UNIT - III

Discrete probn Distribution

- uniform
- Bernoulli
- Binomial
- poisson
- Negative Binomial
- Geometric
- Hyper Geometric

Continuous probn Distribution

- uniform
- Normal
- Exponential
- logarithm

Discrete Probability Distribution:

① Uniform Distribution

$X \sim U(n)$ read as X follows uniform distribution with parameter n

P.M.F $\quad P(x) = \frac{1}{n} \quad x = 1, 2, \cdots n$

mean $\quad \mu = \frac{n+1}{2}$

$$E(x^2) = \frac{(n+1)(2n+1)}{6}$$

$$V(x) = \frac{(n+1)(n-1)}{12}$$

Example problem:

If $X \sim U(6)$ then find its P.M.F, mean, variance,

② Bernoulli Distribution

A random variable X takes the values only '0' and '1' then the Distribution is Bernoulli

P.M.F $P(x) = p^{x} q^{1-x}$; $x = 0, 1$

Mean $\mu = E(x) = p$ $\qquad p+q=1$

$E(x^2) = p$

$V(x) = pq$

3. Binomial Distribution

A random variable x takes the values 0, 1, 2, ... n with its p.m.f $P(x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$

where $0 < p < 1$, $p + q = 1$

$x \sim B(n, p)$ represents Binomial Distr. with parameters n and p

$P(x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$

mean $\mu = np$

variance $\sigma^2 = npq$

Problems on Binomial Distribution

Example 1: Four fair coins are tossed. If the outcomes are assumed to be independent, then find the p.m.f. and c.d.f. of the number of heads obtained.

Solution: Four coins are tossed $\Rightarrow n=4$

Here $p=\frac{1}{2}$, $q=\frac{1}{2}$

Hence $X \sim B(n; p)$

$$X \sim B\left(4; \frac{1}{2}\right)$$

Now its p.m.f $P(x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$

$$P(x) = {}^{4}C_{x} \cdot \left(\frac{1}{2}\right)^{x} \cdot \left(\frac{1}{2}\right)^{4-x}$$

$$= {}^{4}C_{x} \cdot \left(\frac{1}{2}\right)^{4}$$

$$\boxed{P(x) = {}^{4}C_{x} \cdot \frac{1}{16}}$$

Now

$$P(0) = {}^{4}C_{0} \cdot \frac{1}{16} = \frac{1}{16}$$

$$P(1) = {}^{4}C_{1} \cdot \frac{1}{16} = \frac{4}{16}$$

$$P(2) = {}^4C_2 \cdot \frac{1}{16} = \frac{6}{16}$$

$$P(3) = {}^4C_3 \cdot \frac{1}{16} = \frac{4}{16}$$

$$P(4) = {}^4C_4 \cdot \frac{1}{16} = \frac{1}{16}$$

Hence probabi Distribution Table

x	0	1	2	3	4
$P(x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$
$F(x)$	$\frac{1}{16}$	$\frac{5}{16}$	$\frac{11}{16}$	$\frac{15}{16}$	$\frac{16}{16} = 1$

Here $P(x)$: P.M.F

$F(x)$: C.D.F

Example 2: A and B play a game in which their chances of winning are in the ratio 3 : 2. Find A's chance of winning at least three games out of the five games played.

Solution Total games played $n=5$

$P(A) = \frac{3}{5}$ so $p = \frac{3}{5}$, $q = \frac{2}{5}$

$X \sim B(n; p) \Rightarrow X \sim B(5, \frac{3}{5})$

Now $P(x) = {}^nC_x \cdot p^x \cdot q^{n-x}$

$P(x) = {}^5C_x \cdot \left(\frac{3}{5}\right)^x \cdot \left(\frac{2}{5}\right)^{5-x}$

Now we find prob. of A's winning at least 3 out of 5 games

i.e $P(X \geq 3) = P(3) + P(4) + P(5)$

$= {}^5C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^{5-3} + {}^5C_4 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^{5-4}$

$+ {}^5C_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^{5-5}$

$$\boxed{P(X \geq 3) = 0.68}$$

Example 3: The probability of a man hitting a target is $\frac{1}{4}$.

(i) If he fires 7 times, what is the probability of his hitting the target at least twice?

(ii) How many times must he fire so that the probability of his hitting the target at least once is greater than $\frac{2}{3}$?

Solution: The probab.. of man hitting a target is $\frac{1}{4}$ so $p = \frac{1}{4}$, $q = \frac{3}{4}$

(i) he fires 7 times $n = 7$, $p = \frac{1}{4}$

$X \sim B(n; p) \Rightarrow X \sim B(7, \frac{1}{4})$

$P(x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$

$P(x) = {}^{7}C_{x} \cdot \left(\frac{1}{4}\right)^{x} \left(\frac{3}{4}\right)^{7-x}$

Now we find probab.. of his hitting target is at least twice

i.e $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - \left[{}^{7}C_{0} \left(\frac{1}{4}\right)^{0} \left(\frac{3}{4}\right)^{7-0} + {}^{7}C_{1} \left(\frac{1}{4}\right)^{1} \left(\frac{3}{4}\right)^{7-1}\right]$

$= 0.55$

$\therefore \boxed{P(x \geq 2) = 0.55}$

(ii) Now we find How many times must he fire so that the probabli of his hitting the Target at least once is greater Than $\frac{2}{3}$

<u>i.e we find</u> n <u>for $P(x \geq 1)$</u> <u>$> \frac{2}{3}$</u>

$$P(x \geq 1) > \frac{2}{3}$$

$$1 - P(x < 1) > \frac{2}{3}$$

$$1 - P(x = 0) > \frac{2}{3}$$

$$P(x = 0) < \frac{1}{3}$$

$${}^nC_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^n < \frac{1}{3}$$

$$\left(\frac{3}{4}\right)^n < \frac{1}{3}$$

$$n \log \frac{3}{4} < \log \frac{1}{3}$$

$$n < \frac{\log \frac{1}{3}}{\log \frac{3}{4}}$$

$$n > \frac{\log 3}{\log \frac{4}{3}} = 3.8$$

$$n > 3.8 \simeq 4$$

$$n > 4$$

Hence required no. of shots 4

Example 4 : The mean and variance of Binomial Distribution are 4 and $\frac{4}{3}$ respectively. find $P(X \geq 1)$

Solution: $X \sim B(n; p)$

mean $\mu = np = 4$

variance $\sigma^2 = npq = \frac{4}{3}$

$\frac{\text{variance}}{\text{mean}}$ we get $q = \frac{1}{3}$

$p = \frac{2}{3}$

Hence for $np = 4$

$n \times \frac{2}{3} = 4$

$\boxed{n = 6}$

Hence $X \sim B(6; \frac{2}{3})$

$$P(x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

$$\boxed{P(x) = {}^{6}C_{x} \cdot \left(\frac{2}{3}\right)^{x} \cdot \left(\frac{1}{3}\right)^{6-x}}$$

Now we find $P(X \geq 1)$

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - {}^{6}C_{0} \cdot \left(\frac{2}{3}\right)^{0} \cdot \left(\frac{1}{3}\right)^{6-0}$$

$$= 1 - \left(\frac{1}{3}\right)^{6}$$

$$\boxed{P(X \geq 1) = 1 - \left(\frac{1}{3}\right)^{6}}$$

④ Poisson Distribution

$X \sim P(\lambda)$: with parameter $\lambda > 0$

P.m.F $\quad P(x) = \dfrac{e^{-\lambda} \cdot \lambda^{x}}{x!}$; $\lambda > 0$

$x = 0, 1, \ldots$

mean $\mu = \lambda$

variance $\sigma^2 = \lambda$

In Poisson Distribution

$$\boxed{\text{Average} = \text{mean} = \text{variance} = \lambda}$$

Example 6: Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability for each of the following events:

a) **Exactly two messages arrive within one hour.**
b) **No message arrives within one hour.**
c) **At least three messages arrive within one hour.**

Solution: Here $X \sim P(\lambda)$

where $\lambda = 6$

$X \sim P(6)$

P.m.f $P(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$

$$\boxed{P(x) = \frac{e^{-6} \cdot 6^{x}}{x!}} \quad x = 0, 1, 2, \ldots$$

① $P(X=2) = P(2) = \frac{e^{-6} \cdot 6^{2}}{2!} = 18e^{-6}$

② $P(X=0) = P(0) = \frac{e^{-6} \cdot 6^{0}}{0!} = e^{-6}$

③ $P(X \geq 3) = 1 - P(X < 3)$

$= 1 - [P(0) + P(1) + P(2)]$

$$= 1 - \left[\frac{e^{-6} \cdot 6^0}{0!} + \frac{e^{-6} \cdot 6^1}{1!} + \frac{e^{-6} \cdot 6^2}{2!}\right]$$

$$= 1 - 25e^{-6}$$

$$\boxed{P(X \geq 3) = 1 - 25e^{-6}}$$

Example 8: If X and Y are independent Poisson variates such that $P(X = 1) = P(X = 2)$ and $P(Y = 2) = P(Y = 3)$, find the variance of $X - 2Y$.

Given X and Y are poisson variaty

Consider $X \sim P(\lambda)$, $Y \sim P(\mu)$

For $X \sim P(\lambda)$

$$P(X=1) = P(X=2)$$

$$\frac{e^{-\lambda} \cdot \lambda^1}{1!} = \frac{e^{-\lambda} \cdot \lambda^2}{2!}$$

$$2\lambda = \lambda^2$$

$$\lambda^2 - 2\lambda = 0$$

$\lambda = 0$ or $\lambda = 2$

$\boxed{\lambda = 2}$ $\quad \lambda > 0 \quad$ Hence $V(X) = 2$

For $Y \sim P(\mu)$

$$P(Y=2) = P(Y=3)$$

$$\frac{e^{-\mu} \cdot \mu^2}{2!} = \frac{e^{-\mu} \cdot \mu^3}{3!}$$

$$6\mu^2 = 2\mu^3$$

$$\mu^3 - 3\mu^2 = 0$$

$$\mu = 0, 0, 3$$

$\boxed{\mu = 3}$ $\quad \mu > 0 \quad$ Hence $V(Y) = 3$

Now we find the $V(X - 2Y)$

we know that

$$V(aX + bY) = a^2 V(X) + b^2 V(Y)$$

$$V(X - 2Y) = (1)^2 V(X) + (-2)^2 V(Y)$$

$$= (1)(2) + 4(3)$$

$$\boxed{V(X - 2Y) = 14}$$

5 Negative Binomial

$X \sim NB(r; p)$ where r, p are parameters

p.m.f $P(x) = {}^{x+r-1}C_{r-1} \cdot p^r \cdot q^x$

where $x = 0, 1, 2, \ldots$

mean $\mu = \frac{rq}{p}$

variance $\sigma^2 = \frac{rq}{p^2}$

Problem:

Find the probability that there are two daughters before the second son in a family when probability of a son is 0.5

Sol. let x be the no. of daughters before the second son

r = no. of son

Here $x = 2$, $r = 2$

$$P(x) = {}^{x+r-1}C_{r-1}\, p^{r} \cdot q^{x}$$

$$P(2) = {}^{2+2-1}C_{2-1} \cdot \left(\tfrac{1}{2}\right)^{2} \cdot \left(\tfrac{1}{2}\right)^{2}$$

$$= {}^{3}C_{1} \cdot \left(\tfrac{1}{2}\right)^{4} = \frac{3}{16}$$

$$\boxed{P(2) = 3/16}$$

⑥ Geometric Distribution

$X \sim GD(r)$ where r is parameter

P.M.F $P(x) = p \cdot q^x$; $x = 0, 1, 2, \ldots$

mean $\mu = \frac{q}{p}$

variance $\sigma^2 = \frac{q}{p^2}$

Problem:

Find the prob. of there are two daughters before the first son in a family where prob. of son is 0.5

Sol: Here x is no. of daughters before the first son $= 2$

$p = \frac{1}{2}, q = \frac{1}{2}$

$P(x) = p \cdot q^x$, $x = 0, 1, 2, \ldots$

$P(2) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = \frac{1}{8}$

$\boxed{P(2) = \frac{1}{8}}$

7) Hyper Geometric Distribution

A random variable X is said to Hyper Geometric Distribution if its

P.M.F

$$P(x) = \frac{{}^{M}C_{x} \cdot {}^{N-M}C_{n-x}}{{}^{N}C_{n}}$$

where $x = 0, 1, 2 \ldots \min\{n, m\}$

Mean $\mu = \frac{nm}{N}$

Variance $\sigma^2 = \frac{NM(N-M)(N-n)}{N^2(N-1)}$

Problem:

A bag contains 4 white balls, 3 green balls; Three balls are drawn, what is the probability that 2 are white

Sol:

N = Total no. of balls $= 4+3 = 7$

x = no. of white balls * $= 2$

m = white balls * $= 4$

n = green balls $= 3$

$$P(x) = \frac{{}^{m}C_{x} \; {}^{N-m}C_{n-x}}{{}^{N}C_{n}}$$

$$P(2) = \frac{{}^{4}C_{2} \; {}^{3}C_{1}}{{}^{7}C_{3}} = \frac{18}{35}$$

$$\boxed{P(2) = \frac{18}{35}}$$

Example problems in Binomial

(1) The mean of Binomial Distri.. is 3 and variance is $\frac{9}{4}$ Find

(i) n (ii) $P(X \geq 7)$ (iii) $P(X < 10)$

(iv) $P(1 \leq X < 6)$ (v) $P(X \leq 12)$

(2) The probability that the life of a bulb is 100 days is 0.05 Find the probab that out of 6 bulbs (i) At least one

(ii) greater than four

(iii) none

Example of Poisson Distribution

① If the variance of a poisson variate is 3, then find the prob.. that ① $x=0$ ② $0 < x \leq 3$ ③ $1 \leq x < 4$ ④ $x > 1$ ⑤ $x \geq 3$

② If a poisson distribution is such that $P(X=1) \cdot \frac{3}{2} = P(X=3)$ find (i) $P(X \geq 1)$ (ii) $P(X \leq 3)$ (iii) $P(2 \leq X \leq 5)$

Continuous Probability Distribution

Normal Distribution

A random variable X is said to have a normal Distribution, if its p.d.f is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad ; \quad -\infty < x < \infty$$

$$-\infty < \mu < \infty$$

$$\sigma > 0$$

Note: Normal Distribution is represented as $X \sim N(\mu, \sigma^2)$

Standard Normal Distribution

We can transfer any normal distribution to the standard normal distribution with mean $\mu = 0$ and S.D $\sigma = 1$ referred as standard normal distribution.

If we take $z = \frac{x-\mu}{\sigma}$ in p.d.f

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$ we obtain

The standard normal distribution with mean $\mu = 0$, S.D $\sigma = 1$ and its p.d.f

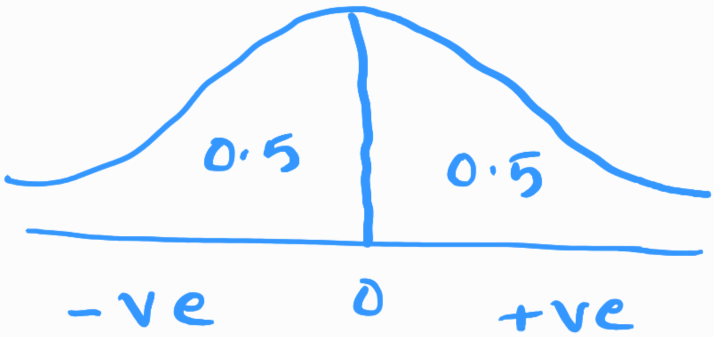
$$P(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

$$-\infty < z < \infty$$

Here z is called the standard normal variate with mean 0 and S.D 1 and in short we say that $P(z)$ is $N(0,1)$

ie $z \sim N(0,1)$

Area of the normal curve is 1



How to find Probability of normal curve

Step ①: change Scale $z = \frac{x-\mu}{\sigma}$

and find z_1 and z_2

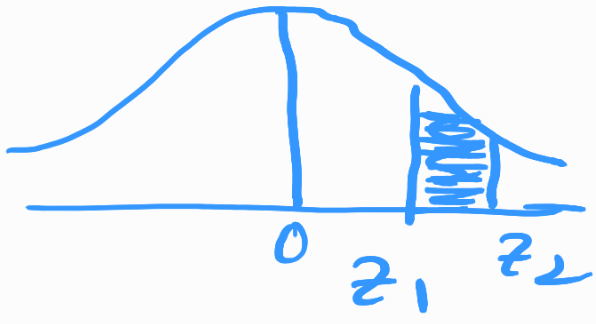
corresponding to the

values of x_1 and x_2 respecd

Step ②: To find

$$P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2)$$

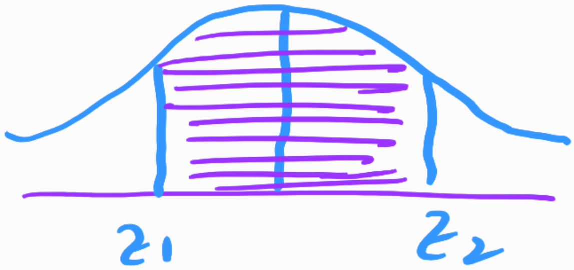
(i) If both z_1 and z_2 are positive (or) negative



Then $P(z_1 \le z \le z_2) = A(z_2) - A(z_1)$

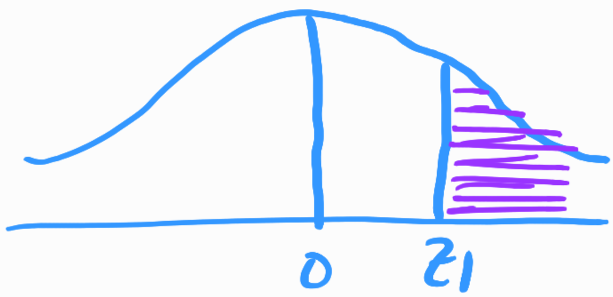
(ii) If z_1 -ve, z_2 +ve ;

$P(z_1 \leq z \leq z_2)$



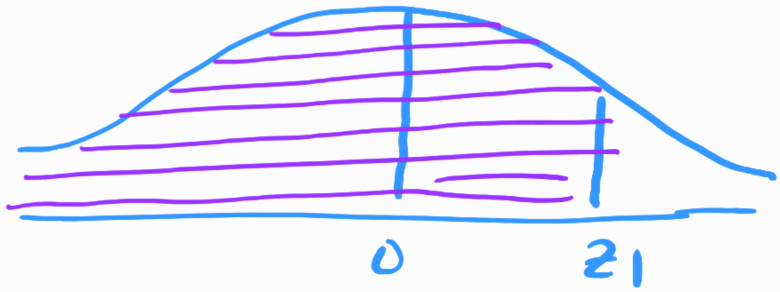
Then $P(z_1 \leq z \leq z_2) = A(z_1) + A(z_2)$

(iii) If $z_1 > 0$: $P(z > z_1)$



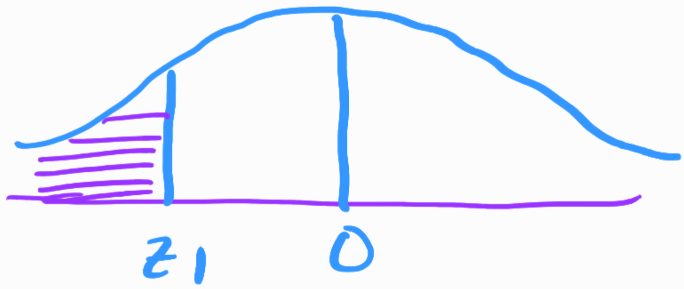
$P(z > z_1) = 0.5 - A(z_1)$

If $z_1 > 0$ $P(z < z_1)$



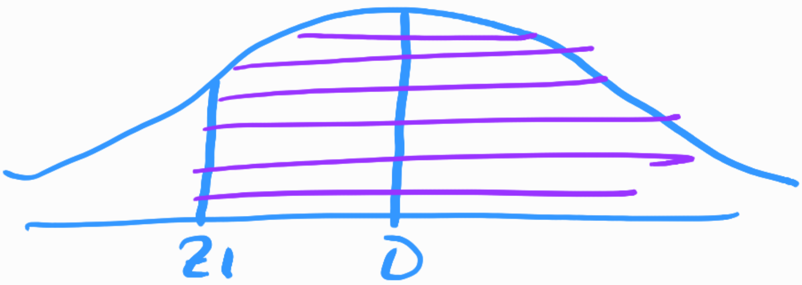
$P(z < z_1) = 0.5 + A(z_1)$

(iv) If $z_1 < 0$ $P(z < z_1)$



$P(z < z_1) = 0.5 - A(z_1)$

If $z_1 < 0$: $P(z > z_1)$

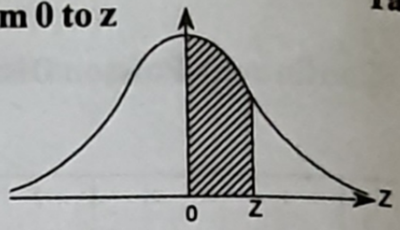


$P(z > z_1) = 0.5 + A(z_1)$

Areas under the Standard Normal Curve from 0 to z

Table – 3

$$Z = \frac{X-\mu}{\sigma}$$



z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1256	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1916	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2649
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4654	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4979	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Problems

① If X is normal variate with mean 30, S.D 5. Find the probability ① $26 \le x \le 40$
② $x \ge 45$

Sol: Given $x \sim N(\mu; \sigma^2)$

$\mu = 30, \quad \sigma = 5$

① $P(26 \le x \le 40)$:-

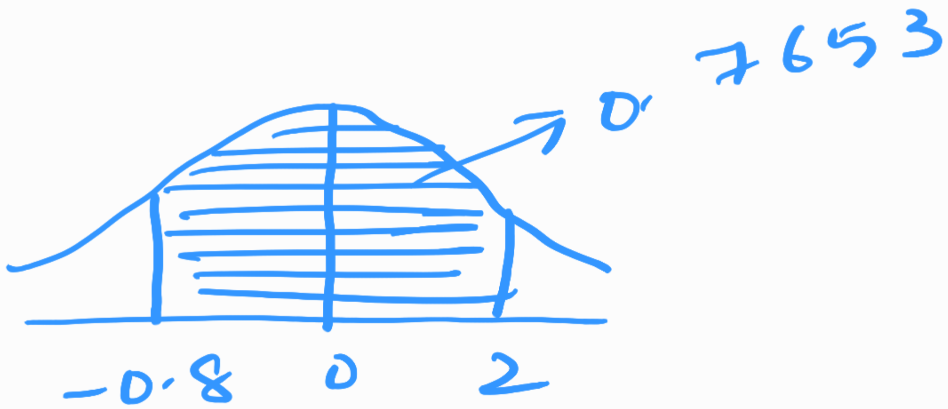
$$P(\underset{x_1}{26} \le x \le \underset{x_2}{40}) = P(z_1 \le z \le z_2)$$

$$z_1 = \frac{x_1 - \mu}{\sigma} \qquad z_2 = \frac{x_2 - \mu}{\sigma}$$

$$= \frac{26 - 30}{5} \qquad = \frac{40 - 30}{5}$$

$$= -0.8 \qquad = 2$$

$\therefore P(26 \le X \le 40) = P(-0.8 \le Z \le 2)$



$A(0.8) + A(2)$ $=$

$0.2881 + 0.4772$ $=$

$= 0.7653$ $=$

$\boxed{6 \le x \le 40) = 0.7653}$

$\therefore P(2$

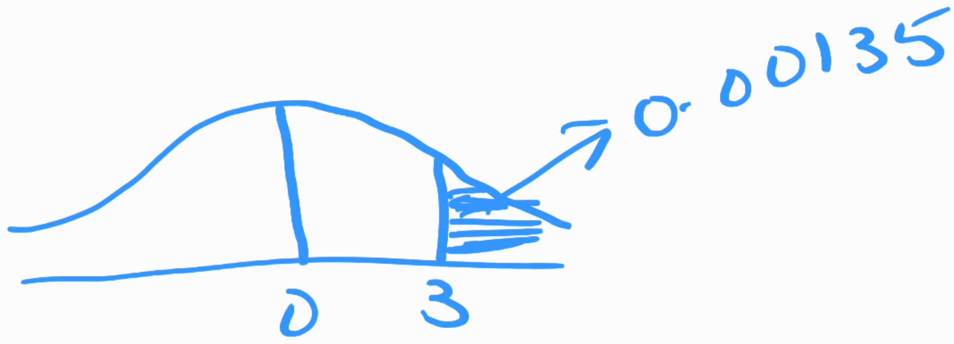
② $P(X \geq 45)$

$$P(\underset{x_1}{X \geq 45}) = P(Z \geq z_1)$$

$$z_1 = \frac{x_1 - \mu}{\sigma}$$

$$= \frac{45-30}{5} = \frac{15}{5} = 3$$

$$\therefore P(X \geq 45) = P(Z \geq 3)$$



$$= 0.5 - A(3)$$

$$= 0.5 - 0.49865$$

$$\boxed{P(X \geq 45) = 0.00135}$$

② If the masses of 300 students are normal Distributed with mean 68 kgs, S.D 3 kgs. how many students have masses

(i) Greater Than 72 kg

(ii) less Than or equal to 64 kg

(iii) Between 65 and 71 kg

Sol. $X \sim N(\mu; \sigma^2)$

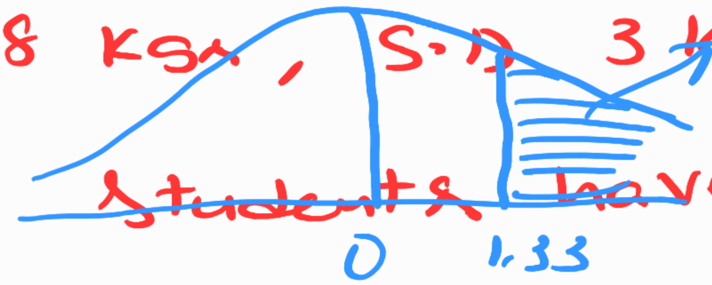
Here $\mu = 68$, $\sigma = 3$

(i) $P(X \geq 72)$

$$P(X \geq \underset{x_1}{72}) = P(Z \geq z_1)$$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{72 - 68}{3} = 1.33$$

② The marks of 300 students are normal distributed with mean 68 kgs, S.D 3 kgs. How many students have marks



(i) Greater than 72 kg

(ii) less than or equal to 64 kg $= 0.5 - A(z_1)$

(iii) Between 65 and 71 kg 0.5 0.0918

Sol. $X \sim N(\mu, \sigma^2)$

$$\boxed{\text{Here } \mu = 68, \ \sigma = 3 \quad P(X \geq 72) = 0.0918}$$

(i) The no. of students with more than 72 kgs $= 300 \times 0.0918$

$$P(X \geq 72) = P(Z \geq z_1) \simeq 28$$

($x_1 = 72$)

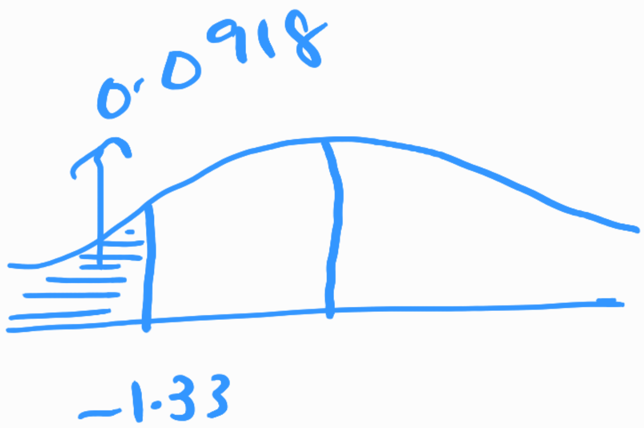
$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{72 - 68}{3} = 1.33$$

(ii) $P(X \le 64)$

$$P(X \le \underset{x_1}{64}) = P(Z \le z_1)$$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{64-68}{3} = -1.33$$

$$P(X \le 64) = P(Z \le -1.33)$$



$$= 0.5 - A(1.33)$$

$$= 0.5 - 0.4082 = 0.0918$$

$$\boxed{P(X \le 64) = 0.0918}$$

Hence the no of students have masses less than or equal to 64 kg

$= 300 \times 0.0918$

$\simeq 28$

③ $P(65 \le X \le 71)$

$P(\underset{x_1}{65} \le X \le \underset{x_2}{71}) = P(z_1 \le Z \le z_2)$

$z_1 = \dfrac{x_1 - \mu}{\sigma}$

$= \dfrac{65 - 68}{3}$

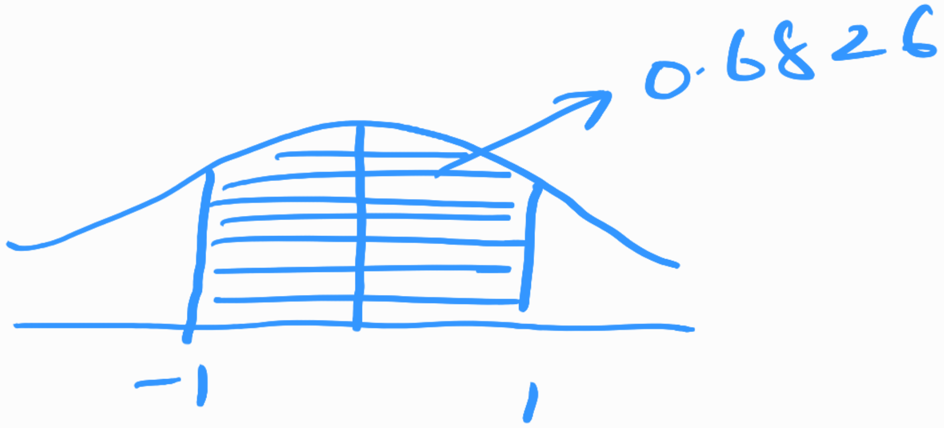
$= -1$

$z_2 = \dfrac{x_2 - \mu}{\sigma}$

$= \dfrac{71 - 68}{3}$

$= 1$

$P(65 \le X \le 71) = P(-1 \le Z \le 1)$



$= A(1) + A(1)$

$= 2A(1)$

$= 2(0.3413)$

$= 0.6826$

$\therefore \boxed{P(65 \le X \le 71) = 0.6826}$

No. of Students Marks between 65 and 71 is $= 300 \times 0.6826$

$\cong 205$

Example 2: If X is normally distributed with mean 12 and standard deviation , then

(a) **Find the probabilities of the following :**

(i) $X \geq 20$

(ii) $X \leq 20$ and

(iii) $0 \leq X \leq 12$

(b) **Find x when $P(X > x) = 0.24$**

(c) **Find x_1 and x_2 when $P(x_1 < X < x_2) = 0.5$ and $P(X > x_2) = 0.25$**

Example 3: The local authorities in a certain city install $10,000$ electric lamps in the streets of the city. If these lamps have an average life of $1,000$ burning hours with a standard deviation of 200 hours, assuming normality, what number of lamps might be expected to fail

(i) **in the first 800 and 1200 burning hours?**

(ii) **between 800 and 1200 burning hours?**

Example 4: A coin is tossed 10 times. Find the probability of getting between 4 and 7 heads inclusive using the (a) binomial distribution and (b) the normal approximation to the binomial distribution.

Example 5:

1. The mean and standard deviation of a normal variate are 8 and 4 respectively, find (*i*) $P(5 \leq x \leq 10)$ (*ii*) $P(x \geq 5)$ **[JNTU 2005 S, JNTU (H) Nov. 2009 (Set No. 1)]** (*iii*) $P(10 \leq x \leq 15)$ (*iv*) $P(x \leq 15)$ **[JNTU 2004 (Set No.2)]**
2. If the mean and standard deviation of a normal distribution are 70 and 16, find $P(38 < x < 46)$. **[JNTU 2005S (Set No.4)]**
3. If X is normally distributed with mean 70 and standard deviation 16, find (*i*) $P(38 \leq X \leq 46)$ (*ii*) $P(82 \leq X \leq 94)$ (*iii*) $P(62 \leq X \leq 86)$
4. Find the probability that a random variable having standard normal distribution will take on a value between 0.87 and 1.28. **[JNTU 2001]**
5. If X is a normally distributed with mean 30 and standard deviation 5, find $P(|X-30|>5)$.

Uniform Distribution

A continuous R.V X is said to follow uniform Distribution if its p.d.f is given by

$$\boxed{f(x) = \frac{1}{b-a}}$$

$\forall x \in [a, b]$

$a \leq x \leq b$

Here $\mu = \frac{b+a}{2}$

$\sigma^2 = \frac{(b-a)^2}{12}$

Exponential Distribution

A continuous R.V X is said to follow an Exponential Distri. If its p.d.f is given by

$$\boxed{f(x) = \lambda e^{-\lambda x}}$$

where $x \geq 0$

Here $\mu = \frac{1}{\lambda}$, $\sigma^2 = \frac{1}{\lambda^2}$