Assignment 3: CS 754, Advanced Image Processing

Due: 14th March before 11:55 pm

Remember the honor code while submitting this (and every other) assignment. All members of the group should work on and understand all parts of the assignment. We will adopt a zero-tolerance policy against any violation.

Submission instructions: You should ideally type out all the answers in Word (with the equation editor) or using Latex. In either case, prepare a pdf file. Create a single zip or rar file containing the report, code and sample outputs and name it as follows: A3-IdNumberOfFirstStudent-IdNumberOfSecondStudent.zip. (If you are doing the assignment alone, the name of the zip file is A3-IdNumber.zip). Upload the file on moodle BEFORE 11:55 pm on 14th March. Late assignments will be assessed a penalty of 50% per day late. Note that only one student per group should upload their work on moodle. Please preserve a copy of all your work until the end of the semester. If you have difficulties, please do not hesitate to seek help from me.

1. Download the paper "The restricted isometry property and its implications for compressed sensing" by Candes from the homework folder `http://www.cse.iitb.ac.in/~ajitvr/CS754/HW3/` and answer the questions mentioned in every orange/yellow-colored blob. Note that there are 23 questions in all and you should include the answer to each question in the proper order in your report. The purpose of this exercise is to have you trace through the proof of a very beautiful theorem. Most of the answers require very simple calculations involving basic vector or matrix properties, but you should show each of them clearly. [2 points each for the first 22 questions and 6 points for the 23rd question]

2. Your job is to implement an algorithm for designing of CS matrices. The algorithm is from the paper 'Learning to Sense Sparse Signals..' by Duarte-Carvajalino which you will find in the homework folder. To this end, you will do as follows:

   (a) Create a random dictionary $\boldsymbol{\Psi}$ of size 64×128 with entries drawn iid from $\mathcal{N}(0,1)$. Implement the procedure from the paper to design $\boldsymbol{\Phi}$ of size 40×64 starting with a Gaussian random $\boldsymbol{\Phi}$ (entries drawn iid from $\mathcal{N}(0,1)$) based on minimizing the mutual coherence of $\boldsymbol{\Phi\Psi}$. We will henceforth refer to the designed matrix as $\boldsymbol{\Phi_d}$ and the original one as $\boldsymbol{\Phi_o}$. Now generate 2000 test signals $\boldsymbol{x_i}$, each of size 64×1, which are random linear combinations of exactly 5 columns from $\boldsymbol{\Psi}$. Reconstruct each of them from their compressive measurements $\boldsymbol{y_i} = \boldsymbol{\Phi_d x_i}$ using the OMP method, for which you may use code from the toolbox on the link below. Compare the average relative reconstruction errors for $\boldsymbol{\Phi_d}$ with those obtained for reconstruction of $\boldsymbol{x_i}$ from $\boldsymbol{y_i} = \boldsymbol{\Phi_o x_i}$. Plot a histogram of the coherence values in the Gram matrix for $\boldsymbol{\Phi_d}$ and $\boldsymbol{\Phi_o}$ - see figure 1 in the paper for reference.

   (b) Download code (KSVD-box v13) for the KSVD algorithm (which we will study later in class) from `http://www.cs.technion.ac.il/~ronrubin/software.html`. You will also need to download the OMP toolbox. Run the code provided to learn a dictionary $\boldsymbol{\Psi}$ with $K = 128$ columns for sparse representation of non-overlapping image patches of size 8×8, extracted from the first 30 (as per the MATLAB command 'dir') training images of the Berkeley Database which you can download from `https://www2.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/BSDS300-images.tgz`. Note, you must convert all images to gray scale (and cast them as double) prior to further processing. Also note that the dictionary size will be 64×128. Display the columns of the dictionary in the form of a grid of small images, where each image has size 8×8 and is obtained by reshaping each column of the dictionary.

(c) Now create 2000 test patches, each obtained as random linear combinations of any 5 columns of $\boldsymbol{\Psi}$ from the previous step. Reconstruct each of them from their compressive measurements $\boldsymbol{y_i} = \boldsymbol{\Phi x_i}$ using the OMP method. Compare the average relative reconstruction errors for the designed $\boldsymbol{\Phi}$ with those obtained for a random $\boldsymbol{\Phi}$. Plot a histogram of the coherence values in the Gram matrix for the original and designed $\boldsymbol{\Phi}$.

(d) Now create an ensemble of non-overlapping 8×8 patches extracted from the next 10 training images (as per the MATLAB command 'dir') from the same database. Again compare the relative reconstruction errors for compressive reconstruction using OMP for $\boldsymbol{\Phi_d}$ as well as $\boldsymbol{\Phi_o}$.

(e) If you had to use this algorithm in an actual compressive sensing application (for example, for the Hitomi video compressive camera), what are the difficulties you would face? In other words, state the limitation of the algorithm given in this paper.

Marking scheme: Succesful implementation of the algorithm as evidence from the coherence values before and after optimization - 25 points. 5 points for plotting the dictionary using KSVD. 5 points for the coherence value plots. 5 points for reporting the relative reconstruction errors. Analysis of the limitations - 10 points.