Computational Physics

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Assignment 3: Solutions

1 Question 7

For convenience, we can assume that a signal have a duration in power of 2. i.e.

$$n = 2^l$$

DFT Formula is

$$\tilde{f}(q) = \frac{1}{\sqrt{n}} \sum_{p=0}^{n-1} f(p) \exp\left(-\frac{2\pi i p q}{n}\right) \tag{1.1}$$

let $\frac{f(p)}{\sqrt{n}} = c(p)$. Separating out the odd and even terms

$$\begin{aligned}\tilde{f}(q) = c(0) + c(2)\exp\left(-\frac{2\pi i q}{n/2}\right) + ... + c(n-2)\exp\left(-\frac{2\pi i (n/2-1)q}{n/2}\right) + \\ \left(c(1) + c(3)\exp\left(-\frac{2\pi i q}{n/2}\right) + ... + c(n-1)\exp\left(-\frac{2\pi i (n/2-1)q}{n/2}\right)\right)\exp\left(\frac{-i2\pi q}{n}\right)\end{aligned} \tag{1.2}$$

In the above expression, each term in the bracket has the form of a $n/2$ length DFT. The first set is a DFT of the even numbers and the second set is that of odd numbers.
Each of the half length transforms can be reduced to 2 quarter length transforms, each of these into 2 eighth-length ones and so on till we we are left with length-2 transforms.

In the first stage we have $\frac{n}{2}$ length -2 transform. Each pair of these transforms are combined by adding on to the another, which is multiplied by a complex exponential factor before addition.

Each pair requires 4 additions and 4 multiplications. Total computation being $\left(8 \times \frac{n}{4} = 2n\right)$, as $n/4$ pairs are there from $n/2$ sets of length 2 each.
The number of times n length can be divided by 2 is $\log_2(n)$.

Hence the total complexity $= 2n \times log_2(n) \sim O(n \log_2 n)$.

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