CS 754 : Advanced Image ProcessingAssignment 5

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April 13, 2017

Q1

Q2

A2.1

Need to show Shift Theorem:

$$R(g(x-x_0, y-y_0)(\rho,\theta) = R(g(x,y))(\rho - x_0cos(\theta) - y_0sin(\theta), \theta) \tag{1}$$

Let $h(x,y) = g(x-x_0, y-y_0)$

$$R(h(x,y))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} h(x,y)\delta(xcos(\theta) + ysin(\theta) - \rho)dxdy$$

$$R(h(x,y))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} g(x-x_0, y-y_0)\delta(xcos(\theta) + ysin(\theta) - \rho)dxdy$$

Put $x_1 = x - x_0$ and $y_1 = y - y_0$

$$R(h(x,y))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} g(x_1, y_1)\delta((x_1+x_0)cos(\theta) + (y_1+y_0)sin(\theta) - \rho)dxdy$$

$$R(h(x,y))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} g(x_1, y_1)\delta(x_1cos(\theta) + y_1sin(\theta) - (\rho - x_0cos(\theta) - y_0sin(\theta))dx_1dy_1 = R(g(x,y)(\rho_1, \theta)$$

where $\rho_1 = \rho - x_0cos(\theta) - y_0sin(\theta)$
Therefore

$$R(h(x,y))(\rho,\theta) = R(g(x-x_0, y-y_0))(\rho,\theta) = R(g(x,y))(\rho_1,\theta)$$

Hence Proved

A2.2

Need to show Rotation Theorem: Let $g^{'}(r,\psi) = g(r, \psi - \psi_0)$.

$$R(g^{'})(\rho,\theta) = R(g)(\rho, \psi_0 - \theta) \tag{2}$$

$$R(g^{'}(r,\psi))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} g^{'}(r,\psi)\delta(\rho - rcos(\psi - \theta))|r|drd\psi$$

$$R(g^{'}(r,\psi))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} g(r,\psi - \psi_0)\delta(\rho - rcos(\psi - \theta))|r|drd\psi$$

Put $\psi_1 = \psi - \psi_0$

$$R(g^{'}(r,\psi))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} g(r,\psi_1)\delta(\rho - rcos(\psi_1 + \psi_0 - \theta)|r|drd\psi_1$$

$$R(g^{'}(r,\psi))(\rho,\theta) = R(g(r,\psi))(\rho, \psi_0 - \theta)$$

A2.3

Need to show Convolution Theorem: Let

$$h(x,y) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} f(x_1,y_1)g(x-x_1,y-y_1)dx_1dy_1$$

$$Rh = Rf * Rg \tag{3}$$

$$Rh = \int_{\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty} f(x_1,y_1)g(x-x_1,y-y_1)\delta(\rho - xcos(\theta) - ysin(\theta))dx_1dy_1dxdy$$

$$Rh = \int_{\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty} R(g)(\rho - x_1cos(\theta) - y_1sin(\theta),\theta)dx_1dy_1$$

$$Rh = \int_{\infty}^{\infty}\int_{\infty}^{\infty}\int_{infty}^{infty} f(x_1,y_1)R(g(\rho-\rho_1,\theta)\delta(\rho_1 - x_1cos(\theta) - y_1sin(\theta))dx_1dy_1d\rho_1$$

$$Rh = \int_{\infty}^{infty} R(f)(\rho_1,\theta)R(g))(\rho-\rho_1,\theta)d\rho_1$$

$$Rh = Rf * Rg$$