CS 754 : Advanced Image Processing
Assignment 5

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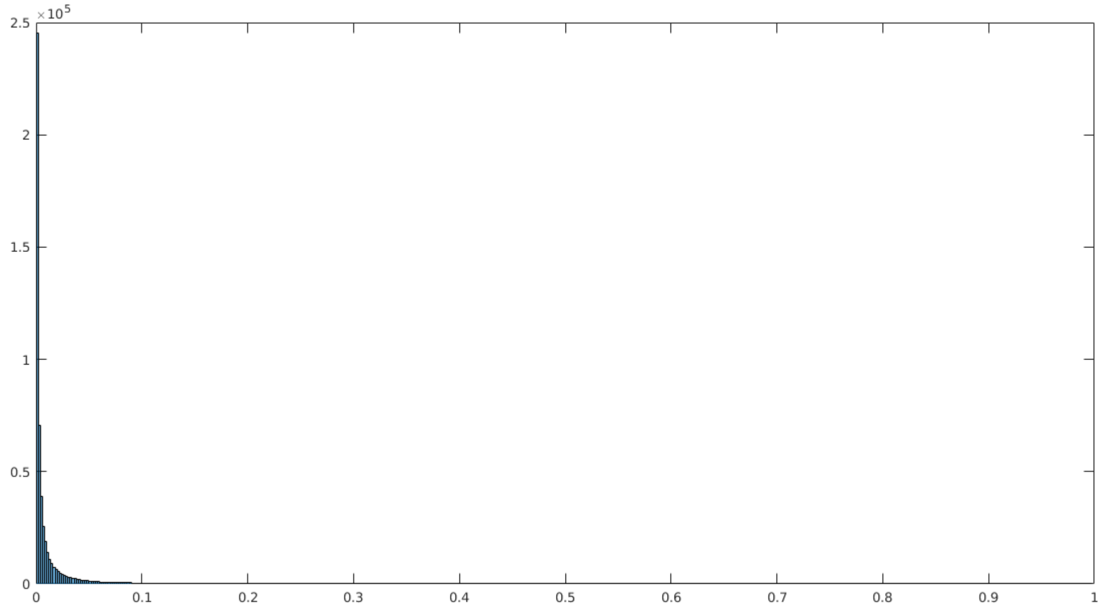
Q1

Mutual Coherence:

$$\mu = 0.996815820660457$$

Histogram of Coherence Values:



Q2

A2.1

Need to show Shift Theorem:

$$R(g(x - x_0, y - y_0)(\rho, \theta) = R(g(x, y))(\rho - x_0cos(\theta) - y_0sin(\theta), \theta) \tag{1}$$

Let $h(x, y) = g(x - x_0, y - y_0)$

$$R(h(x, y))(\rho, \theta) = \int_{\infty}^{\infty} \int_{\infty}^{\infty} h(x, y)\delta(xcos(\theta) + ysin(\theta) - \rho)dxdy$$

$$R(h(x, y))(\rho, \theta) = \int_{\infty}^{\infty} \int_{\infty}^{\infty} g(x - x_0, y - y_0)\delta(xcos(\theta) + ysin(\theta) - \rho)dxdy$$

Put $x_1 = x - x_0$ and $y_1 = y - y_0$

$$R(h(x,y))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} g(x_1,y_1)\delta((x_1+x_0)cos(\theta) + (y_1+y_0)sin(\theta) - \rho)dxdy$$

$$R(h(x,y))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} g(x_1,y_1)\delta(x_1cos(\theta)+y_1sin(\theta)-(\rho-x_0cos(\theta)-y_0sin(\theta))dx_1dy_1 = R(g(x,y)(\rho_1,\theta)$$

where $\rho_1 = \rho - x_0cos(\theta) - y_0sin(\theta)$
Therefore

$$R(h(x,y))(\rho,\theta) = R(g(x-x_0,y-y_0))(\rho,\theta) = R(g(x,y))(\rho_1,\theta)$$

Hence Proved

A2.2

Need to show Rotation Theorem: Let $g^{'}(r,\psi) = g(r,\psi-\psi_0)$.

$$R(g^{'})(\rho,\theta) = R(g)(\rho,\psi_0-\theta) \tag{2}$$

$$R(g^{'}(r,\psi))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} g^{'}(r,\psi)\delta(\rho - rcos(\psi-\theta))|r|drd\psi$$

$$R(g^{'}(r,\psi))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} g(r,\psi-\psi_0)\delta(\rho - rcos(\psi-\theta))|r|drd\psi$$

Put $\psi_1 = \psi - \psi_0$

$$R(g^{'}(r,\psi))(\rho,\theta) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} g(r,\psi_1)\delta(\rho - rcos(\psi_1+\psi_0-\theta)|r|drd\psi_1$$

$$R(g^{'}(r,\psi))(\rho,\theta) = R(g(r,\psi))(\rho,\psi_0-\theta)$$

A2.3

Need to show Convolution Theorem: Let

$$h(x,y) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} f(x_1,y_1)g(x-x_1,y-y_1)dx_1dy_1$$

$$Rh = Rf * Rg \tag{3}$$

$$Rh = \int_{\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty} f(x_1,y_1)g(x-x_1,y-y_1)\delta(\rho - xcos(\theta) - ysin(\theta))dx_1dy_1dxdy$$

$$Rh = \int_{\infty}^{\infty}\int_{\infty}^{\infty} f(x_1,y_1)R(g)(\rho - x_1cos(\theta) - y_1sin(\theta),\theta)dx_1dy_1$$

$$Rh = \int_{\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty} f(x_1,y_1)R(g)(\rho-\rho_1,\theta)\delta(\rho_1 - x_1cos(\theta) - y_1sin(\theta))dx_1dy_1d\rho_1$$

$$Rh = \int_{\infty}^{\infty} R(f)(\rho_1,\theta)R(g))(\rho-\rho_1,\theta)d\rho_1$$

$$Rh = Rf * Rg$$

Q3

We have taken Radon Transform at 18 randomly selected angles in all 3 parts, for both of the images.

A3.1

Filtered Back Projection using the Ram-Lak Projection:

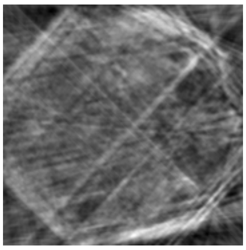


Figure 1: Filtered Back Projection Img1

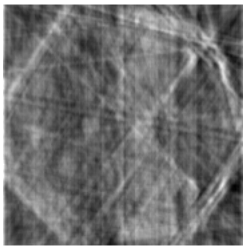


Figure 2: Filtered Back Projection Img2

A3.2

Compressed Sensing Decoupled Tomographic Reconstruction For the l1_ls solver the parameters used were : $\lambda = 0.01$ $rel\_tol = 100$

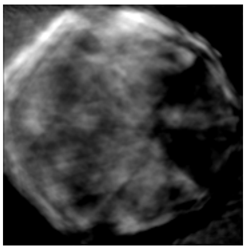


Figure 3: Decoupled Compressed Sensing Tomographic Reconstruction Img1

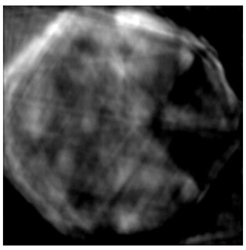


Figure 4: Decoupled Compressed Sensing Tomographic Reconstruction Img2

A3.3

Compressed Sensing Coupled Tomographic Reconstruction For the l1_ls solver the parameters used were : $\lambda = 0.01$ $rel\_tol = 100$

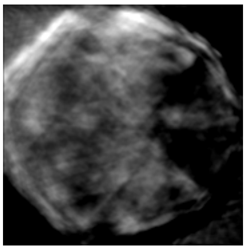


Figure 5: Coupled Compressed Sensing Tomographic Reconstruction Img1

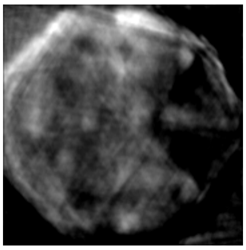


Figure 6: Coupled Compressed Sensing Tomographic Reconstruction Img2

A3.4

Modified Objective function for the case of 3 slices We consider

$$\theta_2 = \theta_1 + \Delta\theta_2$$

and

$$\theta_3 = \theta_1 + \Delta\theta_2 + \Delta\theta_3$$

And $\hat{W}$ is the basis in which the image is sparse.

$$\min_{\hat{\theta}} ||y_t - R_t \hat{W} \hat{\theta}||_{l_2} \tag{4}$$

With

$$\hat{y_t} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \tag{5}$$

$$R_t = \begin{bmatrix} R_1 & & \\ & R_2 & \\ & & R_3 \end{bmatrix}$$

$$\hat{W} = \begin{bmatrix} W & & \\ W & W & \\ W & W & W \end{bmatrix}$$

$$\hat{\theta}_t = \begin{bmatrix} \theta_1 \\ \Delta\theta_2 \\ \Delta\theta_3 \end{bmatrix} \tag{6}$$