Problem Set - 3

Stochastic Gravitational Wave Backgroud from Astrophysical Sources

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Problem 1: Stellar Core Collapse

The empirical model for GW energy spectrum from a core collapse of a star is given by

$$\frac{dE_{\rm GW}}{df} = \frac{G}{c^5} E_\nu^2 \langle q \rangle^2 \left(1 + \frac{f}{a}\right)^2 e^{-2f/b} \tag{1}$$

where E_ν is the energy carried away by neutrinos during the collapse, $\langle q \rangle$ is the averaged neutrino anisotropym and a, b are empirical parameters. Allowed range of parameters a, b

$$5\text{Hz} < a < 150\text{Hz} \tag{2}$$

$$10\text{Hz} < b < 400\text{Hz} \tag{3}$$

The rate of core collapse events will be directly proportional to the SFR (i.e. there is no time delay):

$$R_V(z) = \lambda_{CC} R_*(z) \tag{4}$$

Using the results for GW energy spectrum and rates for core collapse events, we can write the full expression for SGWB energy density spectrum

$$\Omega_{\rm GW}(f) = \frac{8\pi G f \xi}{3H_0^3 c^2} \int_0^\infty dz \frac{R_*(z)}{(1+z)\sqrt{\Omega_{\rm M,0}(1+z)^3 + \Omega_{\Lambda,0}}} \left(1 + \frac{f(1+z)}{a}\right)^2 e^{-2f(1+z)/b} \tag{5}$$

where $\xi = \lambda_{CC} E_\nu^2 \langle q \rangle^2$ captures all uncertainties in one free parameter of the model. Compare the predicted $\Omega_{\rm GW}(f)$ to the detector sensitivities to determine the regions of this empirical parameter space (ξ, a, b) that can be probed by observations.

Problem 2: Magnetars

The magnetars are neutron stars with a strong magnetic field, of the order $B \approx 10^{15}$ Gauss. The GW energy spectrum emitted by a single magnetar is given by

$$\frac{dE_{\rm GW}}{df} = I\pi^2 f^3 \left(\frac{5c^2R^6}{192\pi^2 G I^2}\frac{4\pi B^2}{\mu_0 \epsilon^2} + f^2\right)^{-1} \tag{6}$$

where R is the radius of the magnetar, I is the moment of inertia, and ϵ is the magnetar's ellipticity. The rate of magnetars is again directly proportional to the SFR (i.e. no time delay): $R_V(z) = \lambda_m R_*(z)$, where λ_m effectively captures the fraction of star mass that ends up in magnetar objects. As in the above case, one can compare these predictions to the detector sensitivities to determine which part of the parameter space (λ_m, B, ϵ) can be probed by detectors. For a particular case of poloidal magnetic field

$$\epsilon = \beta \frac{4\pi R^8 B^2}{\mu_0 G I^2} \tag{7}$$

where β is the dimensionless parameter that depends on field geometry and equation of state of the neutron star.