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Random Theory of Variables :-

* A function from the sample space S to the real no. system ~~are~~ is called a Random Variables.

i.e., a mapping $X : S \rightarrow R$ where S is the sample space and R is the Real no. system ~~is called a~~ is called as RV.

* eg: if Two coins are tossed

$$S = \{ \overset{s_1}{HH}, \overset{s_2}{HT}, \overset{s_3}{TH}, \overset{s_4}{TT} \}$$

define $X : S \rightarrow R$ as $X(s) =$ "no. of heads".

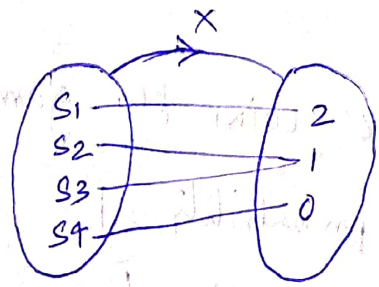
then, $X(s_1) = 2$

$X(s_2) = 1$

$X(s_3) = 1$

$X(s_4) = 0$

Therefore.



Here, X assumes the values {0, 2, 1} which are real no.'s . Therefore X is a Random Variable.

* ## Types of Random Variables :-

There are two types of Random Variables :-

① Discrete ② Continuous

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Discrete RV :-

- The random variable 'x' which assumes only finite values in the given interval i.e., it assumes only the set 0, 1, 2 ... n is called a Discrete RV.
- In other words, a discrete RV is a variable which can only take a countable no of values.

  eg: Rolling a die, tossing a coin

Continuous RV :-

- The random variable 'x' which assumes the infinite value in the given interval is called a continuous RV.
- In other words, a continuous RV is a variable which the date can take infinitely many values.

  eg :- Time, Temperature

* Discrete Probability Distribution

- Let, 'X' be a discrete RV for the sample space of tossing two coins. Then X assumes the values {0, 1, 2}
- Now,

$$P(X=0) = \frac{1}{4}$$

$$P(X=1) = \frac{2}{4} = \frac{1}{2}$$

$$P(X=2) = \frac{1}{4}$$

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X	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Now, Sum of probability = 1, Hence $P(x)$ is called PMF
This is called Discrete probability distribution
(PMF : probability Mass function)

① find the DPD for the sum of the dice if two are rolled

Sol Let 'S' be the sample space of rolling 2 dice

$n(S) = 36$

define $X : S \to R$ as $X(S) =$ "sum of 2 dice"

Hence 'X' assumes the values $\{2, 3, 4, 5, 6 \ldots 12\}$

$P(x=2) = \frac{1}{36}$ $\quad P(x=7) = \frac{6}{36}$ $\quad P(12) = \frac{1}{36}$

$P(x=3) = \frac{2}{36}$ $\quad P(x=8) = \frac{5}{36}$

$P(x=4) = \frac{3}{36}$ $\quad P(x=9) = \frac{4}{36}$

$P(x=5) = \frac{4}{36}$. $\quad P(x=10) = \frac{3}{36}$

$P(x=6) = \frac{5}{36}$. $\quad P(x=11) = \frac{2}{36}$

X	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\cancel{\frac{2}{36}}\ \frac{1}{18}$	$\frac{3}{36}$	$\cancel{\frac{4}{36}}\ \frac{1}{9}$	$\frac{5}{36}$	$\cancel{\frac{6}{36}}\ \frac{1}{6}$	$\frac{5}{36}$	$\cancel{\frac{4}{36}}\ \frac{1}{9}$	$\frac{3}{36}$	$\cancel{\frac{2}{36}}\ \frac{1}{18}$	$\frac{1}{36}$

This is the req. probability distribution.

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FN

(2) Find the DPD for the min. no of the 2 dice i.e., $X : S \to R$ as $X(s) = \min\{a,b\}$ if 2 dice are rolled.

Let 'S' be the sample space of rolling 2 dice.

$n(S) = 36$

Define $X : S \to R$ as $X(s) = \min\{a,b\}$.

Hence X assumes values $\{1, 2, 3, 4, 5, 6\}$

$P(X=1) = \frac{11}{36}$

$P(X=2) = \frac{9}{36}$

$P(X=3) = \frac{\cancel{9}\,7}{36}$

$P(X=4) = \frac{5}{36}$

$P(X=5) = \frac{3}{36}$

$P(X=6) = \frac{1}{36}$.

X	1	2	3	4	5	6
P(x)	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{\cancel{9}\,7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

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Note :—

(8) Consider the discrete probability distribution in the following table :

X	x_1	x_2 .... x_n
P(x)	p_1	p_2 .... p_n

① $\Sigma p_i = 1$ [P(x) is called p.m.f]

② Mean $\mu = \Sigma x_i P(X = x_i) = E(x) =$ Expectation of X

③ Variance, $\sigma^2 = \Sigma (x_i - \mu)^2 p_i$

$$\sigma^2 = \Sigma x_i^2 p_i - \mu^2$$

The positive sq. root of variance is called 'Standard Deviation'.

~~M.D~~ Mean deviation, $MD = \Sigma |x_i - \mu| \, P(X = x_i)$

① A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) find the value of k

(ii) $P(X<6)$

(iii) $P(X \geq 6)$

(iv) $P(0<X<5)$

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Continuous Probability Distribution

Let X be a continuous random variable.
since the continuous random variable 'X' assumes the infinite values in the given interval.
i.e., it assumes the values $(-\infty$ to $\infty)$

Then, the probability function $f(x)$ is defined on $(-\infty, \infty)$ such that,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad (\text{p.d.f})$$

if the above satisfies, then the fn $f(x)$ is called Probability density function.

Mean, $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

Variance, $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

Mean deviation, $M.D = \int_{-\infty} |x-\mu| f(x)\,dx$.

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if $P(a<x<b) = \int_a^b f(x)\,dx$.

Distribution fn: (or) Cummulative distribution fn.

$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$.

x = const
$-\infty$ is the lowest lmt of fn.

(ans in terms of x)

$$\boxed{f(x) = \frac{d}{dx} F(x)}$$