Topic

# ⇒ closure of a set of FD's :-

→ A functional dependency is said to be closure if it cover all attributes.

## Algorithm or procedure :-

Step1 :- Equivate an attribute (or) attributes for which clousure need to be identified ($x^+ = x$).

Step2 :-

Take FD one by one and verify whether LHS is available in X, if so add RHS attributes to X,

Step3 :- Repeat step2 as many times as possible to cover attributes.

Step 4:- Stop procedure after no more attributes added to X and declare X as closure set of attributes.

(OR)

→ **Algorithm to Compute closure of set of FD's F:**

1. $F^+ = F$.
2. Repeat.
   - 2.1 For each FD 'f' in $F^+$
     - 2.1.1. Apply reflexivity and Augmentation rules on 'f'.
     - 2.1.2 Add resulting FD to $F^+$.
   - 2.2. For each pair of FD 'f1' and 'f2' in $F^+$.

2.2.1 if f1 and f2 Can Combined on transitivity.

2.2.2 Then, Add resulting FD to F+.

3. Untill F+ does not change further.

—o—

Ex:- Given Relation

STUDENT ( STUD_NO, STUD_NAME, STUD_PHONE, STUD_STATE, STUD_COUNTRY, STUD_AGE )

Ans:- STUD_NO is unique.

Functional Dependency Set:-

Functional dependency set or FD set of a relation is the set of all FD's present in the relation.

→ For Example FD relation Student in the table

{ STUD_NO → STUD_NAME,
STUD_NO → STUD_PHONE,
STUD_NO → STUD_STATE,

STUD-NO → STUD-COUNTRY,

STUD-NO → STUD-AGE, STUD-STATE → STUD-COUNTRY}

Attribute closure:- Attribute closure of an attribute set can be defined as set of attributes which can be functionally determined from it.

How to find attribute closure of an attribute set:-

→ ADD elements of attribute set to the result set.

→ Recursively add elements to the result set which can be functionally determined from the elements of the result set.

(FD set table:-

(STUD-NO)+ = {STUD-NO, STUD-NAME, STUD-PHONE, STUD-STATE, STUD-COUNTRY, STUD-AGE}

(STUD-STATE)+ = {STUD-STATE, STUD-COUNTRY}.

## How to find Candidate keys and Super keys using Attribute closure:

→ If attribute closure of an attribute set contains all attributes of relation, the attributes set will be super key of the relation.

→ If No subset of this attribute set can functionally determine all attributes of the relation, the set will be Candidate key as well.

→ See the below Example.

→ (STUD_NO, STUD_NAME)$^+$ =

{STUD_NO, STUD_NAME, STUD_PHONE, STUD_STATE, STUD_COUNTRY, STUD_AGE}

→ (STUD_NO)+ = { STUD_NO, STUD_NAME, STUD_PHONE, STUD_STATE, STUD_COUNTRY; STUD_AGE }

→ (STUD_NO, STUD_NAME) will be Super key but not Candidate key because its Subset (STUD_NO)+ is equal to all attributes of the relation. So; STUD_No will be a Candidate key.

— o —

Ex! - (1)



Q). Given Relation R with Schema R(A, B, ~~C~~, D, E, G, H, K) with set of FD's F:

$A \to B$

$B \to DE$

$E \to GH$

$K \to H$

$B \to K$ find $A^+$?.

Ans:-

(a) $A^+ = (A)$

$= (A, B)$ $[A \to B]$

$= (A, B, D, E)$ $[B \to DE]$

$= (A, B, D, E, K)$ $[B \to K]$

$= (A, B, D, E, K, G, H)$ $\{E \to GH$

$= (A, B, D, E, K, G, H)$ $[K \to H]$

Therefore, A+ is closure set.

it is super key because it

Covered all attributes.

(b) $B^+$

$B^+ = B$

$= BDE$ $[B \to DE]$

$\Rightarrow BDEGH$ $[E \to GH]$

$= BDEGHK$ $[B \to K]$

$B^+$ is not super key

because it does not covered

all attributes. (i.e. A) not covered.

(c) $E^+ = E$

$= EGH$ $[E \to GH]$

$E^+$ is not super key

because it does not covered

all attributes (i.e A, B, ~~C~~, D, K)

Ex:- 2

Consider the following FD's.

$AB \to CD$
$AF \to D$
$DE \to F$
$C \to G$
$F \to E$
$G \to A$

⇒ which is following incorrect?.

options. a) $(CF)^+ = \{A, C, D, E, F, G\}$

b) $(BG)^+ = \{A, B, C, D, G\}$

c) $(AF)^+ = \{A, C, D, E, F, G\}$

D) $(AB)^+ = \{A, C, D, F, G\}$

a) $(CF)^+ = \{A, C, D, E, F, G\}$

$(CF)^+ = CF$

$= CFE \quad (F \to E)$

$= CFEG \quad (C \to G)$

$= CFEGD \quad (AF \to D)$

$= CFEGDA \quad (G \to A)$

All attributes are covered.

So Correct statement.

b) $(BG)^+ = \{A, B, C, D, G\}$

$(BG)^+ = BG$

$= BGCD \quad (AB \to CD)$

$\cancel{BGCDF} \quad \cancel{(DE \to F)}$

$= BGCDA \quad (G \to A)$

Correct all attributes are covered.

c) $(AF)^+ = \cancel{AF}\ \{A, C, D, E, F, G\}$

$\times$

$(AF)^+ = AF$

$= AFE \quad (F \to E)$

$= AFED \quad (AF \to D)$

all attributes are not covered

This is incorrect.

D) $(AB)^+ = \{A, C, D, F, G\}$

$\times$

$(AB)^+ = AB$

$= ABCD \quad [AB \to CD]$

$= ABCDG \quad \{C \to G\}$

$[G \to A]$

but already A remain.

all attributes are not covered

This is incorrect.

Ex:- 3

In a Schema with, attributes A, B, C, D, E and set of FD's are.

$A \to B$
$A \to C$
$CD \to E$
$B \to D$
$E \to A$

which of the following FD's not implied by the above set.

options

a) $CD \to AC$
b) $BD \to CD$
c) $BC \to CD$
d) $AC \to BC$

a) $CD \to AC$

$(CD)^+ = CD$
$= CDE \quad (CD \to E)$
$= CDEA \quad (E \to A)$
$= CDEAB \quad (A \to B)$

So, $CD \to AC$

b) $BD \to CD$

$(BD)^+ = BD$

BD (it is not contain CD)

c) $BC \to CD$

$(BC)^+ = BC$

$= BCD \quad (B \to D)$

$BC \to CD$

D) $AC \to BC$

$(AC)^+ = AC$

$= ACB \quad (A \to B)$

$AC \to BC$

Ex (4)

Ex:- R(ABCDEH) what are the Candidate keys for.

A → B

BC → D

E → C

D → A

D option is Candidate Cate
all minimal attributes are satisfied.

options

a) AE, BE

(AE)$^+$ =

(BE)$^+$ =

b) AE, BE, DE

(AE)$^+$ =

(BE)$^+$ =

(DE)$^+$ =

c) AEH, BEH, BCH

D) AEH, BEH, DEH

(AEH)$^+$ = AEH

= AEHB

= AEHBC

= AEHBCD

similarly

(BEH)$^+$ =

(DEH)$^+$ =

Ex:- 5

## Equivalence of FD's:-

A two sets of functional dependencies $F1$ and $F2$ are equivalent if $(F1^+ = F2^+)$

Ex:- Find equivalence of FD's.

| F1: | F2: |
|---|---|
| $A \to C$ | $A \to CD$ |
| $AC \to D$ | $E \to AH$ |
| $E \to AD$ | |
| $E \to H$ | |

$A^+ = A$
$= AC$
$ACD$

$(AC)^+ = ACD$

$(E)^+ = EAD$
$= EADC$
$= EADCH$

$(A)^+ = ACD$

$(E)^+ = EAH$
$= EAHCD$

$\therefore F1 = F2$.

Ex:- 6 not equivalence of F1 != F2

R(ABCDEH)

F1:

A -> C

AC -> D

B -> D

E -> AD

F2:

A -> CD

E -> AH

$(A)^+ =$

$(E)^+ =$

$(A)^+ =$

$(AC)^+ =$

$(B)^+ =$

$(E)^+ =$

$\therefore$ F1 != F2

Grate Bit



Ex! - 6

Consider the relation scheme

$R = \{E, F, G, H, I, J, K, L, M, N\}$

and the set of functional dependencies

$\{E, F\} \rightarrow \{G\}$ [B]

$F \rightarrow I, J$

$E, H \rightarrow K, L$

$K \rightarrow M$

$L \rightarrow N$

options

A) $\{E, F\}$ B) $\{E, F, H\}$

C) $\{E, F, H, K, L\}$ D) $\{E\}$

Sol:-

Finding attribute closure of all given options we get;

A) $\{E, F\}^+ = EFG$

$= EFGij$

B) $\{E, F, H\}^+ = EFH$

$= EFHG$ [$E, F \to G$]

$= EFHGKL$ [$EH \to KL$]

$= EFHGKLM$

$= EFHGKLMN$

$= EFHGKLMNij$

$=$ all attributes

C) $\{E, F, H, K, L\}^+ = EFK, H, L$

$= EFGMHLN$

$= EFGMHLNij$

$= EFGMHLNijKL$

$=$ all attributes.

D) $E^+ = \{E\}$

Sol:-

$\{EFH\}^+$ and

$\{EFHKL\}^+$ results in set of all attributes, but EFH is minimal.

So, it will be Candidate key.

— o —