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## UNIT-I: PROBABILITY

**INTRODUCTION:** In common the term probability refers to the chance of happening (or) non-happening of an event. The use of the word chance in any statement indicates that there is an element of uncertainty about the statement. Probability is a part of our daily life's though many of us may not be conscious of it. Consider the following statements.

1.It may rain today (or) tomorrow?

2.Probably I will get a first class in the first semester?

3.India might draw (or) win the test cricket series against Australia?

There is an element of uncertainty associated with each of the above statements. We have not assigned any numerical value of these statements. If we could provide some numerical value, the statements would become more precise. The theory of probability provides a numerical measure of the element of uncertainty. It enables us to take a decision under conditions of uncertainty with a calculated risk.

In business firms always face uncertain situations and yet they have to take decisions. Sometimes the risk is too high. A wrong decision may involve huge loss to the firm. As such great care has to be exercised before taking a particular decision. In this respect the management is guided by the theory of probability.

- ❖ Probability means the numerical evaluation of chance factor.
- ❖ The limits of probability are 0 & 1. i.e., $0 \le P(E) \le 1$
- ❖ The theory of probability was introduced by Pascal & Fermat in 17$^{th}$ Century.

## BASIC DEFINITIONS OF PROBABILITY

**Experiment**: A physical quantity (or) activity which gives the result is called Experiment.

Examples: 1) Tossing of a coin.

2) Throwing a die.

3) Measuring the length of black board (or) table.

Experiments are classified into two types:

1. Deterministic experiment.
2. Non-Deterministic experiment (or) Random experiment.

**1. Deterministic experiment:** The experiment which gives certain result (or) same result, then the experiment is called Deterministic experiment.

Example: Measuring the length of the table (or) black board.

**2. Non-Deterministic experiment (or) Random experiment**: The experiment is repeated under essential similar conditions and it has several possible outcomes which we know in advance but prediction of an outcome is not possible, such experiment is called random experiment.

Example: Tossing of a coin, Throwing a die.

**1) Outcome:** The result of a random experiment is called outcome.

Example:

1. In tossing of a coin, HEAD and TAIL are possible outcomes.
2. In throwing a die, 1, 2, 3, 4, 5, 6 are possible outcomes.

2) **Sample Space:** The set of all possible outcomes of a random experiment is called Sample Space. It is denoted by 'S'. Sample space is also called Universal set (or) Event Space (or) Possibility set.

Example 1: If a die is thrown, there are 6 possibilities and we say that the sample space associated with the experiment of throwing a die consists of 6 sample points and is denoted as S = {1,2,3,4,5,6}

Example 2: In tossing two coins, the sample space S = {HH, HT, TH, TT}

3) **Sample point:** The number of possible outcomes of the Sample space is called sample point.

Example: In tossing of two coins, the number of sample points are $n(s)=4$.

4) **Trial and Event:** The performance of a random experiment is called a trial and the outcome is an event.

Example: Tossing of a coin to be called a trial and getting Head (or) Tail is called event. Events could be either simple (or) compound.

5) **Simple Event:** An event is said to be simple, if it correspondent to a single outcome.

Example: In throwing of a die the chance of getting 4 is a simple event.

6) **Compound Event:** An event is said to be a compound, if it corresponds to two (or) more possible outcomes.

Example: In throwing a die the chance of getting even number, results in 2, 4, 6.

7) **Exhaustive Events:** The total number of possible outcomes of an experiment is called exhaustive events.

Example 1: In throw of a single die the exhaustive events are 6.

Example 2: In tossing of a single coin the exhaustive events are 2.

**Note:** (a) In general, tossing on 'n' coins, the number of exhaustive events are $2^n$

(b) In general, throwing of 'n' dice the number of exhaustive events are $6^n$

**8) Mutually Exclusive Events:** Two (or) more events are said to be mutually exclusive, if the happening of one event excludes (stops) the happening of all other events in a same experiment is called mutually exclusive events.

Example: In tossing of a coin Head & Tail are mutually exclusive events.

**9) Equally Likely Events:** Two (or) more events are said to be equally likely, if they have same chance of occurrence. i.e., there is no preference of any one event over the other.

Example: In throw of an unbiased coin the coming up of head (or) tail is equally likely events.

**10) Favorable Events:** The number of outcomes which results in the happening of a desired event is called favorable event.

Examples: 1. If two coins are tossed to get exactly one head, the favorable events are {HT, TH}

2. In throwing a dice, the favorable event to get an even number are {2, 4, 6}

3. In a single throw of a die the number of favorable cases of getting an odd number are {1, 3, 5}

**11) Independent Events:** If the happening one event in one trail does not affected by the happening of any other events in another trail, then they are called independent event.

Example: To get a Head in the first toss is independent of getting a Head (or) Tail in the second toss.

**12) Dependent Events:** If the happening one event in one trail does affected by the happening of any other events in another trail, then they are called dependent event.

**13) Certain Event (or) Sure Event (or) Possible Event:** An event which always happens is called a certain event and it is denoted by $P(E)=1$.

Example: Getting head (or) tail in toss of a coin is a certain event.

**14) Impossible Event:** An event which never happens is called an impossible event.

It is denoted by $P(\Phi)=0$.

Example: Getting 7 in a throw of a die is an impossible event.

- **MATHEMATICAL (OR) CLASSICAL (OR) A PRIORI PROBABILITY**

**DEFINITION:** If a random experiment results in 'n' exhaustive, mutually exclusive and equally likely outcomes, out of which 'm' are favorable to the occurrence of an event 'E', then the probability of occurrence of 'E', usually denoted by P(E), is given by

$$P(E)=p=\frac{Number\ \ of\ \ \ favourable\ \ cases}{Total\ \ number\ \ of\ \ exhaustive\ \ cases}=\frac{m}{n}$$

$$\therefore P(E)=p=\frac{n(E)}{n(S)}=\frac{m}{n}$$

$$and\ \ the\ probability\ of\ \ non-occurrence\ of\ \ 'E'$$

$$P\left(\bar{E}\right)=q=\frac{n-m}{n}=\frac{n}{n}-\frac{m}{n}$$

$$=1-\frac{m}{n}$$

$$P\left(\bar{E}\right)=1-P(E) \qquad [\because\ q=1-p\ \ (or)\ \ p+q=1]$$

**Remarks:**

1. Since, m and n are non-negative integers and total probability is 1, hence probability of any event is a number lying between 0 and 1.

   i.e., $0 \le p \le 1$, $0 \le q \le 1$.

2. Probability of happening of the event 'A' is P(A) is also known as the probability of success and usually written as 'p' and the probability of non-happening of 'A' i.e., $P(A^C)$ is known the probability of failure which is usually denoted by 'q'.

3. If P(E)=0, then the event E is Impossible event.

4. If P(E)=1, then the event E is Certain event.

5. E is the complimentary event of E and its probability is $P\left(\overline{E}\right)=1-P(E)$

## ➢ Limitations (or) drawbacks (or) demerits of Mathematical probability:

The mathematical probability breaks down in the following cases:

(1) If the exhaustive number of outcomes (n) of the random experiment is Infinite (or) Unknown.

(2) If the various outcomes of the random experiment are not equally likely.

## ➢ STATISTICAL (OR) EMPIRICAL PROBABILITY

**DEFINITION:** If an experiment is performed repeatedly under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event occurs to the number of trials.

Symbolically, if in 'n' trails an event E happens 'm' times, then the probability of the happing of 'E' denoted by P(E), is given by

$$P(E) = \lim_{n \to \infty} \frac{m}{n}$$

## ➢ AXIOMS OF PROBABILITY: (Axiomatic Probability):

Let 'S' is a sample space and 'A' is an event which is defined on sample space 'S', then P(A) is said to be probability of A if it satisfies the following properties (or) axioms:

**1.Axioms of Positivity:** If 'A' is any event, then $P(A) \geq 0$

**2.Axioms of Certainty:** The probability of an entire sample space is 1. i.e., $P(S) = 1$

**3.Axioms of Union:** If A, B are mutually exclusive (or) disjoint events subset of S, then $P(A \cup B) = P(A) + P(B)$

## ➢ CONDITIONAL PROBABILITY:

i) If the probability of happening the event A when the event B has already happened is called conditional probability of 'A' Given 'B'. It is denoted by $P\left(A/B\right)$.

i.e., $P\left(A/B\right) = \frac{n(A \cap B)}{n(B)} = \frac{P(A \cap B)}{P(B)} \qquad ;if \quad P(B) > 0$

ii) If the probability of happening the event B when the event A has already happened is called conditional probability of ‘B’ given ‘A’. It is denoted by $P\left(B/A\right)$.

$$\text{i.e., } P\left(B/A\right)=\frac{n(A\cap B)}{n(A)}=\frac{P(A\cap B)}{P(A)} \qquad ;if \quad P(A)>0$$

- **Some definitions based on Conditional probability:**

**1) Compound Event:** When two or more events occur in conjunction with each other, their joint occurrence is called “Compound Event”.

Examples:

i) If 2 balls are drawn from a bag containing 4 green, 6 black and 7 white balls, the event of drawing 2 green balls or 2 white balls is a Compound event.

ii) When a die and a coin are tossed the event of getting utmost 4 on the die and head on the coin is a compound event and separately they are independent events.

**NOTE:** Multiplication theorem of probability is also called theorem of compound probability.

**2) Independent Events:** If the occurrence of the event B is not affected by the occurrence or non-occurrence of the event A, then the event B is said to be independent of A and

P (B / A) = P(B), similarly, we define, P (A / B) = P(A).

**3) Mutually Independent or Simply Independent:** If P (A) $\neq$ 0, P (B) $\neq$ 0 and B is independent of A, then A is independent of B. In this case we say that A, B are “Mutually independent or simply independent events”.

**4) Dependent Events:** If the occurrence of the event B is affected by the occurrence of A, then the events A, B are dependent and P (B / A) $\neq$ P (B).

# SOME THEOREMS ON PROBABILITY

Theorem-1: Probability of Impossible event is zero. i.e., $P(\Phi)=0$

Proof: Let S be sample space and $\Phi$ be the null (or) empty set,

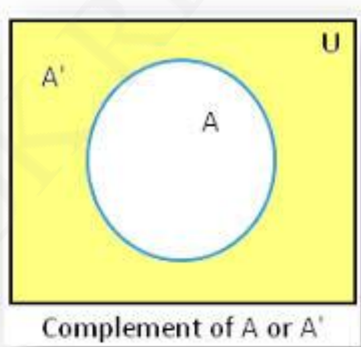
then $S$ & $\Phi$ are two disjoint events and their union is S.

$$\begin{aligned}
&i.e., \quad S\cup\Phi=S \\
&\quad taking\ \ probability\ \ on\ \ both\ \ sides \\
&\quad P(S\cup\Phi)=P(S) \\
&\quad P(S)+P(\Phi)=P(S) \qquad [\because\ Union\ \ axiom] \\
&\quad 1+P(\Phi)=1 \qquad [\because\ Certainty\ \ axiom] \\
&\quad P(\Phi)=1-1 \\
&\quad \therefore\ P(\Phi)=0 \\
&\quad hence\ \ proved
\end{aligned}$$

Theorem-2: Probability of the complementary event of $A^c$ of A is given by

$$P\left(\bar{A}\right)=1-P(A)$$

Proof: Let S be the sample space and A be any event in S.



Complement of A or A'

*From the Venn – diagram*

$A$ & $\overline{A}$ *are two disjoint events and their union is S*

*i.e.,* $A \cup \overline{A} = S$

*taking probability on both sides*

$$P\left(A \cup \overline{A}\right) = P(S)$$

$$P(A) + P\left(\overline{A}\right) = P(S) \qquad [\because \text{Union axiom}]$$

$$P(A) + P\left(\overline{A}\right) = 1 \qquad [\because \textit{certainty axiom}]$$

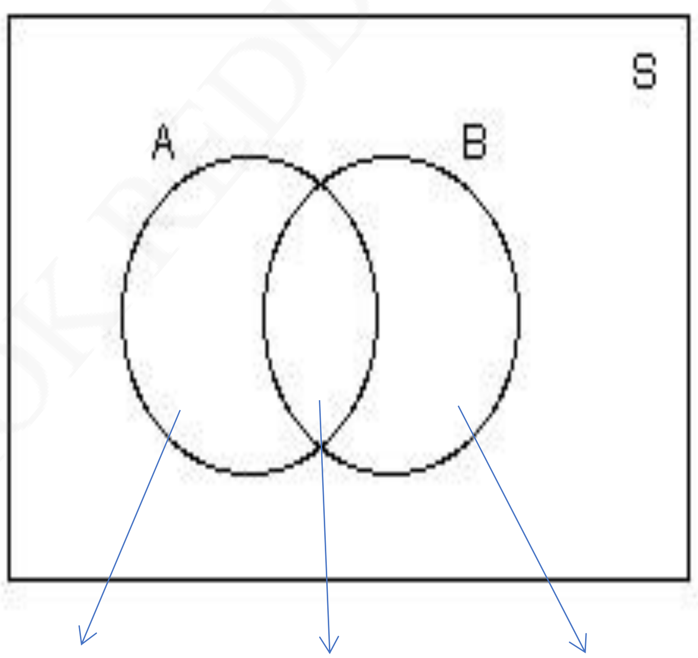
$$\therefore\ P\left(\overline{A}\right) = 1 - P(A)$$

*hence proved*

**Theorem-3:** For any two events A and B defined on a sample space S then

$$P\left(\overline{A} \cap B\right) = P(B) - P(A \cap B)$$

Proof: Let S be the sample space and A and B be any two events defined on S.




$A \cap \overline{B}$ $\qquad$ $A \cap B$ $\qquad$ $\overline{A} \cap B$

From the Venn – diagram

$A \cap B$ & $\bar{A} \cap B$ are two disjoint events and their union is B.

i.e., $B = (A \cap B) \cup (\bar{A} \cap B)$

taking probability on both sides

$$P(B) = P\left[(A \cap B) \cup (\bar{A} \cap B)\right]$$

$$P(B) = P(A \cap B) + P(\bar{A} \cap B) \qquad [\because \text{Union axiom}]$$

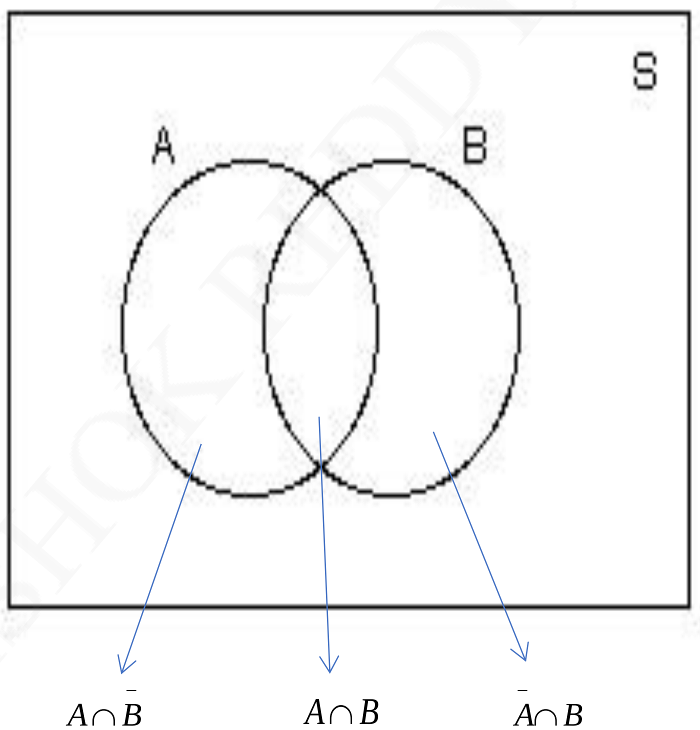
$$\therefore P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

hence proved

**Theorem-4:** For any two events A and B defined on a sample space S then

$$P\left(A \cap \bar{B}\right) = P(A) - P(A \cap B).$$

Proof: Let S be the sample space and A and B be any two events defined on S.



$From\ the\ Venn-diagram$

$A\cap B\ \&\ A\cap \overline{B}$ *are two disjoint events and their union is A.*

$i.e.,\ A=(A\cap \overline{B})\cup(A\cap B)$

taking probability on both sides

$$P(A)=P\left[(A\cap \overline{B})\cup(A\cap B)\right]$$

$$P(A)=P(A\cap \overline{B})+P(A\cap B)\quad (\because Union\ axiom)$$

$$\therefore P(A\cap \overline{B})=P(A)-P(A\cap B)$$

*hence proved*

**Theorem-5:** Let A & B are two events and $B\subseteq A$ then

$$P\left(A\cap \bar{B}\right)=P(A)-P(B)\quad \&\quad P(B)\le P(A)$$

Proof : *Given that* , $B\subseteq A$ *then* $A\cap B=B$

*we know that*, $P(A\cap \bar{B})=P(A)-P(A\cap B)$

$$P(A\cap \bar{B})=P(A)-P(B)$$

$$\therefore\ P(A\cap \bar{B})=P(A)-P(B)$$

*hence proved*

*Since probability can't be negative*

$$P\left(A\cap \bar{B}\right)\ge 0$$

$$P(A)-P(B)\ge 0$$

$$P(A)\ge P(B)\ (or)\ P(B)\le P(A)$$

Theorem-6: Let A & B are two events and $A \subseteq B$ then

$$P\left(\bar{A} \cap B\right) = P(B) - P(A \cap B) \quad \& \; P(A) \leq P(B)$$

$\text{Proof}$ : *Given that* , $A \subseteq B$ *then* $A \cap B = A$

*we know that*, $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$$P(\bar{A} \cap B) = P(B) - P(A)$$

$$\therefore \; P(A \cap \bar{B}) = P(B) - P(A)$$

*hence proved*

*Since probability can't be negative*

$$P\left(\bar{A} \cap B\right) \geq 0$$

$$P(B) - P(A) \geq 0$$

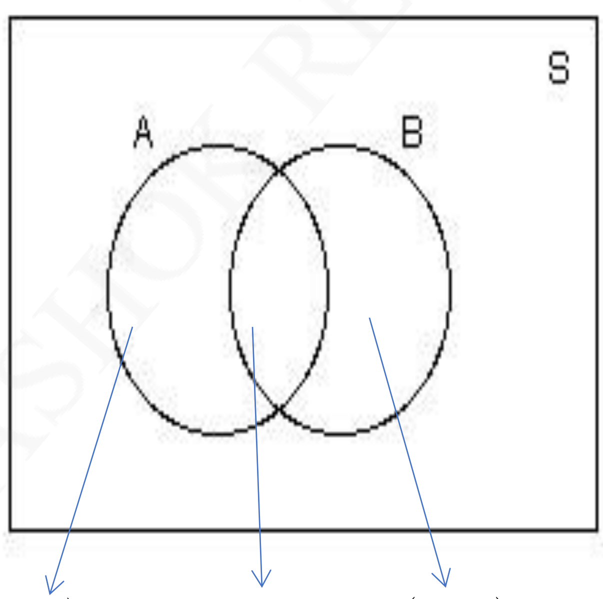
$$P(B) \geq P(A) \;\; (or) \;\; P(A) \leq P(B)$$

**Theorem – 7 : Addition theorem of probability for two events**

*Statement* : *If A & B are two events and are not disjoint events, then*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof: Let A & B are any two events defined on Sample space (S).




$(A \cap \bar{B})$ $(A \cap B)$ $(\bar{A} \cap B)$

$\Pr oof$ :

*From the Venn – diagram*

$A$ & $\left(\bar{A}\cap B\right)$ *are any two disjoint events and their union is* $A\cup B$

*i.e.,* $A\cup B = A\cup\left(\bar{A}\cap B\right)$

*taking probability on both sides*

$$P\left(A\cup B\right)=P\left(A\cup\left(\bar{A}\cap B\right)\right)$$

$$P\left(A\cup B\right)=P\left(A\right)+P\left(\bar{A}\cap B\right) \qquad \left[\because \text{ Union axiom}\right]$$

$$P\left(A\cup B\right)=P\left(A\right)+P\left(B\right)-P\left(A\cap B\right) \quad \left[\because \; P\left(\bar{A}\cap B\right)=P\left(B\right)-P\left(A\cap B\right)\right]$$

$$\therefore\; P\left(A\cup B\right)=P\left(A\right)+P\left(B\right)-P\left(A\cap B\right)$$

*hence proved*

In Case: If A & B are mutually exclusive (or) disjoint events, then

$$P\left(A\cup B\right)=P\left(A\right)+P\left(B\right)$$

Proof: Given that A & B two disjoint events (or) mutually exclusive events,

then $P\left(A\cap B\right)=0$

By using addition theorem

$$P\left(A\cup B\right)=P\left(A\right)+P\left(B\right)-P\left(A\cap B\right)$$

$$\therefore P\left(A\cup B\right)=P\left(A\right)+P\left(B\right) \qquad (\because P\left(A\cap B\right)=0)$$

*NOTE : Addition theorem for three events*

*In case : If A, B & C are three events, then*

$$P\left(A\cup B\cup C\right)=P\left(A\right)+P\left(B\right)+P\left(C\right)-P\left(A\cap B\right)-P\left(B\cap C\right)-P\left(C\cap A\right)+P\left(A\cap B\cap C\right)$$

*in case A, B & C are three disjoint events then*

$$\therefore\; P\left(A\cup B\cup C\right)=P\left(A\right)+P\left(B\right)+P\left(C\right) \qquad \left[\begin{array}{c}\because \text{ Total probability is always unity} \\ P\left(A\right)+P\left(B\right)+P\left(C\right)=1\end{array}\right]$$

**Theorem – 8 : Multiplication theorem of probability for two events**

*Statement :* Let $A$ & $B$ are two events, then

$$P(A\cap B)=P(A)P\left(B/A\right) \quad ; \; P(A)>0$$

(OR)

$$P(A\cap B)=P(B)P\left(A/B\right) \quad ; \; P(B)>0$$

*Proof :* Let $S$ be a Sample Space with exhaustive events $n(S)$.

$A$ & $B$ are any two events in Sample Space $S$.

$n(A), n(B)$ & $n(A\cap B)$ are favourable events in $S$.

The definition of probability :

$$P(A)=\frac{n(A)}{n(S)} \quad , \quad P(B)=\frac{n(B)}{n(S)} \quad , \quad P(A\cap B)=\frac{n(A\cap B)}{n(S)}$$

Let us consider,

$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}$$

dividing & multiply with $\frac{n(A)}{n(A)}$ on R.H.S

$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}\times\frac{n(A)}{n(A)}$$

$$P(A\cap B)=\frac{n(A\cap B)}{n(A)}\times\frac{n(A)}{n(S)}$$

$$P(A\cap B)=P\left(B/A\right)P(A) \qquad \left[\begin{array}{l} \text{by using conditional probability} \\ \because \; P\left(B/A\right)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{\frac{n(A\cap B)}{n(S)}}{\frac{n(A)}{n(S)}}=\dfrac{n(A\cap B)}{n(A)} \end{array}\right]$$

$$\therefore P(A\cap B)=P(A)P\left(B/A\right)$$

hence proved

(OR)

*Let us consider,*

$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}$$

*dividing & multiply with* $\frac{n(B)}{n(B)}$ *on R.H.S*

$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}\times\frac{n(B)}{n(B)}$$

$$P(A\cap B)=\frac{n(A\cap B)}{n(B)}\times\frac{n(B)}{n(S)}$$

$$P(A\cap B)=P\left(A/B\right)P(B) \qquad \left[\begin{array}{l} \text{by using conditional probability} \\ \because\ P\left(A/B\right)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{\frac{n(A\cap B)}{n(S)}}{\frac{n(B)}{n(S)}}=\dfrac{n(A\cap B)}{n(B)} \end{array}\right]$$

$$\therefore\ P(A\cap B)=P(B)P\left(A/B\right)$$

*hence proved*

## BAYE'S THEOREM

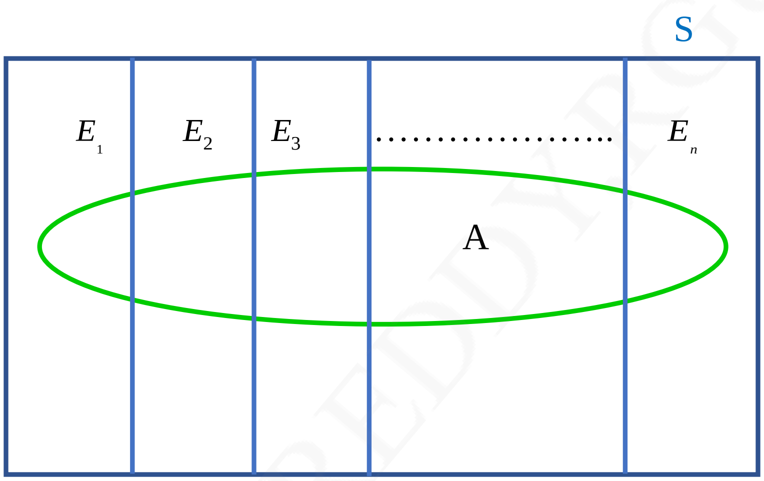
STATEMENT : Let $E_1, E_2, E_3 .......E_n$ are $n$ disjoint events in the sample space $(S)$ and $'A'$ is another in the same sample space $(S)$ such that,

$$A \subseteq \left( \bigcup_{i=1}^{n} E_i \right) \text{ then, } P\left( E_i / A \right) = \frac{P(E_i) P\left( A / E_i \right)}{\sum_{i=1}^{n} P(E_i) P\left( A / E_i \right)}$$

Proof :

Given that $E_1, E_2, E_3 .......E_n$ are $n$ disjoint events.

i.e., $E_i \cap E_j = \Phi \qquad (\forall \quad i \neq j)$



From the Venn-diagram

$$S = E_1 \cup E_2 \cup E_3 \cup ............ \cup E_n$$

now we have to find

$$A \cap S = A \cap \left(E_1 \cup E_2 \cup E_3 \cup ............ \cup E_n\right)$$

$$A \cap S = \left(A \cap E_1\right) \cup \left(A \cap E_2\right) \cup \left(A \cap E_3\right) \cup ........ \cup \left(A \cap E_n\right) = A \qquad \left[\because A \cap S = A\right]$$

*here,* $\left(E_1 \cap A\right), \left(E_2 \cap A\right), \left(E_3 \cap A\right)........\left(E_n \cap A\right)$ are n disjoint events & their union is A.

*i.e.,* $A = \left(E_1 \cap A\right) \cup \left(E_2 \cap A\right) \cup \left(E_3 \cap A\right) \cup ...... \cup \left(E_n \cap A\right)$

taking probability on both sides

$$P\left(A\right) = P\left[\left(E_1 \cap A\right) \cup \left(E_2 \cap A\right) \cup \left(E_3 \cap A\right) \cup ...... \cup \left(E_n \cap A\right)\right]$$

$$P\left(A\right) = P\left(E_1 \cap A\right) + P\left(E_2 \cap A\right) + P\left(E_3 \cap A\right) + ...... + P\left(E_n \cap A\right)$$

$$P\left(A\right) = \sum_{i=1}^{n} P\left(E_i \cap A\right)......................(1)$$

Let us consider L.H.S from the statement

*i.e.,* $P\left(E_i/A\right)$

now we have to apply conditional probability:

$$P\left(E_i/A\right) = \frac{P\left(E_i \cap A\right)}{P\left(A\right)} \qquad \left[\because P\left(A/B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)}\right]$$

$$P\left(E_i/A\right) = \frac{P\left(E_i \cap A\right)}{\sum_{i=1}^{n} P\left(E_i \cap A\right)} \qquad \left[\because \textit{from equation}.....(1)\right]$$

$$P\left(E_i/A\right) = \frac{P\left(E_i\right)P\left(A/E_i\right)}{\sum_{i=1}^{n} P\left(E_i \cap A\right)} \qquad \left[\begin{array}{c}\because \textit{multiplication theorem for two events} \\ \textit{i.e., } P\left(A \cap B\right) = P\left(A\right)P\left(B/A\right)\end{array}\right]$$

$$P\left(E_i/A\right) = \frac{P\left(E_i\right)P\left(A/E_i\right)}{\sum_{i=1}^{n} P\left(E_i\right)P\left(A/E_i\right)} \qquad \left[\because P\left(E_i \cap A\right) = P\left(E_i\right)P\left(A/E_i\right)\right]$$

*hence proved*

## PROBLEMS ON BAYE'S THEOREM

(1) A Bolt Factory Machines A, B &C manufacture 20%,30% & 50% of the total of their output and 6%,3% & 2% are defective. A bolt is drawn at random and found to be defective. Find the probabilities that it is manufactured from i) Machine-A ii) Machine-B iii) Machine-C.

*Solution*:

Let $E_1, E_2, E_3$ be the events that the bolts manufactured by Machine – A, Machine – B & Machine – C respectively.

$$\text{Then, } P(E_1) = 20\% = \frac{20}{100} = 0.2$$

$$P(E_2) = 30\% = \frac{30}{100} = 0.3$$

$$P(E_3) = 50\% = \frac{50}{100} = 0.5$$

*Let 'A' be the event of getting defective bolt.*

$P\left(A/E_1\right)$, Probability that the Machine – A, Produces defective bolt.

$$\text{i.e., } P\left(A/E_1\right) = 6\% = \frac{6}{100} = 0.06$$

$P\left(A/E_2\right)$, Probability that the Machine – B, Produces defective bolt.

$$\text{i.e., } P\left(A/E_2\right) = 3\% = \frac{3}{100} = 0.03$$

$P\left(A/E_3\right)$, Probability that the Machine – C Produces defective bolt.

$$\text{i.e., } P\left(A/E_3\right) = 2\% = \frac{2}{100} = 0.02$$

*Now, we have to apply Baye's theorem*

$(i)$ Pr*obability that the defective bolt is manufactured from Machine – A*

$$i.e.,\ P\left(E_1/A\right)=\frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right)+P(E_2)P\left(A/E_2\right)+P(E_3)P\left(A/E_3\right)}$$

$$=\frac{0.2\times0.06}{0.2\times0.06+0.3\times0.03+0.5\times0.02}$$

$$P\left(E_1/A\right)=0.39$$

$(ii)$ Pr*obability that the defective bolt is manufactured from Machine – B*

$$i.e.,\ P\left(E_2/A\right)=\frac{P(E_2)P\left(A/E_2\right)}{P(E_1)P\left(A/E_1\right)+P(E_2)P\left(A/E_2\right)+P(E_3)P\left(A/E_3\right)}$$

$$=\frac{0.3\times0.03}{0.2\times0.06+0.3\times0.03+0.5\times0.02}$$

$$P\left(E_2/A\right)=0.29$$

$(iii)$ Pr*obability that the defective bolt is manufactured from Machine – C*

$$i.e.,\ P\left(E_3/A\right)=\frac{P(E_3)P\left(A/E_3\right)}{P(E_1)P\left(A/E_1\right)+P(E_2)P\left(A/E_2\right)+P(E_3)P\left(A/E_3\right)}$$

$$=\frac{0.5\times0.02}{0.2\times0.06+0.3\times0.03+0.5\times0.02}$$

$$P\left(E_3/A\right)=0.32$$

(2) Suppose 5 men out of 100 and 25 women out of 10,000 are colour blind.
A colour blind person is chosen at random. What is the probability of the person being i) a male ii) a female?

(Assume male and female to be in equal numbers)

Solution:

Let $E_1$ denote blind person of Male

Let $E_2$ denote blind person of Female.

Given that, male & female to be equal numbers.

i.e., $P(E_1)=\frac{1}{2}=0.5$

$P(E_2)=\frac{1}{2}=0.5$

Now we have to apply Baye's theorem

Let 'A' denotes a colour blind person

i.e., $P\left(A/E_1\right)=\frac{5}{100}=0.05$

$P\left(A/E_2\right)=\frac{25}{10000}=0.0025$

(i) Probability of the colour blind being a male:

$$P\left(E_1/A\right)=\frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right)+P(E_2)P\left(A/E_2\right)}$$

$$=\frac{0.5\times0.05}{0.5\times0.05+0.5\times0.0025}$$

$$\therefore\ P\left(E_1/A\right)=0.95$$

(ii) Probability of the colour blind being a female:

$$P\left(E_2/A\right)=\frac{P(E_2)P\left(A/E_2\right)}{P(E_1)P\left(A/E_1\right)+P(E_2)P\left(A/E_2\right)}$$

$$=\frac{0.5\times0.0025}{0.5\times0.05+0.5\times0.0025}$$

$$\therefore\ P\left(E_2/A\right)=0.05$$

(3) *Of the three men, the chances that a politician, a businessman (or) an academician will be appointed as a vice – chancellor (V.C) of a University are 0.5, 0.3, 0.2 respectively. Probability that research is promoted by these persons if they are appointed as V.C are 0.3, 0.7, 0.8 respectively.*

(i) *Determine the probability that research is promoted?*

(ii) *If research is promoted, what is the probability that V.C is an academician?*

*Solution:*

Let $E_1, E_2, E_3$ be the events that a politician, business man & an academician.

i.e., $P(E_1) = 0.5$

$P(E_2) = 0.3$

$P(E_3) = 0.2$

The probabilities that research is promoted, if they are appointed as V.Cs are

i.e., $P\left(A/E_1\right) = 0.3,\ P\left(A/E_2\right) = 0.7\ \&\ P\left(A/E_3\right) = 0.8$

(i) The probability that the research is promoted.

$$P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_3)P\left(A/E_3\right)$$

$$= 0.5\times 0.3 + 0.3\times 0.7 + 0.2\times 0.8$$

$$= 0.52$$

(ii) The probability that the research is promoted when the V.C is an academician.

$$P\left(E_3/A\right) = \frac{P(E_3)P\left(A/E_3\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_3)P\left(A/E_3\right)}$$

$$= \frac{0.2\times 0.8}{0.5\times 0.3 + 0.3\times 0.7 + 0.2\times 0.8}$$

$$= \frac{0.16}{0.52}$$

$$\therefore P\left(E_3/A\right) = 0.307$$

(4) *A businessman goes to hotels $X$, $Y$, $Z$, 20%, 50%, 30% of the time respectively.*
*It is known that 5%, 4%, 8% of the rooms in $X$, $Y$, $Z$ hotels have faulty plumbing's.*
*What is the probability that business man's room having faulty plumbing is assigned to hotel $Z$?*

*Solution:*

Let $E_1$ denote a businessman goes to hotel $X$

Let $E_2$ denote a businessman goes to hotel $Y$

Let $E_3$ denote a businessman goes to hotel $Z$

$$i.e., P(E_1) = 20\% = \frac{20}{100} = 0.20$$

$$P(E_2) = 50\% = \frac{50}{100} = 0.50$$

$$P(E_3) = 30\% = \frac{30}{100} = 0.30$$

*Let 'A' be event that the hotel room has faulty plumbing*

$$i.e., P\left(A/E_1\right) = 5\% = \frac{5}{100} = 0.05$$

$$P\left(A/E_2\right) = 4\% = \frac{4}{100} = 0.04$$

$$P\left(A/E_3\right) = 8\% = \frac{8}{100} = 0.08$$

*The probability that the businessman's room having faulty plumbing is assigned to hotel Z*

$$P\left(E_3/A\right) = \frac{P(E_3)P\left(A/E_3\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_3)P\left(A/E_3\right)}$$

$$= \frac{0.30 \times 0.08}{0.20 \times 0.05 + 0.50 \times 0.04 + 0.30 \times 0.08}$$

$$\therefore\ P\left(E_3/A\right) = 0.44$$

(5) First box contains 2 black, 3 red, 1 white balls; Second box contains 1 black, 1 red, 2 white balls and third box contains 5 black, 3 red, 4 white balls. Of these a box is selected at random. From it a red ball is randomly drawn. If the ball is red, find the probability that it is from i) First box ii) Second box iii) Third box.

Solution:

Let $E_1, E_2, E_3$ be the first, second & third boxes.

$$i.e., P(E_1)=\frac{1}{3}, P(E_2)=\frac{1}{3}, P(E_3)=\frac{1}{3}$$

Let 'A' be event of drawing a red ball from a box

$$i.e., P\left(A/E_1\right)=\frac{3}{6}=\frac{1}{2}$$

$$P\left(A/E_2\right)=\frac{1}{4}$$

$$P\left(A/E_3\right)=\frac{3}{12}=\frac{1}{4}$$

$\therefore$ By Baye's theorem, the required probability

(i) A red ball drawn from first box

$$P\left(E_1/A\right)=\frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right)+P(E_2)P\left(A/E_2\right)+P(E_3)P\left(A/E_3\right)}$$

$$=\frac{\frac{1}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{4}}=\frac{1}{2}$$

$$\therefore\ P\left(E_1/A\right)=0.5$$

(ii) A red ball drawn from second box

$$P\left(E_2/A\right)=\frac{P(E_2)P\left(A/E_2\right)}{P(E_1)P\left(A/E_1\right)+P(E_2)P\left(A/E_2\right)+P(E_3)P\left(A/E_3\right)}$$

$$=\frac{\frac{1}{3}\times\frac{1}{4}}{\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{4}}=\frac{1}{4}$$

$$\therefore\ P\left(E_2/A\right)=0.25$$

(iii) A red ball drawn from third box

$$P\left(E_3/A\right)=\frac{P(E_3)P\left(A/E_3\right)}{P(E_1)P\left(A/E_1\right)+P(E_2)P\left(A/E_2\right)+P(E_3)P\left(A/E_3\right)}$$

$$=\frac{\frac{1}{3}\times\frac{1}{4}}{\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{4}}=\frac{1}{4}$$

$$\therefore\ P\left(E_3/A\right)=0.25$$

(6) *A manufacturer firm produces steel pipes in three plants, with daily production volume of* 500, 1000 & 2000 *units respectively. According to past experience, It is known that the fraction of defective outputs produced by these plants are respectively* 0.005, 0.008 & 0.010. *If a pipe is selected from day's total production and found to be defective, find out*

(*i*) *from which plant the pipe came*

(*ii*) *what is the probability that it came from the first plant?*

*Solution:*

*Let* $E_1, E_2, E_3$ *be the events that a pipe is manufactured by* in plant 1,2 & 3 respectively.

$$\text{Then, } P(E_1) = \frac{500}{3500} = \frac{1}{7}$$

$$P(E_2) = \frac{1000}{3500} = \frac{2}{7}$$

$$P(E_3) = \frac{2000}{3500} = \frac{4}{7}$$

*Let 'A' be the event that defective pipe is drawn*

$$\text{i.e., } P\left(A/E_1\right) = 0.005$$

$$P\left(A/E_2\right) = 0.008$$

$$P\left(A/E_3\right) = 0.010$$

*Baye's theorem gives*

$$P\left(E_1/A\right) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_3)P\left(A/E_3\right)}$$

$$= \frac{\frac{1}{7}\times 0.005}{\frac{1}{7}\times 0.005 + \frac{2}{7}\times 0.008 + \frac{4}{7}\times 0.010}$$

$$\therefore\ P\left(E_1/A\right) = \frac{5}{61} = 0.08$$

$$P\left(E_2/A\right) = \frac{P(E_2)P\left(A/E_2\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_3)P\left(A/E_3\right)}$$

$$= \frac{\frac{2}{7}\times 0.008}{\frac{1}{7}\times 0.005 + \frac{2}{7}\times 0.008 + \frac{4}{7}\times 0.010}$$

$$\therefore\ P\left(E_2/A\right) = \frac{16}{61} = 0.26$$

$$P\left(E_3/A\right) = \frac{P(E_3)P\left(A/E_3\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_3)P\left(A/E_3\right)}$$

$$= \frac{\frac{4}{7}\times 0.010}{\frac{1}{7}\times 0.005 + \frac{2}{7}\times 0.008 + \frac{4}{7}\times 0.010}$$

$$\therefore\ P\left(E_3/A\right) = \frac{40}{61} = 0.66$$

*Thus it is most probable that the defective pipe has been drawn from the output of the third plant*.

(7) In a certain college ,25% of boys and 10% of girls are studying statistics.The girls constitute 60% of the students.

(i)what is the probability that statistics is being studied?

(ii) if a student is selected at random and is found to be studying statistics,find the probability that the student is a girl (iii) a boy?

Solution:

Let $E_1$ denote the event of selecting a girl.

Let $E_2$ denote the event of selecting a boy.

$$\text{i.e.,} P(E_1) = 60\% = \frac{60}{100} = 0.60$$

$$P(E_2) = 40\% = \frac{40}{100} = 0.40$$

Let 'A' be event that the statistics student.

$$i.e., P\left(E_1/A\right) = 10\% = \frac{10}{100} = 0.10$$

$$P\left(E_2/A\right) = 25\% = \frac{25}{100} = 0.25$$

(i) The probability that the statistics is studied.

$$\sum_{i=1}^{2}\left(E_i/A\right) = P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right)$$

$$= 0.60\times0.10 + 0.40\times0.25$$

$$\therefore \sum_{i=1}^{2}\left(E_i/A\right) = 0.16$$

(ii) The probability of statistics student is a girl.

$$P\left(E_1/A\right) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right)}$$

$$= \frac{0.60\times0.10}{0.60\times0.10+0.40}$$

$$= \frac{0.06}{0.16}$$

$$\therefore P\left(E_1/A\right) = 0.375$$

(iii) The probability of statistics student is a boy.

$$P\left(E_2/A\right) = \frac{P(E_2)P\left(A/E_2\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right)}$$

$$= \frac{0.40\times0.25}{0.40\times0.25+0.60\times0.10}$$

$$= \frac{0.1}{0.16}$$

$$\therefore P\left(E_2/A\right) = 0.625$$

**(8) There are two boxes. In box I, 11 cards are there numbered 1 to 11 and in the box II, 5 cards are there numbered 1 to 5. A box is chosen and a card is drawn. If the card shows an even number then another card is drawn from the same box. If card shows an odd number another card is drawn from the other box. Find the probability that (i) both are even (ii) both are odd (iii) if both are even, what is the probability that they are from box I.**

*Solution*:

*Number of cards in box I* $= 11$

*Number of cards with even numbers in box I* $= 5$

*Number of cards with odd numbers in box I* $= 6$

*Number of cards in box II* $= 5$

*Number of cards with even numbers in box II* $= 2$

*Number of cards with odd numbers in box II* $= 3$

$\therefore$ *The probability of choosing any one box is* $= \dfrac{1}{2}$

*(i) Let E = the event that both the cards are even.*

*For this a box is chosen and a card is picked, if the first card is even then the second card is also picked from the same box and that card is also even.*

*Let* $E_1 =$ *both the cards are from box I.*

$$\therefore\ P(E_1) = \frac{1}{2}\times\frac{5}{11}\times\frac{4}{10} = \frac{1}{11}$$

*Let* $E_2 =$ *both the cards are from box II.*

$$\therefore\ P(E_2) = \frac{1}{2}\times\frac{2}{5}\times\frac{1}{4} = \frac{1}{20}$$

$$\text{Then, } P(E) = P(E_1) + P(E_2)$$

$$= \frac{1}{11}+\frac{1}{20}$$

$$\therefore P(E) = \frac{31}{220}$$

*Hence, the probability that both the cards are even is* $\dfrac{31}{220}$ .

$(ii)$ *Let $E =$ the event that both the cards are odd.*

*For this a box is chosen and a card is picked, if the first card is odd then the second card is also picked from another box and that card is also odd.*

*Let $E_1 =$ first card is from box I and second card is odd from box II.*

$$\therefore P(E_1) = \frac{1}{2}\times\frac{6}{11}\times\frac{3}{5}=\frac{9}{55}$$

*Let $E_2 =$ first card is odd from box II and second card is odd from box I.*

$$\therefore P(E_2) = \frac{1}{2}\times\frac{6}{11}\times\frac{3}{5}=\frac{9}{55}$$

*Then,* $P(E) = P(E_1) + P(E_2)$

$$\therefore P(E) = \frac{9}{55}+\frac{9}{55}=\frac{18}{55}$$

*Hence, the probability that both the cards are odd is* $\frac{18}{55}$.

$(iii)$ *The probability that both cards are even and from box I is*

$$= \frac{1}{2}\times\frac{5}{11}\times\frac{4}{10}=\frac{1}{11}$$

*The probability that both cards are even and from box II is*

$$= \frac{1}{2}\times\frac{2}{5}\times\frac{1}{4}=\frac{1}{20}$$

*By using Baye's theorem,*

*The probability that if both cards are even then they are from* box I *is*

$$=\frac{\frac{1}{2}\times\frac{5}{11}\times\frac{4}{10}}{\frac{1}{2}\times\frac{5}{11}\times\frac{4}{10}+\frac{1}{2}\times\frac{2}{5}\times\frac{1}{4}}=\frac{\frac{1}{11}}{\frac{1}{11}+\frac{1}{20}}=\frac{20}{31}$$

*Hence, the probability that both are even and are from box I is* $\frac{20}{31}$.

(9) **In a factory, machine 'A' produces 40% of the output and machine 'B' produces 60%.**
**On the average, 9 items in 1000 produced by 'A' are defective and 1 item in 250 produced by 'B' is defective.**
**An item drawn at random from a day's output is defective. What is the probability that it was produced by A (or) B?**

**Solution:**

*Let $E_1$ be the event Output produced by machine – A*

*Let $E_2$ be the event Output produced by machine – B*

*Output produced by machine A is 40%, then the probability of machine – A is*

$$\therefore P(E_1) = \frac{40}{100} = 0.40$$

*Output produced by machine B is 60%, then the probability of machine – B is*

$$\therefore P(E_2) = \frac{60}{100} = 0.60$$

*Let 'A' be the event of getting the item as defective item.*

$P\left(A/E_1\right)$ = *Probability that items produced by '$E_1$' are defective* = $\frac{9}{1000}$ = 0.009

$P\left(A/E_2\right)$ = *Probability that items produced by '$E_2$' are defective* = $\frac{1}{250}$ = 0.004

*Now, we have to apply Baye's theorem*

(i) *Probability that the defective item is produced by Machine – A*

$$\text{i.e., } P\left(E_1/A\right) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right)}$$

$$= \frac{0.40\times 0.009}{0.40\times 0.009 + 0.60\times 0.004}$$

$$\therefore P\left(E_1/A\right) = 0.6$$

(ii) *Probability that the defective item is produced by Machine – B*

$$\text{i.e., } P\left(E_2/A\right) = \frac{P(E_2)P\left(A/E_2\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right)}$$

$$= \frac{0.60\times 0.004}{0.40\times 0.009 + 0.60\times 0.004}$$

$$\therefore P\left(E_2/A\right) = 0.4$$

## SOME PROBLEMS ON PROBABILITY

(1) What is the chance that non-leap year selected at random will contains?

i)53 Sundays    ii) Not 53 Sundays

Solution:

In non-leap year which consists of 365 days.

There are 52 complete weeks and one day is more (or) over.

That may be {SUN, MON, TUE, WED, THU, FRI, SAT}

i.e., Total number of exhaustive events are n(S) = n = 7

**i) 53 Sundays: -** Let us consider 'E' is an event of happening of 53 Sundays in a non-leap year.

i.e.,

n(E) = m = The number of favourable events

={SUN}=1

Required Probability,

$$P(E) = \frac{\text{The number of favourable events}}{\text{Total number of exhaustive events}} = \frac{n(E)}{n(S)}$$

$$= \frac{m}{n} = \frac{1}{7}$$

**ii)Not 53 Sundays: -** Let us consider 'E' is an event of happening of not 53 Sundays in a non-leap year.

i.e.,

n(E) = m = The number of favourable events

= {MON, TUE, WED, THU, FRI, SAT} = 6

Required Probability,

$$P(E) = \frac{\text{The number of favourable events}}{\text{Total number of exhaustive events}} = \frac{n(E)}{n(S)}$$

$$= \frac{m}{n} = \frac{6}{7}$$

(2) What is the chance that leap year selected at random will contains?

i)53 Sundays     ii) Not 53 Sundays

Solution:

In a leap year which consists of 366 days.

There are 52 complete weeks and two days are more (or) over.

That may be {(SUN, MON), (MON, TUE), (TUE, WED) (WED, THU), (THU, FRI), (FRI, SAT), (SAT, SUN)}

i.e.,

Total number of exhaustive events are n(S) = n = 7

**i) 53 Sundays: -** Let us consider 'E' is an event of happening of 53 Sundays in a leap year.

i.e.,

n(E) = m = The number of favourable events

= {(SUN, MON), (SAT, SUN)} = 2

Required Probability,

$$P(E) = \frac{\text{The number of favourable events}}{\text{Total number of exhaustive events}} = \frac{n(E)}{n(S)}$$

$$= \frac{m}{n} = \frac{2}{7}$$

**ii)Not 53 Sundays: -** Let us consider 'E' is an event of happening of not 53 Sundays in a leap year.

i.e.,

n(E) = m = The number of favourable events =

{(MON, TUE), (TUE, WED) (WED, THU), (THU, FRI), (FRI, SAT)} = 5

Required Probability,

$$P(E) = \frac{\text{The number of favourable events}}{\text{Total number of exhaustive events}} = \frac{n(E)}{n(S)}$$

$$= \frac{m}{n} = \frac{5}{7}$$

(3) What is the probability for a leap year to have 52 Mondays & 53 Sundays?

Solution:

In a leap year which consists of 366 days.

There are 52 complete weeks and two days are more (or) over.

That may be {(SUN, MON), (MON, TUE), (TUE, WED) (WED, THU), (THU, FRI), (FRI, SAT), (SAT, SUN)}

i.e., Total number of exhaustive events are n(S) = n = 7

Let us consider 'E' is an event of happening of 52 Monday & 53 Sundays in a leap year.

i.e., n(E) = m = The number of favourable events

= {(SAT, SUN)} = 1

Required Probability,

$$P(E) = \frac{\text{The number of favourable events}}{\text{Total number of exhaustive events}} = \frac{n(E)}{n(S)}$$

$$= \frac{m}{n} = \frac{1}{7}$$

(4) If $A, B$ are events with $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$ & $P(A\cup B)=\frac{1}{2}$, Then find

(i) $P(A\cap B)$ (ii) $P(\bar{A}\cap B)$ (iii) $P(A\cap\bar{B})$ (iv) $P(A\cup\bar{B})$ (v) $P(\bar{A}\cup B)$

(vi) $P(A/B)$ (vii) $P(B/A)$ (viii) $P(\bar{A}/B)$ (ix) $P(A/\bar{B})$

*Solution:* Given that $A, B$ are events with $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$ & $P(A\cup B)=\frac{1}{2}$

By using Addition theorem for two events

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

Now we have to find $P(\bar{A})=1-P(A)=1-\frac{1}{3}=\frac{2}{3}$

$$P(\bar{B})=1-P(B)=1-\frac{1}{4}=\frac{3}{4}$$

(i) $P(A\cap B)=P(A)+P(B)-P(A\cup B)=\frac{1}{3}+\frac{1}{4}-\frac{1}{2}=\frac{1}{12}$

(ii) $P(\bar{A}\cap B)=P(B)-P(A\cap B)=\frac{1}{4}-\frac{1}{12}=\frac{1}{6}$

(iii) $P(A\cap\bar{B})=P(A)-P(A\cap B)=\frac{1}{3}-\frac{1}{12}=\frac{1}{4}$

(iv) $P(A\cup\bar{B})=P(A)+P(\bar{B})-P(A\cap\bar{B})=\frac{1}{3}+\frac{3}{4}-\frac{1}{4}=\frac{5}{6}$

(v) $P(\bar{A}\cup B)=P(\bar{A})+P(B)-P(\bar{A}\cap B)=\frac{2}{3}+\frac{1}{4}-\frac{1}{6}=\frac{3}{4}$

(vi) $P(A/B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{\frac{1}{12}}{\frac{1}{4}}=\frac{1}{3}$

(vii) $P(B/A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{\frac{1}{12}}{\frac{1}{3}}=\frac{1}{4}$

(viii) $P(\bar{A}/B)=\dfrac{P(\bar{A}\cap B)}{P(B)}=\dfrac{\frac{1}{6}}{\frac{1}{4}}=\frac{2}{3}$

(x) $P(A/\bar{B})=\dfrac{P(A\cap\bar{B})}{P(\bar{B})}=\dfrac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$

(5) If $P\left(A^C\right)=\frac{3}{8}, P\left(B^C\right)=\frac{1}{2}$ & $P\left(A\cap B\right)=\frac{1}{4}$ ,then find

$i) P\left(A/B\right)$ $ii) P\left(B/A\right)$ $iii) P\left(A^C/B^C\right)$ & $iv) P\left(B^C/A^C\right)$

*Solution:*

Now we have to find

$$P(A)=1-P\left(A^C\right)=1-\frac{3}{8}=\frac{5}{8}$$

$$P(B)=1-P\left(B^C\right)=1-\frac{1}{2}=\frac{1}{2}$$

$$i) P\left(A/B\right)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$$

$$ii) P\left(B/A\right)=\frac{P(A\cap B)}{P(A)}=\frac{\frac{1}{4}}{\frac{5}{8}}=\frac{2}{5}$$

$$iii) P\left(A^C/B^C\right)=\frac{P\left(A^C\cap B^C\right)}{P\left(B^C\right)}=\frac{1-P(A\cup B)}{1-P(B)}=\frac{1-\left[P(A)+P(B)-P(A\cap B)\right]}{1-P(B)}$$

$$\left[\begin{array}{c}\because P(A\cup B)+P\left(A^C\cap B^C\right)=1\\ (or)\\ \because P(A\cup B)+P\left(\overline{A\cup B}\right)=1\end{array}\right]$$

$$=\frac{1-\left[\frac{5}{8}+\frac{1}{2}-\frac{1}{4}\right]}{1-\frac{1}{2}}=\frac{1-\frac{7}{8}}{1-\frac{1}{2}}=\frac{\frac{1}{8}}{\frac{1}{2}}=\frac{1}{4}$$

$$iv) P\left(B^C/A^C\right)=\frac{P\left(A^C\cap B^C\right)}{P\left(A^C\right)}=\frac{1-P(A\cup B)}{1-P(A)}=\frac{1-\left[P(A)+P(B)-P(A\cap B)\right]}{1-P(A)}$$

$$=\frac{1-\left[\frac{5}{8}+\frac{1}{2}-\frac{1}{4}\right]}{1-\frac{5}{8}}=\frac{1-\frac{7}{8}}{1-\frac{5}{8}}=\frac{\frac{1}{8}}{\frac{3}{8}}=\frac{1}{3}$$

(6) *If* $A, B \& C$ *are mutually exclusive events (or) disjoint events associated with random experiment.*

*If* $P(B)=\frac{3}{2}P(A)$ & $P(C)=\frac{1}{2}P(B)$ *then find* $P(A)$.

*Solution:*

*Given that* $P(B)=\frac{3}{2}P(A)$ & $P(C)=\frac{1}{2}P(B)$

$A, B \& C$ *are mutually exclusive events (or) disjoint events*

$$i.e.,\ P(A\cup B\cup C)=1$$

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)$$

$$1=P(A)+\frac{3}{2}P(A)+\frac{1}{2}P(B)$$

$$1=P(A)+\frac{3}{2}P(A)+\frac{1}{2}\times\frac{3}{2}P(A)$$

$$1=P(A)+\frac{3}{2}P(A)+\frac{3}{4}P(A)$$

$$1=\frac{4P(A)+6P(A)+3P(A)}{4}$$

$$1=\frac{13P(A)}{4}$$

$$4=13P(A)$$

$$\therefore P(A)=\frac{4}{13}$$

(7) A chance of a problem solved by a person $A$ is 3 to 5, by solving the same problem by person $B$ is 4 to 9. Find the probability that the problem is solved.

Solution: Let us consider $A$ & $B$ are two persons.

So, $A$ & $B$ are independent events, then $\bar{A}$ & $\bar{B}$ are also independent

Given that,

$$P(A)=\frac{3}{5} \ \& \ P(B)=\frac{4}{9}$$

Now, we have to find

$$P(\bar{A})=1-P(A)=1-\frac{3}{5}=\frac{2}{5}$$

$$P(\bar{B})=1-P(B)=1-\frac{4}{9}=\frac{5}{9}$$

Problem is solved, if either $A$ (or) $B$ solves the problems.

By using Addition theorem

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$P(A\cup B)=P(A)+P(B)-P(A)P(B) \quad [\because A \ \& \ B \text{ are two independent events}]$$

$$P(A\cup B)=\frac{3}{5}+\frac{4}{9}-\frac{3}{5}\times\frac{4}{9}$$

$$=\frac{3}{5}+\frac{4}{9}-\frac{12}{45}$$

$$=\frac{27+20-12}{45}$$

$$P(A\cup B)=\frac{35}{45}=\frac{7}{9}$$

(OR)

$$P(\text{Problem is solved})=1-P(\text{Problem is not solved})$$

$$P(A\cup B)=1-P\left(\overline{A\cup B}\right) \quad \left[\because P(A\cup B)+P\left(\overline{A\cup B}\right)=1\right]$$

$$=1-P\left(\bar{A}\cap\bar{B}\right) \quad \left[\because P\left(\overline{A\cup B}\right)=P\left(\bar{A}\cap\bar{B}\right)\right]$$

$$=1-P\left(\bar{A}\right)P\left(\bar{B}\right) \quad \left[\begin{array}{l}\because \bar{A} \ \& \ \bar{B} \text{ are two independent events} \\ P\left(\bar{A}\cap\bar{B}\right)=P\left(\bar{A}\right)P\left(\bar{B}\right)\end{array}\right]$$

$$=1-\frac{2}{5}\times\frac{5}{9}$$

$$=1-\frac{10}{45}=\frac{35}{45}$$

$$\therefore P(A\cup B)=\frac{7}{9}$$

(8) *The probability of three students to solve a problem in the statistics are* $\frac{1}{2}, \frac{1}{3}$ & $\frac{1}{4}$ *respectively. Find the probability of the problem to be solved.*

*Solution:*

*Let us consider, A, B & C are three students.*

*So,* $A, B$ & $C$ *are independent events, then* $\bar{A}, \bar{B}$ & $\bar{C}$ *are also independent events.*

*Given that,*

$$P(A)=\frac{1}{2}\ ,\ P(B)=\frac{1}{3}\ \&\ P(C)=\frac{1}{4}$$

*Now, we have to find*

$$P\left(\bar{A}\right)=1-P(A)=1-\frac{1}{2}=\frac{1}{2}$$

$$P\left(\bar{B}\right)=1-P(B)=1-\frac{1}{3}=\frac{2}{3}$$

$$P\left(\bar{C}\right)=1-P(C)=1-\frac{1}{4}=\frac{3}{4}$$

*Problem is solved if, either A (or) B (or) C solves the problem.*

$\therefore$ P(*Problem is solved*) = 1 − P(*Problem is not solved*)

$$P(A\cup B\cup C)=1-P\left(\overline{A\cup B\cup C}\right)$$

$$=1-P\left(\bar{A}\cap\bar{B}\cap\bar{C}\right) \qquad \left[\begin{array}{l}\because \textit{By using Demorgan Law's}\\ P\left(\overline{A\cup B\cup C}\right)=P\left(\bar{A}\cap\bar{B}\cap\bar{C}\right)\\ P\left(\overline{A\cap B\cap C}\right)=P\left(\bar{A}\cup\bar{B}\cup\bar{C}\right)\end{array}\right]$$

$$=1-P\left(\bar{A}\right)P\left(\bar{B}\right)P\left(\bar{C}\right)$$

$$=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}$$

$$=1-\frac{1}{4}$$

$$P(A\cup B\cup C)=\frac{3}{4}$$

(9) *The probability of* Four *students* A,B,C & D *to solve a problem in the* Mathematics *are* $\frac{1}{3},\frac{2}{5},\frac{1}{5}\ \&\ \frac{1}{4}$ *respectively.Find the probability of the problem to be solved.*

*Solution :*

*Let us consider , A, B, C & D are three students.*

*So , A , B , C & D are independent events, then* $\bar{A},\bar{B},\bar{C}\ \&\ \bar{D}$ *are also independent events.*

*Given that,*

$$P(A)=\frac{1}{3}\ ,P(B)=\frac{2}{5}\ ,P(C)=\frac{1}{5}\ \&\ P(D)=\frac{1}{4}$$

*Now, we have to find*

$$P\left(\bar{A}\right)=1-P(A)=1-\frac{1}{3}=\frac{2}{3}$$

$$P\left(\bar{B}\right)=1-P(B)=1-\frac{2}{5}=\frac{3}{5}$$

$$P\left(\bar{C}\right)=1-P(C)=1-\frac{1}{5}=\frac{4}{5}$$

$$P\left(\bar{D}\right)=1-P(D)=1-\frac{1}{4}=\frac{3}{4}$$

*Problem is solved if ,either A (or) B (or) C (or) D solves the problem.*

$\therefore$ P(*Problem is solved*) = 1 − P(Pr*oblem is not solved*)

$$P(A\cup B\cup C\cup D)=1-P\left(\overline{A\cup B\cup C\cup D}\right)$$

$$=1-P\left(\bar{A}\cap\bar{B}\cap\bar{C}\cap\bar{D}\right) \qquad \left[\begin{array}{c}\because \textit{By using Demorgan Law's}\\ P\left(\overline{A\cup B\cup C\cup D}\right)=P\left(\overline{A}\cap\overline{B}\cap\overline{C}\cap\overline{D}\right)\\ P\left(\overline{A\cap B\cap C\cap D}\right)=P\left(\overline{A}\cup\overline{B}\cup\overline{C}\cup\overline{D}\right)\end{array}\right]$$

$$=1-P\left(\bar{A}\right)P\left(\bar{B}\right)P\left(\bar{C}\right)P\left(\bar{D}\right)$$

$$=1-\frac{2}{3}\times\frac{3}{5}\times\frac{4}{5}\times\frac{3}{4}$$

$$=1-\frac{6}{25}$$

$$\therefore P(A\cup B\cup C\cup D)=\frac{19}{25}$$

(10) From 25 tickets marked with first 25 numbers. Find Chance that

i) It is multiple of 5 (or) 7.

ii) It is multiple of 3 (or) 7.

*Solution:*

*Given that, 25 tickets are marked with first 25 numbers.*

*Total number of exhaustive events are* $S = \{1, 2, 3, 4, 5 \ldots\ldots\ldots\ldots 23, 24, 25\}$

*i.e.,* $n(S) = n = 25$

(i) *Let* $E_1$ *be the event that multiple of* 5.

*Number of favourable events are* $E_1 = \{5, 10, 15, 20, 25\}$

*i.e.,* $n(E_1) = m = 5$

$$\text{Probability of } E_1 \text{ event is } P(E_1) = \frac{\text{Number of favourable events}}{\text{Total number of exhaustive events}} = \frac{n(E_1)}{n(S)} = \frac{m}{n} = \frac{5}{25}$$

*Let* $E_2$ *be the event that multiple of* 7.

*Number of favourable events are* $E_2 = \{7, 14, 21\}$

*i.e.,* $n(E_2) = m = 3$

$$\text{Probability of } E_2 \text{ event is } P(E_2) = \frac{\text{Number of favourable events}}{\text{Total number of exhaustive events}} = \frac{n(E_2)}{n(S)} = \frac{m}{n} = \frac{3}{25}$$

*Here, multiple of 5 & multiple of 7 are mutually exclusive (or) disjoint events.*

*then,* $P(E_1 \cap E_2) = 0$

Required Probability,

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) \qquad [\because \text{ by using Union Axiom}]$$

$$= \frac{5}{25} + \frac{3}{25}$$

$$\therefore P(E_1 \cup E_2) = \frac{8}{25}$$

*Solution* :

Given that, 25 tickets are marked with first 25 numbers.

Total number of exhaustive events are $S=\{1,2,3,4,5...............23,24,25\}$

i.e., $n(S)=n=25$

(ii) Let $E_1$ be the event that multiple of 3.

Number of favourable events are $E_1=\{3,6,9,12,15,18,21,24\}$

i.e., $n(E_1)=m=8$

$$\text{Probability of } E_1 \text{ event is } P(E_1)=\frac{\text{Number of favourable events}}{\text{Total number of exhaustive events}}=\frac{n(E_1)}{n(S)}=\frac{m}{n}=\frac{8}{25}$$

Let $E_2$ be the event that multiple of 7.

Number of favourable events are $E_2=\{7,14,21\}$

i.e., $n(E_2)=m=3$

$$\text{Probability of } E_2 \text{ event is } P(E_2)=\frac{\text{Number of favourable events}}{\text{Total number of exhaustive events}}=\frac{n(E_2)}{n(S)}=\frac{m}{n}=\frac{3}{25}$$

Here, multiple of 3 & multiple of 7 are not mutually exclusive (or) not disjoint events.

Let $E_1$ & $E_2$ be the event that multiple of 21. $[\because L.C.M \text{ of } 3,7=21]$

Number of favourable events $E_1\cap E_2=\{21\}$

i.e., $n(E_1\cap E_2)=m=1$

$$\text{Probability of } E_1\cap E_2 \text{ event is } P(E_1\cap E_2)=\frac{\text{Number of favourable events}}{\text{Total number of exhaustive events}}=\frac{n(E_1\cap E_2)}{n(S)}=\frac{m}{n}=\frac{1}{25}$$

Required Probability,

$$P(E_1\cup E_2)=P(E_1)+P(E_2)-P(E_1\cap E_2) \qquad [\because \text{ by using Addition Theorem}]$$

$$=\frac{8}{25}+\frac{3}{25}-\frac{1}{25}$$

$$\therefore P(E_1\cup E_2)=\frac{10}{25}=\frac{2}{5}$$

(11) Three Students A, B & C are in a running race have the same probability of winning and each is Twice being likely to win has C. Find the probability that B (or) C Wins.

*Solution:*

$$Let\ A, B \,\&\, C\ are\ mutually\ exclusive (or)\ disjoint\ events$$

$$consider, The\ probability\ of\ C\ winning\ is\ 'x'$$

$$i.e.,\ P(C) = x$$

$$The\ probability\ of\ A\ winning\ is\ '2x'$$

$$i.e.,\ P(A) = 2x$$

$$The\ probability\ of\ B\ winning\ is\ '2x'$$

$$i.e.,\ P(B) = 2x$$

$$The\ three\ students\ are\ winning$$

$$i.e.,\quad P(A \cup B \cup C) = 1$$

$$P(A) + P(B) + P(C) = 1 \qquad [\because A, B \,\&\, C\ are\ disjoint\ events]$$

$$2x + 2x + x = 1$$

$$5x = 1$$

$$\therefore x = \frac{1}{5}$$

$$now,\ we\ have\ to\ find$$

$$P(B \cup C) = P(B) + P(C)$$

$$= 2x + x \qquad [\because B \,\&\, C\ are\ disjoint\ events]$$

$$= 3x$$

$$\therefore P(B \cup C) = 3 \times \frac{1}{5} = \frac{3}{5} \qquad \left[\because x = \frac{1}{5}\right]$$

(12) The probability of a horse 'A' winning a race is $\frac{1}{5}$ and probability of a horse 'B' winning a race is $\frac{1}{4}$.

What is the probability that (i) Either of them will win (ii) None of them will win.

Solution:

Let $E_1$ be the event 'A' horse winning. i.e., $P(E_1)=\frac{1}{5}$

Let $E_2$ be the event 'B' horse winning. i.e., $P(E_2)=\frac{1}{4}$

Now, we have to find

$$P(\overline{E_1})=1-P(E_1)=1-\frac{1}{5}=\frac{4}{5}$$

$$P(\overline{E_2})=1-P(E_2)=1-\frac{1}{4}=\frac{3}{4}$$

Here, $E_1, E_2$ are mutually exclusive (or) disjoint events.

(i) Either of them will win:

$$P(E_1 \cup E_2)=P(E_1)+P(E_2)$$

$$=\frac{1}{5}+\frac{1}{4}=\frac{9}{20}$$

(ii) None of them will win:

$$P(\overline{E_1 \cup E_2})=1-P(E_1 \cup E_2)$$

$$=1-\frac{9}{20}=\frac{11}{20}$$

(13) ‘A’ can hit a target 3 times in 5 shots, ‘B’ hits target 2 times in 5 shots, ‘C’ hits target 3 times in 4 shots. Find the probability of the target being hit when all of them try. And also find i) Two shots hits ii) At least two shots hits iii) At most two shots hits.

Solution:

Let $P(A)$ be the probability of 'A' hitting the target

Let $P(B)$ be the probability of 'B' hitting the target

Let $P(C)$ be the probability of 'C' hitting the target

Given that,

$$P(A)=\frac{3}{5},P(B)=\frac{2}{5},P(C)=\frac{3}{4}$$

now, we have to find

$$P(\overline{A})=1-P(A)=1-\frac{3}{5}=\frac{2}{5}$$

$$P(\overline{B})=1-P(B)=1-\frac{2}{5}=\frac{3}{5}$$

$$P(\overline{C})=1-P(C)=1-\frac{3}{4}=\frac{1}{4}$$

$$\therefore P(\text{All of them hits})=P(A\cup B\cup C)=1-P(\overline{A\cup B\cup C})$$

$$=1-P(\overline{A}\cap\overline{B}\cap\overline{C}) \quad [\because \text{By using Demorgan Law}]$$

$$=1-P(\overline{A})P(\overline{B})P(\overline{C})$$

$$=1-\frac{2}{5}\times\frac{3}{5}\times\frac{1}{4}=1-\frac{6}{100}=\frac{94}{100}=\frac{47}{50}$$

$$\therefore P(\text{All of them hits})=P(A\cap B\cap C)=\frac{47}{50}$$

$i)$ *Two shots hits* :

$$P(\text{ Two shots hits}) = P(A\cap B\cap \overline{C}) + P(A\cap \overline{B}\cap C) + P(\overline{A}\cap B\cap C)$$

$$= P(A)P(B)P(\overline{C}) + P(A)P(\overline{B})P(C) + P(\overline{A})P(B)P(C) \quad \left[\begin{array}{l}\because A, B, C \,\&\, \overline{A}, \overline{B}, \overline{C} \\ \text{are independent events}\end{array}\right]$$

$$= \frac{3}{5}\times\frac{2}{5}\times\frac{1}{4}+\frac{3}{5}\times\frac{3}{5}\times\frac{3}{4}+\frac{2}{5}\times\frac{2}{5}\times\frac{3}{4} = \frac{9}{20}$$

$ii)$ *At least two shots hits* :

$$P(\text{At least two shots hits}) = P(\text{Two shots hits}) + P(\text{Three shots hits})$$

$$= P(A\cap B\cap \overline{C}) + P(A\cap \overline{B}\cap C) + P(\overline{A}\cap B\cap C) + P(A\cap B\cap C)$$

$$= P(A)P(B)P(\overline{C}) + P(A)P(\overline{B})P(C) + P(\overline{A})P(B)P(C) + P(A)P(B)P(C)$$

$$= \frac{3}{5}\times\frac{2}{5}\times\frac{1}{4}+\frac{3}{5}\times\frac{3}{5}\times\frac{3}{4}+\frac{2}{5}\times\frac{2}{5}\times\frac{3}{4}+\frac{3}{5}\times\frac{2}{5}\times\frac{3}{4} = \frac{63}{100}$$

$iii)$ *At most two shots hits* :

$$P(\text{At most two shots hits}) = P(\text{No shots hits}) + P(\text{One shots hits}) + P(\text{Two shots hits})$$

$$= P(\overline{A}\cap \overline{B}\cap \overline{C}) + P(\overline{A}\cap \overline{B}\cap C) + P(\overline{A}\cap B\cap \overline{C}) + P(A\cap \overline{B}\cap \overline{C}) +$$
$$P(A\cap B\cap \overline{C}) + P(A\cap \overline{B}\cap C) + P(\overline{A}\cap B\cap C)$$

$$= P(\overline{A})P(\overline{B})P(\overline{C}) + P(\overline{A})P(\overline{B})P(C) + P(\overline{A})P(B)P(\overline{C}) + P(A)P(\overline{B})P(\overline{C}) +$$
$$P(A)P(B)P(\overline{C}) + P(A)P(\overline{B})P(C) + P(\overline{A})P(B)P(C)$$

$$= \frac{2}{5}\times\frac{3}{5}\times\frac{1}{4}+\frac{2}{5}\times\frac{3}{5}\times\frac{3}{4}+\frac{2}{5}\times\frac{2}{5}\times\frac{1}{4}+\frac{3}{5}\times\frac{3}{5}\times\frac{1}{4}+\frac{3}{5}\times\frac{2}{5}\times\frac{1}{4}+\frac{3}{5}\times\frac{3}{5}\times\frac{3}{4}+\frac{2}{5}\times\frac{2}{5}\times\frac{3}{4}+\frac{3}{5}\times\frac{2}{5}\times\frac{3}{4}$$

$$= \frac{82}{100}$$

(14) In a class consists there are 10 boys and 5 girls. A committee of 4 students is to be selected from the class. Find the probability for the committee to contain

i) at least 3 girls ii) exactly 2 girls iii) 4 boys are selected.

Solution:

A Committee of 4 students out of 15 can be formed in $15_{c_4}$ ways.

i.e., $n = 15_{c_4} = 1365$

i) at least 3 girls

Let E be the event of forming a committee with at least 3 girls.

Now, the committee can have 1 boy & 3 girls (or) No boy & 4 girls.

So, the number of ways of forming the committee.

The number of favourable ways to E event =

P (1 Boy & 3 Girls) + P (0 Boy & 4 Girls)

$$\therefore \text{ The number of favourable ways to E event}$$

$$m = 10_{C_1} \times 5_{C_3} + 10_{C_0} \times 5_{C_4}$$

$$= 10 \times 10 + 1 \times 5$$

$$= 100 + 5$$

$$= 105$$

$$\therefore \text{Required Probability}$$

$$\therefore P(E) = \frac{m}{n} = \frac{105}{1365} = \frac{1}{13} = 0.08$$

$ii)$ *exactly* $2$ *girls*

*Let* $E$ *be the event of forming a committee with exactly* $2$ *girls.*

$$\therefore \text{ The number of favourable ways to E event}$$

$$m = 10_{C_2} \times 5_{C_2}$$

$$= 45 + 10$$

$$= 55$$

$$\therefore \text{Required Probability}$$

$$P(E) = \frac{m}{n} = \frac{55}{1365} = \frac{11}{273} = 0.04$$

$iii)$ *4 boys are selected*

*Let E be the event of forming a committee with 4 Boys are selected*

$\therefore$ *The number of favourable ways to E event i.e.,* $m = 10_{C_4} = 45$

$\therefore$ Required Probability

$$\therefore P(E) = \frac{m}{n} = \frac{45}{1365} = \frac{9}{91} = 0.03$$

(15) Six boys and six girls sit in a row at random. What is the probability that?
(i) The six girls sit together? (ii) The boys and girls sit alternatively?

Solution:

The total number of students = 6 boys + 6 girls = 12

The number of ways of seating 6 boys & 6 girls in a row = n = 12!

(i) Let E be the event of seating 6 boys are to be seated in a row.
For this, treat all 6 girls as one unit along with 6 boys and we have seat in a row, of course in every arrangement 6 girls among themselves are to be seated in all possible ways.

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 girls | 1 boy | 2 boy | 3 boy | 4 boy | 5 boy | 6 boy |

$$\therefore \quad n(E) = m = 7!6!$$

*Hence* , Required *probability*

$$P(E) = \frac{m}{n} = \frac{7!6!}{12!} = \frac{1}{132}$$

(ii) Let E be the event of seating 6 boys and 6 girls alternately.
For this we can start with a Boy (or) a Girl.
The arrangement can be two types.
[ G B G B G B G B G B G B] (or) [ B G B G B G B G B G B G]

$$\therefore \quad n(E) = m = 2\times 6!\times 6!$$

$$Hence\ ,\mathrm{Re}quired\ probability$$

$$P(E) = \frac{m}{n} = \frac{2\times 6!\times 6!}{12!} = \frac{1}{462}$$

(16) What is the probability that 4S's appear consecutively in the word MISSISSIPPI assuming that the letters are arranged at random.

Solution:

The word MISSISSIPPI contains 11 letters and it consists of M-1, I-4, S-4, P-2.

The number of all possible arrangement with all the letters.

$$\therefore\ n(S) = n = \frac{11!}{1!4!4!2!} = 34650$$

Let E be the event of getting arrangements in which all 4S's appear consecutively.

In this type of arrangements, we have M-1, 4S's-1, I-4, P-2.

Total = 8 letters

$$\therefore\ n(E) = m = \frac{8!}{1!1!4!2!} = 840$$

$$\mathrm{Re}quired\ \ probability$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{m}{n} = \frac{840}{34650} = \frac{4}{165} = 0.02$$

(17) *Two aero planes bomb a target in succession. The probability of each correctly scoring a hit is 0.3 & 0.2 respectively. The second will bomb only if the first misses the target. Find the probability that (i) target is hit (ii) both fails to score hits.*

*Solution*:

*Let A be the event of first plane hitting the target*

*And B be the event of second plane hitting the target*

*The probability of first plane hitting the target* $= P(A) = 0.3$

*The probability of second plane hitting the target* $= P(B) = 0.2$

$$P(\bar{A}) = 1 - P(A) = 1 - 0.3 = 0.7$$

$$P(\bar{B}) = 1 - P(B) = 1 - 0.2 = 0.8$$

(i) $P(\text{target is hit}) = P\left[(A \text{ hits (or)}) (A \text{ fails and } B \text{ hits})\right]$

$$= P\left[A \cup (\bar{A} \cap B)\right]$$

$$= P(A) + P(\bar{A} \cap B) \quad [\because \text{By using Union axiom}]$$

$$= P(A) + P(\bar{A})P(B) \quad \left[\begin{array}{l}\because A \,\&\, B \text{ are independent} \\ P(\bar{A} \cap B) = P(\bar{A})P(B)\end{array}\right]$$

$$= 0.3 + (0.7)(0.2)$$

$$= 0.44$$

(ii) $P(\text{both fails}) = P\left[(A \text{ fails and } B \text{ fails})\right]$

$$= P\left[(\bar{A} \cap \bar{B})\right]$$

$$= P(\bar{A})P(\bar{B}) \quad \left[\begin{array}{l}\because A \,\&\, B \text{ are independent} \\ P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})\end{array}\right]$$

$$= 0.7 \times 0.8$$

$$= 0.56$$

(ii) $P(\text{both fails}) = P\left[(A \text{ fails and } B \text{ fails})\right]$

$$= P\left[(\bar{A} \cap \bar{B})\right]$$

$$= P(\bar{A})P(\bar{B}) \quad \left[\begin{array}{l}\because A \,\&\, B \text{ are independent} \\ P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})\end{array}\right]$$

$$= 0.7 \times 0.8$$

$$= 0.56$$

(18) Box ‘A’ contains 5 red and 3 white marbles and box ‘B’ contains 2 red and 6 white marbles. If a marble is drawn from each box, what is the probability that they are both of same colour?

*Solution* :

$E_1 =$ *The event that the marble is drawn from box A and is red.*

$$\therefore\ P(E_1) = \frac{1}{2}\times\frac{5}{8} = \frac{5}{16}$$

$E_2 =$ *The event that the marble is drawn from box* B *and is red.*

$$\therefore\ P(E_1) = \frac{1}{2}\times\frac{2}{8} = \frac{2}{16}$$

$E_3 =$ *The event that the marble is drawn from box A and is* white.

$$\therefore\ P(E_3) = \frac{1}{2}\times\frac{3}{8} = \frac{3}{16}$$

$E_4 =$ *The event that the marble is drawn from box* B *and is* white.

$$\therefore\ P(E_4) = \frac{1}{2}\times\frac{6}{8} = \frac{6}{16}$$

$\therefore$ *The probability that both the marbles are red is*

$$P(E_1 \cap E_2) = P(E_1)P(E_2)$$

$$= \frac{5}{16}\times\frac{2}{16} = \frac{5}{128}$$

$\therefore$ *The probability that both the marbles are white is*

$$P(E_3 \cap E_4) = P(E_3)P(E_4)$$

$$= \frac{3}{16}\times\frac{6}{16} = \frac{9}{128}$$

Re*quired probability*

*The probability that the marbles are of same colour is*

$$= P(E_1 \cap E_2) + P(E_3 \cap E_4)$$

$$= \frac{5}{128} + \frac{9}{128} = \frac{14}{128} = \frac{7}{64} = 0.11$$

(19) A bag contains 4 green, 6 black and 7 white balls. A ball is drawn at random. What is the probability that it is either a green (or) black ball?

Solution:

Let S be the sample space associated with the drawing of ball from a bag containing 4 green,6 black and 7 white balls.

One ball can be drawn out of 17 balls in $17_{C_1} = 17 \ \ ways$

i.e., $Total\ number\ of\ exhaustive\ events\ are\ n(S) = n = 17_{C_1} = 17$

Let ‘$E_1$’ denote the event of drawing a green ball.

‘$E_2$’ denote the event of drawing a black ball.

$The\ number\ of\ favourable\ event\ E_1 = n(E_1) = m = 4_{C_1} = 4$

$The\ number\ of\ favourable\ event\ E_2 = n(E_2) = m = 6_{C_1} = 6$

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{m}{n} = \frac{4}{17}$$

$$\therefore P(E_2) = \frac{n(E_2)}{n(S)} = \frac{m}{n} = \frac{6}{17}$$

$E_1, E_2\ are\ mutually exclusive\ (or)\ dijo\text{int}\ events.$

$i.e.,\ P(E_1 \cap E_2) = \Phi$

$\text{Re}\,quired\ probability,$

$\therefore \text{Pr}\,obability\ of\ drawing\ either\ a\ green\ (or)\ black\ ball$

$$i.e.,\ P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

$$= \frac{4}{17} + \frac{6}{17} = \frac{10}{17}$$

(20) A bag contains 8 red and 6 blue balls. Two drawing of each 2 balls are made. Find the Probability that the first drawing gives two red balls and second drawing gives 2 blue balls, If the balls drawn first are replaced before the second draw.

Solution:

Total number of balls = 8 red + 6 blue =14 balls

Two balls can be drawn out of 14 balls in $14_{C_2} = 91 \ \ ways$

Let $E_1$ be the event of drawing 2 red balls in the first draw from the bag containing 8 red and 6 blue balls.

$$\therefore P(E_1) = \frac{8_{C_2}}{14_{C_2}} = \frac{28}{91}$$

Let $E_2$ be the event of drawing 2 blue balls in the second draw from the bag containing 8 red and 6 blue balls.

$$\therefore P(E_2) = \frac{6_{C_2}}{14_{C_2}} = \frac{15}{91}$$

now, $E_1 \cap E_2$ = Event of drawing 2 red balls in the first draw and another drawing of 2 blue balls in the second draw after the balls are replaced.

$$\therefore P(E_1 \cap E_2) = P(E_1)P(E_2) \quad [\because E_1, E_2 \text{ are independent}]$$

$$= \frac{28}{91} \times \frac{15}{91} = \frac{60}{1183} = 0.05$$

(21) Two marbles are drawn from a box containing 10 red,15 oranges, 20 blue and 30 white marbles, with replacement being made after each draw. Find the probability that (i) both are white (ii) first is red and second is white.

Solution:

Total number of marbles in the box = 10 red +15 oranges + 20 blue + 30 white =75

One ball can be drawn out of 75 balls in $75_{C_1} = 75$

(i) Let $E_1$ be the event of the first drawn marble is white.

$$\therefore P(E_1) = \frac{30_{C_1}}{75_{C_1}} = \frac{30}{75} = \frac{2}{5}$$

Let $E_2$ be the event of the second drawn marble is also white.

$$\therefore P(E_2) = \frac{30_{C_1}}{75_{C_1}} = \frac{30}{75} = \frac{2}{5}$$

*The probability that both marbles are white (with replacement)*

$$P(\text{Both are white}) = P(E_1 \cap E_2) = P(E_1)P\left(E_2/E_1\right) \quad [\because \text{By using multiplication theorem}]$$

$$= P(E_1)P(E_2) \quad \left[\begin{array}{l} \because P\left(E_2/E_1\right) = \dfrac{P(E_1 \cap E_2)}{P(E_1)} = \dfrac{P(E_1)P(E_2)}{P(E_1)} = P(E_2) \\ \because \quad E_1, E_2 \text{ are independent events} \end{array}\right]$$

$$= \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

(ii) Let $E_1$ be the event of the first drawn marble is red.

$$\therefore P(E_1) = \frac{10_{C_1}}{75_{C_1}} = \frac{10}{75} = \frac{2}{15}$$

Let $E_2$ be the event of the white marble in the second drawn when the first drawn is replaced.

$$\therefore P(E_2) = \frac{30_{C_1}}{75_{C_1}} = \frac{30}{75} = \frac{2}{5}$$

*The probability that the first marble is red and second marble is white (with replacement)*

$$P(\text{first marble is red and second marble is white})$$

$$= P(E_1 \cap E_2) = P(E_1)P\left(E_2/E_1\right) \quad [\because \text{By using multiplication theorem}]$$

$$= P(E_1)P(E_2) \quad \left[\begin{array}{l} \because P\left(E_2/E_1\right) = \dfrac{P(E_1 \cap E_2)}{P(E_1)} = \dfrac{P(E_1)P(E_2)}{P(E_1)} = P(E_2) \\ \because \quad E_1, E_2 \text{ are independent events} \end{array}\right]$$

$$= \frac{2}{15} \times \frac{2}{5} = \frac{4}{75}$$

(22) A 5-digit number is formed by using digits 1,2,3,4,5 without repetition. What is the probability that the number is divisible by 4?

Solution:

A 5-digit number is formed by without repetition equals to 5! Ways.

The total number of exhaustive events are n(S) = n = 5! = 120

If the number is divisible by 4, the last two digits must be

Case(i):

| | | | | |
|---|---|---|---|---|
| - | - | - | 1 | 2 |

Case(ii):

| | | | | |
|---|---|---|---|---|
| - | - | - | 2 | 4 |

Case(iii):

| | | | | |
|---|---|---|---|---|
| - | - | - | 3 | 2 |

Case(iv):

| | | | | |
|---|---|---|---|---|
| - | - | - | 5 | 2 |

In case – (i), (ii), (iii) & (iv) rest 3-digits can be filled in 3! Ways = 6

The number of favourable cases are (m) = 6+6+6+6 = 24

Required probability,

$$\therefore P(E) = \frac{m}{n} = \frac{24}{120} = \frac{1}{5} = 0.20$$

(23) Two digits are selected at random from the digits 1 through 9.

(i) If the sum is odd, what is the probability that 2 is one of the numbers selected?

(ii) If 2 is one of the digits selected, what is the probability that the sum is odd?

Solution:

The given set consists of five odd digits (1,3,5,7,9) and four even digits (2,4,6,8)

(i) To get the Sum is Odd.

Odd + Even = Odd

Even + Odd = Odd

The total number of events to get the sum is Odd

$$\begin{Bmatrix} (1,2)(1,4)(1,6)(1,8)(3,2)(3,4)(3,6)(3,8)(5,2)(5,4)(5,6)(5,8)(7,2)(7,4)(7,6)(7,8)(9,2)(9,4)(9,6)(9,8) \\ (2,1)(2,3)(2,5)(2,7)(2,9)(4,1)(4,3)(4,5)(4,7)(4,9)(6,1)(6,3)(6,5)(6,7)(6,9)(8,1)(8,3)(8,5)(8,7)(8,9) \end{Bmatrix}$$

$$\therefore\ n(S) = n = 40$$

Number of elements having one of the number 2 are

$$\{(1,2)(3,2)(5,2)(7,2)(9,2)(2,1)(2,3)(2,5)(2,7)(2,9)\}$$

The number of favourable events

$$\therefore n(E) = m = 10$$

$\text{Required Probability},$

$$\therefore P(E) = \frac{m}{n} = \frac{10}{40} = \frac{1}{4}$$

(ii) If 2 is one of the digits selected, what is the probability that the sum is odd.

The total number of possible events are

$$\{(2,1)(2,3)(2,4)(2,5)(2,6)(2,7)(2,8)(2,9)(1,2)(3,2)(4,2)(5,2)(6,2)(7,2)(8,2)(9,2)\}$$

$$\therefore\ n(S) = n = 16$$

Number of elements having one of the number 2 are

$$\{(1,2)(3,2)(5,2)(7,2)(9,2)(2,1)(2,3)(2,5)(2,7)(2,9)\}$$

The number of favourable events

$$\therefore n(E) = m = 10$$

$\text{Required Probability},$

$$\therefore P(E) = \frac{m}{n} = \frac{10}{16} = \frac{5}{8}$$

(24) There are 12 cards numbered 1 to 12 cards in a box, if two cards are selected, what is the probability that sum is odd. if

(i) With replacement

(ii) Without replacement

Solution:

The cards having even numbers = {2,4,6,8,10,12}

The cards with odd numbers = {1,3,5,7,9,11}

(i) With replacement: Suppose we select a card place it back and select one card.

This can be happening in $12 \times 12 = 144 \quad ways$

To have an odd sum, (i) the first card has to be odd and the second card has to be even (or) (ii) the first card has to be even and the second card has to be odd

The number of favourable ways = $6 \times 6 + 6 \times 6 = 36 + 36 = 72$

$\text{Required Probability}$

$$\therefore\ P(Sum\ is\ \ Odd) = \frac{m}{n} = \frac{72}{144} = \frac{1}{2}$$

(ii)Without replacement: Suppose we select a card place it not back and select one card.

This can be happening in $12_{P_2} = 132 \quad ways$

To have an odd sum, (i) the first card has to be odd and the second card has to be even (or) (ii) the first card has to be even and the second card has to be odd

The number of favourable ways = $6\times6+6\times6 = 36+36 = 72$

$\text{Required Probability},$

$$\therefore\ P(Sum\ is\ Odd) = \frac{m}{n} = \frac{72}{132} = \frac{6}{11}$$

(25) Two cards are selected at random from 10 cards numbered 1 to10.Find the probability that the sum is even if

(i) The two cards are drawn together.

(ii) The two cards are drawn one after the other with replacement.

(iii) The two cards are drawn one after the other without replacement.

Solution:

(i)The two cards are drawn together:

The cards having even numbers = {2,4,6,8,10}

The cards with odd numbers = {1,3,5,7,9}

$Two\ cards\ are\ drawn\ at\ a\ time\ out\ of\ 10\ cards\ in\ 10_{C_2}\ ways = 45\ ways$

$\therefore The\ total\ number\ of\ exhaustive\ events\ are\ n = 45$

$Two\ cards\ can\ be\ selected\ from\ 5\ even\ cards\ in\ 5_{C_2}\ ways = 10\ ways$

$Two\ cards\ can\ be\ selected\ from\ 5\ odd\ cards\ in\ 5_{C_2}\ ways = 10\ ways$

$\therefore The\ number\ of\ favourable\ events\ are\ m = 10+10 = 20$

$\text{Required Probability},$

$$\therefore\ P(The\ two\ cards\ are\ drawn\ together) = \frac{m}{n} = \frac{20}{45} = \frac{4}{9}$$

(ii) With replacement: Suppose we select a card place it back and select one card.

This can be happening in $10\times 10=100\ \ ways$

To have an Even Sum, (i) the first card has to be odd and the second card has to be odd (or) (ii) the first card has to be even and the second card has to be even

The number of favourable ways = $5\times 5+5\times 5=25+25=50$

$$\text{Required Probability},$$

$$\therefore\ P(Sum\ is\ \ Even)=\frac{m}{n}=\frac{50}{100}=\frac{1}{2}$$

(iii)Without replacement: Suppose we select a card place it not back and select one card.

This can be happening in $10_{P_2}=90\ \ ways$

To have an Even sum, (i) the first card has to be odd and the second card has to be odd (or) (ii) the first card has to be even and the second card has to be even

The number of favourable ways = $5\times 5+5\times 5=25+25=50$

$$\text{Required Probability},$$

$$\therefore\ P(Sum\ is\ \ Even)=\frac{m}{n}=\frac{50}{90}=\frac{5}{9}$$

(26) In a box contains 4 granite stones, 5 sand stones, 6 bricks of identical size and shape. Out of them 3 are chosen at random. Find the chance that

(i) They belongs to different varieties.

(ii) They belongs to same varieties.

(iii)They are all granite stones.

(iv) 2 of the granite stones.

(v) None is Bricks.

*Solution* :

The total number of stones $= 4+5+6=15$

So, from these 15 stones, we choose 3 stones are in $15_{C_3}$ ways

i.e., The number of exhaustive events $n(S) = n = 15_{C_3} = 455$ ways

(i) Let $E_1$ be the event of chance that they are belongs to different variety.

The number of favourable events to the event $E_1$ is

$$n\left(E_1\right) = m = 4_{C_1} \times 5_{C_1} \times 6_{C_1} = 120 \text{ ways}$$

Required Probability

$$P\left(E_1\right) = \frac{n(E_1)}{n(S)} = \frac{m}{n} = \frac{120}{455} = 0.2638$$

(ii) Let $E_2$ be the event of chance that they are belongs to same variety.

The number of favourable events to the event $E_2$ is

$$n\left(E_2\right) = m = 4_{C_3} + 5_{C_3} + 6_{C_3} = 34 \text{ ways}$$

Required Probability

$$\therefore\ P\left(E_2\right) = \frac{n(E_2)}{n(S)} = \frac{m}{n} = \frac{34}{455} = 0.0748$$

(iii) Let $E_3$ be the event of chance that they are belongs to all are granite stones.

The number of favourable events to the event $E_3$ is

$$n\left(E_3\right) = m = 4_{C_3} = 4 \text{ ways}$$

Required Probability

$$\therefore\ P\left(E_3\right) = \frac{n(E_3)}{n(S)} = \frac{m}{n} = \frac{4}{455} = 0.0088$$

(iv) Let $E_4$ be the event of chance that they are belongs to 2 granite stones.

The number of favourable events to the event $E_3$ is

$$n\left(E_4\right) = m = 4_{C_2} \times 11_{C_1} = 66 \text{ ways}$$

Required Probability

$$\therefore\ P\left(E_4\right) = \frac{n(E_4)}{n(S)} = \frac{m}{n} = \frac{66}{455} = 0.1451$$

(v) Let $E_5$ be the event of chance that they are belongs to None is bricks stones.

The number of favourable events to the event $E_3$ is

$$n\left(E_5\right) = m = 9_{C_3} = 84 \text{ ways}$$

Required Probability

$$\therefore\ P\left(E_5\right) = \frac{n(E_5)}{n(S)} = \frac{m}{n} = \frac{84}{455} = 0.1847$$

## PROBLEMS ON COIN

(1) Two coins are tossed. Find the probability of getting (i) exactly one Head (ii)exactly two Heads (iii) at least one Head (iv) at most one Head.

Solution:

If two coins are tossed at a time, then the possible outcomes are

S = {HH, HT, TH, TT}

The total number of exhaustive events are (n) = 4

(i) Let $E_1$ be the event of getting exactly one Head. i.e., {HT, TH}

The number of favourable events are (m) = 2

$$\therefore P(E_1) = \frac{The\ number\ of\ \ favourable\ events}{The\ total\ number\ of\ \ exhaustive\ events} = \frac{m}{n} = \frac{2}{4} = \frac{1}{2}$$

(ii) Let $E_2$ be the event of getting exactly two Heads. i.e., {HH}

The number of favourable events are (m) = 1

$$\therefore P(E_2) = \frac{The\ number\ of\ \ favourable\ events}{The\ total\ number\ of\ \ exhaustive\ events} = \frac{m}{n} = \frac{1}{4}$$

(iii) Let $E_3$ be the event of getting at least One Head.

i.e., {HH, HT, TH}

The number of favourable events are (m) = 3

$$\therefore P(E_3) = \frac{The\ number\ of\ \ favourable\ events}{The\ total\ number\ of\ \ exhaustive\ events} = \frac{m}{n} = \frac{3}{4}$$

(iv) Let $E_4$ be the event of getting at most one Head.

i.e., {HT, TH, TT}

The number of favourable events are (m) = 3

$$\therefore P(E_4) = \frac{The\ number\ of\ \ favourable\ events}{The\ total\ number\ of\ \ exhaustive\ events} = \frac{m}{n} = \frac{3}{4}$$

(2) If three coins are tossed at a time, then the probability of getting (i) exactly one Head (ii) exactly two Heads (iii) at least two Heads (iv) at most two Heads (v) at least one Head (vi) at most one Head.

Solution:

If three coins are tossed at a time, then the possible outcomes are

S = {HHH, HHT, HTH, THH, HTT, THT, HTT, TTT}

The total number of exhaustive events are (n) = 8

(i) Let $E_1$ be the event of getting to exactly one Head.

i.e., {HTT, THT, TTH}

The number of favourable events are (m) = 3

$$\therefore P(E_1) = \frac{\text{The number of favourable events}}{\text{The total number of exhaustive events}} = \frac{m}{n} = \frac{3}{8}$$

(ii) Let $E_2$ be the event of getting exactly two Heads.

i.e., {HHT, HTH, THH}

The number of favourable events are (m) = 3

$$\therefore P(E_2) = \frac{\text{The number of favourable events}}{\text{The total number of exhaustive events}} = \frac{m}{n} = \frac{3}{8}$$

(iii) Let $E_3$ be the event of getting at least two Heads.

i.e., {HHH, HHT, HTH, HHT}

The number of favourable events are (m) = 4

$$\therefore P(E_3) = \frac{\text{The number of favourable events}}{\text{The total number of exhaustive events}} = \frac{m}{n} = \frac{4}{8} = \frac{1}{2}$$

(iv) Let $E_4$ be the event of getting to at most two Heads.

i.e., {HHT, HTH, THH, HTT, THT, TTH, TTT}

The number of favourable events are (m) = 7

$$\therefore P(E_4) = \frac{\text{The number of favourable events}}{\text{The total number of exhaustive events}} = \frac{m}{n} = \frac{7}{8}$$

(v) Let $E_5$ be the event of getting at least one Head.

i.e., {HHH, HHT, HTH, HHT, HTT, THT, TTH}

The number of favourable events are (m) = 7

$$\therefore P(E_5) = \frac{\text{The number of favourable events}}{\text{The total number of exhaustive events}} = \frac{m}{n} = \frac{7}{8}$$

(vi) Let $E_6$ be the event of getting at most one Head.

i.e., {HTT, THT, TTH, TTT}

The number of favourable events are (m) = 4

$$\therefore P(E_6) = \frac{\text{The number of favourable events}}{\text{The total number of exhaustive events}} = \frac{m}{n} = \frac{4}{8} = \frac{1}{2}$$

# PROBLEMS ON DICE

(1) A die is thrown. Find the probability of getting (i) a number is even (ii) a number is odd (iii) a composite number.

Solution:

In a die is thrown at a time, then the possible outcomes are

S = {1,2,3,4,5,6}

i.e., The total number of exhaustive events are n(S) = n = 6

(i) Let $E_1$ be the event that event of getting a number is even.

i.e., {2,4,6}

The number of favourable events are (m) = 3

$$\therefore P(E_1) = \frac{\text{The number of favourable events}}{\text{The total number of exhaustive events}} = \frac{m}{n} = \frac{3}{6} = \frac{1}{2}$$

(ii) Let $E_2$ be the event that event of getting a number is odd.

i.e., {1,3,5}

The number of favourable events are (m) = 3

$$\therefore P(E_2) = \frac{\text{The number of favourable events}}{\text{The total number of exhaustive events}} = \frac{m}{n} = \frac{3}{6} = \frac{1}{2}$$

(iii) Let $E_3$ be the event that event of getting a Composite number.

i.e., {4,6}

The number of favourable events are (m) = 2

$$\therefore P(E_3) = \frac{\text{The number of favourable events}}{\text{The total number of exhaustive events}} = \frac{m}{n} = \frac{2}{6} = \frac{1}{3}$$

(2) A die is thrown, If the number is odd. What is the probability that it is prime?

Solution:

In a die is thrown at a time, If the number is odd,

then the possible outcomes are S = {1,3,5}

i.e., The total number of exhaustive events are n(S) = n = 3

Let E be the event of getting a Prime number.

i.e., E = {3,5}

The number of favourable events are n(E)= m = 2

$$\therefore P(E) = \frac{\text{The number of favourable events}}{\text{The total number of exhaustive events}} = \frac{m}{n} = \frac{n(E)}{n(S)} = \frac{2}{3}$$

(3) Two unbiased dice are thrown. Find the probability that (i) Both dice show the same number (ii) The first dice show '6' (iii) The total of the number on the dice is '8' (iv) a sum '10' (v) The total of the number on dice is '13' (vi) The total of the number on the dice is any number from 2 to 12 (both inclusive).

*Solution* :

*In two dice are thrown at a time, then the number of possible outcomes are*

$$S=\begin{Bmatrix}(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)\\(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)\\(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\end{Bmatrix}$$

*i.e., The total number of exhaustive events are* $n(S) = n = 36$

$(i)$ *Let* $E_1$ *be the event of getting both dice shows the same number*

*The number of favourable events to the event '*$E_1$*'*

$$E_1=\{(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}$$

*i.e.,* $m = 6$

Re*quired* Pr*obability*

$$\therefore\ P(E_1)=\frac{m}{n}=\frac{6}{36}=\frac{1}{6}$$

$(ii)$ *Let* $E_2$ *be the event of getting* the first dice *shows the number* '6'

*The number of favourable events to the event '*$E_2$*'*

$$E_2=\{(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$$

*i.e.,* $m = 6$

Re*quired* Pr*obability*

$$\therefore\ P(E_2)=\frac{m}{n}=\frac{6}{36}=\frac{1}{6}$$

$(iii)$ *Let* $E_3$ *be the event of getting* the *total of the number* on *the dice is* '8'

*The number of favourable events to the event '*$E_3$*'*

$$E_2=\{(2,6)(3,5)(4,4)(5,3)(6,2)\}$$

*i.e.,* $m = 5$

Re*quired* Pr*obability*

$$\therefore\ P(E_3)=\frac{m}{n}=\frac{5}{36}$$

$(iv)$ Let $E_4$ be the event of getting a sum '10' on the two dice.

The number of favourable events to the event '$E_4$'

$$E_4 = \{(4,6)(5,5)(6,4)\}$$

i.e., $m = 3$

Required Probability

$$\therefore\ P(E_4) = \frac{m}{n} = \frac{3}{36} = \frac{1}{12}$$

$(v)$ Let $E_5$ be the event of getting the total of the value number on the dice is '13'.

The number of favourable events to the event '$E_5$'

$$E_5 = \Phi$$

i.e., $m = 0$

Required Probability

$$\therefore\ P(E_5) = \frac{m}{n} = \frac{0}{36} = 0 \qquad [\because \text{ Impossible event}]$$

$(vi)$ Let $E_6$ be the event of getting the total of the number on the dice is any number from 2 to 12(both inclusive).

The number of favourable events to the event '$E_6$'

$$E_6 = \begin{Bmatrix} (1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2)(2,3)(2,4)(2,5)(2,6) \\ (3,1)(3,2)(3,3)(3,4)(3,5)(3,6)(4,1)(4,2)(4,3)(4,4)(4,5)(4,6) \\ (5,1)(5,2)(5,3)(5,4)(5,5)(5,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6) \end{Bmatrix}$$

i.e., $m = 36$

Required Probability

$$\therefore\ P(E_6) = \frac{m}{n} = \frac{36}{36} = 1 \qquad [\because \text{ Certain event}]$$

(4) If two dice are thrown. What is the probability that (i) Sum is either 10 (or) 11? (ii)Sum is either 7 (or) 12 (iii)Sum is less than 5 (iv) Sum is less than equal 3 (v) Sum is greater than 8 (vi) Sum is greater than equal 9 (vii) Sum which is perfect square.

*Solution* :

*In two dice are thrown at a time, then the number of possible outcomes are*

$$S = \begin{Bmatrix} (1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2)(2,3)(2,4)(2,5)(2,6) \\ (3,1)(3,2)(3,3)(3,4)(3,5)(3,6)(4,1)(4,2)(4,3)(4,4)(4,5)(4,6) \\ (5,1)(5,2)(5,3)(5,4)(5,5)(5,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6) \end{Bmatrix}$$

*i.e., The total number of exhaustive events are* $n(S) = n = 36$

In two dice are thrown at a time, then the probabilities are

| Sum(X) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P(x) | 1/36 | 2/36 | 3/36 | 4/36 | 5/36 | 6/36 | 5/36 | 4/36 | 3/36 | 2/36 | 1/36 |

$(i)\ P(10 \cup 11) = P(10) + P(11)$ $\qquad [\because\ 10 \,\&\, 11 \textit{ are disjoint events}]$

$$= \frac{3}{36} + \frac{2}{36} = \frac{5}{36}$$

$(ii)\ P(7 \cup 12) = P(7) + P(12)$ $\qquad [\because\ 7 \,\&\, 12 \textit{ are disjoint events}]$

$$= \frac{6}{36} + \frac{1}{36} = \frac{7}{36}$$

$(iii)\ P(X < 5) = P(2) + P(3) + P(4)$ $\qquad [\because\ 2, 3 \,\&\, 4 \textit{ are disjoint events}]$

$$= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \frac{6}{36} = \frac{1}{6}$$

$(iv)\ P(X \le 3) = P(2) + P(3)$ $\qquad [\because\ 2 \,\&\, 3 \textit{ are disjoint events}]$

$$= \frac{1}{36} + \frac{2}{36} = \frac{3}{36} = \frac{1}{12}$$

$(v)\ P(X > 8) = P(9) + P(10) + P(11) + P(12)$ $\qquad [\because\ 9,10,11 \,\&\, 12 \textit{ are disjoint events}]$

$$= \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{10}{36} = \frac{5}{18}$$

$(vi)\ P(X \ge 9) = P(9) + P(10) + P(11) + P(12)$ $\qquad [\because\ 9,10,11 \,\&\, 12 \textit{ are disjoint events}]$

$$= \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{10}{36} = \frac{5}{18}$$

$(vii)$ *Let E be the event of getting* a sum which is a perfect square on throwing a pair of dice..

Then the required sum is either 4 (or) 9.

*The number of favourable events to the event 'E'*

$$E = \{(1,3)(2,2)(3,1)(3,6)(4,5)(5,4)(6,3)\}$$

$i.e., \quad m =$

Required Probability

$$\therefore \; P(E) = \frac{m}{n} = \frac{7}{36}$$

$(OR)$

$$(vii) \; P(4 \cup 9) = P(4) + P(9) \qquad [\because \; 4 \,\&\, 9 \; are \; disjoint \; events]$$

$$= \frac{3}{36} + \frac{4}{36} = \frac{7}{36}$$

(5) Two dice are thrown. Let A be the event that the sum of the points on the faces is 9. Let B be the event that at least one number is 6.

*Find* (i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A^C \cup B^C)$

*Solution*:

*In two dice are thrown at a time, then the number of possible outcomes are*

$$S = \begin{Bmatrix} (1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2)(2,3)(2,4)(2,5)(2,6) \\ (3,1)(3,2)(3,3)(3,4)(3,5)(3,6)(4,1)(4,2)(4,3)(4,4)(4,5)(4,6) \\ (5,1)(5,2)(5,3)(5,4)(5,5)(5,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6) \end{Bmatrix}$$

*i.e., The total number of exhaustive events are* $n(S) = n = 36$

*Let* $A$ *be the event of getting* a sum '9' *on the two dice.*

*The number of favourable events to the event* '$A$'

$$A=\{(3,6)(4,5)(5,4)(6,3)\}$$

$$i.e., \quad m=4$$

Required Probability

$$\therefore\ P(A)=\frac{m}{n}=\frac{4}{36}$$

*Let* $B$ *be the event of getting* at least *one number is* '6'

*The number of favourable events to the event* '$B$'

$$A=\{(1,6)(2,6)(3,6)(4,6)(5,6)(6,6)(6,1)(6,2)(6,3)(6,4)(6,5)\}$$

$$i.e., \quad m=11$$

Required Probability

$$\therefore\ P(B)=\frac{m}{n}=\frac{11}{36}$$

*Now, we have to find*

$$(i)\ A\cap B=\{(3,6)(6,3)\} \qquad [\because A \,\&\, B \text{ are not disjoint events}]$$

$$i.e., \quad m=2$$

Required Probability

$$\therefore\ P(A\cap B)=\frac{m}{n}=\frac{2}{36}=\frac{1}{18}$$

$$(ii)\ P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=\frac{4}{36}+\frac{11}{36}-\frac{2}{36}$$

$$=\frac{13}{36}$$

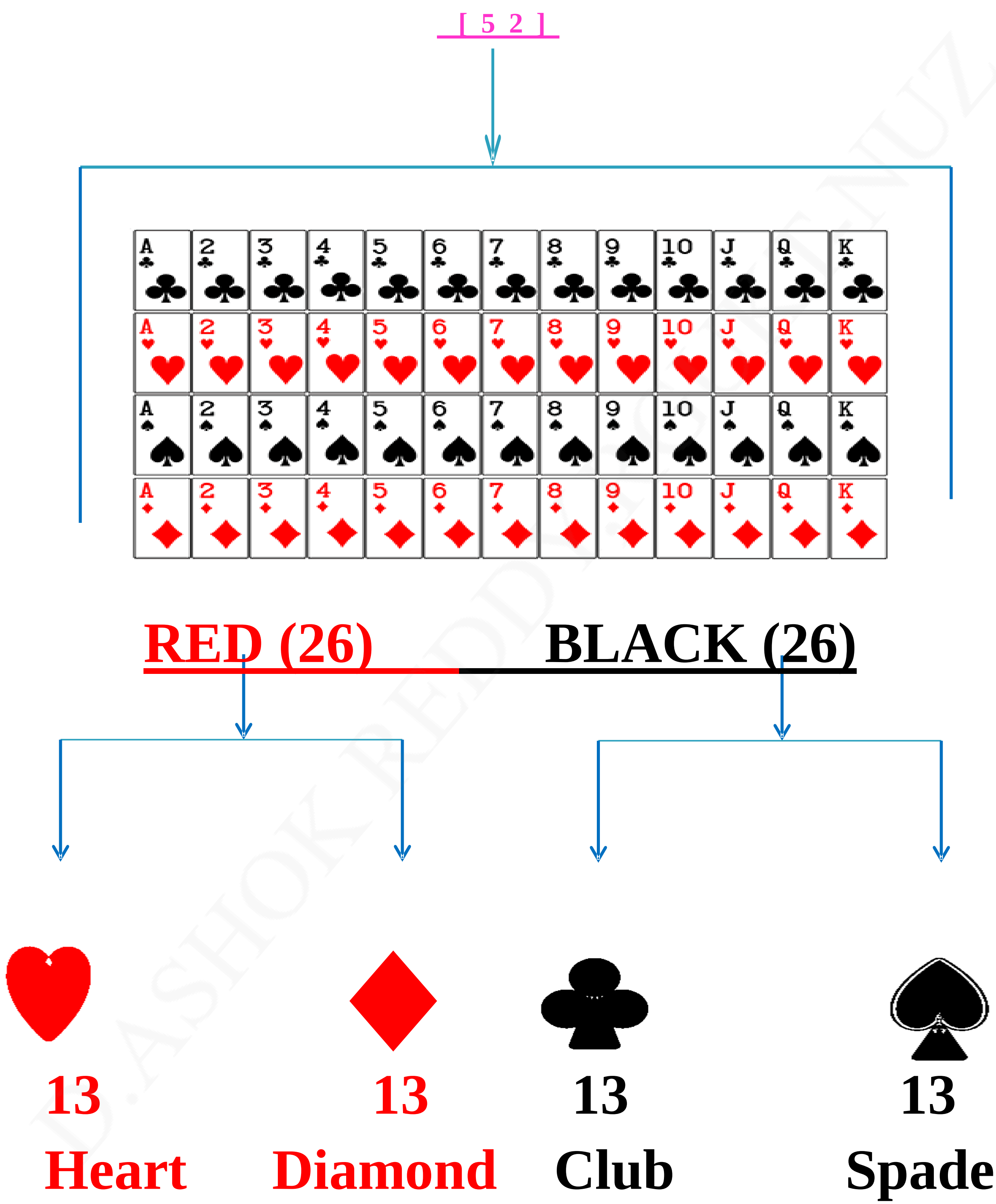
$$(iii)\ P\left(A^C\cup B^C\right)=P\left(\overline{A}\cup\overline{B}\right)$$

$$=P\left(\overline{A\cap B}\right) \qquad \left[\begin{array}{l}\because \text{ by using Demorgan Law's} \\ P\left(\overline{A}\cup\overline{B}\right)=P\left(\overline{A\cap B}\right)\end{array}\right]$$

$$=1-P(A\cap B) \qquad \left[\because\ P(A\cap B)+\left(\overline{A\cap B}\right)=1\right]$$

$$=1-\frac{1}{18}=\frac{17}{18}$$

## PROBLEMS ON PACK OF CARDS



**Playing card pack (or) deck contains 52 cards**

**52 cards are in 4 suits, 13 cards in each suit**

SUIT (or) SET: Ace,2,3,4,5,6,7,8,9,10, J, Q, K (13 cards)

NUMBER CARDS: 2,3,4,5,6,7,8,9,10 (9 cards)

LETTER CARDS: Ace(A), Jack(J), Queen(Q), King(K) (4 cards)

FACE (or) HONOUR CARDS: J, Q, K (3 cards)

(1) Four cards are drawn at random from a pack of 52 cards. Find the probability that

(i) They are a king, a queen, a jack & ace.
(ii) Two are kings & Two are queens.
(iii) Two are black & Two are red.
(iv) There are two cards of hearts & two cards of diamonds.
(v) There is one card of each suit.
(vi) All are diamonds.

*Solution:*

*Fours cards are drawn from a well shuffled pack of* $52$ *cards in* $52_{C_4}$ *ways*

$\therefore$ *The total number of exhaustive events* $n(S) = n = 270725$

$(i)$ *Let* $E_1$ *be the event of that among four cards one is king, one is queen, one is jack & one is ace.*

*The number of favourable events* $n(E_1) = m = 4_{C_1} \times 4_{C_1} \times 4_{C_1} \times 4_{C_1}$

$= 4 \times 4 \times 4 \times 4 = 256$

$\therefore$ Required Probablity

$$P(E_1) = \frac{m}{n} = \frac{256}{270725} = 0.0009472$$

$(ii)$ *Let* $E_2$ *be the event of that two are queen & two are kings*

*The number of favourable events* $n(E_2) = m = 4_{C_2} \times 4_{C_2}$

$= 6 \times 6 = 36$

$\therefore$ Required Probablity

$$P(E_2) = \frac{m}{n} = \frac{36}{270725} = 0.0001329$$

$(iii)$ *Let* $E_3$ *be the event of that two are black & two are red*

*The number of favourable events* $n(E_3) = m = 26_{C_2} \times 26_{C_2}$

$$= 325 \times 325 = 105625$$

$\therefore$ Required Probablity

$$P(E_3) = \frac{m}{n} = \frac{105625}{270725} = 0.3902$$

$(iv)$ *Let* $E_4$ *be the event of that among four cards two are hearts & two are diamonds*

*The number of favourable events* $n(E_4) = m = 13_{C_2} \times 13_{C_2}$

$$= 78 \times 78 = 6084$$

$\therefore$ Required Probablity

$$P(E_4) = \frac{m}{n} = \frac{6084}{270725} = 0.0224$$

$(v)$ *Let* $E_5$ *be the event of consisting of one card of each suit.*

*The number of favourable events* $n(E_5) = m = 13_{C_1} \times 13_{C_1} \times 13_{C_1} \times 13_{C_1}$

$$= 13 \times 13 \times 13 \times 13 = 28561$$

$\therefore$ Required Probablity

$$P(E_5) = \frac{m}{n} = \frac{28561}{270725} = 0.1055$$

$(vi)$ *Let* $E_6$ *be the event of all are diamonds.*

*The number of favourable events* $n(E_6) = m = 13_{C_4}$

$$= 715$$

$\therefore$ Required Probablity

$$P(E_6) = \frac{m}{n} = \frac{715}{270725} = 0.002642$$

(2) A card is drawn from a well shuffled pack of cards. What is the probability of a card being either queen (or) king?

*Solution* :

A cards is drawn from a well shuffled pack of 52 cards in $52_{C_1}$ ways

$\therefore$ The total number of exhaustive events $n(S) = n = 52$

Let $E_1$ be the event of drawing a queen card

The number of favourable events $n(E_1) = m = 4_{C_1}$

$= 4$

$\therefore$ Required Probablity

$$P(E_1) = \frac{m}{n} = \frac{4}{52} = \frac{1}{13}$$

Let $E_2$ be the event of drawing a king card

The number of favourable events $n(E_2) = m = 4_{C_1}$

$= 4$

$\therefore$ Required Probablity

$$P(E_2) = \frac{m}{n} = \frac{4}{52} = \frac{1}{13}$$

here, $E_1$ & $E_2$ are disjoint (or) mutually exclusive events

$$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

$$= \frac{1}{13} + \frac{1}{13}$$

$$\therefore P(E_1 \cup E_2) = \frac{2}{13}$$

(3) A card is drawn from a well shuffled pack of cards. What is the probability of a card being either spade (or) ace?

*Solution* :

$A \ cards$ is $drawn \ from \ a \ well \ shuffled \ pack \ of \ 52 \ cards \ in \ 52_{C_1} \ ways$

$\therefore The \ total \ number \ of \ exhaustive \ events \ n(S) = n = 52$

$Let \ E_1 \ be \ the \ event \ of \ drawing \ a \ Spade \ card$

$The \ number \ of \ favourable \ events \ n(E_1) = m = 13_{C_1}$

$= 13$

$\therefore \text{Re}quired \ \text{Pr}obablity$

$$P(E_1) = \frac{m}{n} = \frac{13}{52}$$

$Let \ E_2 \ be \ the \ event \ of \ drawing \ a \ Ace \ card$

$The \ number \ of \ favourable \ events \ n(E_2) = m = 4_{C_1}$

$= 4$

$\therefore \text{Re}quired \ \text{Pr}obablity$

$$P(E_2) = \frac{m}{n} = \frac{4}{52}$$

$here, \ E_1 \ \& \ E_2 \ are \ not \ disjo\text{int} \ (or) \ not \ mutually \ exclusive \ events$

$Let \ E_1 \cap E_2 \ be \ the \ event \ of \ drawing \ spade \ \& \ ace \ card.$

$$P(E_1 \cap E_2) = \frac{1}{52}$$

$$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$\therefore P(E_1 \cup E_2) = \frac{4}{13}$$

(4) One card is drawn from a regular deck of 52 cards. What is the probability of a card being red (or) a king?

*Solution:*

*A cards is drawn from a well shuffled pack of* 52 *cards in* $52_{C_1}$ *ways*

$\therefore$ *The total number of exhaustive events* $n(S) = n = 52$

*Let* $E_1$ *be the event of drawing a red card*

*The number of favourable events* $n(E_1) = m = 26_{C_1}$

$= 26$

$\therefore$ Required Probablity

$$P(E_1) = \frac{m}{n} = \frac{26}{52}$$

*Let* $E_2$ *be the event of drawing a king card*

*The number of favourable events* $n(E_2) = m = 4_{C_1}$

$= 4$

$\therefore$ Required Probablity

$$P(E_2) = \frac{m}{n} = \frac{4}{52}$$

*here,* $E_1$ & $E_2$ *are not disjoint (or) not mutually exclusive events*

*Let* $E_1 \cap E_2$ *be the event of drawing red & king card.*

$$P(E_1 \cap E_2) = \frac{2}{52}$$

$$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52}$$

$$= \frac{28}{52}$$

$$\therefore P(E_1 \cup E_2) = \frac{7}{13}$$

(5) *Two cards are drawn succession from a pack of 52 cards. Find the probability that first is king & second is queen. If the first card is (i) replaced (with replacement) (ii) not replaced (without replacement)*

*Solution:*

A cards is drawn from a well shuffled pack of 52 cards in $52_{C_1}$ ways

$\therefore$ The total number of exhaustive events $n(S) = n = 52$

Let $E_1$ be the event of drawing a king card

The number of favourable events $n(E_1) = m = 4_{C_1}$

$= 4$

$\therefore$ Required Probablity

$$P(E_1) = \frac{m}{n} = \frac{4}{52}$$

Let $E_2$ be the event of drawing a queen card

The number of favourable events $n(E_2) = m = 4_{C_1}$

$= 4$

$\therefore$ Required Probablity

$$P(E_2) = \frac{m}{n} = \frac{4}{52}$$

here, $E_1$ & $E_2$ are independent events

(i) replaced (with replacement)

when the card is drawn first card is replaced, we have

$$\therefore P(E_1 \cap E_2) = P(E_1) \times P(E_2)$$

$$= \frac{4}{52} \times \frac{4}{52}$$

$$= \frac{1}{13} \times \frac{1}{13}$$

$$\therefore P(E_1 \cap E_2) = \frac{1}{169}$$

(ii) not replaced (without replacement)

when the card is drawn first card is not replaced, we have

$$\therefore P(E_1 \cap E_2) = P(E_1) \times P(E_2)$$

$$= \frac{4}{52} \times \frac{4}{51}$$

$$= \frac{1}{13} \times \frac{4}{51}$$

$$\therefore P(E_1 \cap E_2) = \frac{4}{663}$$

(6) *Two cards are drawn succession from a pack of 52 cards. Find the probability that they are both Aces. If the first card is (i) replaced (with replacement) (ii) not replaced (without replacement)*

*Solution*:

*A cards* is *drawn from a well shuffled pack of* 52 *cards in* $52_{C_1}$ *ways*

$\therefore$ *The total number of exhaustive events* $n(S) = n = 52$

*Let* $E_1$ *be the event of drawing a ace card*

*The number of favourable events* $n(E_1) = m = 4_{C_1}$

$= 4$

$\therefore$ Required Probablity

$$P(E_1) = \frac{m}{n} = \frac{4}{52}$$

*Let* $E_2$ *be the event of drawing a ace card*

*The number of favourable events* $n(E_2) = m = 4_{C_1}$

$= 4$

$\therefore$ Required Probablity

$$P(E_2) = \frac{m}{n} = \frac{4}{52}$$

*here,* $E_1$ & $E_2$ *are independent events*

(*i*)*replaced* (*with replacement*)

*when the card is drawn first card is replaced, we have*

$$\therefore P(E_1 \cap E_2) = P(E_1)P(E_2)$$

$$= \frac{4}{52} \times \frac{4}{52}$$

$$= \frac{1}{13} \times \frac{1}{13}$$

$$\therefore P(E_1 \cap E_2) = \frac{1}{169}$$

(*ii*)*not replaced* (*without replacement*)

*when the card is drawn first card is not replaced, we have*

$$\therefore P(E_1 \cap E_2) = P(E_1)P(E_2)$$

$$= \frac{4}{52} \times \frac{3}{51}$$

$$= \frac{1}{13} \times \frac{1}{17}$$

$$\therefore P(E_1 \cap E_2) = \frac{1}{221}$$