

DESIGN AND ANALYSIS OF ALGORITHMS

UNIT-1

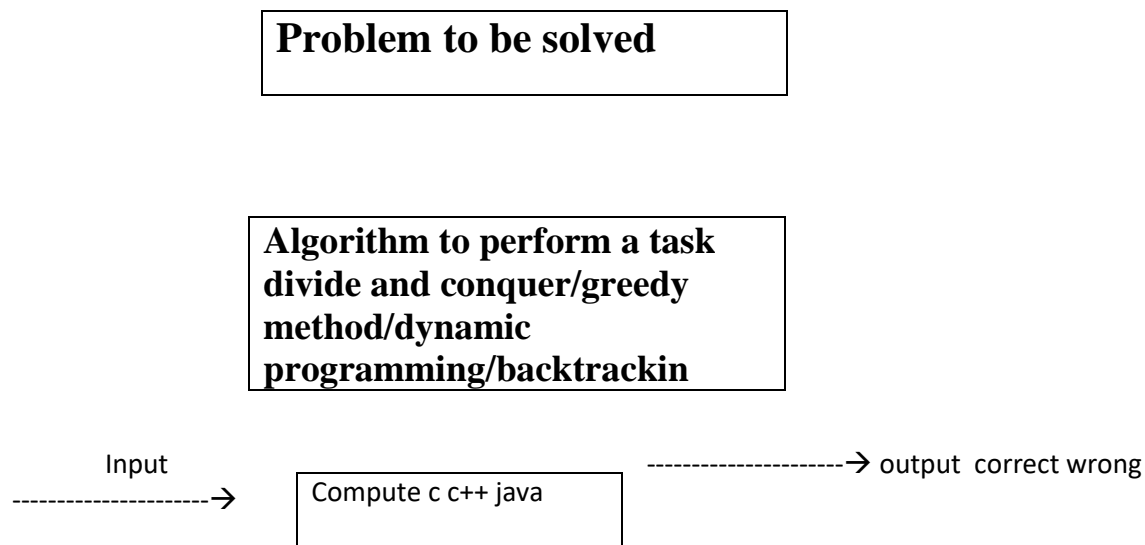
Algorithm: it is step by step procedure to solve a particular task or problem.

Or

It is a finite number of steps which can be used to solve a particular task or problem in step by step procedure.

Abu jafar mohammed ibn musa al khwarizmi in 780dc.

Notation of an algorithm:



Properties of an algorithm:

- 1.input:
- 2.output
- 3.definiteness
- 4.effectiveness
- 5.finiteness

1.input: algorithm may take zero or more number of inputs

```
Void main()
{
printf("WELCOME");
}
```

----→ welcome

```
Void main()
{
Int a,b,c;
Printf("enter a,b values");
Scanf("%d%d",&a,&b);
C=a+b;
Printf("c=",&c);
}
```

----→a

----→b

---→c

2. Output: an algorithm should produce at least one output. If there is no output it is simply not an algorithm.

3. Definiteness: each and every statement should be clearly stated.
2+5=7 yes

10+2=12yes
10-5=5yes
10/0= infine wrong

4. effectiveness: each and every statement should effective. An algorithms does't contain unnecessary statements.

```
Void main()
{
Int a,b,c;//necessary statement
Printf("enter a,b values");// may or maynot
Scanf("%d%d", &a,&b);//necessary statement
C=a+b;//necessary statement
Scanf("%d",&c);// un necessary statement
Printf("c=",&c);// necessary statement
}
```

5. Finiteness: an algorithm must be terminating at any particular point. If there is no terminating point simply it is not an algorithm.

Ex:

```
For(int i=0;i<=5;i++)
{

}
```

Output: 0 1 2 3 4

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Pseudo code for expressing algorithm

Pseudo code is an artificial and informal language which can be used to implement an algorithm.

Pseudo code is text based language which can be used to implement an algorithm.

Algorithm is basically sequence of instructions written in simple English language.

Algorithm can be represented in two ways

- 1.flow chart-----graphical representation of an algorithm
- 2.pseudo code -----it is text based representation of an algorithm by using some programming constraints.

Representation of an algorithms using pseudo code

- 1.each algorithm must starting with head and body

Algorithm algorithm-name(parameter1,parameter2..)

The body may represents with open brace and closed with closed braces.
{-----→ begin

}----→ending

Ex:

algorithm add(a,b,c)

Begin

Ending

2.every statement must ending with (;) delimiter

3.single line comments are written as ‘//’ beginning of comment

4.identifiers must starting with letters but not the digits

Ex: value-----yes

9value-----wrong

5.if your assigning a value to the variable we can use assignment operator(:=)

Syntax:

Variable :=value;

Ex: a:=10;

6.there are some operators are

Ex: <, <=, >, >=

7. The input and output statements can be declared as

In c language we can use printf() statement but where as in algorithm we can use write

Printf -----for output -----algorithm---→ write

Scanf-----for input= algorithm-----→ read

8. the conditional statements such as if-then-else can be declared as

```
If(condition)
Begin
    Write(" a is large");
Ending
```

```
Else
Begin
    Write("b is large");
ending
```

9. for loop can be declared as

Program:

```
For(int a=0;a<=9;a++)
{
Printf("welcome");
}
```

Algorithm:

```
For variable:=value1 to value2 do
Begin
```

```
Write("welcome");
```

```
Ending
```

10. the while loop can be declared as

Syntax:

```
While(condition)
begin
```

```
Statement1;  
Statement2;  
Ending
```

11. the switch case can be declared as

```
Switch(value)  
Begin  
Case1: statement;  
Break;  
Case2:statement;  
Break;  
Ending
```

12.functions can be declared as

```
Returntype function-name(parameters)  
Begin
```

```
Statements;
```

```
Ending
```

1.implement addition of two numbers algorithm using pseudo code

```
Algorithm addition(a,b,c)  
Begin  
Write(“enter a, b vales”);  
Read(“read a, b values”);  
C:=adding a,b values;  
Write(“display c values”);  
Ending
```

2.implementing whether the given number is even or odd algorithm
using pseudo code($\text{value} \% 2 == 0$)

Algorithm evenodd(a)

Begin

 Write("enter a value");

 Read("read a value");

 If(condition)

 Begin

 Write("given number is even");

 Ending

 Else

 Begin

 Write("given number is odd");

 Ending

ending

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Approaches of an Algorithm

1. Priori analysis
2. Posterior analysis

1. Priori analysis: before executing the algorithm we will give behaviour or performance of an algorithm. It is not giving exact result. Less accurate.

2. Posterior analysis: after executing the algorithm we will measure the execution time or performance of an algorithm. It will give exact results or accurate results.

Best case: which algorithm will take less amount of time to complete the task or problem that is best algorithm. Which problem will take less number of steps to complete that is best case.

Worst case: which algorithm will take max amount of time to complete the task or problem that is worst algorithm. Which problem will take max number of steps to complete that is called worst case.

Average case: which algorithm will take average amount of time to complete the task or problem that is average algorithm. Which problem will take average number of steps to complete that is called average case.

Ex: Linear Search (Sequential Search)

Array=

2	4	5	6	8	9	10	11	13	14
---	---	---	---	---	---	----	----	----	----

Problem: search the element 2 in an array using posterior analysis

Best case:

Sol: searching starting from zero index

A[0] -----→ 2 and our searching element is 2

Element 2 is in the a[0]. So we can solve the problem within a single step

Time complexity $O(1)$

Average case:

Problem: searching the element 8 is in the array or not

Sol:

First a[0] compare with 8 (2=8) not matching

A[1] compare with (4=8) not matching

A[2] compare with (5=8) not matching

A[3] compare with (5=8) not matching

A[4] compare with (8=8) both are matching

So the time complexity is $O(n/2)$

Worst case:

Problem. Searching the element 14 whether the element is in the array or not.

Sol:

A[0]-----→ 2 not matching

A[1]-----→ 4 not matching

A[2]-----→ 5 not matching

A[3]-----→ 6 not matching

A[4]-----→ 8 not matching

A[5]-----→ 9 not matching

A[6]-----→ 10 not matching

A[7]-----→11 not matching

A[8]-----→13 not matching

A[9]-----→14 matching means searching element is in the array index a[9]

Time complexity $O(n)$

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Performance of an algorithm

The efficiency of an algorithm can be decided by measuring the performance of an algorithm.

We can measure the performance of an algorithm by using two factors

1. Space complexity
2. Time Complexity

1. Space Complexity: The amount of memory taken by the algorithm to complete the task or problem that is called space complexity.

The amount of memory required by an algorithm during the execution of the algorithm.

Space complexity means space to store only data values but not the space to store the algorithm itself.

Ex: Int a,b a=2bytes b=2bytes inside the ram

Space complexity can be calculated by using the equation

$$S(p)=C+SP$$

Where S(P)=space of problem

C=constant variables or fixed variables---→ means those are not dependent variables

Sp=instance variables or variable parts---→ means those are depends on some other factors.

Ex1:

Algorithm display()

Begin

 Write(“welcome”);

End

$$S(p)=C+SP$$

No variables are declared in this algorithm so c=0 and sp=0

S(p)=0 no memory is required for this algorithm

Ex2:

Algorithm add(a,b,c)

Begin

 Write(“enter a,b values”);

 Read(“read a, b values”);

 C=add a, b values;

 Write(“display c value”);

Ending

$$S(p)=C+SP$$

A variable occupy 1 memory space

B variable occupy 1 memory space

C variable occupy 1 memory space

There is no instance variables so SP=0

Space complexity $S(p) = C + SP$
 $= 3 + 0$
 $S(p) = 3$

Ex3:

Algorithm add(x,n)

Begin

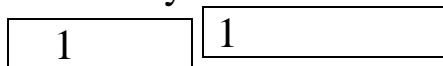
Sum:=0;

For i:=1 to n do

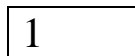
Sum:=sum+x[i];

Ending

Sum 2 bytes i



N



X[i] requiry n memory space



Space complexity $S(p) = C + SP$
 $= 3 + n$

Ex4:

Algorithm display(i,n)

Begin

For i:=1 to n do

begin

Write("welcome");

end

End

Space complexity $S(p) = C + SP$

I occupy 1 memory space

N occupy 1 memory space

There is no instance variables $sp=0$

$S(p) = 2 + 0 \rightarrow 2$ memory spaces

Ex5:

Algorithm multiply(i,n,a,b,c)

Begin

For i:=1 to n do

Begin

C:=a[i]+b[i];

Ending

Ending

I occupy 1 memory space

N occupy 1 memory space

C variable occupy 1 memory space

A[i] occupy n memory space because it is depends up on n value

B[i] occupy n memory space because it is depends up on n value

Space complexity $S(p) = 3 + 2n$ memory spaces

2. Time complexity: the amount of time taken by an algorithm to complete the task or problem.

Time complexity is measured by using frequency count. Frequency count means how many number of times that the statements are executed.

Ex1:

s.no	algorithm	Frequency count
1	Algorithm display()	0
2	begin	0
3	Write("welcome")	1
4	end	0

Frequency count=0+0+1+0=1

Time complexity= O(1)

Ex2:

s.no	Algorithm	Frequency count
1	Algorithm display(i,n)	0
2	begin	0
3	For i:=1 to n do	N+1
4	begin	0
5	Write("welcome")	n
6	end	0
7	End	0

Frequency count=0+0+n+1+0+n+0+0
=2n+1

Time complexity=O(2n+1)

Note: when you want to calculate time complexity first we eliminate the constants. And we will take the upper polynomial time

Time complexity=O(n)

Ex4:

s.no	Algorithm	Frequency count
1	Algorithm add(a,b,c,i,j,n)	0
2	begin	0
3	For i:=1 to n do	N+1
4	Begin	0
5	For j:= 1 to n do	N(N+1)
6	Begin	0
7	C[i,j]:=a[i,j]+b[i,j]	N*n
8	end	0
9	end	0
10	end	0

$$\begin{aligned}\text{Frequency count} &= 0+0+n+1+0+n(n+1)+0+n^2+0+0+0 \\ &= n+1+n^2+n+n^2+1 \\ &= 2n^2+2n+2\end{aligned}$$

$$\text{Time complexity} = O(2n^2+2n+1)$$

Note: when you want to calculate time complexity first we eliminate the constants. And we will take the upper polynomial time

$$\text{Time complexity} = O(n^2+n)$$

$$= O(n^2)$$

Ex:

s.no	Algorithm	Frequency count
1	Algorithm mul(a,b,c,i,j,n)	0
2	begin	0
3	For i:=1 to m do	M+1
4	Begin	0
5	For j:= 1 to n do	M(N+1)
6	Begin	0
7	C[i,j]:=a[i,j]*b[i,j]	M*n
8	end	0
9	end	0
10	end	0

$$\begin{aligned}\text{Frequency count} &= 0+0+m+1+0+m(n+1)+0+mn+0+0+0 \\ &= m+1+mn+m+mn \\ &= 2mn+2m+1\end{aligned}$$

$$\text{Time complexity} = O(2mn+2m+1)$$

Note: when you want to calculate time complexity first we eliminate the constants. And we will take the upper polynomial time

$$\begin{aligned}\text{Time complexity} &= O(mn+m) \\ &= O(mn)\end{aligned}$$

Analysis of linear search

```
int main()
{
    int array[100], search, c, n;
    printf("Enter number of elements in array\n");-----1
    scanf("%d", &n);-----1
    printf("Enter %d integer(s)\n", n);-----1
    for (c = 0; c < n; c++)-----n+1
        scanf("%d", &array[c]);-----n
    printf("Enter a number to search\n"); -----1
    scanf("%d", &search); -----1
    for (c = 0; c < n; c++)-----n+1
    {
        if (array[c] == search)
        {
            printf("%d is present at location %d.\n", search, c+1);-----n
            break;
        }
    }
    if (c == n)
        printf("%d isn't present in the array.\n", search);-----1
    return 0;
}
```

Analysis of linear search

space complexity $S(P) = 3 + n$
search=1, n=1, c=1 sp=n

time complexity = $4n + 8$
 $= O(4n + 8)$
 $= O(n)$

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Asymptotic Notations

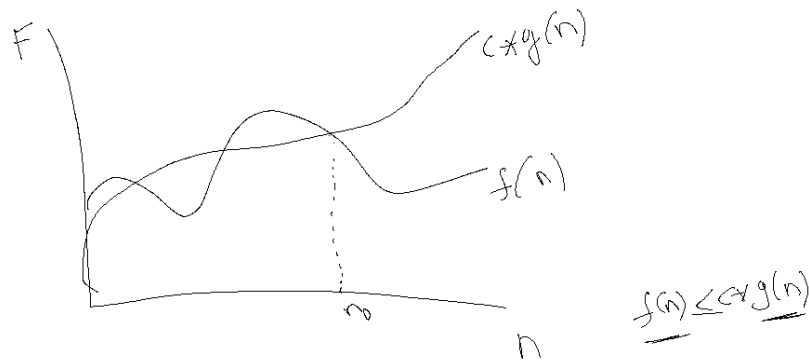
To choose the best algorithm we need to check efficiency of an algorithm. Efficiency of an algorithm is depends upon time complexity.

Asymptotic notation is a shorthand way to represent the time complexity of an algorithm. Using asymptotic notations we can give best worst and average.

1. Big oh Notation
2. Omega Notation
3. Theta Notation
4. Little on Notation
5. Little omega Notation

1. Big oh Notation: it is represented by “O”. It is method of representing upper bound of an algorithm running time. Using big oh notation we can give longest amount time taken by the algorithm to complete.

Definition: let $f(n)$ and $g(n)$ are two non-negative functions. And there exists an integer n_0 and constant c such that $c > 0$ and for all integers $n > n_0$, then $\mathbf{F(n) \leq c * g(n)}$. Then $f(n) = O(g(n))$



EX: consider the function $f(n)=2n+2$ and $g(n)=n^2$ we have to find constant c such that $f(n) \leq c \cdot g(n)$

Sol:

$N=1$ and $c=1$

$$F(n) \leq c \cdot g(n)$$

$$2n+2 \leq c \cdot n^2$$

$$2(1)+2 \leq 1 \cdot (1)^2$$

$$4 \leq 1 \text{-----} \rightarrow \text{false}$$

$N=2$ and $c=1$

$$2(2)+2 \leq 1 \cdot (2)^2$$

$$6 \leq 4 \text{-----} \rightarrow \text{false}$$

$N=3$ and $c=1$

$$2(3)+2 \leq 1 \cdot (3)^2$$

$$8 \leq 9 \text{-----} \rightarrow \text{true}$$

$N=4$ and $c=1$

$$2(4)+2 \leq 1*(4)^2$$

$$10 \leq 16 \text{-----} \rightarrow \text{true}$$

$N=5$ and $c=1$

$$2(5)+2 \leq 1*(5)^2$$

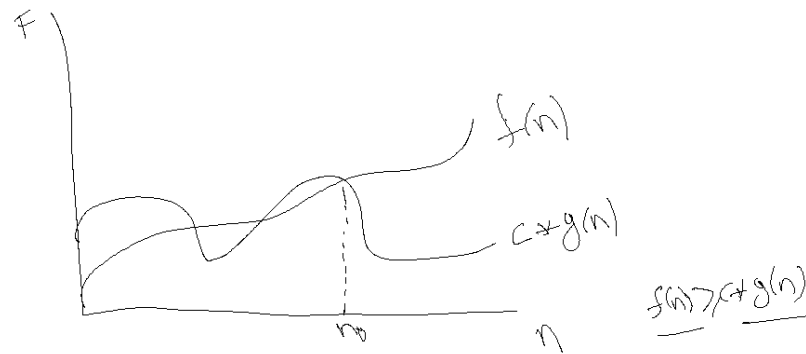
$$12 \leq 25 \text{-----} \rightarrow \text{true}$$

Where $n=3$ and $c=1$ we are satisfying the Big oh notation

$$\begin{aligned} \text{So time complexity } f(n) &= O(g(n)) \\ &= O(n^2) \end{aligned}$$

2. Omega Notation: it is denoted by " Ω ". It is method of representing lower bound of an algorithm running time. Using the omega notation we can calculate shortest amount of time taken by the algorithm to complete.

Definiton: let $f(n)$ and $g(n)$ are two non-negative functions and there exists an integer and constant where $c > 0$ and $n > n_0$ then $f(n) \geq c * g(n)$
Then $f(n) = \Omega(g(n))$.



Example: let $f(n)=3n+3$ and $g(n)=2n+5$ and calculate c and n values and satisfy the omega notation.

Sol:

$N=1$ and $c=1$ then

$$F(n) \geq c * g(n)$$

$$3n+3 \geq c * 2n+5$$

Substitute the n , c values

$$3(1)+3 \geq 1 * 2(1)+5$$

$$6 \geq 7 \text{-----} \rightarrow \text{false}$$

$N=2$ and $c=1$

$$3n+3 \geq c * 2n+5$$

$$3(2)+3 \geq 1 * 2(2)+5$$

$$9 \geq 9 \text{-----} \rightarrow \text{true}$$

$N=3$ and $c=1$

$$3n+3 \geq c * 2n+5$$

$$3(3)+3 \geq 1 * 2(3)+5$$

$$12 \geq 11 \text{-----} \rightarrow \text{true}$$

$$N=4 \text{ and } c=1$$

$$3(4)+3 \geq 1*2(4)+5$$

$$16 \geq 13 \text{-----} \rightarrow \text{true}$$

Where $c=1$ and $n=2$ we can satisfy the condition $f(n) \geq c*g(n)$

$$\begin{aligned} \text{Then time complexity} &= \Omega(g(n)) \\ &= \Omega(2n+5) \\ &= \Omega(n) \end{aligned}$$

Ex: let $F(n)=2n+5$ and $g(n)=n$ and calculate c and n values such that $f(n) \geq c*g(n)$

Sol:

$$N=1 \text{ and } c=1 \text{ then}$$

$$2(1)+5 \geq 1*1$$

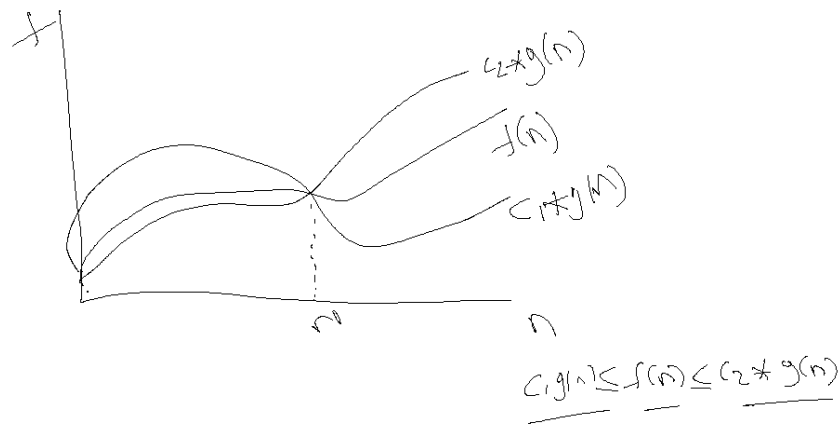
$$7 \geq 1 \text{-----} \rightarrow \text{true}$$

Where $n=1$ and $c=1$ we are satisfying the condition $f(n) \geq c*g(n)$

$$\text{Time complexity} = \Omega(n)$$

3. Theta Notation: it is denoted by “ Θ ”. It is a method of representing average bound of an algorithm running time. Means we can calculate average amount of time taken by the algorithm to complete.

Definition: let there exists two non-negative function $f(n)$ and $g(n)$ and there exists constants c_1 , c_2 and integer n , then $c_1 < c_2$ and $n > n_0$ such that $c_1 * g(n) \leq f(n) \leq c_2 * g(n)$.



prob: $f(n)=2n+8$ $g(n)=n$ then calculate c_1, c_2 and n values such that satisfy theta notation.

Sol: $c_1 * g(n) \leq f(n) \leq c_2 * g(n)$

$C_1 < c_2$

$C_1=2$ and $c_2=7$ and $n=1$

$C_1 * n \leq 2n+8 \leq c_2 * n$

Substitute the values

$2*1 \leq 2(1)+8 \leq 7(1)$

$2 \leq 10 \leq 7$ -----false

$N=2$

$2*2 \leq 2(2)+8 \leq 7(2)$

$4 \leq 12 \leq 14$ -----true

$N=3$

$2*3 \leq 2(3)+8 \leq 7(3)$

$6 \leq 14 \leq 21$ -----true

Where $c_1=2$ and $c_2=7$ and $n>1$ we can satisfy the theta notation now

Time complexity = $\omega(g(n))$
= $\omega(n)$

4. little oh notation: it is denoted by "o" let $f(n)$ and $g(n)$ are two non-negative function then

Lim $f(n)/g(n) = 0$ then such that $f(n) = o(g(n))$
 $n \rightarrow \infty$

5. little omega notation: it is denoted by " ω " let $f(n)$ and $g(n)$ are two non-negative functions such that

Lim $g(n)/f(n) = 0$ such that $f(n) = \omega(g(n))$
 $n \rightarrow \infty$

prob: determine $f(n) = 12n^2 + 6n$ is $O(n^3)$

sol: big oh notation condition is

$f(n) \leq c * g(n)$
 $f(n) = 12n^2 + 6n$
 $g(n) = n^3$

$n=1$ and $c=2$

$12n^2 + 6n \leq c * n^3$
 $12(1)^2 + 6(1) \leq 2 * (1)^3$
 $18 \leq 2$ ----- false

$N=2$ and $c=2$

$12(2)^2 + 6(2) \leq 2 * (2)^3$

$$48+12 \leq 2*8$$
$$60 \leq 16 \text{-----false}$$

$$N=3$$

$$12(3)^2+6(3) \leq 2*(3)^3$$

$$108+18 \leq 2*27$$
$$126 \leq 54 \text{----false}$$

$$N=4$$

$$12(4)^2+6(4) \leq 2*(4)^3$$
$$192+24 \leq 2*64$$
$$216 \leq 128 \text{-----false}$$

$$N=5$$

$$12(5)^2+6(5) \leq 2*(5)^3$$

$$12*25+30 \leq 2*125$$
$$330 \leq 250$$

$$N=6$$

$$12(6)^2+6(6) \leq 2*(6)^3$$
$$12*36+36 \leq 2*216$$

$$432+36 \leq 432$$
$$468 \leq 432$$

$$N=7$$

$$12(7)^2+6(7) \leq 2*(7)^3$$
$$12*49+42 \leq 2*343$$

$$588+42 \leq 686$$

$$630 \leq 686 \text{---TRUE}$$

Where $c=2$ and $n=7$ we are satisfy the big oh notation such that

$$\begin{aligned}\text{Time complexity} &= O(g(n)) \\ &= O(n^3)\end{aligned}$$

or

$$N=3 \text{ and } c=5$$

$$12(3)^2 + 6(3) \leq 5*(3)^3$$

$$126 \leq 5*27$$

$$126 \leq$$

$$\text{Prob: } f(n) = 4n^2 - 64n + 288 = \Omega(n^2)$$

$$\text{Sol: omega notation } f(n) \geq c*g(n)$$

$$\text{LHS } f(n) = 4n^2 - 64n + 288$$

$$N=1 \text{ } c=5 \text{ then}$$

$$F(n) = 4(1)^2 - 64(1) + 288$$

$$= 4 - 64 + 288$$

$$= 228$$

$$\text{RHS } c*g(n)$$

$$= C*n^2$$

$$= 5*(1)^2$$

$$= 5$$

$$\text{LHS} \geq \text{RHS}$$

$$F(n) \geq c*g(n)$$

$$F(n) = \Omega(g(n)) \text{ is proved}$$

15-12-2021

Analysis of Insertion sort

Insertion sort works similar to the sorting of playing play cards. It is assumed that the first card is already in sorted order. We can select an unsorted card then compare with sorted card.

If the unsorted card is greater than the sorted card then we can place at right hand side.

If the unsorted card is less than the sorted card then we can place at left hand side.

Algortihm:

- 1.assume that the first element is in sorted order
- 2.select the next element and sorted it separately[compare with current element to its predecessor]
- 3.if the element is greater than the predecessor then we can we place at right hand side
- 4.if the element is less than the predecessor then we can place at left hand side(swapping condition)
5. repeat the steps 2 to 4 up to all the elements are placing in the sorted order.

3	6	2	1	8
-----sorted--		←-----unsorted sub array-----→		

Ex:

12	31	25	8	32	17
----	----	----	---	----	----

--sorted- | ←-----unsorted sub-array-----→

Step-1: assume the first element in the array is sorted sub-array means the element is in sorted order.

Step-2: select next element from the unsorted sub-array and compare with elements in the sorted sub-array

Step-3: now 31 is greater than 12 that means 31 is placing at right hand side ($31 > 12$) means 12 is placing at correct position.

Now 12 and 31 is in sorted sub-array

12	31	25	8	32	17
----	----	----	---	----	----

---sorted sub-array--→ | ←-----unsorted sub-array-----→

Step-4: now we can select the next element from the unsorted sub-array. And compare with all the elements in the sorted sub-array.

Now 25 is less than 31 ($25 < 31$) then we can placed at left hand side.

Swap 25 with 31

12	31	25	8	32	17
----	----	----	---	----	----

After swapping

12	25	31	8	32	17
----	----	----	---	----	----

Now again 25 is comparing with element 12 now 25 is greater than 12 ($25 > 12$) means 25 is placing at right hand side. Now the sorted array is

12	25	31	8	32	17
----	----	----	---	----	----

←-----sorted sub-array--→ | ←-----unsorted sub-array-----→

Step-5: again select the next element from the unsorted sub-array. Now 8 is comparing with all the elements in the sorted sub-array.

Now 8 is compare with element 31, now 8 is less than 31($8 < 31$) then we can placed at left hand side. Apply swapping now 8 is swapping with element 31

12	25	31	8	32	17
----	----	----	---	----	----

After swapping the array is

12	25	8	31	32	17
----	----	---	----	----	----

Step-6: Now again 8 is comparing with element 25. Now $8 < 25$ means we can placed at left hand side. We can apply swapping condition

Now before swapping the array is

12	25	8	31	32	17
----	----	---	----	----	----

After swapping the array is

12	8	25	31	32	17
----	---	----	----	----	----

Step-7: Now again element 8 is comparing with element 12 which is less than the element 12. ($8 < 12$) means 8 is placing at left hand side. We can swapping condition. 8 is swapping with element 12

12	8	25	31	32	17
----	---	----	----	----	----

After swapping the array is

8	12	25	31	32	17
---	----	----	----	----	----

←-----sorted sub-array-----→ | ←unsorted sub-array→

Step-8: we can select next element from unsorted sub-array. And comparing with all the elements in the sorted sub-array.

Now 32 is comparing with its predecessor 31. Now 32 is greater than 31 ($32 > 31$) now we can placed at right hand side. That means 31 is placing at right position. Again 32 is comparing with 25 ($32 > 25$) we can placed at right hand side only. 32 is comparing with element 12 ($32 > 12$) we can placed at right hand side only. Again 32 is comparing with element 8 ($32 > 8$) we can placed at right hand side only.

After than

8	12	25	31	32	17
---	----	----	----	----	----

←-----sorted sub-array-----→ | <un sorted>

Step-9: now again we can select the next element from unsorted sub-array. And compare with all the elements in the sorted sub-array. Now 17 is comparing with its predecessor element 32. Now 17 is less than 32 ($17 < 32$) we can placed at left hand side. Apply swapping condition now 17 is swapping with element 32

8	12	25	31	32	17
---	----	----	----	----	----

After swapping

8	12	25	31	17	32
---	----	----	----	----	----

Step-10: now element 17 is comparing with element 31. 17 is less than 31 ($17 < 31$) now we can placed at left hand side. Swapping condition is apply. Now 17 is swapping with element 31

8	12	25	31	17	32
---	----	----	----	----	----

After swapping

8	12	25	17	31	32
---	----	----	----	----	----

Step-11: now again the element 17 is comparing with the element 25.
Now 17 is less than 25($17 < 25$). Now 17 is swapping with element 25

8	12	25	17	31	32
---	----	----	----	----	----

After swapping

8	12	17	25	31	32
---	----	----	----	----	----

Step-12: now again element 17 is comparing with the element 12.
Now 17 is greater than 12. Now we can placed at right hand side.

8	12	17	25	31	32
---	----	----	----	----	----

Now all the elements in the array are placing in sorted order

Algorithm:

Algorithm insertion(i,n,a[])

Begin

For i:=1 to n

Temp:=a[i]

J:=i-1

While(j>=0&&a[j]>temp)

Begin

A[j+1]=a[j]

J=j-1;


```

End while
A[j+1]=temp
End for

```

End

Algorithm:

Algorithm insertion(i,j,n,a[])

```

{
For(i=1;i<n;i++)
{
Temp=a[i];
J=i-1;
While(j>=0&& a[j]>temp)-----compare whether it is greater or less than
{
A[j+1]=a[j]
J=j-1
}
A[j+1]=temp-----storing statement
}-----ending for loop
}

```

Ex:

5	4	10	1	6	2
Sorted	←-----unsorted sub-array -----→				

Sol:

Step-1: i=1

For(i=1;1<6;i++) -----true

Temp=a[i]=a[1]

Temp=4

J=i-1

J=1-1=0

While(j>=0&&a[j]>temp)

While(0>=0&&a[0]>temp)

While(0>=0&&5>4)-----true

{

A[j+1]=a[j] → a[0+1]=a[0] → a[1]=a[0]

A[1]=a[0] that means a[0] is swapping with a[1]

4	5	10	1	6	2
---	---	----	---	---	---

J=j-1 → 0-1

J=-1

}

Again while loop is executed

While(j>=0&&a[j]>temp)

While(-1>=0&&a[-1]>temp) -----false

Out of while loop is executed

A[j+1]=temp

A[-1+1]=temp

A[0]=temp

That means element 4 is placed at a[0]

4	5	10	1	6	2
---	---	----	---	---	---

---sorted-----→ | < ---- unsorted sub-array-----→

Step-2: increment i value $i=i++=i+1=1+1=2$

```
For(i=2;2<6;i++)
{
Temp=a[i]
Temp=a[2]
Temp=10
J=i-1
J=2-1
J=1
While(j>=0&& a[j]>temp)
While(1>=0&& a[1]>temp)
While(1>=0&& 5>10)-----false
```

If while loop is false then out of while loop is executed.

```
A[j+1]=temp
A[1+1]=10
A[2]=10
```

Again i value is incremented

Step-3: $i=i+1=2+1=3$

4	5	10	1	6	2
---	---	----	---	---	---

```
For(i=3;3<6;i++)
{
Temp=a[i]
Temp=a[3]
Temp=1

J=i-1
J=3-1
J=2
```

```

While(j>=0&&a[j]>temp)
While(2>=0&&a[2]>1)
While(2>=0&&10>1)-----ture
{
A[j+1]=a[j]
A[2+1]=a[2]
A[3]=a[2] means a[2] is swapping with a[3]

```

4	5	1	10	6	2
---	---	---	----	---	---

```

J=j-1
J=2-1
J=1
}

```

Again same while loop is executed

```

While(1>=0&&a[j]<temp)
While(1>=0&&a[1]>1)
While(1>=0&&5>1)-----true
{
A[j+1]=a[j]
A[1+1]=a[1]
A[2]=a[1] means a[1]is swpping with a[2]

```

4	1	5	10	6	2
---	---	---	----	---	---

```

J=j-1
J=1-1=0
J=0
}

```

Again same while loop is executed

```

While(j>=0&&a[j]>temp)
While(0>=0&&a[0]>temp)

```

```

While(0>=0&&4>1)-----true
{
A[j+1]=a[j]
A[0+1=a[0]
A[1]=a[0] means =a[0] is swpping with a[1]

```

1	4	5	10	6	2
---	---	---	----	---	---

```

J=j-1
J=0-1
J=-1
}

```

```

Again while loop[
While(j>=0&&a[j]>temp)
While(-1>=0&&a[-1]>1)----false

```

Out of while loop is executed

```

A[j+1]=temp
A[-1+1]=temp
A[0]=temp -----means 1 is stored in a[0]

```

Step-3; i is incremented i=i+1

1	4	5	10	6	2
---	---	---	----	---	---

```

I=3+1=4
For (i=4;4<6;i++)
{
Temp=a[i]=a[4]=6
Temp=6
J=i-1
J=4-1=3
While(j>=0&&a[j]>temp)

```

```

While(3>=0&&a[3]>6)
While(3>=0&&10>6)-----true
{
A[j+1]=a[j]
A[3+1]=a[3]
A[4]=a[3] swap a[3] with a[4]

```

1	4	5	6	10	2
---	---	---	---	----	---

```

J=j-1=3-1=2
}

```

Again while loop is executed

```

While(j>=0&&a[j]>temp)
While(2>=0&&a[2]>6)
While(2>=0&&5>6)-----false

```

Out of while loop is executed

```

A[j+1]=temp
A[2+1]=temp
A[3]=temp
A[3]=6

```

Step-4: i value is incremented

1	4	5	6	10	2
---	---	---	---	----	---

```

I=i+1=4+1=5
For(i=5;5<6;i++)-----true
{
Temp=a[i]=a[5]=2
J=i-1=5-1=4
While(4>=0&&a[4]>2)
While(4>=0&&10>2)-----true

```

```
{
A[j+1]=a[j]
A[4+1]=a[4]
A[5]=a[4]
```

1	4	5	6	2	10
---	---	---	---	---	----

```
J=j-1
J=4-1
J=3
}
Again while loop executed
```

```
While(3>=0&&a[3]>temp)
While(3>=0&&6>2)-----true
{
A[j+1]=a[j]
A[4]=a[3]
```

1	4	5	2	6	10
---	---	---	---	---	----

```
J=j-1
J=3-1
J=2
}
```

```
Again while loop executed
While(2>=0&&a[2]>temp)
While(2>=0&&5>2)-----true
{
A[j+1]=a[j]
A[2+1]=a[2]
A[3]=a[2] swap a[2] with a[3]
```

1	4	2	5	6	10
---	---	---	---	---	----

```
J=j-1
```

```
J=2-1
J=1
}
```

Again while loop is executed

```
While (j>0&& a[j]>temp)
While (1>=0&& a[1]>2)
While (1>=0&& 4>2) -----true
{
A [1+1] =a [1]
A [2] =a [1] swap a [1] with a [2]
```

1	2	4	5	6	10
---	---	---	---	---	----

```
J=j-1
J=1-1=0
}
```

Again while loop is executed

```
While (j>=0&& a[j]>temp)
While (0>=0&& a[0]>2)
While (0>=0&& 1>2) ----false
Means 2 is placed in correct position
Out of for loop is executed
```

```
A[j+1]=temp
A[0+1]=temp
A[1]=temp
Element 2 is stored in a[1]
```

Finally i values is incremented

Step-5: $i=i+1=5+1=6$

For($i=6; 6 < 6; i++$)-----false

Finally all the elements are in sorted order ascending order

1	2	4	5	6	10
---	---	---	---	---	----