Problem Set - 1

Stochastic Astrophysical Foreground from Compact Binary Mergers

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Problem 1:

Stochastic Gravitational Wave Background (SGWB) energy density spectrum in terms of the normalized energy density of GWs:

$$\Omega_{\rm GW}(f) = \frac{f}{\rho_{c,0} H_0} \int_0^\infty dz \frac{R_V(z)}{(1+z)\sqrt{\Omega_{\rm M,0}(1+z)^3 + \Omega_{\Lambda,0}}} \left.\frac{dE_{\rm GW}}{df}\right|_{f(1+z)} \tag{1}$$

where $\rho_{c,0}$, $\Omega_{\rm M,0}$, and $\Omega_{\Lambda,0}$ are the critical energy density, matter density, and the cosmological constant or vacuum density of the universe respectively. $R_V(z)$ is the rate in comoving volume measured in the source frame time, and $dE_{\rm GW}/df$ is the energy spectrum of gravitational waves emitted by a single source (evaluated in the frame of the source).

Apply it first to the compact binary coalescence model. Assume that all binaries have the same chirp mass, $\mathcal{M}$ (in terms of component masses:$=(m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$) and that their merger rate follows exactly the star formation rate (i.e. ignore the time delay between formation and merger of the binary). In this case, energy spectrum can be written in weak field approximation

$$\frac{dE}{df} = \frac{(\pi G)^{2/3}\mathcal{M}^{5/3}}{3} f^{-1/3} \tag{2}$$

and star formation rate (SFR)

$$R_*(z) = \nu \frac{a e^{b(z-z_m)}}{a - b + b e^{a(z-z_m)}} \tag{3}$$

where $\nu = 0.146 M_\odot/\mathrm{yr}/\mathrm{Mpc}^3$, $a = 2.80, b = 2.46, z_m = 1.72$. You may start by computing the SGWB frequency spectrum using python, matlab, or another platform you are comfortable with. Plot the SGWB energy density spectrum, $\Omega_{\rm GW}(f)$, as a function of frequency.

Problem 2:

Compare your results with the sensitivity of the upcoming LIGO/Virgo detectors, or with the future 3rd generation detectors. The sensitivity of the detectors for SGWB, $\Omega_n(f)$, can be estimated given the detector's noise power spectral density ($S_n(f)$) using the formula:

$$\Omega_n(f) = \frac{\pi c^2 f^3}{4\rho_{c,0} G} S_n(f) = \frac{2\pi^2}{3H_0^2} f^3 S_n(f) \tag{4}$$

Problem 3:

If time permits, assume that binaries have a chirp mass distribution, $P_c(\mathcal{M})$, instead of having a single value of chirp mass (as assumed in problem 1). Then energy spectrum from the source can written as:

$$\frac{dE}{df} = \frac{(\pi G)^{2/3}}{3} f^{-1/3} \int d\mathcal{M} \mathcal{M}^{5/3} P_c(\mathcal{M}) \tag{5}$$

Play with different mass distributions to see how much this impacts the overall $\Omega_{\rm GW}(f)$.