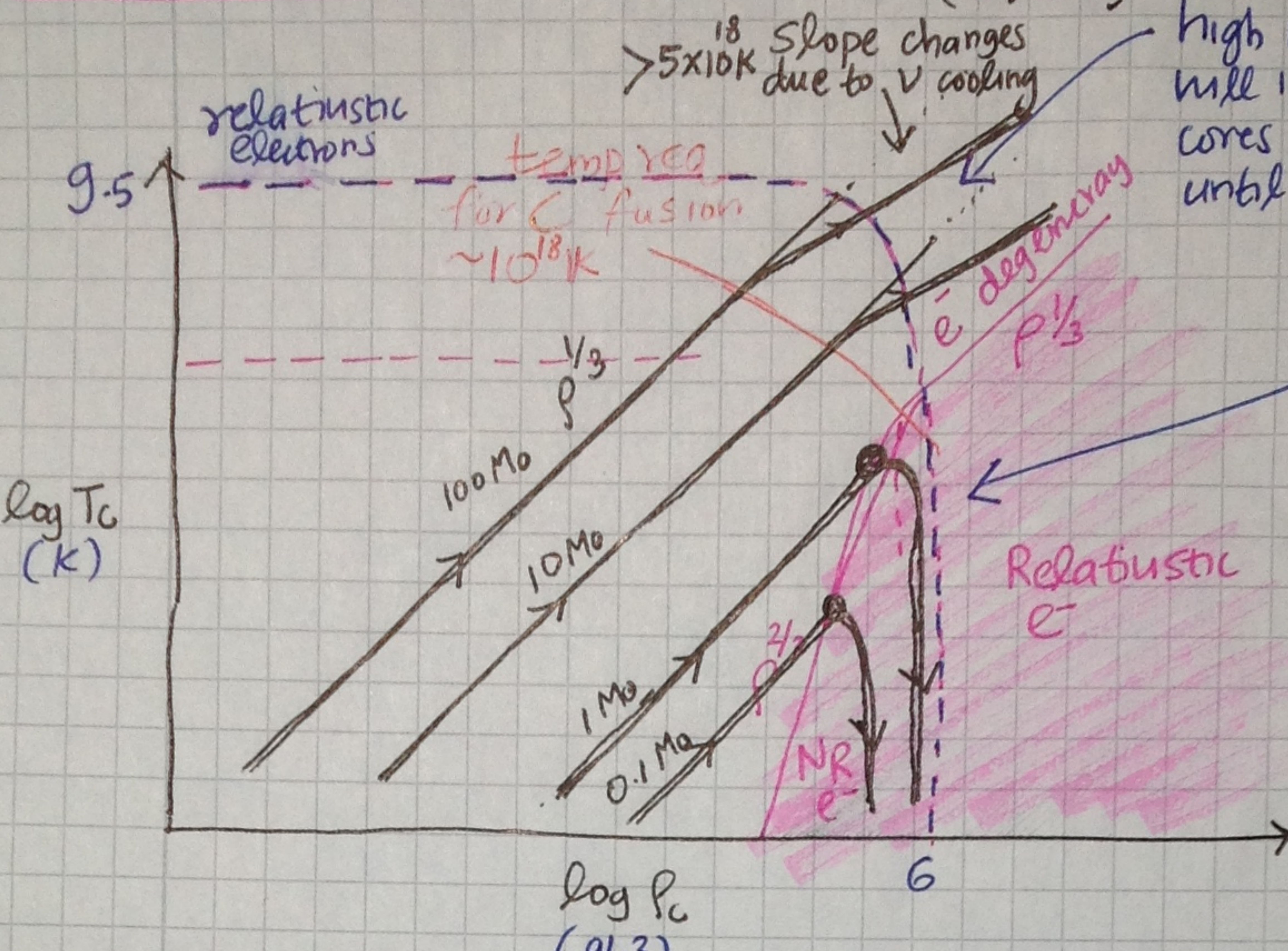
STELLAR COLLAPSE & SUPERNOVAE

($L_\nu \gg L$)

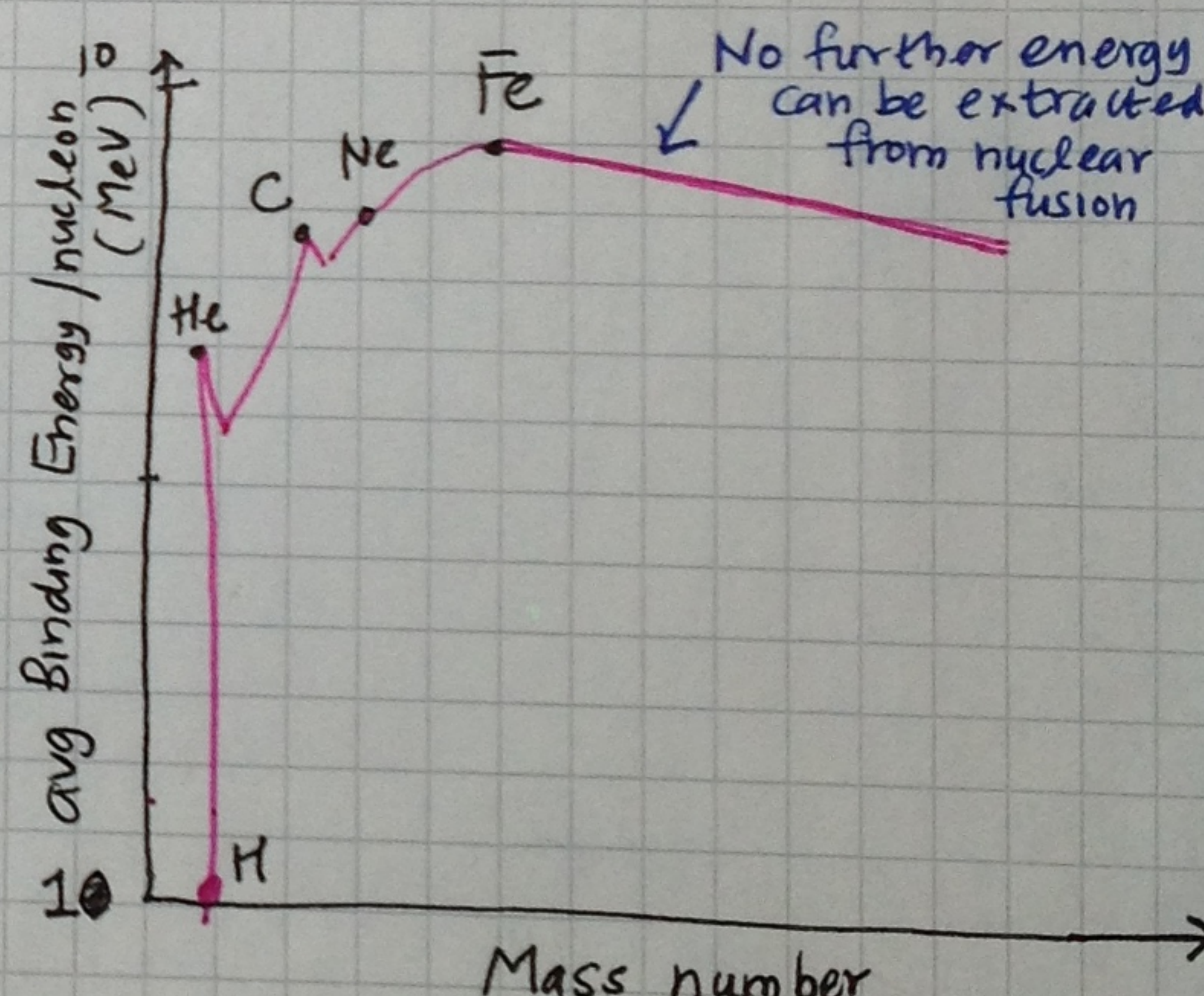


high mass stars (> 8 $M_\odot$) will ignite carbon at their cores (and heavier elements) until an Fe core is formed.

low & intermediate mass stars (~0.8 - 8 $M_\odot$) form a degenerate CO core (after He burning). Shed the envelope by stellar wind. Core becomes a CO WD.

~~Very~~ very low mass WDs ($M < 0.45 M_\odot$) have He core and Heavy WDs ($M > 1.2 M_\odot$) have ONe core

Pre-supernova structure of a massive star (~15 $M_\odot$):



Collapse of the Iron Core:

At densities ($\rho > 10^9$ g/cm^3) and temp ($T_c > 10^{9.5}$ K) exist in the core, electrons are relativistic. Even though there is a considerable degeneracy, the collapse cannot be stopped ($P \propto \rho^{1/3}$).

Two processes that further accelerate the collapse:

I) Electron capture: $p + e^- \rightarrow n + \nu_e$ (inverse β-decay)

The composition becomes increasingly neutron-rich. (neutronization). Reduction of the electron deg. pressure $\rightarrow$ collapse.

2) Photo-disintegration: If $T_c \sim 10^{10}$ K, energy of photons can break heavy nuclei into lighter ones. In particular

$$^{56}Fe + \overset{\swarrow 100\,MeV}{\gamma} \rightarrow 13\ ^{4}He + 4n \qquad \text{(endothermic process)}$$

The required energy is absorbed from radn. (ultimately from the internal energy of the star). As a result, the pressure drops → core collapses.

Collapse is extremely rapid $t_{ff} \sim (G\rho)^{-1/2} \sim 10^{-2}$ s. $(\rho \sim 10^{10}\ g/cm^3)$

Temp & Pressure will increase - but not enough to stop the collapse until ~~nuk~~ nuclear densities are reached.

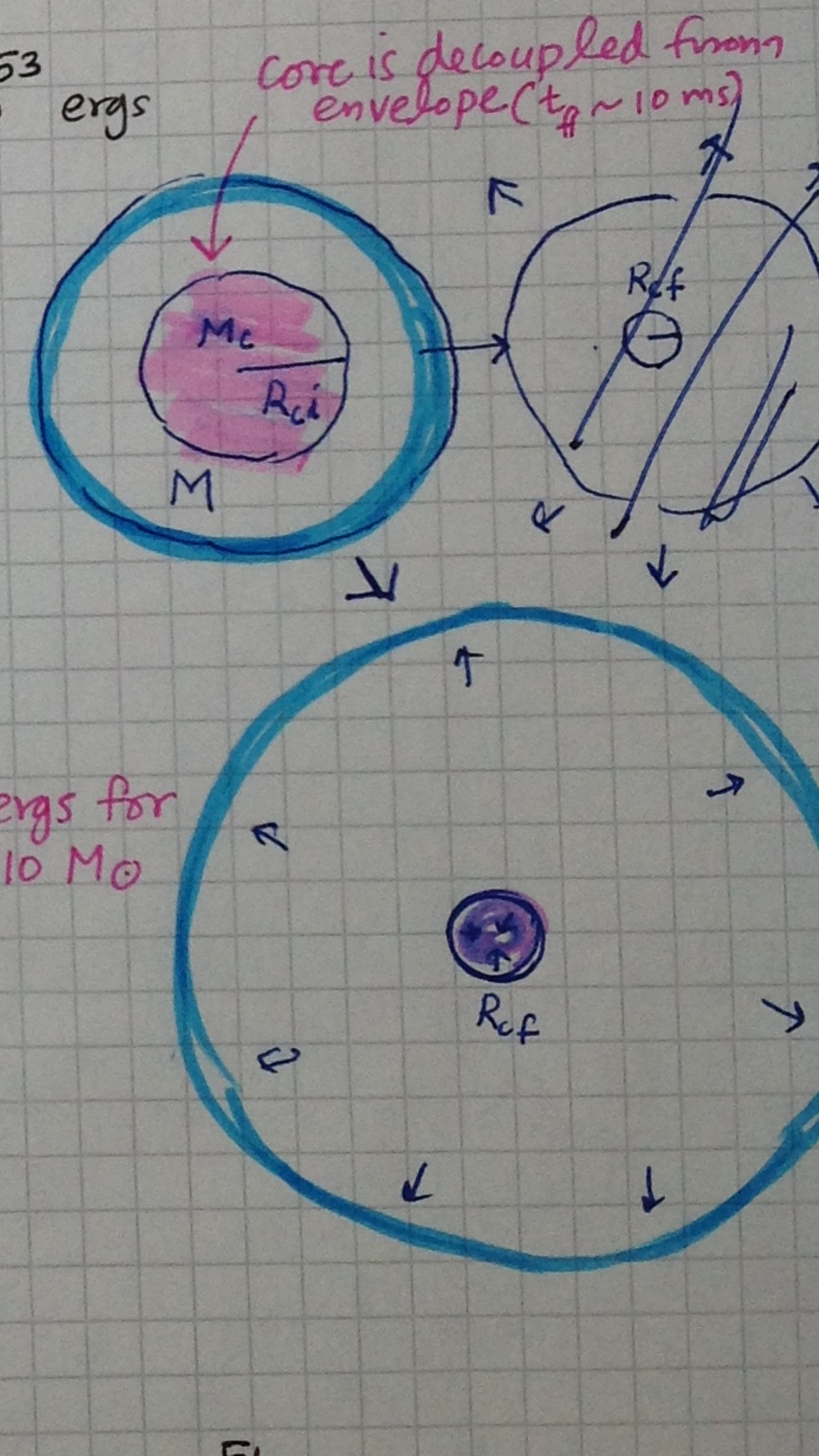
When the collapsing core reaches nuclear densities ($10^{14}\ g/cm^3$), the neutrons become degenerate → EOS 'stiffens'. This terminate the collapse at $R \approx 20$ km.

Supernova explosion:

The grav. energy released by the SN

$$E_{gr} \approx -\underbrace{\frac{GM_c^2}{R_{ci}}}_{\substack{\text{initial PE} \\ (R_c \sim 3000\,km)}} + \underbrace{\frac{GM_c^2}{R_{cf}}}_{\substack{\text{final PE} \\ (R_c \sim 20\,km)}} \approx \frac{GM_c^2}{R_{cf}} \approx 3 \times 10^{53}\ \text{ergs}$$

($M_c \sim 1.4\ M_\odot$)



The energy necessary to expel the envelope

$$E_{env} = \underbrace{\int_{M_c}^{M} \frac{Gm\,dm}{r}}_{\sim 10^{50}\ \text{erg}} \ll \frac{GM^2}{R_{ci}} \approx 10^{53}\ \text{ergs for } M = 10\ M_\odot$$

($\sim 10^{50}$ erg when one considers the realistic mass distribution)

Only a small fraction of the energy released is needed ~~from~~ for blowing the envelope.

$$\boxed{\begin{array}{c} E_{env} \ll E_{gr} \\ \sim 10^{50}\ \text{erg} \ll \sim 10^{53}\ \text{ergs} \end{array}}$$

From observations, the envelope seems to get $\sim 10^{51}$ ergs ($M_{ejecta} \sim 10\ M_\odot$, $v_{ejecta} \sim 10^4$ km/s). Energy ~~g~~ went ~~to~~ into the photons is negligible (10^{49} ergs, $\sim 10^8\ L_\odot$ for several months).

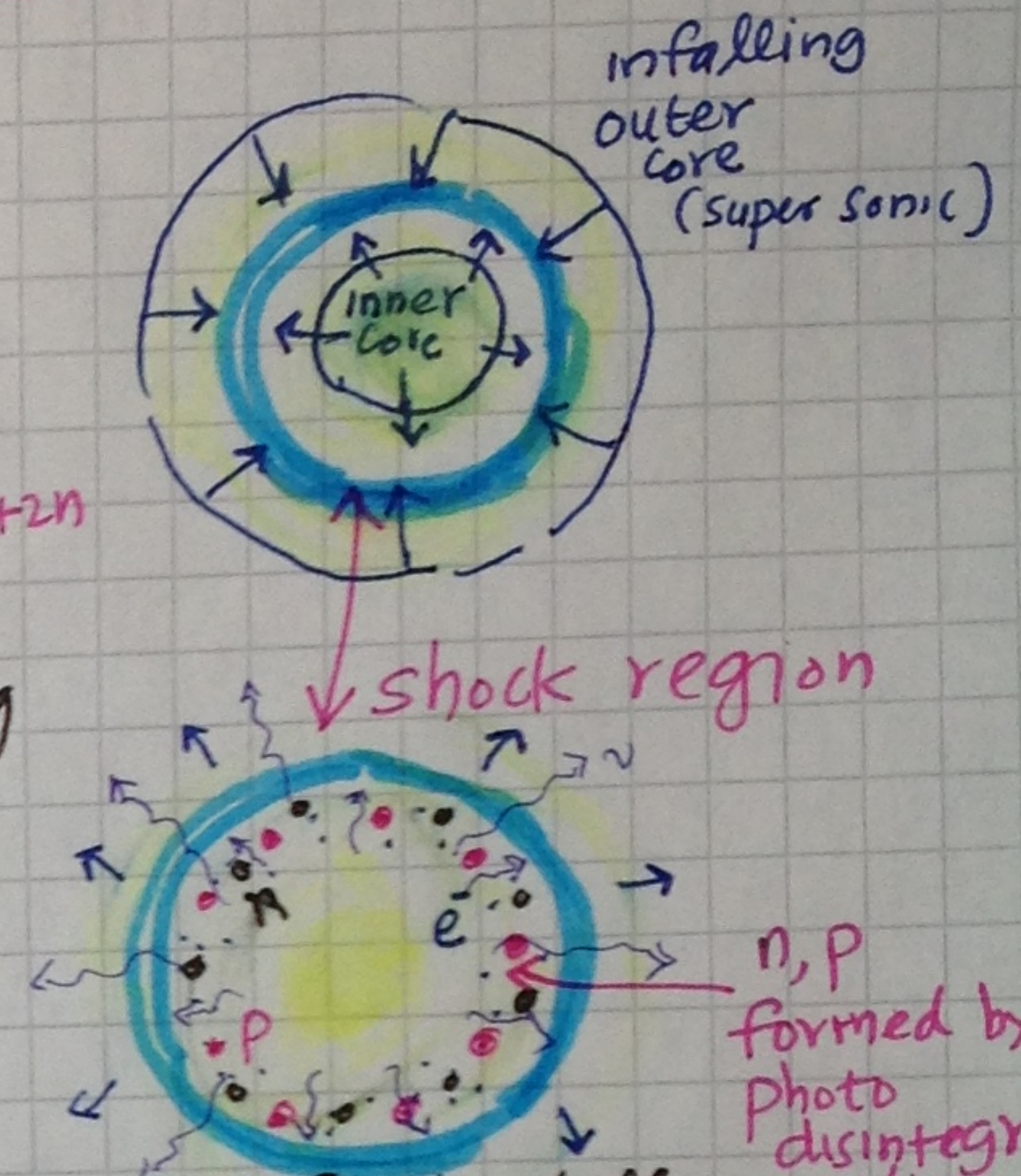
Thus $KE_{env} \sim 10^{-2}\ E_{gr}$

SN Mechanism

The exact mechanism by which the grav. PE of the collapsing star is transferred to the exploding envelope is ~~poorly known~~. not well understood. Possible mechanisms include:

At nuclear densities ($\rho_{nuc} \sim 3\times10^{14}\ g/cm^3$) the EOS stiffens (neutron degeneracy + repulsive force between nucleons at close separations). When core is compressed further, it bounces back - core bounce.

- prompt hydrodynamic explosion: The outer core is collapsing supersonically. Inner core bounces back. → 'super elastic' collission. Creation of a ~~shock~~ shock wave. If the outer core is not too massive, the shock can emerge ~~out~~ at the outer region and cause an ~~expansion~~ explosion:



- Neutrino-driven delayed-explosion: ~~As~~ the shock wave ~~p~~ loses its energy in photo-disintegrating the Iron nuclei into protons & neutrons. (rea. ~9 MeV/nucleon)) - disintegration of 1.4 M☉ or 1.7×10^{57} nucleons require $\sim10^{52}$ ergs

  $4He + \gamma \rightarrow 2p + 2n$

  Also, e^- capture ($p + e^- \rightarrow n + \nu$) on the free protons will create neutrinos. These neutrinos can carry away ~90% of the availabe energy. Shock stalls.

  Summary: Additional mechanism required to revive the shock for higher mass cores

  Neutrino heating: At sufficient densities the collapsing core will become opaque to neutrinos. A small fraction of the ν energy gets deposited into the matter → heat up the matter and revive the shock. → explosion.

- Other possible mechanisms: Oscillations in the proto-neutron star getting acoustically coupled to the envelope, instabilities created by a combination of magnetic fields and rotation, etc. An area of active research.

  Neutrinos from SN 1987A was observed. ~~Evi~~ Evidence of neutronization happening in the core.

Remnants of the core

Stars with initial mass $\lesssim$ 20 M☉ $\xrightarrow{likely}$ NSs as their remnants after collapse

For the more massive stars, the BE of the envelope increases → weak explosion → fall back to the proto-neutron star. If ~~the~~ $M_{PNS} > M^{NS}_{max}$, it collapses to a BH.

Homology Relations:

1) From the stellar structure eqns

$$\frac{dx}{dm} = \frac{M}{4\pi R^3 \rho(x)} \frac{1}{x^2} \Rightarrow \rho(x) \propto \frac{M}{R^3}$$

↑ dimensionless quantities (independent of the mass of the star)

$\bar{\rho}$ avg density

ie $\rho(x) \propto \bar{\rho}$

~~This~~ ie $\boxed{\rho_c \propto \rho(x) \propto \bar{\rho}}$ — (22)

the relation has to be true for any homological shell, including the central one (hence ρ_c)

2) From $\cancel{\frac{dP}{dm}} = \frac{dp}{dm} = \left(-\frac{GM^2}{4\pi R^4 P_c}\right)\frac{m}{x^4} \Rightarrow \boxed{P_c \propto \frac{M^2}{R^4} \propto P(x)}$ — (22)

3) ~~For~~ From the ideal gas EOS

$$P = \frac{\mathcal{R}}{\mu} \rho T$$

$\mathcal{R}$ ← gas const

μ ← mean molecular weight

$$T \propto \frac{P\mu}{\rho} \propto \frac{M^2}{R^4}\frac{R^3}{M}\mu = \mu \frac{M}{R}$$

$$\boxed{T(x) \propto T_c \propto \mu \frac{M}{R}} \quad \text{— (23)}$$

Mass-temperature reln for a star described by ideal gas EOS

4) From the radiative equilibrium

$$\frac{dt}{dm} = \left(\frac{3K}{64\pi^2 ac}\right)\left(\frac{ML}{T_c^4 R^4}\right)\frac{l}{t^3 x^4}$$

$$\boxed{L \propto \frac{1}{K}\frac{R^4 T_c^4}{M} \propto \frac{1}{K}\frac{\mu^4 M^3}{\cancel{M^3}}} \quad \text{— (24)}$$

Mass-Luminosity reln for a radiative, homologous star with const opacity

If we assume a specific form of the energy generation rate: $\epsilon = \epsilon_0 \rho T^\nu$ (good approx. for main sequence stars)

$$\frac{dl}{dm} = \frac{M}{L}\epsilon_0 \rho T^\nu$$

Substituting for ρ, T and L from (22), (23), (24)

$$\boxed{R \propto K^{-(\nu+3)} M^{\frac{\nu-1}{\nu+3}} \mu^{\frac{\nu-4}{\nu+3}}} \quad \text{— (25)}$$

Mass-radius relation for a (MS) star (homologous) with const opacity & energy gen given by $\epsilon \propto \rho T^\nu$

For MS stars (~~ν=4~~)

$T_c \propto M^{4/\nu+3}$ ~~or~~ $\propto M^{0.57}$ for $\nu = 4$ (P-P chain)

$\propto M^{0.27}$ for $\nu = 18$ (CNO)

Homology relations for MS stars $\left(\epsilon \propto \rho T^{\nu} \right)$

Plugging ~~in~~ (25) in (23)

$$\boxed{T_c \propto M^{\frac{4}{\nu+3}}} \quad \text{or} \propto M^{0.57} \quad \text{for p-p chain } (\nu = 4)$$
$$\propto M^{0.21} \quad \text{for CNO cycle } (\nu = 16)$$

Plugging (25) in (22)

$$\boxed{\rho_c \propto M^{1-3\left(\frac{\nu-1}{\nu+3}\right)}} \quad \text{or} \quad \rho_c \propto M^{-0.3} \quad \text{for p-p}$$
$$\rho_c \propto M^{-1.4} \quad \text{for CNO}$$

Plugging

$$L \propto M^3 \propto \cancel{T_c}^{\frac{\nu+3}{4}}$$