CS 754 : Advanced Image ProcessingAssignment 2

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Q2

A2.3

We have the relation :

$$E_u = \sum_{t=1}^{T} C_t \cdot F_t$$

Consider

$$E_1 = C_1 \cdot F_1$$

and suppose that we want to construct it as a matrix product, then we can write it as

$$E_1 = \phi_1 f_1$$

where $\phi_1 = diag(C_1)$ and $f_1 = vec(F_1)$ Hence

$$E_u = [\phi_1|\phi_2|...|\phi_T] \begin{bmatrix} f_1 \\ f_2 \\ .... \\ f_T \end{bmatrix}$$

Therefore

$$x = \begin{bmatrix} f_1 \\ f_2 \\ .... \\ f_T \end{bmatrix}$$

$$y = vec(Eu)$$

$$A = [\phi_1|\phi_2|...|\phi_T]$$

Q3

A3

Coherence is defined as :

$$\mu(\phi, \psi) = \sqrt{(n)} * \max_{i,j \in 0,1,..,n-1} |{\phi^{i}}^{t} \psi_j|$$

(a) Upper Bound: Consider a row of ϕ matrix to be exactly same as one of the columns of ψ the inner product would be 1, because both matrices are unit normalized, and result in $\mu_{max}(\phi, \psi) = \sqrt{(n)}$.

(b) Lower Bound: Let g be a row of the ϕ matrix, which is unit normalized. g can be written with basis vectors as columns of ψ. That is

$$g = \sum_{k=1}^{n} \alpha_k \psi_k$$

Also since g is unit norm,

$$\sum_{k=1}^{n} \alpha_k^2 = 1$$

When we take the inner product of g with ψ_k, only one α will remain. Now consider the coherence of g and ψ. Clearly

$$\mu(g, \psi) = \sqrt{n} * max(\alpha_j)_{j=1}^{n}$$

where α_j is corresponds to the jth coefficient. Now we also know that

$$max(\alpha_j)_{j=1}^{n} \geq avg(\alpha_j)_{j=1}^{n}$$

and equality occurs when all α_j are equal. Hence to get the minimum coherence we need all α_j to be equal and using the previous constraint on unit norm we get

$$\alpha_j = \frac{1}{\sqrt{n}}$$

. This gives

$$\mu_{min}(g, \psi) = 1$$

This is the bound for all rows, and hence we have obtained

$$\mu_{min}(\phi, \psi) = 1$$

Q4

A4