STELLAR EVOLUTION

Nuclear reaction time scale

$$t_{nuc} \approx 10^{10} \left(\frac{M}{M_\odot}\right)^{-2.5} \text{yr} \quad -(1)$$

high mass stars will evolve faster

Structure of the stars will evolve in this time scale

From the numerical soln of stellar structure eqn

$$\frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot}\right)^{3.5}$$

$\epsilon = 6 \times 10^{18}$ ergs

ϵ ← energy released per g of H consumed

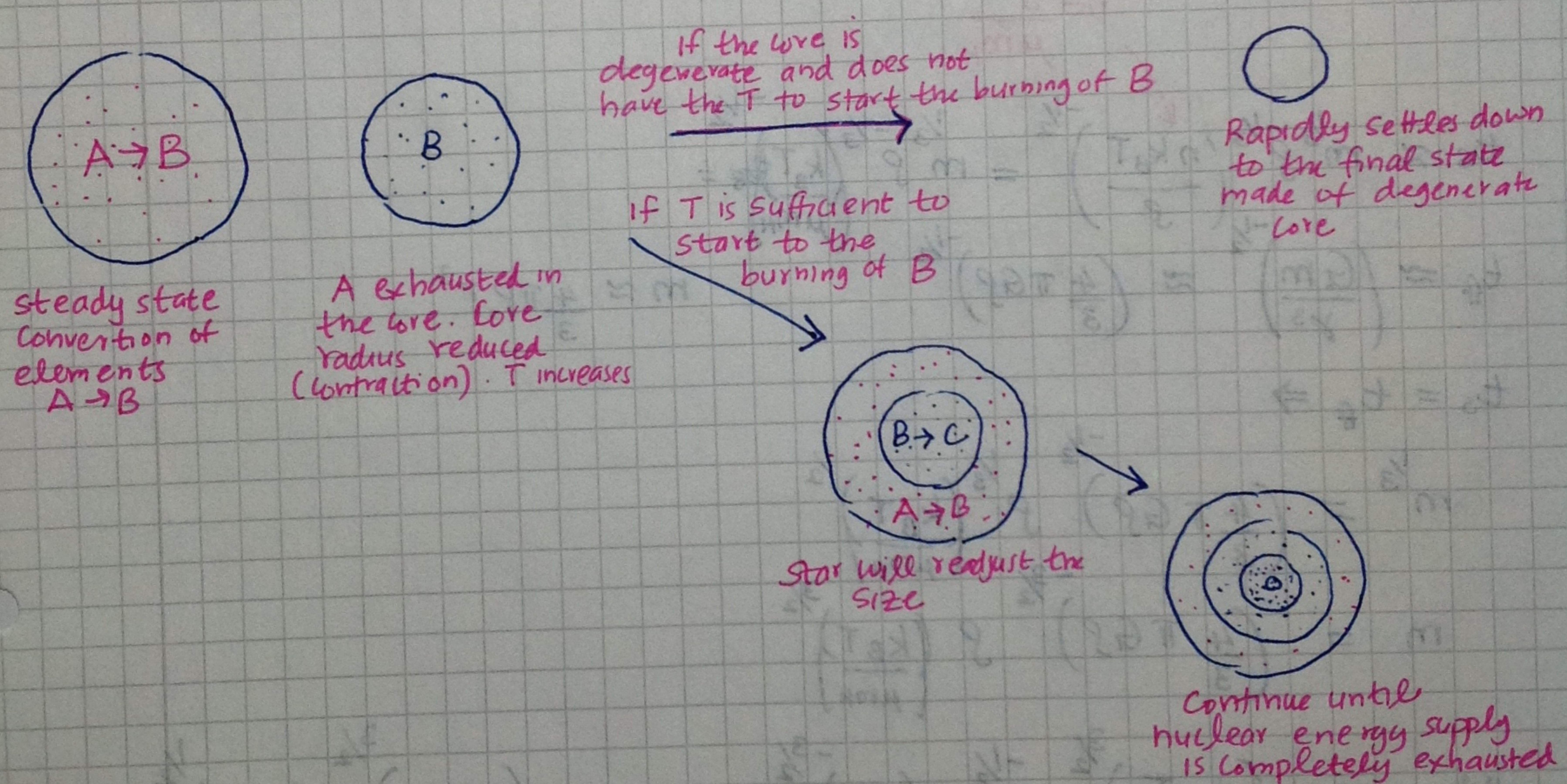
$$t_{nuc} \approx 0.1\, \epsilon \frac{X M}{L}$$

0.1 ← assume that 10% of the original mass of hydrogen undergoes fusion

$$\approx 10^{10} \left(\frac{M}{M_\odot}\right)\left(\frac{L}{L_\odot}\right)^{-1} \text{yr}$$

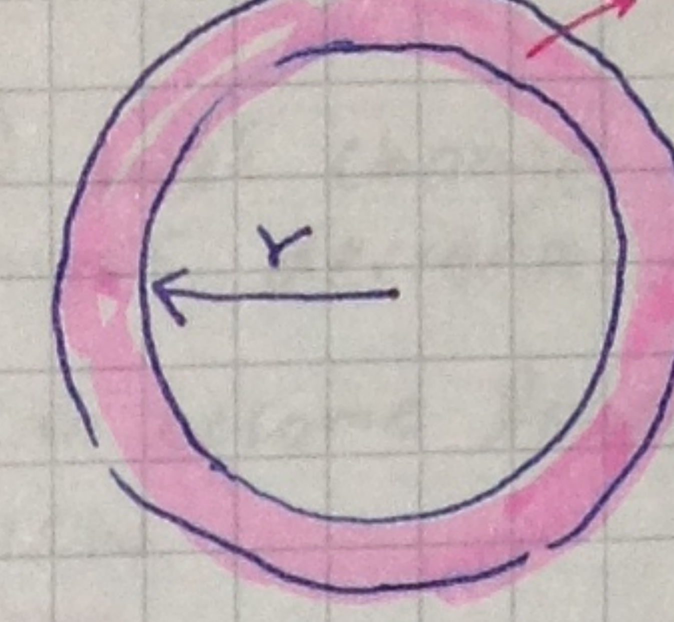
$$\approx 10^{10} \left(\frac{M}{M_\odot}\right)^{-2.5} \text{yr}$$

Stellar evolution overview



Pre-main sequence collapse:

We discuss the simplified model of star formation by the collapse of a spherical ~~mass~~ cloud of gas



mass of the shell $= dm$

~~dm~~ d area $= dA$

The EOS of a shell of radius $r(m,t)$ enclosing a mass of $m(r)$ is

$$\cancel{dm\frac{\partial^2 r}{\partial t^2} = -\frac{Gm(r)\rho(r)}{r^2} - \frac{\partial P}{\partial r}\frac{1}{\rho}}$$

$$\frac{\partial^2 r}{\partial t^2} = -\frac{Gm(r)}{r^2} - 4\pi r^2\frac{\partial P}{\partial m(r)} \quad -(2)$$

gravity (if dominated, the cloud will contract)

pressure force (if dominated, the cloud will expand)

The net force on the shell

$$dm\frac{\partial^2 r}{\partial t^2} = \frac{-Gm(r)\,dm}{r^2} - \frac{\partial P}{\partial r}\,dr\,dA$$

$$\frac{\partial^2 r}{\partial t^2} = -\frac{Gm(r)}{r^2} - \frac{\partial P}{\partial r}\frac{dr}{dm}4\pi r^2$$

$$= -\frac{Gm(r)}{r^2} - 4\pi r^2\frac{\partial P}{\partial m(r)}$$

Characteristic time scales for expansion/collapse

$$\boxed{t_s \approx \left(\frac{R}{c_s}\right) \propto \left(\frac{m}{\rho}\right)^{1/3}\left(\frac{P}{\rho}\right)^{-1/2} \propto m^{1/3}\rho^{-1/3}T^{-1/2}} \quad -(3)$$

expansion

speed of sound $\sim\left(\frac{P}{\rho}\right)^{1/2}$

ideal gas $P \propto \rho T$

$$\boxed{t_{ff} \approx \left(\frac{GM}{r^3}\right)^{-1/2} \propto (G\rho)^{-1/2}} \quad -(4)$$

free fall

From virial theorem

$$v \sim \left(\frac{GM}{r}\right)^{1/2}$$

$$t_{ff} \sim \frac{v}{a} = \left(\frac{GM}{r}\right)^{1/2}\left(\frac{GM}{r^2}\right)^{-1/2}$$

Here a = GM/r^2

If $t_{ff} \ll t_s \rightarrow$ Collapse under self gravity.

Alternatively, the condition for collapse is,

$$\frac{3}{2}Nk_BT \ll -\frac{3}{5}\frac{GM^2}{R} \qquad R = \left(\frac{3M}{4\pi\rho_0}\right)^{1/3}$$

thermal KE $\ll$ grav potential energy of the Cloud of mass M & radius R

$N = \frac{M}{\mu m_H}$

or $M \gg M_J$ where

$$\boxed{M_J \simeq \left(\frac{5k_BT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}} \quad -(5)$$

M_J JEAN'S MASS

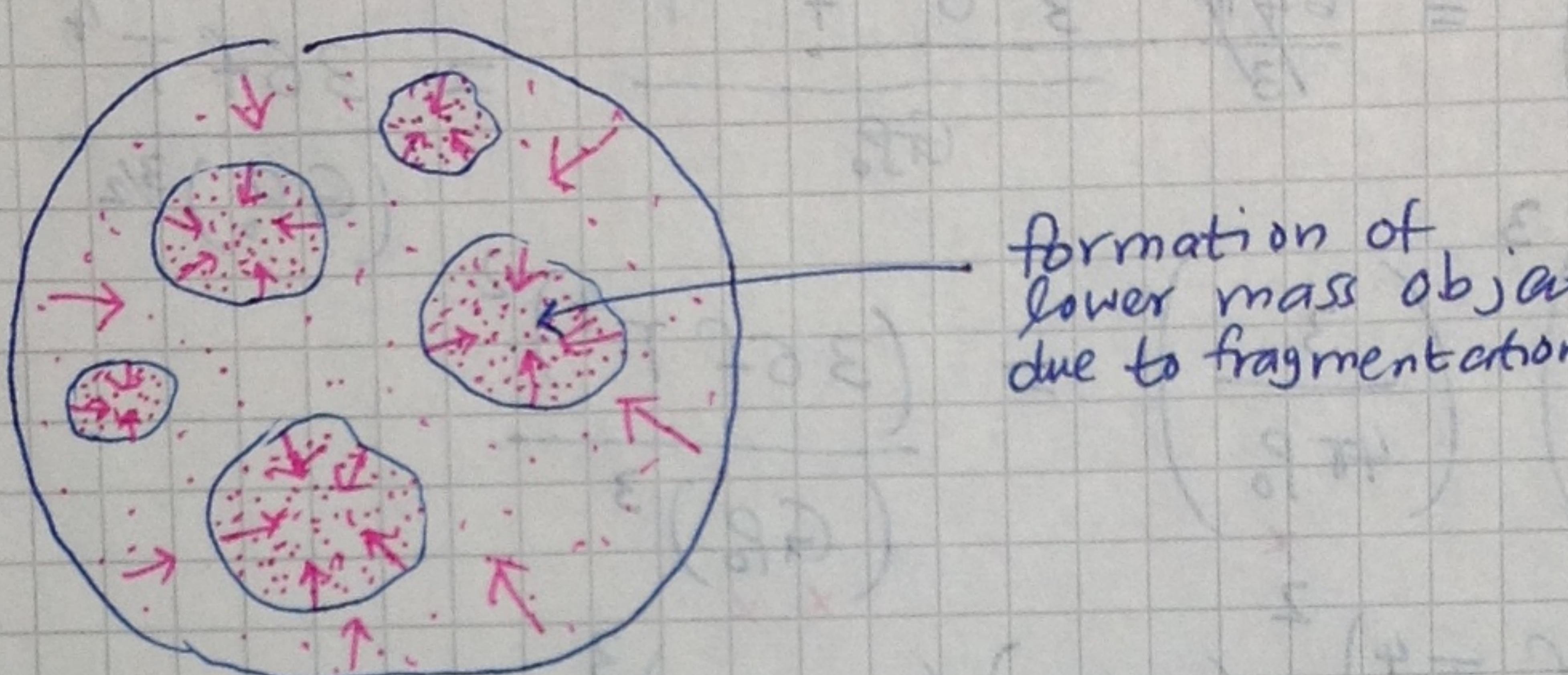
mean molecular weight

avg density

$$M_J \simeq 10^5 M_\odot \left(\frac{T}{100K}\right)^{3/2} \left(\frac{\rho_0}{10^{-24}\ g\,cm^{-3}}\right)^{-1/2} \mu^{-3/2} \quad — (6)$$

typical values for ISM

~~Whe~~ During the collapse, the physical parameters (eg T) will change. Hence the Jeans mass will also change. If the Jeans mass decreases as the cloud collapses, then the subregion in the cloud can become locally ustable and will collapse further → FRAGMENTATION



$$M_J \propto T^{3/2} \rho^{-1/2}$$

keeps increasing, for a collapsing cloud

the increase in grav P.E. can increase the temp. However, if it is radiated is radiated away (ie $t_{cool} \ll t_{ff}$) the cloud can contract isothermally.

ie $M_J \propto \rho^{-1/2}$

mostly satisfied.

At some point during the contraction, the cloud will become optically thick to its own radn. ~~After~~ After this, collapse will be ~~isot~~ adiabatic.

For ideal gas with $\nabla_{ad} = 2/5$

$$T \propto P^{2/5} \propto \rho^{2/3}$$

during adiabatic compression

$$\therefore M_J \propto T^{3/2} \rho^{-1/2} \propto \rho^{1/2} \Rightarrow \text{Jeans mass will grow with time.}$$

The smallest mass scale that can form due to fragmentation will correspond to the M_J at the moment when the cloud transitions from isothermal to adiabatic evolution.

To calculate this, we use the following condn. To maintain isothermality, the rate of energy radiated,

P.E. of the cloud

$$A \approx \frac{E}{t_{ff}} \simeq \frac{GM^2}{R}(G\rho)^{1/2} = \left(\frac{3}{4\pi}\right)^{1/2} \frac{G^{3/2} M^{5/2}}{R^{5/2}} \quad —(7)$$

~~totale~~ rate of energy radiated

free fall time

This is the rate of energy produced due to collapse

The energy loss rate from the cloud fragment due to thermal radn. is

$$B = (4\pi R^2)(\sigma T^4) f \qquad \text{—— (8)}$$

a factor < 1

For isothermal collapse, $B \gg A$. B = A Transition ~~for~~ from isothermal to adiabatic evolution happens when $B \simeq A$.

ie when, $$M^5 = \left(\frac{64\pi^3}{3}\right)\left(\frac{\sigma^2 f^2 T^8 R^9}{G^3}\right) \qquad \text{— (9)}$$

$$M^{5/2} = 4\pi R^2 \sigma T^4 f \frac{R^{5/2}}{G^{3/2}} \left(\frac{4\pi}{3}\right)^{1/2}$$

$$\boxed{M^5 = \frac{(4\pi)^3}{3} \frac{\sigma^2 f^2 T^8 R^9}{G^3}}$$

The fragmentation will reach its limit when $M_J = M_{trans}$ where

$$M_{trans} = \left(\frac{64\pi^3}{3} \frac{\sigma^2 f^2 T^8 R^9}{G^3}\right)^{1/5} \qquad \text{— (10)}$$

$$\frac{4}{3}\pi R^3 \rho_0 = M$$

$$R^3 = \frac{3M}{4\pi\rho_0}$$

$$M_J^5 = \frac{1}{3} \frac{\sigma^2 f^2 T^8}{\rho_0^3} \frac{9M^3}{G^3}$$

$$M^2 = \frac{9\sigma^2 f^2 T^8}{\rho_0^3 G^3}$$

$$\cancel{M = \frac{3^{1/2}\sigma^{2/3} f^{2/3} T^{8/3}}{G\rho_0}}$$

$$M = \frac{3\sigma f T^4}{(G\rho_0)^{3/2}}$$

In order to estimate the numerical value of this mass limit, we can express R in terms of the mean density $\left(R^3 = \frac{3M}{4\pi\rho_0}\right)$

Then we can express ρ_0 in terms of the mean molecular weight μ and temp by setting $M_{trans} = M_J$ (see eqn 5).

This way

$$M_J = \left(\frac{3 \times 5^9}{64\pi^3}\right)^{1/4} \frac{1}{(\sigma G^3)^{1/2}} \left(\frac{k_B}{\mu m_H}\right)^{9/4} f^{-1/2} T^{1/4}$$

$$M_J = 0.02 \frac{T^{1/4}}{f^{1/2} \mu^{9/4}}$$

For $T \lesssim 10^3$ K, $f \approx 0.1$, we get $M \approx 0.4 M_\odot$. Thus, collapse of the cloud can lead to fragments of $\gtrsim M_\odot$; not significantly below.

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~~$$\text{or } M = \frac{3^{1/2}\sigma^{2/3} f^{2/3} T^{8/3}}{G\rho_0}$$~~

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Schematic stellar evolution

We will look at the evolution of the center of the star – its most 'evolved' part. The center is characterized by T_c, P_c, ρ_c and the chemical composition. These quantities are related through the EOS. Hence the evolution of the star can be represented by a track in the (P_c, ρ_c) diagram or (T_c, ρ_c) diagram.

From the homologous relations, for a star in hydr. equilibrium

$$\boxed{P_c = C\, G M^{2/3} \rho_c^{4/3}} \quad (11)$$

$P_c \propto \frac{M^2}{R^4}$ $\qquad \rho_c \propto \frac{M}{R^3}$

$P_c \propto \frac{M^2}{\cancel{R^4}}\left(\frac{M}{\rho_c}\right)^{-4/3}$

For a star that evolves homologously $P_c \propto \rho_c^{4/3}$. (independent of the EOS). Assuming quasi-steady state (slow evolution).

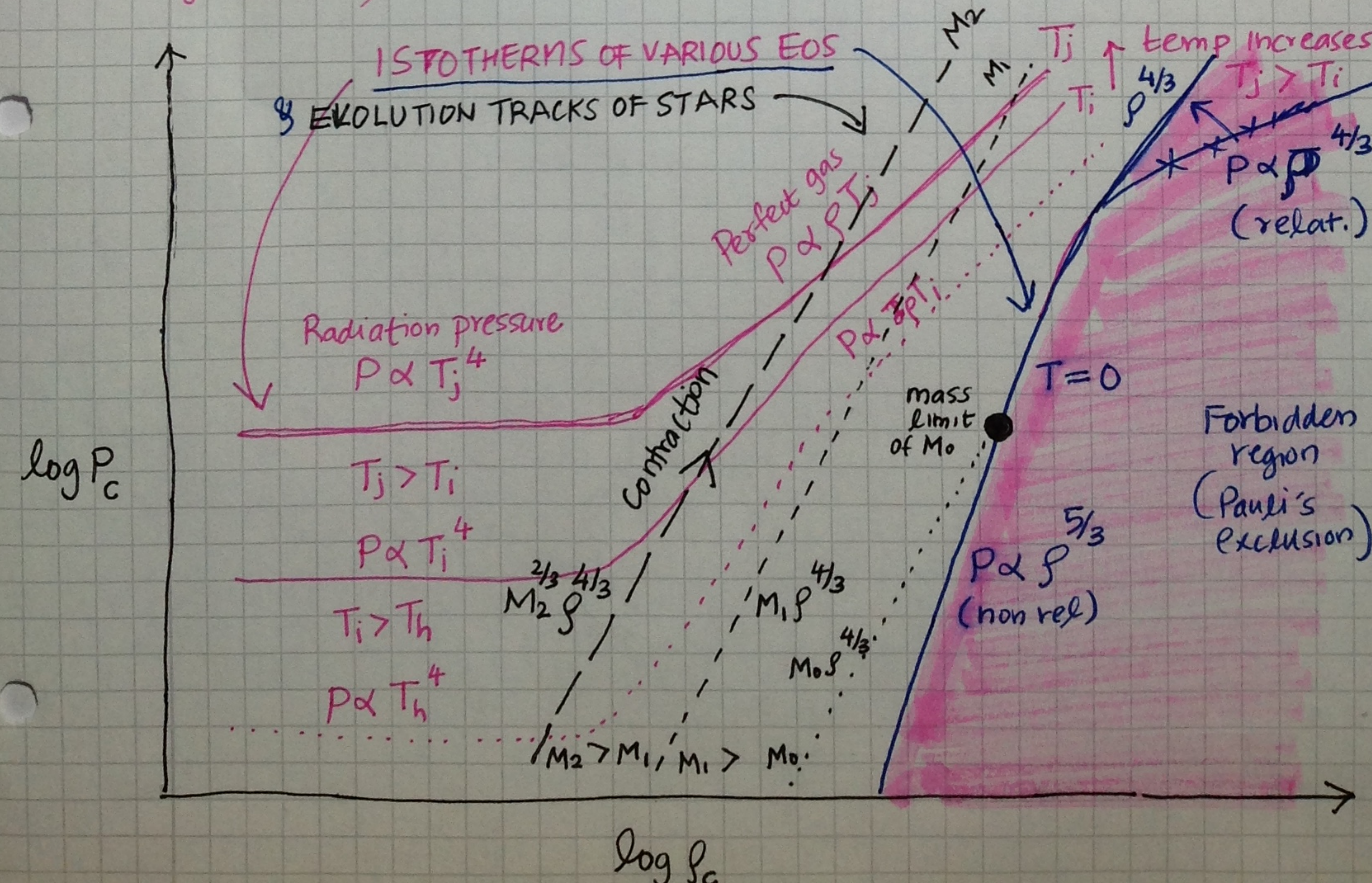
By using an EOS, we can also derive the evolution of T_c. For eg

Isotherm is a line of const P

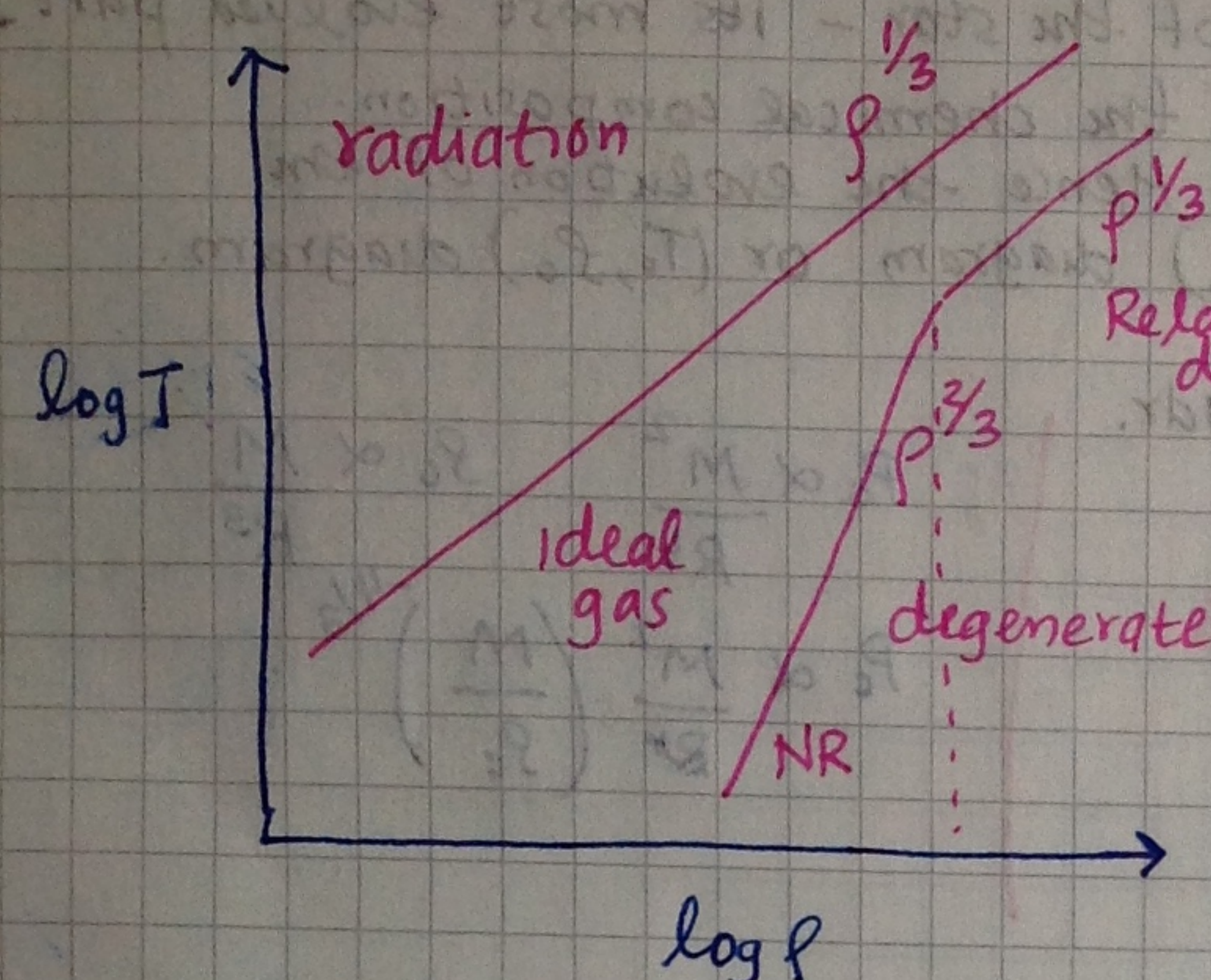
- $P = \frac{1}{3} a T^4$ (radiation dominated) $\Rightarrow$ isotherm is a line of const ρ
- (Classical) ideal gas: $P = \frac{\mathcal{R}}{\mu}\rho T \Rightarrow$ isotherm has $P \propto \rho$
- Non relatinstic electron degeneracy: $P = K_{NR}\left(\rho/\mu_e\right)^{5/3} \Rightarrow P \propto \rho^{5/3}$
  (for $T \to 0$ and 'low' densities)
- Relatinstic electron degeneracy: $P = K_{ER}\left(\rho/\mu_e\right)^{4/3} \Rightarrow P \propto \rho^{4/3}$
  (for $T \to 0$ and high densities)

num density
$n_e = \frac{1}{\mu_e}\frac{\rho}{m_u}$
AMU

mean molecular weight per free electron $\mu_e \approx 2/(1+X)$



EOS domination regimes



① $P_{rad} = P_{gas} \Rightarrow$

$$\frac{1}{3} a T^4 = \frac{\mathcal{R}}{\mu} \rho T$$

$$\Rightarrow T = \left(\frac{3}{a} \frac{\mathcal{R}}{\mu} \rho \right)^{1/3} = 3.2 \times 10^{7} \mu^{-1/3} \rho^{1/3}$$

(in cgs units)

② $P_{gas} = P_{e,NR}$ (non relatinstic electron deg pressure)

$$\frac{\mathcal{R}}{\mu} \rho T = K_{NR} \left(\rho / \mu_e \right)^{5/3}$$

$$T = \frac{K_{NR}}{\mathcal{R}} \frac{\mu}{\mu_e^{5/3}} \rho^{2/3} = 1.21 \times 10^{5} \frac{\mu}{\mu_e^{5/3}} \rho^{2/3}$$

(cgs)

③ $P_{gas} = P_{e,Rel} \Rightarrow$

$$\frac{\mathcal{R}}{\mu} \rho T = K_{ER} \left(\rho / \mu_e \right)^{4/3}$$

$$T = \frac{K_{ER}}{\mathcal{R}} \frac{\mu}{\mu_e^{4/3}} \rho^{1/3}$$

- For a star of given mass obeying ideal EoS, contraction (higher ρ_c) increases T_c.
- Low-mass stars (eg $M_\odot$) have a max. achievable ~~con~~ ρ_c and P_c. The end point of their evolution (a completely degenerate state – white dwarf).
- ~~High m~~ Stars with a critical mass ($M > M_{crit}$) will miss the degenerate region (EoS and evolution tracks have the same power law 4/3). Star continues to contract and getting hotter (until it gets supported by the neutron degeneracy pressure).

Chandrasekhar mass M_{ch}

Similarly, we can look at the evolution of stars in the $T_c - \rho_c$ plane

For a slowly contracting star in hydro. equil. and ideal gas EoS

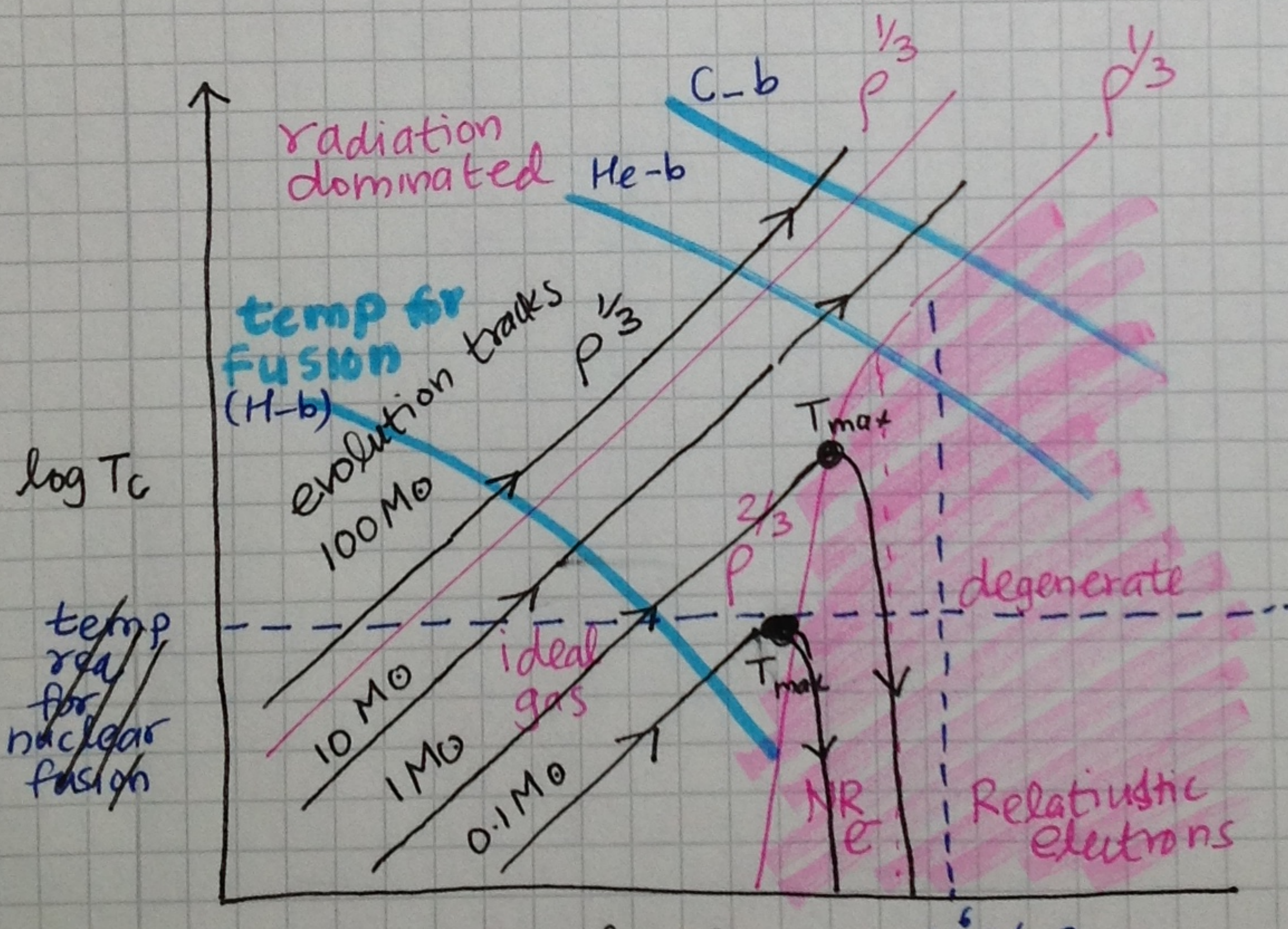
$$T_c = \frac{CG}{\mathcal{R}} \mu M^{2/3} \rho_c^{1/3} \quad \text{— (12)}$$

gas const $\frac{k_B}{m_H}$

$$\frac{\mathcal{R}}{\mu} \rho_c T_c = C \; G M^{2/3} \rho_c^{4/3}$$

ideal gas EoS pressure

from (11)



- high-mass stars – radn dominated.
- For $M > M_{ch}$ contraction continues. Temp continues to increase
- For $M < M_{ch}$, a max temp is reached. Then the star cools down at const density, when degenerate e^- provide the pressure.

Stars with $M < M_{ch}$ approaches a density where non-relativistic electron deg dominates. Inside this region,

$$\rho_c = \left(\frac{CG}{K_{NR}}\right)^3 \mu_e^5 M^2 \quad \text{—(13)}$$

evolutionary track becomes independent of T_c. Also $\rho_c \propto M^2$

NR deg. pressure = P_c from (11)

$$K_{NR} \left(\frac{\rho_c}{\mu_e}\right)^{5/3} = C \; G M^{2/3} \rho_c^{4/3}$$

$$\rho_c^{1/3} = \frac{\mu_e^{5/3} C}{K_{NR}} G M^{2/3}$$

From ~~eq~~ (12) and (13)

$$T_{c_{max}} = \frac{c^2 G^2}{4 \mathcal{R} K_{NR}} \mu \mu_e^{5/3} M^{4/3} \propto M^{4/3}$$

For $M > M_{ch}$ the star tracks will miss the degenerate region. and T_c continues to increase as $\rho^{1/3}$.

- Also, only gas spheres with sufficiently high masses will ~~have~~ attain ~~to~~ temperatures sufficient for nuclear burning. (remember that $\epsilon \simeq \epsilon_0 \rho^\lambda T^\nu$, with $\nu = 4$ for p-p chain, $\nu = 18$ for CNO cycle, ~~etc~~ The min mass reaquired for hydrogen ~~byr~~ burning $\sim 0.15 M_\odot$.
- Less massive objects will become partially degenerate before reaching the required temp and will continue to cool and contract. (brown dwarfs).
- Also very high mass stars are dominated by radn pressure. They are ~~margi~~ hydrodynamically (marginally) unstable - lose mass easily due to radn.
- As a result of the above two restrictions, stars exist only in a narrow ~~ra~~ range $\sim$ $0.1 - 10^2 M_\odot$