $n = 2$

$\boxed{f(n) \le c_2 * g(n)} \Rightarrow \frac{1}{2}(2^2) \le 2^2$ is TRUE

As both the cases are true, it proves that

$1 + 2 + 3 + \dots + n \in \Theta(n^2).$