Д.З.6 2022.11.8

N.1

а) $E(X) = 300$

по неравенстве Маркова $P(x > A) \leq \frac{E(X)}{A}$

$P(x > 400) \leq \frac{300}{400} = 0.75$

Ответ: $P(x > 400) \leq 0.75$

б) $1 = P(x > A) + P(x \leq A)$

$1 - P(x \leq A) \leq \frac{E(X)}{A}$

$P(x \leq A) \geq 1 - \frac{E(X)}{A}$

$P(x \leq 500) \geq 1 - \frac{300}{500} = 0.4$

Ответ: $P(x \leq 500) \geq 0.4$

N.2

$n = 1600$ Схема Бернулли

$p = 0.3$

$\varepsilon = 50$

Неравенство Чебышева

$\forall \varepsilon > 0 \quad P(|X - E(X)| \geq \varepsilon) \leq \frac{D(X)}{\varepsilon^2}$

$E(X) = np = 480$

$D(X) = np(1-p) = 336$

$P(|X - 480| < 50) \geq 1 - \frac{336}{50^2} = 0.8656$

Ответ: 0.8656

N.3

$D(x) = 1 = \sigma^2 \quad x \in \{9, 5, 7, 7, 4, 10\}$

$$\bar{x} = \frac{\sum^{n} x_i}{n} = \frac{9+5+7^{*}+7+4+10}{6} = 7$$

$\alpha = 0{,}01$ по таблице $z_\alpha = 2{,}58$

$1 - \frac{\alpha}{2} = 0.995$

$$\Delta = \frac{\sigma}{\sqrt{n}} z_\alpha = \frac{1}{\sqrt{6}} 2.58 \approx 1{,}05$$

Ответ: доверительный ~~интер~~ интервал

$(\bar{x} - \Delta, \bar{x} + \Delta) = (5.95;\ 8.05)$

N.4

$x_i \sim N(\mu, \sigma^2) \qquad \hat{\mu},\ \sigma^2 - ?$

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}} = \frac{1}{(\sqrt{2\pi\sigma^2})^n} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$$

$$\ln L = \ln \frac{1}{(\sqrt{2\pi\sigma^2})^n} - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2$$

$$(\ln L)'_\mu = \frac{1}{\sigma^2} \sum_{i=1}^{n} (x_i - \mu) = 0 \Rightarrow \frac{1}{\sigma^2} \frac{\bar{x} - \mu}{n} = 0 \Rightarrow \hat{\mu} = \bar{x}$$

$$(\ln L)'_{\sigma^2} = \left[\left(-\frac{n}{2}\right)(\ln 2\pi + \ln \sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2\right]' = 0$$

$$\Rightarrow -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{i=1}^{n} (x_i - \mu)^2 = 0$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

Ответ: $\hat{\mu} = \bar{x}$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$