Comparison of functions:

In general order of functions

$$\boxed{1 < \log n < \sqrt{n} < n\log n < n^2 < n^3 < \ldots 2^n < 3^n \ldots n^n}$$

even though though functions order is same but functions will behave differ after some large value of 'n'

Ex:

	$f(n)=n^2$	$g(n)=2^n$	
$n=1$	1	2	$f(n) < g(n)$
$n=2$	4	4	$f(n) = g(n)$
$n=3$	9	8	$f(n) > g(n)$

$\therefore$ so we can't say just by looking which one is greater, so we need to check for large value of 'n' how it will behave, then only we can say which one is bigger or smaller.

In order to check we may apply log and simplify.

Logarithmic properties:

1. $\log_a^a = 1$
2. $\log(ab) = \log a + \log b$
3. $\log(a/b) = \log a - \log b$
4. $\log a^n = n\log a$
5. $a^{\log_c^b} = b^{\log_c^a}$
6. If $a^b = n$ then $b = \log_a^n$

Steps:

1. In order to compare two functions if any common term is both side cancel it out.
2. If we place 'n' large value, If value can be predictable and behaviour same then just based value we can judge.
3. If n't apply log and simplify for large 'n' values we will get constant, so that we can judge.

Eg1:

$f(n) = n^2 \quad g(n) = n^3$

$\cancel{n^2} \qquad \cancel{n^2} * n \qquad$ [$\because$ cancel common term]

$\boxed{1 < n}$

$\therefore f(n) < g(n) \qquad \therefore \boxed{n^2 < n^3}$

Eg2:

$f(n) = 2^n \qquad g(n) = n^2$

apply log on both sides

$\log(2^n)$	$\log(n^2)$
$\Rightarrow n \log_2 2$	$\Rightarrow 2 \log_2 n$
$\Rightarrow n$	$\Rightarrow 2 \log_2 n$

$\boxed{2^n > n^2}$

Assume $n = 2^{10}$

$2^{10} \qquad \Rightarrow 2 \log_2 2^{10}$

$2^{10} \qquad 2 * 10 \log_2 2 \qquad [\because \log a^b = b \log a]$

$2^{10} > 20 \quad \Rightarrow \therefore f(n) > g(n)$

Eg 3: 3^n 2^n

apply log on both sides

$\log(3^n)$ | $\log(2^n)$

$\Rightarrow \cancel{n}\log 3$ | $\Rightarrow \cancel{n}\log 2$

Commonterm 'n'

$\log 3 > \log 2$

$\therefore \boxed{3^n > 2^n}$

Eg 4:

n^2 $n\log n$

$\cancel{n} * n$ $\cancel{n} * \log n$

Cancel the common term

n $\log n$

Assume $n = 2^{1024}$

$\Rightarrow 2^{1024}$ $\log 2^{1024}$

$2^{1024} > 1024$

$\boxed{\therefore n^2 > n\log n}$

Eg 5:

n $(\log n)^{100}$

apply log both sides

$\log n$ | $\log[(\log n)^{100}]$

$\log n$ | $100 * \log\log n$

Assume $n = 2^{2^{10}}$ [here loglog so I am taking $n = 2^{2^{10}}$]

Eg 3: 3^n 2^n

apply log on both sides

$\log(3^n)$ | $\log(2^n)$

$\Rightarrow \cancel{n}\log 3$ | $\Rightarrow \cancel{n}\log 2$

common term 'n'

$\log 3 > \log 2$

$\therefore \boxed{3^n > 2^n}$

Eg 4: n^2 $n\log n$

$\cancel{n} * n$ $\cancel{n} * \log n$

Cancel the common term

n $\log n$

Assume $n = 2^{1024}$

$\Rightarrow 2^{1024}$ $\log 2^{1024}$

$2^{1024} > 1024$

$\boxed{\therefore n^2 > n\log n}$

Eg 5:

n $(\log n)^{100}$

apply log both sides

$\log n$ | $\log[(\log n)^{100}]$

$\log n$ | $100 * \log\log n$

Assume $n = 2^{2^{10}}$ [here loglog so I am taking $n = 2^{2^{10}}$]

2^{10} | $100 * \log\log_2 2^{2^{10}}$

2^{10} | $100 * \log_2 2^{10}$

$1024 > 1000$

$$\boxed{\therefore \quad n > (\log n)^{100}}$$

Eg 6: $n^{\log n}$ | $n \log n$

apply log on both sides

$\log(n^{\log n})$ | $\log(n \log n)$

$\log n \log n$ | $\log n + \log\log n$

Let $n = 2^{2^{10}}$

$\log_2 2^{2^{10}} \cdot \log_2 2^{2^{10}}$ $\quad$ $\log_2 2^{2^{10}} + \log\log_2 2^{2^{10}}$

$2^{10} * 2^{10}$ $\quad$ $2^{10} + \log_2 2^{10}$

$2^{20} > 2^{10} + 10$

$2^{20} > 1034$

$$\therefore \boxed{n^{\log n} > n \log n}$$

Eg 7: $\sqrt{\log n}$ $\quad$ $\log\log n$

apply log on both sides

$\log((\log n)^{1/2})$ $\quad$ $\log(\log\log n)$

$\frac{1}{2}\log\log n$ $\quad$ $\log\log\log n$

Let $n = 2^{2^{10}}$

$\frac{1}{2}\log\log_2 2^{2^{10}}$ $\log\log\log_2 2^{2^{10}}$

$\frac{1}{2}\log_2 2^{10}$ $\log\log_2 2^{2^{10}}$

$\frac{1}{2} * 10$ $\log 10$

$5 > 3.5$

$\boxed{\therefore \sqrt{\log n} > \log\log n}$

Eg8: $n^{\sqrt{n}}$ $n^{\log n}$

apply log both sides

$\log(n^{\sqrt{n}})$ $\log(n^{\log n})$

$\Rightarrow \sqrt{n}\,\cancel{\log n}$ $\Rightarrow \log n\,\cancel{\log n}$

cancel common term

$\Rightarrow \sqrt{n}$ $\log n$

apply log on both sides

$\log(n^{1/2})$ $\log\log n$

$\Rightarrow \frac{1}{2}\log n$ $\Rightarrow \log\log n$

Let $n = 2^{1024}$

$\Rightarrow \frac{1}{2}\log_2 2^{1024}$ $\Rightarrow \log\log_2 2^{1024}$

$\Rightarrow \frac{1}{2} \times 1024$ $\Rightarrow \log_2 2^{10}$

$512 > 10$

$\boxed{\therefore n^{\sqrt{n}} > n^{\log n}}$

Eg 9:

$$f(n) = \begin{cases} n^3 & 0 < n < 10{,}000 \\ n^2 & n \geq 10{,}000 \end{cases}$$

$$g(n) = \begin{cases} n & 0 < n < 100 \\ n^3 & n > 100 \end{cases}$$

Range	0-99	100-9999	> 10,000
f(n)	n^3	n^3	n^2
g(n)	n	n^3	n^3

∴ After 10,000 for large value of 'n'

$f(n) = n^2$ (behaves)

$g(n) = n^3$

$$\boxed{n^2 < n^3}$$

Eg 10:

$f(n) = 2n \qquad g(n) = 3n$

Here both are equal we will get Asymptotically O(n) for both functions.

Eg 4: $f(n) = 2^n \qquad g(n) = 2^{2n}$

apply log

$\log(2^n)$	$\log(2^{2n})$
$n \cancel{\log 2}$	$2n \cancel{\log 2}$

$n < 2n$

$$\therefore \boxed{2^n < 2^{2n}}$$

Assignments

1) Compare the following functions

a) $f(n) = n^2 \log n$ $\quad g(n) = n(\log n)^{10}$

b) $f(n) = 3n^{\sqrt{n}}$ $\quad g(n) = 2^{\sqrt{n}\log n}$

c) $f(n) = n^{\log n}$ $\quad g(n) = 2^{\sqrt{n}}$

d) $f(n) = n^{\log n}$ $\quad g(n) = 2^{n}$

2) State whether the following statements are True or false?

a) $(n+k)^m = O(n^m)$ $\quad$ k is constant?

b) $2^{n+1} = O(2^n)$

c) $2^{2n} = O(2^n)$

d) $\sqrt{\log n} \geq O(\log\log n)$

e) $n^{\log n} > O(2^n)$