```
Question 4
relaxation method(w=1.1):

A=
[[ 3 -1  1]
 [ 3  6  2]
 [ 3  3  7]]
b= [1 0 4]
x0= [0. 0. 0.]
r0= [1 0 4]

iteration:  1
r_ 1 = 1.0
x_ 1 = 0.3666666666666667
r_ 2 = -1.1
x_ 2 = -0.2016666666666667
r_ 3 = 3.505
x_ 3 = 0.5507857142857143

iteration:  2
r_ 1 = -0.8524523809523812
x_ 1 = 0.054100793650793566
r_ 2 = -0.05387380952380916
x_ 2 = -0.2115435317460317
r_ 3 = 0.6168282142857144
x_ 3 = 0.6477158622448981

solution:x=
[ 0.05410079 -0.21154353  0.64771586]
---------------
A=
[[10 -1  0]
 [-1 10 -2]
 [ 0 -2 10]]
b= [9 7 6]
x0= [0. 0. 0.]
r0= [9 7 6]

iteration:  1
r_ 1 = 9.0
x_ 1 = 0.99
r_ 2 = 7.99
x_ 2 = 0.8789000000000001
r_ 3 = 7.7578000000000005
x_ 3 = 0.8533580000000001

iteration:  2
r_ 1 = -0.021100000000000563
x_ 1 = 0.987679
r_ 2 = 0.9053949999999986
x_ 2 = 0.97849345
r_ 3 = -0.5765931000000002
x_ 3 = 0.789932759

solution:x=
[0.987679   0.97849345 0.78993276]
---------------
A=
[[10  5  0  0]
 [ 5 10 -4  0]
 [ 0 -4  8 -1]
 [ 0  0 -1  5]]
b= [  6  25 -11 -11]
x0= [0. 0. 0. 0.]
```

```
r0= [  6  25 -11 -11]

iteration:  1
r_ 1 = 6.0
x_ 1 = 0.66
r_ 2 = 21.7
x_ 2 = 2.387
r_ 3 = -1.452
x_ 3 = -0.19965000000000002
r_ 4 = -11.19965
x_ 4 = -2.4639230000000003

iteration:  2
r_ 1 = -12.535
x_ 1 = -0.7188500000000001
r_ 2 = 3.9256499999999974
x_ 2 = 2.8188215
r_ 3 = -0.5914369999999991
x_ 3 = -0.28097258749999987
r_ 4 = 1.0386424125000033
x_ 4 = -2.2354216692499995

solution:x=
[-0.71885     2.8188215  -0.28097259 -2.23542167]
---------------
A=
[[ 4  1  1  0  1]
 [-1 -3  1  1  0]
 [ 2  1  5 -1 -1]
 [-1 -1 -1  4  0]
 [ 0  2 -1  1  4]]
b= [6 6 6 6 6]
x0= [0. 0. 0. 0. 0.]
r0= [6 6 6 6 6]

iteration:  1
r_ 1 = 6.0
x_ 1 = 1.6500000000000001
r_ 2 = 7.65
x_ 2 = -2.805
r_ 3 = 5.505
x_ 3 = 1.2111
r_ 4 = 6.0561
x_ 4 = 1.6654275
r_ 5 = 11.155672500000001
x_ 5 = 3.0678099375000007

iteration:  2
r_ 1 = -2.0739099375000016
x_ 1 = 1.0796747671874996
r_ 2 = -4.211852732812501
x_ 2 = -1.2606539979687497
r_ 3 = 3.779041901093751
x_ 3 = 2.0424892182406253
r_ 4 = 1.1997999874593752
x_ 4 = 1.9953724965513282
r_ 5 = -3.7028150323732056
x_ 5 = 2.049535803597369

solution:x=
[ 1.07967477 -1.260654    2.04248922  1.9953725   2.0495358 ]
```

```
Question 5
relaxation method(w= 1.3 ):

A=
[[ 3 -1  1]
 [ 3  6  2]
 [ 3  3  7]]
b= [1 0 4]
x0= [0. 0. 0.]
r0= [1 0 4]

iteration:  1
r_ 1 = 1.0
x_ 1 = 0.43333333333333335
r_ 2 = -1.3
x_ 2 = -0.2816666666666667
r_ 3 = 3.545
x_ 3 = 0.6583571428571429

iteration:  2
r_ 1 = -1.2400238095238096
x_ 1 = -0.10401031746031753
r_ 2 = 0.6853166666666666
x_ 2 = -0.13318138888888892
r_ 3 = 0.10307511904761935
x_ 3 = 0.6774996649659865

solution:x=
[-0.10401032 -0.13318139  0.67749966]
---------------
A=
[[10 -1  0]
 [-1 10 -2]
 [ 0 -2 10]]
b= [9 7 6]
x0= [0. 0. 0.]
r0= [9 7 6]

iteration:  1
r_ 1 = 9.0
x_ 1 = 1.1700000000000002
r_ 2 = 8.17
x_ 2 = 1.0621
r_ 3 = 8.1242
x_ 3 = 1.056146

iteration:  2
r_ 1 = -1.637900000000002
x_ 1 = 0.9570729999999998
r_ 2 = -0.551635000000001
x_ 2 = 0.99038745
r_ 3 = -2.5806851
x_ 3 = 0.720656937

solution:x=
[0.957073   0.99038745 0.72065694]
---------------
A=
[[10  5  0  0]
 [ 5 10 -4  0]
 [ 0 -4  8 -1]
 [ 0  0 -1  5]]
b= [  6  25 -11 -11]
x0= [0. 0. 0. 0.]
```

```
r0= [  6  25 -11 -11]

iteration:  1
r_ 1 = 6.0
x_ 1 = 0.78
r_ 2 = 21.1
x_ 2 = 2.7430000000000003
r_ 3 = -0.027999999999998693
x_ 3 = -0.004549999999999788
r_ 4 = -11.00455
x_ 4 = -2.861183

iteration:  2
r_ 1 = -15.515
x_ 1 = -1.2369500000000004
r_ 2 = 3.7365499999999976
x_ 2 = 3.2287515
r_ 3 = -0.9097770000000018
x_ 3 = -0.15238876250000008
r_ 4 = 3.1535262375000013
x_ 4 = -2.04126617825

solution:x=
[-1.23695     3.2287515  -0.15238876 -2.04126618]
---------------
A=
[[ 4  1  1  0  1]
 [-1 -3  1  1  0]
 [ 2  1  5 -1 -1]
 [-1 -1 -1  4  0]
 [ 0  2 -1  1  4]]
b= [6 6 6 6 6]
x0= [0. 0. 0. 0. 0.]
r0= [6 6 6 6 6]

iteration:  1
r_ 1 = 6.0
x_ 1 = 1.9500000000000002
r_ 2 = 7.95
x_ 2 = -3.4450000000000003
r_ 3 = 5.545
x_ 3 = 1.4417
r_ 4 = 5.9467
x_ 4 = 1.9326775
r_ 5 = 12.399022500000001
x_ 5 = 4.0296823125

iteration:  2
r_ 1 = -3.8263823125000016
x_ 1 = 0.7064257484374996
r_ 2 = -7.002951751562502
x_ 2 = -0.41038757432291595
r_ 3 = 3.7513958899479185
x_ 3 = 2.4170629313864587
r_ 4 = 0.9823911055010424
x_ 4 = 2.251954609287839
r_ 5 = -9.13284577925555
x_ 5 = 1.0615074342419462

solution:x=
[ 0.70642575 -0.41038757  2.41706293  2.25195461  1.06150743]
```

```
Question 7
Jacobi method:

A=
[[ 3 -1  1]
 [ 3  6  2]
 [ 3  3  7]]
b= [1 0 4]
x0= [0. 0. 0.]

iteration:  1
x_ 1 =1/ 3 (- 0.0 + 1 )= 0.3333333333333333
x_ 2 =1/ 6 (- 0.0 + 0 )= 0.0
x_ 3 =1/ 7 (- 0.0 + 4 )= 0.5714285714285714

iteration:  2
x_ 1 =1/ 3 (- 0.5714285714285714 + 1 )= 0.14285714285714288
x_ 2 =1/ 6 (- 2.142857142857143 + 0 )= -0.35714285714285715
x_ 3 =1/ 7 (- 1.0 + 4 )= 0.42857142857142855

solution:x=
[ 0.14285714 -0.35714286  0.42857143]
---------------
A=
[[10 -1  0]
 [-1 10 -2]
 [ 0 -2 10]]
b= [9 7 6]
x0= [0. 0. 0.]

iteration:  1
x_ 1 =1/ 10 (- 0.0 + 9 )= 0.9
x_ 2 =1/ 10 (- 0.0 + 7 )= 0.7
x_ 3 =1/ 10 (- 0.0 + 6 )= 0.6

iteration:  2
x_ 1 =1/ 10 (- -0.7 + 9 )= 0.97
x_ 2 =1/ 10 (- -2.1 + 7 )= 0.9099999999999999
x_ 3 =1/ 10 (- -1.4 + 6 )= 0.74

solution:x=
[0.97 0.91 0.74]
---------------
A=
[[10  5  0  0]
 [ 5 10 -4  0]
 [ 0 -4  8 -1]
 [ 0  0 -1  5]]
b= [  6  25 -11 -11]
x0= [0. 0. 0. 0.]

iteration:  1
x_ 1 =1/ 10 (- 0.0 + 6 )= 0.6
x_ 2 =1/ 10 (- 0.0 + 25 )= 2.5
x_ 3 =1/ 8 (- 0.0 + -11 )= -1.375
x_ 4 =1/ 5 (- 0.0 + -11 )= -2.2

iteration:  2
x_ 1 =1/ 10 (- 12.5 + 6 )= -0.65
x_ 2 =1/ 10 (- 8.5 + 25 )= 1.65
x_ 3 =1/ 8 (- -7.8 + -11 )= -0.4
x_ 4 =1/ 5 (- 1.375 + -11 )= -2.475

solution:x=
```

```
[-0.65   1.65  -0.4   -2.475]
---------------
A=
[[ 4  1  1  0  1]
 [-1 -3  1  1  0]
 [ 2  1  5 -1 -1]
 [-1 -1 -1  4  0]
 [ 0  2 -1  1  4]]
b= [6 6 6 6 6]
x0= [0. 0. 0. 0. 0.]

iteration:  1
x_ 1 =1/ 4 (- 0.0 + 6 )= 1.5
x_ 2 =1/ -3 (- 0.0 + 6 )= -2.0
x_ 3 =1/ 5 (- 0.0 + 6 )= 1.2
x_ 4 =1/ 4 (- 0.0 + 6 )= 1.5
x_ 5 =1/ 4 (- 0.0 + 6 )= 1.5

iteration:  2
x_ 1 =1/ 4 (- 0.7 + 6 )= 1.325
x_ 2 =1/ -3 (- 1.2 + 6 )= -1.5999999999999999
x_ 3 =1/ 5 (- -2.0 + 6 )= 1.6
x_ 4 =1/ 4 (- -0.7 + 6 )= 1.675
x_ 5 =1/ 4 (- -3.7 + 6 )= 2.425

solution:x=
[ 1.325 -1.6    1.6    1.675  2.425]
```

Question 8

Gauss Seidal method:

```
A=
[[ 3 -1  1]
 [ 3  6  2]
 [ 3  3  7]]
b= [1 0 4]
x0= [0. 0. 0.]

iteration:  1
x_ 1 =1/ 3 (- 0.0 - 0.0 + 1 )= 0.3333333333333333
x_ 2 =1/ 6 (- 1.0 - 0.0 + 0 )= -0.16666666666666666
x_ 3 =1/ 7 (- 0.5 - 0.0 + 4 )= 0.5

iteration:  2
x_ 1 =1/ 3 (- 0.0 - 0.6666666666666666 + 1 )= 0.11111111111111112
x_ 2 =1/ 6 (- 0.33333333333333337 - 1.0 + 0 )= -0.22222222222222224
x_ 3 =1/ 7 (- -0.33333333333333337 - 0.0 + 4 )= 0.619047619047619

solution:x=
[ 0.11111111 -0.22222222  0.61904762]
---------------
A=
[[10 -1  0]
 [-1 10 -2]
 [ 0 -2 10]]
b= [9 7 6]
x0= [0. 0. 0.]

iteration:  1
x_ 1 =1/ 10 (- 0.0 - 0.0 + 9 )= 0.9
x_ 2 =1/ 10 (- -0.9 - 0.0 + 7 )= 0.79
x_ 3 =1/ 10 (- -1.58 - 0.0 + 6 )= 0.758

iteration:  2
x_ 1 =1/ 10 (- 0.0 - -0.79 + 9 )= 0.9789999999999999
x_ 2 =1/ 10 (- -0.9789999999999999 - -1.516 + 7 )= 0.9495000000000001
x_ 3 =1/ 10 (- -1.8990000000000002 - 0.0 + 6 )= 0.7899

solution:x=
[0.979  0.9495 0.7899]
---------------
A=
[[10  5  0  0]
 [ 5 10 -4  0]
 [ 0 -4  8 -1]
 [ 0  0 -1  5]]
b= [  6  25 -11 -11]
x0= [0. 0. 0. 0.]

iteration:  1
x_ 1 =1/ 10 (- 0.0 - 0.0 + 6 )= 0.6
x_ 2 =1/ 10 (- 3.0 - 0.0 + 25 )= 2.2
x_ 3 =1/ 8 (- -8.8 - 0.0 + -11 )= -0.2749999999999999
x_ 4 =1/ 5 (- 0.2749999999999999 - 0.0 + -11 )= -2.255

iteration:  2
x_ 1 =1/ 10 (- 0.0 - 11.0 + 6 )= -0.5
x_ 2 =1/ 10 (- -2.5 - 1.0999999999999996 + 25 )= 2.6399999999999997
x_ 3 =1/ 8 (- -10.559999999999999 - 2.255 + -11 )= -0.33687500000000004
x_ 4 =1/ 5 (- 0.33687500000000004 - 0.0 + -11 )= -2.267375
```

```
solution:x=
[-0.5       2.64     -0.336875 -2.267375]
---------------
A=
[[ 4  1  1  0  1]
 [-1 -3  1  1  0]
 [ 2  1  5 -1 -1]
 [-1 -1 -1  4  0]
 [ 0  2 -1  1  4]]
b= [6 6 6 6 6]
x0= [0. 0. 0. 0. 0.]

iteration:  1
x_ 1 =1/ 4 (- 0.0 - 0.0 + 6 )= 1.5
x_ 2 =1/ -3 (- -1.5 - 0.0 + 6 )= -2.5
x_ 3 =1/ 5 (- 0.5 - 0.0 + 6 )= 1.1
x_ 4 =1/ 4 (- -0.10000000000000009 - 0.0 + 6 )= 1.525
x_ 5 =1/ 4 (- -4.574999999999999 - 0.0 + 6 )= 2.64375

iteration:  2
x_ 1 =1/ 4 (- 0.0 - 1.24375 + 6 )= 1.1890625
x_ 2 =1/ -3 (- -1.1890625 - 2.625 + 6 )= -1.5213541666666668
x_ 3 =1/ 5 (- 0.856770833333333 - -4.168749999999999 + 6 )= 1.8623958333333335
x_ 4 =1/ 4 (- -1.5301041666666666 - 0.0 + 6 )= 1.8825260416666667
x_ 5 =1/ 4 (- -3.022578125 - 0.0 + 6 )= 2.2556445312499998

solution:x=
[ 1.1890625  -1.52135417  1.86239583  1.88252604  2.25564453]
---------------
```

```
Question 9
Jacobi method
A=
[[ 3 -1  1]
 [ 3  6  2]
 [ 3  3  7]]
b= [1 0 4]
x0= [0. 0. 0.]

iteration:  1
x_ 1 =1/ 3 (- 0.0 + 1 )= 0.3333333333333333
x_ 2 =1/ 6 (- 0.0 + 0 )= 0.0
x_ 3 =1/ 7 (- 0.0 + 4 )= 0.5714285714285714

realtive change under l_infty norm: inf

iteration:  2
x_ 1 =1/ 3 (- 0.5714285714285714 + 1 )= 0.14285714285714288
x_ 2 =1/ 6 (- 2.142857142857143 + 0 )= -0.35714285714285715
x_ 3 =1/ 7 (- 1.0 + 4 )= 0.42857142857142855

realtive change under l_infty norm: 0.625

iteration:  3
x_ 1 =1/ 3 (- 0.7857142857142857 + 1 )= 0.07142857142857144
x_ 2 =1/ 6 (- 1.2857142857142856 + 0 )= -0.21428571428571427
x_ 3 =1/ 7 (- -0.6428571428571428 + 4 )= 0.6632653061224489

realtive change under l_infty norm: 0.5476190476190476

iteration:  4
x_ 1 =1/ 3 (- 0.8775510204081632 + 1 )= 0.040816326530612256
x_ 2 =1/ 6 (- 1.5408163265306123 + 0 )= -0.2568027210884354
x_ 3 =1/ 7 (- -0.4285714285714285 + 4 )= 0.6326530612244898

realtive change under l_infty norm: 0.06410256410256414

iteration:  5
x_ 1 =1/ 3 (- 0.8894557823129252 + 1 )= 0.03684807256235826
x_ 2 =1/ 6 (- 1.3877551020408165 + 0 )= -0.23129251700680276
x_ 3 =1/ 7 (- -0.6479591836734694 + 4 )= 0.6639941690962099

realtive change under l_infty norm: 0.04953917050691236

iteration:  6
x_ 1 =1/ 3 (- 0.8952866861030127 + 1 )= 0.03490443796566245
x_ 2 =1/ 6 (- 1.4385325558794946 + 0 )= -0.23975542597991575
x_ 3 =1/ 7 (- -0.5833333333333335 + 4 )= 0.6547619047619049

realtive change under l_infty norm: 0.013904134650566943

iteration:  7
x_ 1 =1/ 3 (- 0.8945173307418206 + 1 )= 0.035160889752726465
x_ 2 =1/ 6 (- 1.4142371234207 97 + 0 )= -0.2357061872367995
x_ 3 =1/ 7 (- -0.6145529640427599 + 4 )= 0.6592218520061086

realtive change under l_infty norm: 0.006811555791147518

solution:x=
[ 0.03516089 -0.23570619  0.65922185]
```

```
Question 10
Gauss Seidal method:

A=
[[ 3 -1  1]
 [ 3  6  2]
 [ 3  3  7]]
b= [1 0 4]
x0= [0. 0. 0.]

iteration:  1
x_ 1 =1/ 3 (- 0.0 - 0.0 + 1 )= 0.3333333333333333
x_ 2 =1/ 6 (- 1.0 - 0.0 + 0 )= -0.16666666666666666
x_ 3 =1/ 7 (- 0.5 - 0.0 + 4 )= 0.5

realtive change under l_infty norm: inf

iteration:  2
x_ 1 =1/ 3 (- 0.0 - 0.6666666666666666 + 1 )= 0.11111111111111112
x_ 2 =1/ 6 (- 0.33333333333333337 - 1.0 + 0 )= -0.22222222222222224
x_ 3 =1/ 7 (- -0.33333333333333337 - 0.0 + 4 )= 0.619047619047619

realtive change under l_infty norm: 0.4444444444444444

iteration:  3
x_ 1 =1/ 3 (- 0.0 - 0.8412698412698412 + 1 )= 0.05291005291005294
x_ 2 =1/ 6 (- 0.15873015873015883 - 1.238095238095238 + 0 )= -0.2328042328042328
x_ 3 =1/ 7 (- -0.5396825396825395 - 0.0 + 4 )= 0.6485260770975056

realtive change under l_infty norm: 0.09401709401709399

iteration:  4
x_ 1 =1/ 3 (- 0.0 - 0.8813303099017384 + 1 )= 0.039556563366087184
x_ 2 =1/ 6 (- 0.11866969009826156 - 1.2970521541950113 + 0 )= -0.23595364071554548
x_ 3 =1/ 7 (- -0.5891912320483749 - 0.0 + 4 )= 0.6555987474354821

realtive change under l_infty norm: 0.020590520590520626

iteration:  5
x_ 1 =1/ 3 (- 0.0 - 0.8915523881510276 + 1 )= 0.03614920394965748
x_ 2 =1/ 6 (- 0.10844761184897245 - 1.3111974948709642 + 0 )= -0.2366075177866561
x_ 3 =1/ 7 (- -0.6013749415109958 - 0.0 + 4 )= 0.6573392773587138

realtive change under l_infty norm: 0.005197324475921185

solution:x=
[ 0.0361492  -0.23660752  0.65733928]
```