

CS 754 : Advanced Image Processing Assignment 1

Meet Udeshi - 14D070007

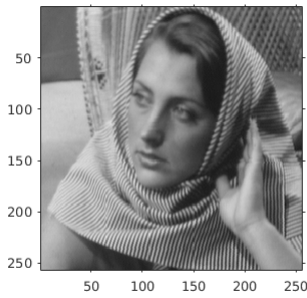
Arka Sadhu - 140070011

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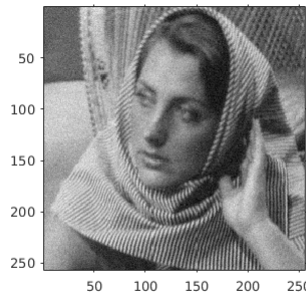
Q1

A1.1

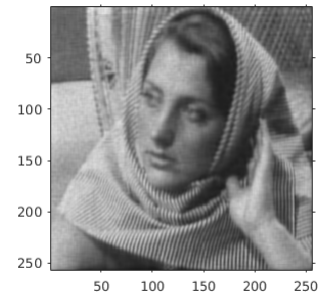
Parameters chosen : $\lambda =$



(a) Original Image Barbara



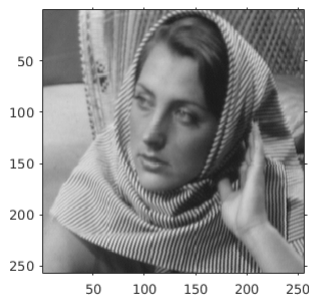
(b) Noisy Image Barbara



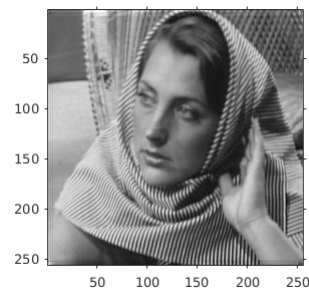
(c) Denoised Image Barbara

Figure 1: Figures for Q1

A1.2



(a) Original Image Barbara



(b) Reconstructed Image from a Random Matrix Φ Barbara

Figure 2: Figures for Q2

Q3

A3.1

Assuming the image is of size $N \times N$, and there are M filters. From equation (6) we note that, there are 4 terms to be considered. The first term is : $\rho(f_{ik} \cdot I_1)$. To convert it into the form of $\rho(A_j \cdot v - b_j)$ where v = vectorized image I_1 , we need to construct A_j such that its rows denote the derivative filter, which on multiplication with v , will result in the derivative image. Every possible $f_{i,k}$ is represented by one row of A_j , such that $A_j \cdot v$ will give a column vector containing every term of the summation. We will have to redefine $\rho_j(\vec{x})$ as $\sum_k \rho(x_k)$.

For all other terms A_j will be constructed in the same way. Only for the third and fourth term, we will take $f_{i,k}$ for only $i \in S_1/S_2$. b_j will be calculated as $f_{i,k} \cdot I$ for second and third term. Effectively $b_j = A_j \cdot I$. For first and fourth term b_j will be 0.

Size of A_j for first and second term is $N^2 M \times N^2$, and for third and fourth term it will be $S_{(1|2)} M \times N^2$.

A3.2

In equation (6) the first two terms are obtained from prior, while the last two terms are obtained from likelihood.

That is, the prior terms are:

$$\sum_{i,k} \rho(f_{i,k} \cdot I_1)$$

$$\sum_{i,k} \rho(f_{i,k} \cdot I_1 - f_{i,k} \cdot I)$$

The likelihood terms are :

$$\lambda \sum_{i \in S_1,k} \rho(f_{i,k} \cdot I - f_{i,k} \cdot I_1)$$

$$\lambda \sum_{i \in S_2,k} \rho(f_{i,k} \cdot I_1)$$

The prior used in this paper is the mixed Laplacian model, given by:

$$Pr(I) \approx \prod_{i,k} Pr(f_{i,k} \cdot I)$$

Approx sign is used, since we are assuming independence of derivative filters over space which is strictly not true.

The likelihood used in this paper is that the gradient of I_1 at all locations S_1 agree with the gradients of the input image I and similarly gradient of I_2 at all locations S_2 agree with the gradients of the input image I . That is,

At all points S_1 :

$$\nabla I_1 = \nabla I$$

At all points S_2

$$\nabla I_2 = \nabla I$$

A3.3

Consider the third term :

$$\lambda \sum_{i \in S_1,k} \rho(f_{i,k} \cdot I - f_{i,k} \cdot I_1)$$

This can be reduced to

$$\lambda \sum_{i \in S_{1,k}} \rho(f_{i,k} \cdot I_2)$$

We have implicitly assumed that at points S_1 the gradient of I_2 is approximately zero (since the user will predominantly pick edges of I_1 and they are highly unlikely to coincide with the edges of I_2). Hence we expect this term to be even more sparse than the second term, which is

$$\sum_{i,k} \rho(f_{i,k} \cdot I_2)$$

Since we have seen Gaussian model doesn't even fit

$$\sum_{i,k} \rho(f_{i,k} \cdot I_2)$$

It is definitely not expected to fit the sparser term (which is the third term). In fact we can use a sparser model (instead of current mixed Laplacian) but we definitely cannot use Gaussian likelihood.

Similar argument can be provided for the fourth term.