Assignment 5: CS 754, Advanced Image Processing

Due: 14th April before 11:55 pm

Remember the honor code while submitting this (and every other) assignment. All members of the group should work on and understand all parts of the assignment. We will adopt a zero-tolerance policy against any violation.

Submission instructions: You should ideally type out all the answers in Word (with the equation editor) or using Latex. In either case, prepare a pdf file. Create a single zip or rar file containing the report, code and sample outputs and name it as follows: A5-IdNumberOfFirstStudent-IdNumberOfSecondStudent.zip. (If you are doing the assignment alone, the name of the zip file is A5-IdNumber.zip). Upload the file on moodle BEFORE 11:55 pm on 14th April. Late assignments will be assessed a penalty of 50% per day late. Note that only one student per group should upload their work on moodle. Please preserve a copy of all your work until the end of the semester. If you have difficulties, please do not hesitate to seek help from me.

1. Let $\boldsymbol{R}$ be an operator that denotes the Radon transform of an image for a set of angles $S = \{0 : 12 : 168\}$. Let $\boldsymbol{U}$ denote the 2D-DCT matrix for an image of size 32×32. Determine the mutual coherence of $\boldsymbol{RU}$ without physically constructing the matrix $\boldsymbol{R}$, but using the MATLAB function 'radon'. Plot a histogram of the coherence values as well. [30 points]

2. Prove the following properties of the Radon transform:

   (a) Shift theorem: $R(g(x - x_0, y - y_0))(\rho, \theta) = R(g(x, y))(\rho - x_0 \cos\theta - y_0 \sin\theta, \theta)$.

   (b) Rotation theorem: Let $g'(r, \psi) = g(r, \psi - \psi_0)$. Then prove that $R(g')(\rho, \theta) = R(g)(\rho, \psi_0 - \theta)$.

   (c) Prove that the Radon transform of the convolution of two functions is equal to the convolution of their Radon transforms. [10+10+15=35 points]

3. In this task, you will you use the well-known package L1_LS from `https://stanford.edu/~boyd/l1_ls/` for the purpose of tomographic reconstruction. The homework folder contains images of two slices taken from an MR volume of the brain. Create measurements by parallel beam tomographic projections at any 18 randomly angles chosen from a uniform distribution on $[0, \pi)$. Use the MATLAB function 'radon' for this purpose. Now perform tomographic reconstruction using the following method: (a) filtered back-projection using the Ram-Lak filter, as implemented in the 'iradon' function in MATLAB, (b) independent CS-based reconstruction for each slice by solving an optimization problem of the form $E(x) = \|y - Ax\|^2 + \lambda\|x\|_1$, (c) a coupled CS-based reconstruction that takes into account the similarity of the two slices using the model given in slide 46 of the lectures notes on tomography. For parts (b) and (c), use the aforementioned package from Stanford. For part (c0, make sure you should a different random set of 18 angles for each of the two slices. The tricky part is careful creation of the forward model matrix A or a function handle representing that matrix, as well as the corresponding adjoint operator A^T. Use the 2D-DCT basis for the image representation.

   Modify the objective function from slide 46 of the lecture notes for the case of three similar slices. Carefully define all terms in the equation. [5+10+10+10 = 35 points]