UNIT-5 [part-2]

Branch And Bound

* The backtracking Algorithm is effective for decision problems, but it is not designed for optimization problems. This draw back is rectified in case of branch and bound technique.

* In this algo we will use bounding function i.e similar to backtracking.

* The main difference b/w backtracking and branch and bound technique is. If we get a solution then we will terminate the search procedure in backtracking,

* where as in branch and bound technique, we will continue the searching until we will get an optimal solution.

S.No	Backtracking method	Branch And Bound method
1	In this technique, the solution is obtained using DFS	1. In this technique DFS, BFS, can be used to obtain the solution
2	Backtracking Approach provides solutions to decision problem	2. Branch And Bound technique is used to solve optimization problems
3	There is a possibility of obtaining bad solutions	3. No bad solutions are obtained.
4	A state space tree is not searched completely instead, the process of searching	4. State space tree generated using branch and bound method is searched completely,

	terminates as soon as the solution is obtained	since there is a possibility of obtaining an optimum solution at any point in the state space tree.
5.	Backtracking technique is used on problems like graph coloring, sum of subsets, n-queens etc	5. Branch and Bound method is applied to the problems like TSP, job sequencing with deadlines, 0/1 knapsack etc.

Travelling saly person problem

* If there are n cities and cost of travelling from one city to other city is given. A salesman has to start from any one of the city and has to visit all the cities exactly once. And has to return to the starting place with shortest distance or minimum cost

* Assume that every tour starts and ends at vertex 1. To use least cost branch and bound to search the travelling salesperson state space tree,

Reduced cost matrix:- A row or column is said to be reduced if it contains atleast one zero and all remaining entries are non-negative. A matrix is reduced if every row and column is reduced.

1) If a constant t is choosen to be minimum entry in row i or column j then subtracting it from all entries in row i (column j) will introduce a zero into a row i (column j)

2) the total amount subtracted from the columns and rows is lower bound on the length of a minimum cost tour and can be used as the

$\hat{c}(x)$ value for the root of state space tree.

* with every node in state space tree, we Associate a reduced cost matrix.

* let A be the reduced cost matrix for node R. let S be the child of R such that the edge (R,S) corresponds to including edge (i,j) in the tour.

* If S is not a leaf node then the reduced cost matrix for node S can be obtained as follows.

1) change all entries in row i and column j of A to ∞

2) set $A(j,i)$ to ∞. To prevent the use of edge (j,i).

3) Apply row reduction and column reduction except for rows and columns containing ∞.

4). the total cost for node S can be calculated as.

$$\hat{c}(S) = \hat{c}(R) + A(i,j) + r$$

where r is the total Amount subtracted in rows and columns.

Q:- solve the TSP problem using LCBB (3)

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & 20 & 30 & 10 & 11 \\ 2 & 15 & \infty & 16 & 4 & 2 \\ 3 & 3 & 5 & \infty & 2 & 4 \\ 4 & 19 & 6 & 18 & \infty & 3 \\ 5 & 16 & 4 & 7 & 16 & \infty \end{array}$$

Sol:-

Row Reduction :- select minimum value in a row and subtract with all the values in a row

$$\begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \begin{matrix} 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{matrix} \Rightarrow \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix}$$

21 total

* in first row 10 is minimum value, so subtract with all the values in a row
* in second row 2 is minimum value, so subtract with remaining values in a row
* in third row 2 is minimum value, so subtract with remaining values in a row
* in 4th row 3 is minimum value, so subtract with remaining in a row
* In 5th row 4 is minimum values, so subtract with remaining values in a row.

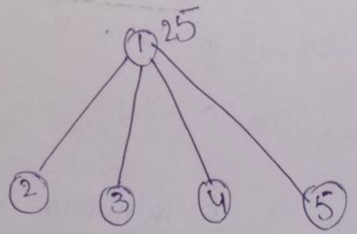
Column Reduction:- select the minimum value in a column and subtract with each value in a column.

$$\begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} \Rightarrow \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

$$1 \quad 0 \quad 3 \quad 0 \quad 0$$

* In first column 1 is minimum value, subtract with remaining values in a column. And soon for remaining colums in a matrix.

* Total Amount of subtraction $\underline{r = 21 + 4 = 25}$



consider the path (1,2): change all entries of first row and second colum of reduced matrix to ∞, And set A(2,1) to ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

Apply row reduction And column reduction

$r = 0$

$\hat{c}(S) = \hat{c}(R) + A(i,j) + r$

$\hat{c}(2) = \hat{c}(1) + A(1,2) + 0$

$= 25 + 10 + 0$

$\hat{c}(2) = 35$

Consider the path (1,3): change all Entries of first Row And third column of reduced matrix to ∞. And set $A(3,1)$ to ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

11

Applying row & column reduction. And total Amount of subtraction $r = 11 + 0 = 11$

$\hat{c}(3) = \hat{c}(1) + A(1,3) + r$

$= 25 + 17 + 11$

$= 53$

Consider the path (1,4) :- change All Entries of first row and 4th column to ∞. And set A(4,1) to ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

Apply row & colum reduction All rows & Columns minimum values are 0. So Total cost of subtraction

$r = 0 + 0 = 0$

$$\hat{c}(4) = \hat{c}(1) + A(1,4) + r$$
$$= 25 + 0 + 0$$
$$= 25$$

Consider the path (1,5) :- change All Entries of first row and 5th column to ∞. And set A(5,1) to ∞

$$\begin{array}{c|c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} & \begin{array}{c} \\ 2 \\ 0 \\ 3 \\ 0 \end{array} \\ \begin{array}{ccccc} 0 & 0 & 0 & 0 & \end{array} & \end{array}$$

Apply row & column reduction so. second ~~column~~ Row minimum value is 2. So subtract All values.

* 3rd row minimum value is 3. so subtract Remaing values

* there is no minimum valey in All columns.

* So the reduced matrix is

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

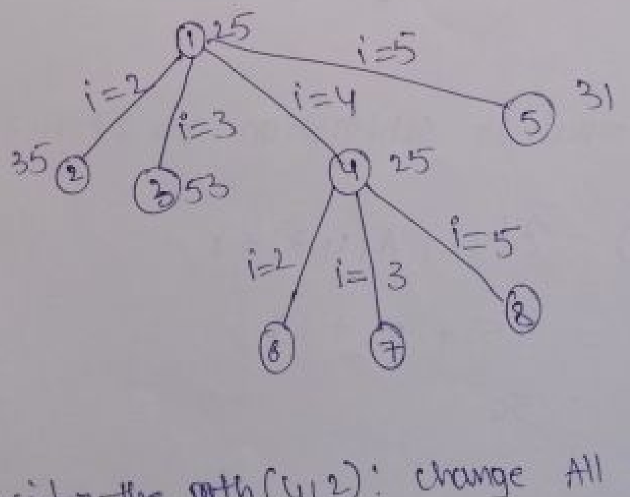
Total Amount of subtraction $r = 5+0 = 5$

$$\hat{c}(5) = \hat{c}(1) + A(1,5) + r$$
$$= 25 + 1 + 5$$
$$= 31$$

* Since the minimum cost is 25, select node 4.

The matrix obtained for path (1,4) is considered as reduced cost matrix.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$



Consider the path (4,2): change All entries of fourth row And 2nd column to ∞. And set $A(2,1)$ to ∞.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

Apply row & column reduction to to each row & column.

$\gamma = 0 + 0 = 0$

$$\hat{c}(2) = \hat{c}(4) + A(4,2) + \gamma$$
$$= 25 + 3 + 0$$
$$= 28$$

consider the path (4,3):- change All Entries of fourth row And third column to ∞. And set (3,1) to ∞

$$\begin{array}{cc} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} & \begin{array}{c} \\ \\ 2 \\ \\ \\ \end{array} \\ \begin{array}{ccccc} 11 & & & & \end{array} & \end{array} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix}$$

total Amount of subtraction $\gamma = 2 + 11 = 13$

$$\hat{c}(3) = \hat{c}(4) + A(4,3) + \gamma$$
$$= 25 + 12 + 13$$
$$= 50$$

Consider the path (4,5):- Change All Entries of Fourth row and fifth Column to ∞. And set A(5,1) to ∞.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \Rightarrow \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix} \end{array} \begin{array}{c} \\ 11 \\ \\ \\ \\ \end{array}$$

* Apply row & column reduction method. in second row minimum value is 11. so subtract Remaining value in row with value 11. So matrix is

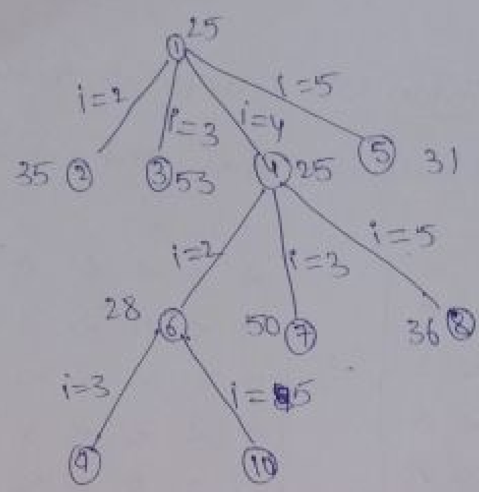
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

total Amount of subtraction

$r = 11+0 = 11$

$$\hat{c}(5) = \hat{c}(4) + A(4,5) + r$$
$$= 25 + 0 + 11$$
$$= 36$$

* So minimum cost is 28, select node 2.



* The matrix obtained for path (4,2) is considered as reduced cost matrix

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

consider the path (2,3) :- change All entries of second row and 3^{rd} column to ∞. And set $A(3,1)$ to ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{bmatrix} 2$$

11

Apply row & column reduction in 3^{rd} row minimum value 2. So subtract will remaining values in a row.

* 1^{st} column min value 11 is subtracted with remaining values

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$$

total Amount of subtraction $\gamma = 2+11$

$\gamma = 13$

$$\hat{c}(3) = \hat{c}(2) + A(2,3) + r$$
$$= 28 + 11 + 13$$
$$= 52$$

Consider the path (2,5):- change All Entries of 2nd row & 5th column to ∞. And set $A(5,1)$ to ∞.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

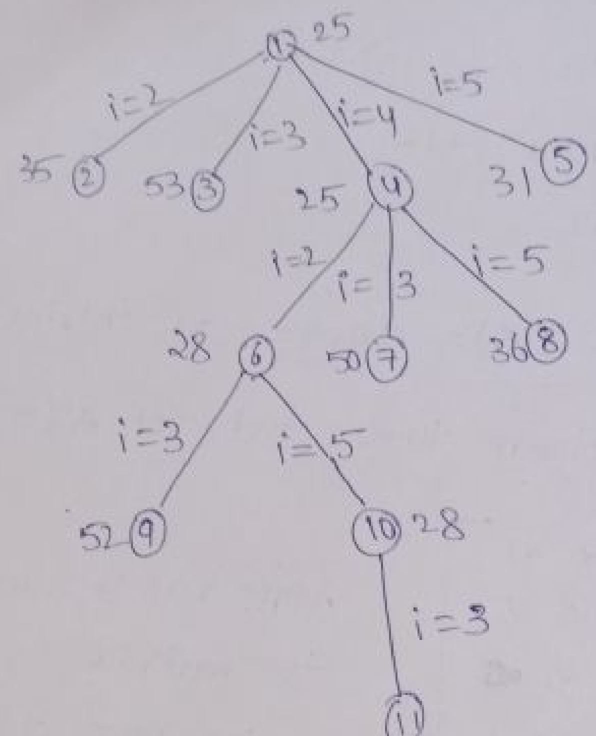
Apply row & column reduction to matrix

$r = 0 + 0 = 0$

$$\hat{c}(5) = \hat{c}(2) + A(2,5) + r$$
$$= 28 + 0 + 0$$
$$= 28$$

* Since the minimum cost is 28, select node is 5.
* the matrix obtained for path (2,5) is considered as reduced cost matrix.

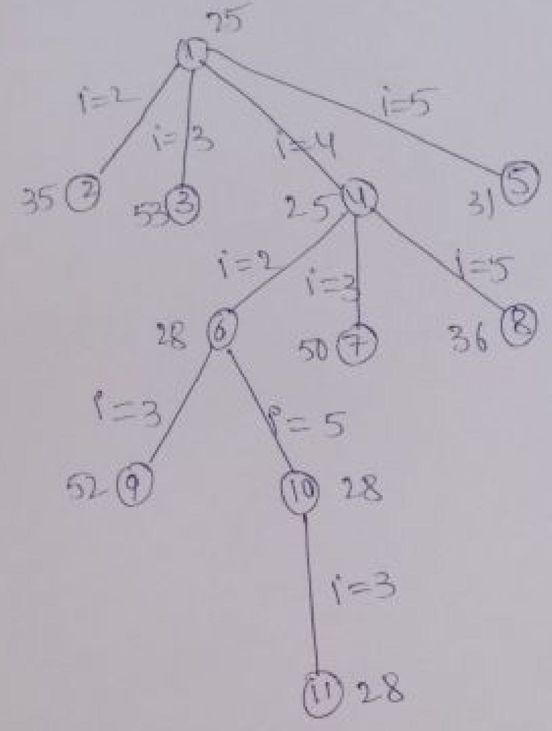
$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$



Consider the path (5,3):- change All entries of 5th row & 3rd column to ∞. And set A(3,1) to ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

Apply row & column reduction so $r = 0$



The path is $1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 3 \rightarrow 1 =$

minimum cost $= 10+6+2+7+3 = \underline{28}$

0/1 Knapsack problem

* The 0/1 Knapsack problem states that, there are n objects given and capacity of knapsack is m. then select some objects to fill the Knapsack in such a way that it should not exceed the capacity of Knapsack and maximum profit can be earned.

* The 0/1 Knapsack problem can be stated as

$$\max Z = P_1x_1 + P_2x_2 + \dots + P_nx_n$$

subject to $w_1x_1 + w_2x_2 + \dots + w_nx_n \le m$, $\underline{x_i = 0 \text{ or } 1}$

* A Branch And Bound techniwe is used to find solution to the Knapsack problem. But we can't directly Apply the Branch and Bound tehniwe to the knapsack problem.

* Because the Branch And Bound deals only the minimizing problems. we modify the Knapsack problem to the minimization problem. The modified problem is

$$\min Z = -P_1x_1 - P_2x_2$$

subject to $w_1x_1 + w_2x_2 + \dots w_nx_n \le m$, $x_i = 0 \text{ or } 1$

* let $\hat{c}(x)$ and $\hat{u}(x)$ are the two cost functions such that $\hat{c}(x) \le c(x) \le \hat{u}(x)$, satisfying the requirements where $c(x) = -\Sigma p_i x_i$.

* The $c(x)$ is the cost function for Answer node x, which lies b/w two functions called <u>lower and upper bounds</u> for the cost function $c(x)$.

* the search begins at the root node. initially we compute the lower and upper bound at root node called $\hat{c}(1)$ and $\hat{u}(1)$. consider the first variable x_1 to take a decision. the x_1 takes values 0 or 1.

* compute the lower & upper bound in each case of the variable. select the node whose cost is minimum.

$$c(x) = \min\{c(\text{lchild}(x)), c(\text{rchild}(x))\}$$

$$c(1) = \min\{c(2), c(3)\}$$

* the problem can be solved by making a sequence of decisions on the variable $x_1, x_2, \ldots x_n$ level wise.

* the path from root to the leaf node whose height is maximum is selected and is the solution space for the knapsack problem.

Q1 Draw a portion of state space tree generated by LCBB by following Knapsack problem $n=5$, $m=12$ (10)

$(P_1 P_2 P_3 P_4 P_5) = (10, 15, 6, 8, 4)$

$(w_1 w_2 w_3 w_4 w_5) = (4, 6, 3, 4, 2)$

Sol:- convert the profits to negative $(P_1 P_2 P_3 P_4 P_5) = (-10, -15, -6, -8, -4)$ calculate the lower bound and upper bound for each node.

place first item in the bag i.e, 4 remaining weight is

$12 - 4 = 8$

place second item in the bag i.e 6, Remaining bag weight is

$8 - 6 = 2$

since fractions are not allowed in calculation of upper bound, so we can't place third and fourth item in the bag. place fifth item

$2 - 2 = 0$

$\therefore$ profit earned $= -10 - 15 - 2 = -29 =$ upper bound

To ~~place the~~ calculate the lower bound, place third item in a bag since fractions are Allowed.

$\therefore$ lower bound $= -10 - 15 - \frac{2}{3} * 6 = -29$

$\therefore \; \hat{c}(1) = -29, \; u(1) = -29$

* for node 2, $x_1 = 1$ means we should place first item in the bag.

place first item in the bag i.e $= 4$ Remaining weight is

$12 - 4 = 8$

place second item in the bag ie 6 Remaining weight is

$8-6=2$

when we want to calculate upper bound we can't place fraction of items in the bag, so third and fourth item can't be placed. next we have to placed 5th item in the bag.

$2-2=0$

$\therefore$ profit earned $=-10-15-4=-29=$ upperbound

$\therefore \hat{u}(2)=-10-15-4=-29$

* when we want to calculate lowerbound we can place fraction of items in the bag. so now we can third item in the bag.

place first item in the bag i.e 4, Remaining weight

$12-4=8$

place second item in the bag i.e 6 Remaining weight

$8-6=2$

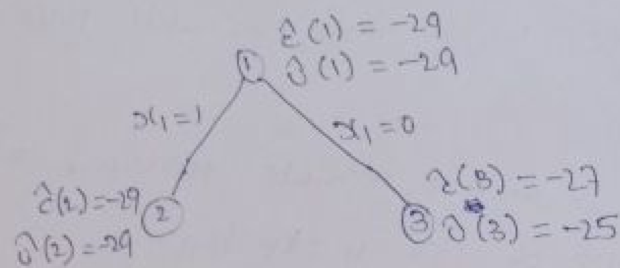
place third item in the bag ie $=3$ Remaining weight

$2-2/3=1.3$

$\therefore$ lowerbound $\hat{c}(2)=-10-15-\frac{2}{3}*6$

$\hat{c}(2)=-10-15-4=-29$

$\therefore \hat{u}(2)=-29,\ \hat{c}(2)=-29$



∴ select the minimum of lowerbound i.e

$$\min\{\hat{c}(2), \hat{c}(3)\} = \min\{-29, -27\}$$
$$= -29$$

∴ choose the node 2.

∴ first object is selected $x_1 = 1$

For node 4 $(x_2 = 1)$:- place 2^{nd} item in the bag

* when we want to calculate upperbound fraction can't be Allowed.
* place first item in the bag i.e = 4, Remaining weight

$$12 - 4 = 8$$

* place second item in the bag i.e = 6, Remaining weight

$$8 - 6 = 2$$

* fraction can't be placed, so we can't place 3^{rd} & 4^{th} item in the bag, we can place only 5^{th} item in the bag

$$2 - 2 = 0$$

∴ upperbound $\hat{u}(4) = -10 - 15 - 4 = -29$

* when we want to calculate lowerbound fraction can be Allowed.

for node 3 $(x_1=0)$:- that means we can't place first item in the bag.

* when we want to calculate upperbound we can't place fraction of item in the bag.

* so place second item in the bag i.e 6. Remaining weight

$$12-6=6$$

* place third item in the bag i.e 3, Remaing weight

$$6-3=3$$

* we can't place fraction of item in the bag, so we can't place 4th item in the bag. place 5th item in the bag

$$3-2=1$$

$\therefore$ profit earned $= -15-6-4 = -25$

$$\therefore \hat{u}(3) = -25$$

* when we want to calculate lowerbound we can place fraction of items in the bag

* place second item in the bag i.e 6, Remaining weight

$$12-6=6$$

* place third item in the bag i.e 3, Remaing weight

$$6-3=3$$

* place fourth item in the bag i.e 4, Remaining weight

$$3-\frac{3}{4}=2.25$$

$\therefore \hat{c}(3) = -15-6-\frac{3}{4}*8 = -27$

* place first item in the bag ie 4, Remaining weight

$12-4=8$

* place second item in the bag i.e 6, Remaing weight

$8-6=2$

* place third item in the bag i.e. 3, Remaining weight

$2-2/3=1.3$

$\therefore \hat{c}(4) = -10-15-\frac{2}{3}*6 = -29$

$\therefore \hat{c}(4) = -29, \quad \hat{u}(4) = -29$

for node 5 $(x_2=0)$:- that means we can't place second item in the bag, then calculate upper bound & lower bound.

* place first item in the bag i.e = 4, Remaining weight

$12-4=8$

* place third item in the bag i.e = 3 Remaining weight

$8-3=5$

* place 4th item in the bag i.e = 4 Remaining weight

$5-4=1$

$\therefore$ upper bound $\hat{u}(5) = -10-6-8 = -24$

$\therefore \hat{u}(5) = -24$

* when we calculate lower bound, place fraction of item in the bag

* place first item in the bag i.e = 4 Remaing weight

$12-4=8$

* place third item in the bag. i.e = 3 Remaining weight

$8-3=5$

* place 4th item in the bag i.e 4, Remaining weight

$5-4=1$

* place 5th item in the bag i.e 2, Remaining weight

$1-\frac{1}{2} = 0.5$

$\therefore \hat{c}(5) = -10-6-8-4\times\frac{1}{2} = -26$

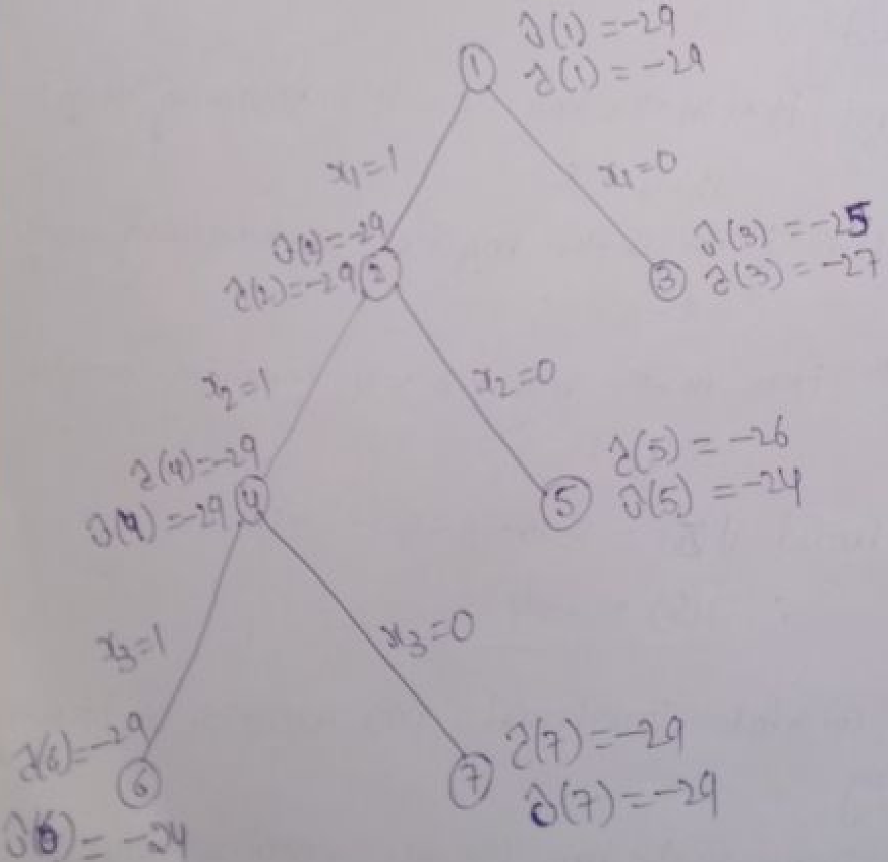
$\therefore \hat{c}(5) = -26$

$\therefore$ select minimum of lower bound

$\min\{\hat{c}(4), \hat{c}(5)\} = \{-29, -26\} = -29$

$\therefore$ node 4 is selected

$\therefore$ second object is selected $x_2=1$



for node 6 ($x_3=1$) :- we can place 3^{rd} item in the bag

upperbound:- fractions can't be Allowed

place first item in the bag i.e 4, Remaining weight

$12-4=8$

place ~~second~~ third item in the bag i.e 6, ''

$8-\cancel{3}=5$

place 4^{th} item in the bag i.e 4 ''

$5-4=1$

$\therefore \hat{u}(6) = -10-6-8 = -24$

lowerbound :- fractions can be Allowed

place first item in the bag i.e 4, Remaining.

$12-4=8$

place second item in the bag i.e 6 ''

$8-6=2$

place third item in the bag i.e 3 ''

$2-\frac{2}{3}$

$\therefore \hat{c}(6) = -10-15-\frac{2}{3}*6 = -29$

$\therefore \hat{c}(6) = -29, \hat{u}(6) = -29$

for node 7 ($x_3=0$) :- can't place 3^{rd} item in the bag

upperbound :- fractions can't be Allowed

place first item in the bag i.e 4, Remaining weight

$12-4=8$

place second item in the bag i.e 6, Remaining weight

$8-6=2$

place 5^{th} " " 2 "

$2-2=0$

$\therefore \hat{u}(7) = -10-15-4 = -29$

lowerbound:- fraction can be Allowed

place first item in the bag i.e 4, Remaining weight

$12-4=8$

place second item in the bag i.e 6, "

$8-6=2$

place 4^{th} item in the bag i.e 4 "

$2-2/4$

$\therefore \hat{c}(7) = -10-15-\cancel{8}\frac{2}{4} = -29$

$\therefore \hat{c}(7) = -29, \hat{u}(7) = -29$

$\therefore$ select minimum lowerbound, lowerbounds are same, select minimum of upper bounds

$\therefore \min\{\hat{u}(6), \hat{u}(7)\} = \min\{-24, -29\} = -29$

$\therefore$ node 7 is selected

$\therefore$ third object is not selected $x_3 = 0$

Consider 4^{th} variable

for node 8 $(x_4 = 1)$:- place 4^{th} item in the bag

upper bound:- fractions can't be Allowed

* place ~~first~~ 2^{nd} item in the bag i.e 6, Remaining weight

$12 - 6 = 6$

* place 4^{th} item in the bag i.e 4, " "

$6 - 4 = 2$

* place 5^{th} item in the bag i.e 2, $2 - 2 = 0$

$\therefore \hat{u}(8) = -4 - 15 - 8 = -27$

lower bound:- fractions can be Allowed

* place 1^{st} item in the bag i.e 4, Remaining weight

$12 - 4 = 8$

* place 2^{nd} item in the bag i.e 6, " "

$8 - 6 = 2$

* place 3^{rd} item in the bag i.e 3, " "

$2 - \frac{2}{3} =$

$\therefore \hat{c}(8) = -10 - 15 - \frac{2}{3} \times 6 = -29$

for node 9 $(x_4 = 0)$:- can't place 4^{th} item in the bag

upper bound:- fractions can't Allowed

* place 1^{st} item in the bag i.e 4, Remaing weight

$12 - 4 = 8$

* place 2^{nd} item in the bag i.e = 6, " "

$8 - 6 = 2$

* place 5^{th} item in the bag i.e 2, " "

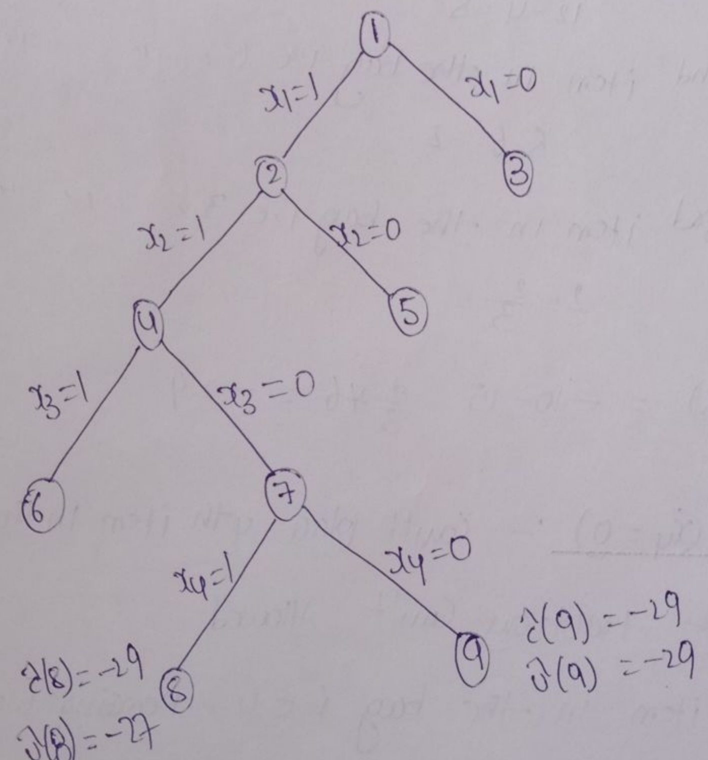
$2 - 2 = 0$

$\therefore \hat{u}(9) = -10-15-4 = -29$

Lowerbound :- fractions can be Allowed

* place 1st item in the bag i.e 4, Remaining weight
$12-4=8$
* place 2nd item in the bag i.e 6, " "
$8-6=2$
* place 3rd item in the bag i.e 3, " "
$\frac{2}{3}$

$\therefore \hat{c}(9) = -10-15-\frac{2}{3} * 6 = -29$



$\therefore$ Since the lowerbounds are same, select the minimum of upperbounds.

$\therefore \min\{\hat{u}(8), \hat{u}(9)\} = \min\{-27, -29\}$
$= -29$

$\therefore$ select 9th node. so $x_4 = 0$

consider 5^{th} variable

for node 10 ($x_5=1$) :- place 5^{th} item in the bag

upper bound:- fraction can't be Allowed

* place 1^{st} item in the bag i.e 4, Remaining weight

$12-4=8$

* place 2^{nd} item in the bag i.e 6, " "

$8-6=2$

* place 5^{th} item in the bag i.e 2, " "

$2-2=0$

$\therefore \hat{u}(10) = -10-15-4=-29$

lower bound:- fraction be Allowed

* place first item in the bag i.c 4, Remaing weight

$12-4=8$

* place 2^{nd} item in the bag i.c 6, " "

$8-6=2$

* place 3^{rd} item in the bag i.e 3, " "

$2-\frac{2}{3}$

$\therefore \hat{c}(10) = -10-15-\frac{2}{3}\times 6=-29$

for node 11 ($x_5=0$) :- 5^{th} item can't be placed in the bag

upper bound :- fraction can't be Allowed

* place 1^{st} item in the bag i.c 4, Remaining weight

$12-4=8$

* place 2^{nd} item in the bag i.e 6, " "

$8-6=2$

$\therefore \hat{u}(11) = -10-15=-25$

Lowerbound :- fractions can be Allowed

* place 1st item in the bag i.e 4, Remaining weight

$12-4=8$

* place 2nd item in the bag i.e 6, " "

$8-6=2$

* place 3rd item in the bag i.e 3, " "

$2-\frac{2}{3}=$

$\therefore \hat{c}(11) = -10-15-\frac{2}{3}*6 = -29$

∴ select minimum of Lowerbounds.

$\therefore \min\{\hat{c}(10), \hat{c}(11)\} = \min\{-29, -29\}$

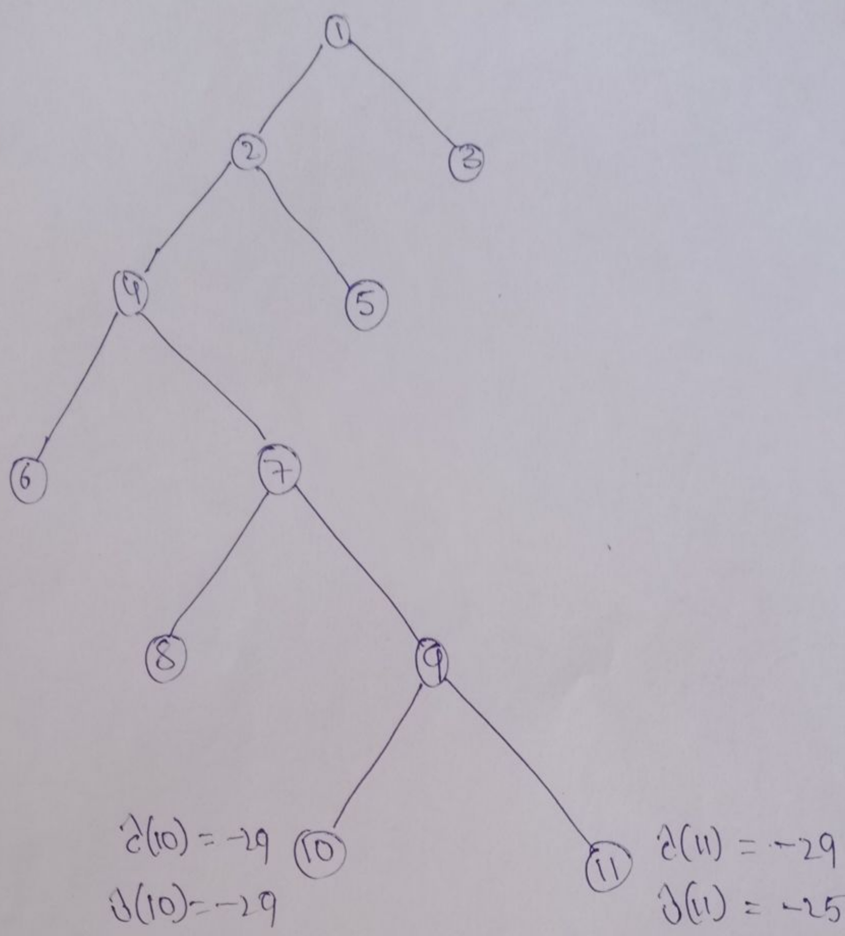
∴ since lower bounds are same, so select minimum of upperbounds

$\therefore \min\{u(10), u(11)\} = \min\{-29, -25\}$

$= -29$

∴ node 10 is selected

∴ fifth object is selected $x_5 = 1$



∴ The path is 1 – 2 – 4 – 7 – 9 – 10

∴ The solution for 0/1 Knapsack problem is

$(x_1, x_2\ x_3\ x_4\ x_5) = (1, 1, 0, 0, 1)$

maximum profit $= 10 + 15 + 4 = 29$

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Q:- Draw the portion of state space tree generated by LC Knap and FIFO Knap for the Knapsack instance $n=4$, $m=15$, $(p_1\ p_2\ p_3\ p_4) = (10, 10, 12, 18)$
$(w_1\ w_2\ w_3\ w_4) = (2, 4, 6, 9)$.

FIFO Branch And Bound Solution

* The problem is to find the most valuable subset of the items that fit in the knapsack where the given (n) items are of known weights w_i and values v_i

$$n = 4, \; m = 15, \; (v_1 v_2 v_3 v_4) = (10, 10, 12, 18)$$
$$(w_1 w_2 w_3 w_4) = (2, 4, 6, 9)$$

* The FIFOBB Algorithm proceeds with node 1 as the root node and makes it E-node. And hence nodes 2 and 3 are produced and therefore node 2 and 3 are sent to queue

first out ← | n_2 | n_3 | | | ... ← in (first)

* As n_2 is staying first in the queue, it becomes E-node and it produces node 4 and 5 as children. And node 4 and 5 are sent to queue.

out ← | n_3 | n_4 | n_5 | | | ... ← in

* As n_3 is staying first in the queue, it becomes E-node, And it produces node 6 and 7 as childerens And node 6 and 7 are sent to queue

out ← | n_4 | n_5 | n_6 | n_7 | . | ... ← in