B-Tree

Introduction of B-Tree:

B-Tree is a self-balancing search tree. In most of the other self-balancing search trees (like AVL and Red-Black Trees), it is assumed that everything is in main memory. To understand the use of B-Trees, we must think of the huge amount of data that cannot fit in main memory. When the number of keys is high, the data is read from disk in the form of blocks. Disk access time is very high compared to main memory access time. The main idea of using B-Trees is to reduce the number of disk accesses. Most of the tree operations (search, insert, delete, max, min, ..etc) require O(h) disk accesses where h is the height of the tree. B-tree is a fat tree. The height of B-Trees is kept low by putting maximum possible keys in a B-Tree node. Generally, a B-Tree node size is kept equal to the disk block size. Since h is low for B-Tree, total disk accesses for most of the operations are reduced significantly compared to balanced Binary Search Trees like AVL Tree, Red-Black Tree, ..etc.

Properties of B-Tree:

1) All leaves are at same level.

2) A B-Tree is defined by the term *minimum degree* 't'. The value of t depends upon disk block size.

3) Every node except root must contain at least t-1 keys. Root may contain minimum 1 key.

4) All nodes (including root) may contain at most 2t – 1 keys.

5) Number of children of a node is equal to the number of keys in it plus 1.

6) All keys of a node are sorted in increasing order. The child between two keys k1 and

k2 contains all keys in the range from k1 and k2.

7) B-Tree grows and shrinks from the root which is unlike Binary Search Tree. Binary Search Trees grow downward and also shrink from downward.

8) Like other balanced Binary Search Trees, time complexity to search, insert and delete is O(Logn).

Insertion:

1) Initialize x as root.

2) While x is not leaf, do following

..**a)** Find the child of x that is going to be traversed next. Let the child be y.

..**b)** If y is not full, change x to point to y.

..**c)** If y is full, split it and change x to point to one of the two parts of y. If k is smaller than mid key in y, then set x as the first part of y. Else second part of y. When we split y, we move a key from y to its parent x.

3) The loop in step 2 stops when x is leaf. x must have space for 1 extra key as we have been splitting all nodes in advance. So simply insert k to x.

Note that the algorithm follows the Cormen book. It is actually a proactive insertion algorithm where before going down to a node, we split it if it is full. The advantage of splitting before is, we never traverse a node twice. If we don't split a node before going down to it and split it only if a new key is inserted (reactive), we may end up traversing all nodes again from leaf to root. This happens in cases when all nodes on the path from the root to leaf are full. So when we come to the leaf node, we split it and move a key up. Moving a key up will cause a split in parent node (because the parent was already full). This cascading effect never happens in this proactive insertion algorithm.

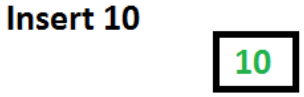
There is a disadvantage of this proactive insertion though, we may do unnecessary

splits

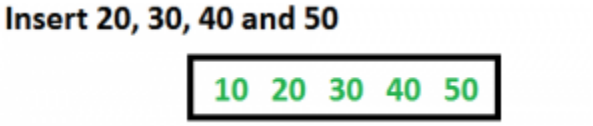
Example:

Let us understand the algorithm with an example tree of minimum degree ‘t’ as 3 and a sequence of integers 10, 20, 30, 40, 50, 60, 70, 80 and 90 in an initially empty B-Tree.

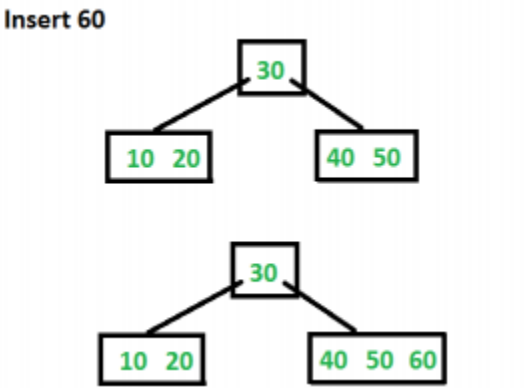
Initially root is NULL. Let us first insert 10.



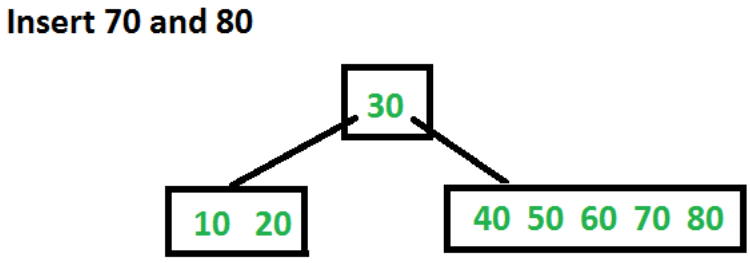
Let us now insert 20, 30, 40 and 50. They all will be inserted in root because the maximum number of keys a node can accommodate is 2*t – 1 which is 5.



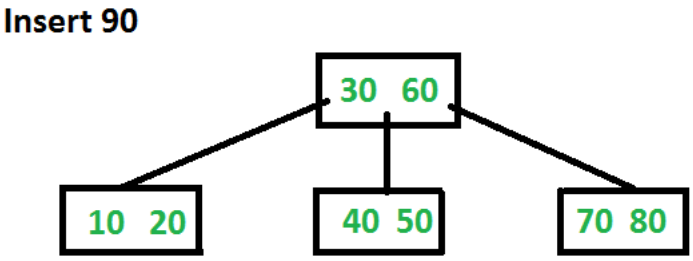
Let us now insert 60. Since root node is full, it will first split into two, then 60 will be inserted into the appropriate child.



Let us now insert 70 and 80. These new keys will be inserted into the appropriate leaf without any split.



Let us now insert 90. This insertion will cause a split. The middle key will go up to the parent.



Deletion process:

1. If the key k is in node x and x is a leaf, delete the key k from x.

2. If the key k is in node x and x is an internal node, do the following.

a) If the child y that precedes k in node x has at least t keys, then find the predecessor k0 of k in the sub-tree rooted at y. Recursively delete k0, and replace k by k0 in x. (We can find k0 and delete it in a single downward pass.)

b) If y has fewer than t keys, then, symmetrically, examine the child z that follows k in node x. If z has at least t keys, then find the successor k0 of k in the subtree rooted at z. Recursively delete k0, and replace k by k0 in x. (We can find k0 and delete it in a single downward pass.)

c) Otherwise, if both y and z have only t-1 keys, merge k and all of z into y, so that x loses both k and the pointer to z, and y now contains 2t-1 keys. Then free z and recursively delete k from y.

3. If the key k is not present in internal node x, determine the root x.c(i) of the appropriate subtree that must contain k, if k is in the tree at all. If x.c(i) has only t-1 keys, execute step 3a or 3b as necessary to guarantee that we descend to a node containing at least t keys. Then finish by recursing on the appropriate child of x.

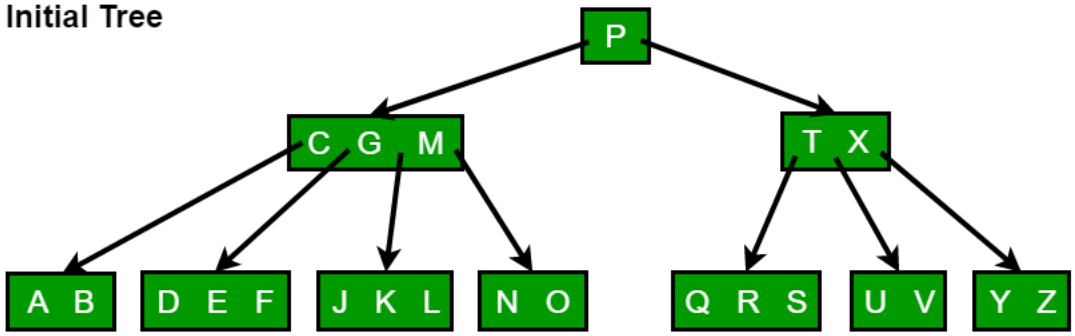
a) If x.c(i) has only t-1 keys but has an immediate sibling with at least t keys, give x.c(i) an extra key by moving a key from x down into x.c(i), moving a key from x.c(i) 's immediate left or right sibling up into x, and moving the appropriate child pointer from the sibling into x.c(i).

b) If x.c(i) and both of x.c(i)'s immediate siblings have t-1 keys, merge x.c(i) with one sibling, which involves moving a key from x down into the new merged node to become the median key for that node.

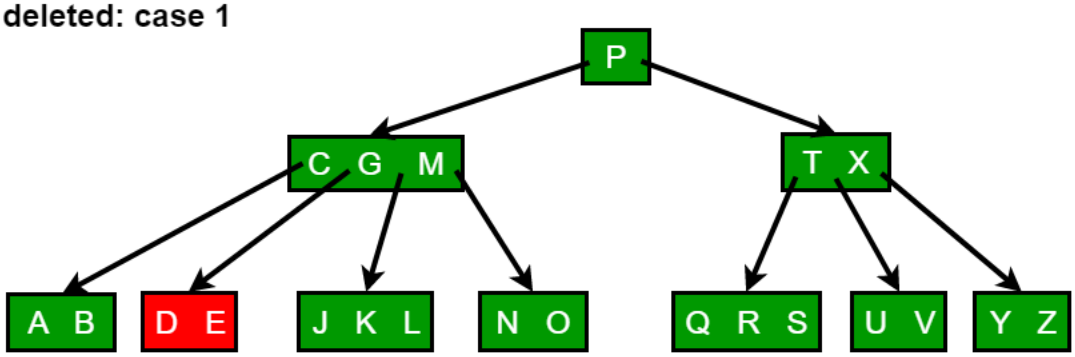
Since most of the keys in a B-tree are in the leaves, deletion operations are most often used to delete keys from leaves. The recursive delete procedure then acts in one downward pass through the tree, without having to back up. When deleting a key in an internal node, however, the procedure makes a downward pass through the tree but may have to return to the node from which the key was deleted to replace the key with its predecessor or successor (cases 2a and 2b).

The following figures explain the deletion process.

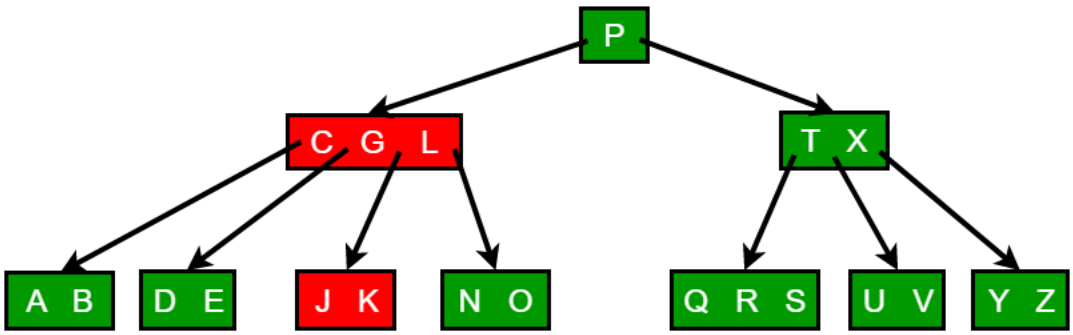
(a) Initial Tree



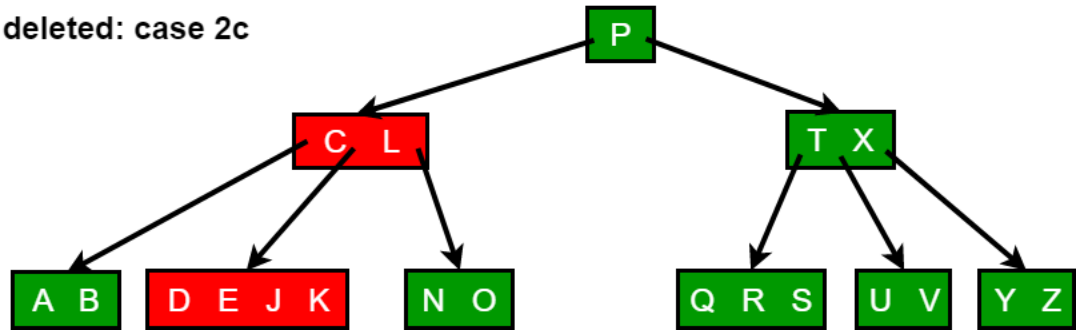
(b) F deleted: case 1



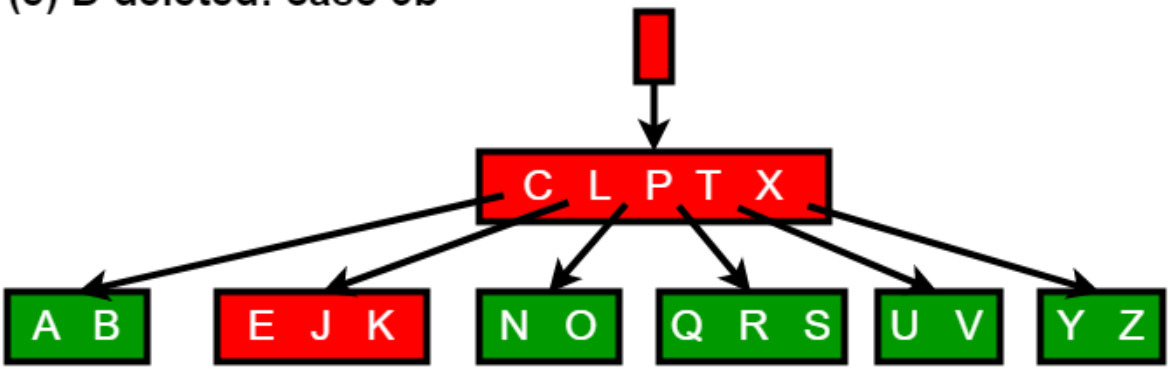
(c) M deleted: case 2a



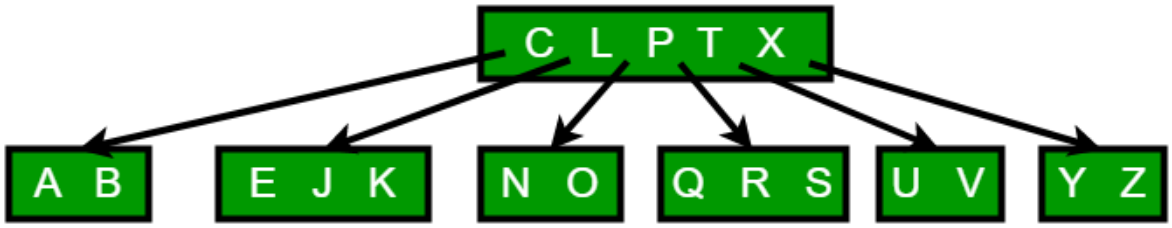
(d) G deleted: case 2c



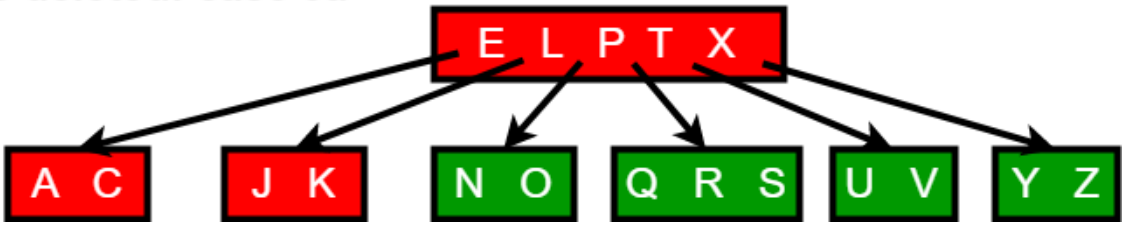
(e) D deleted: case 3b



(e') tree shrinks in height



(f) B deleted: case 3a



Problems:

1. **Insert 40,30,20,19,16,17,34,29,44,66,39,23,46,90,101 ,9,5,3,2,11,22,33**

   m=5

2. **Insert**

   60,33,4,22,1,5,6,29,46,12,34,56,21,67,43,88,90,34,52,54,66,123,90,55,78

   M=3

3. **consider 1st question delete elements 39,23,101,9,40,19,33,17**