CS 754 : Advanced Image ProcessingAssignment 1

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February 1, 2017

Q1

A1.1

In the folder q1, run the file denoising.m to get the images.

Parameters chosen:

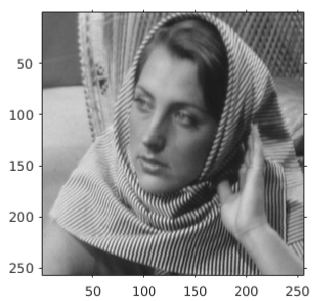
$$\lambda = 0.5$$

$$\alpha = max(eig(A' * A))$$

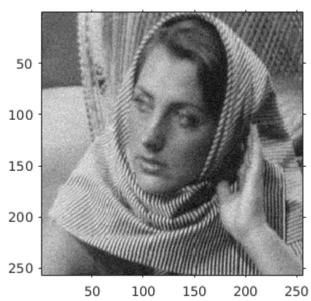
$$\sigma = 10$$

$$\hat{\lambda} = \lambda * (2 * \sigma^2)$$

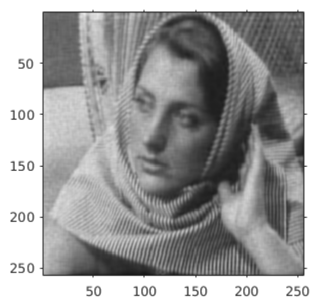
To compensate for the different intensity of the input and output image we have added the difference of the DC coefficients of the input and output image and added it to the output image.



(a) Original Image Barbara



(b) Noisy Image Barbara



(c) Denoised Image Barbara

Figure 1: Figures for Q1a

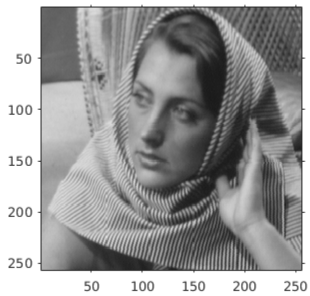
A1.2

We have created ϕ as a random matrix whose entries are drawn iid from $\mathcal{N}(0,1)$. Parameters chosen :

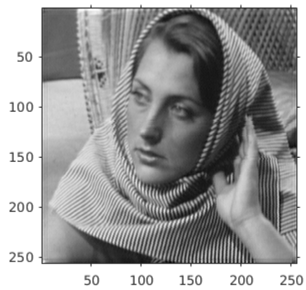
$$\lambda = 1$$

$$\alpha = max(eig(A' * A))$$

The resultant output image was converted from float to uint8, by first converting it into range of $[0, 1]$ and then scaling by 255. This resulted in slight contrast stretching of the image.



(a) Original Image Barbara



(b) Reconstructed Image Barbara

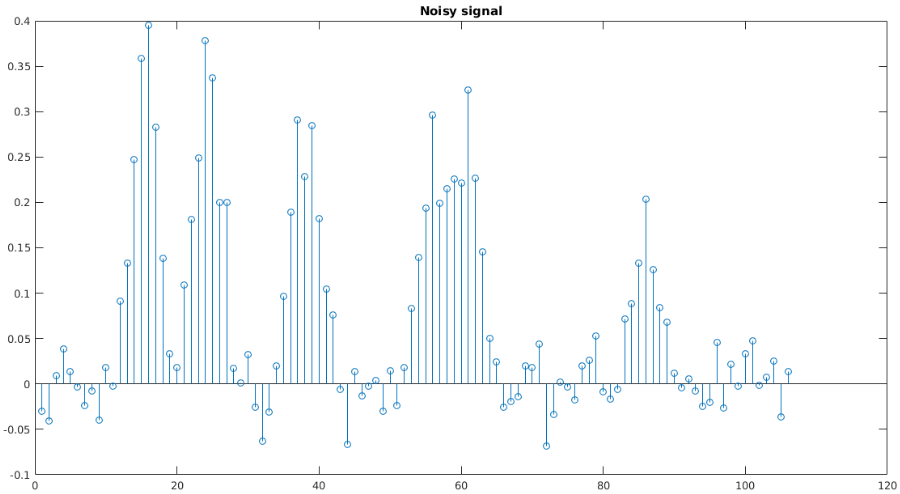
Figure 2: Figures for Q1b

A1.3

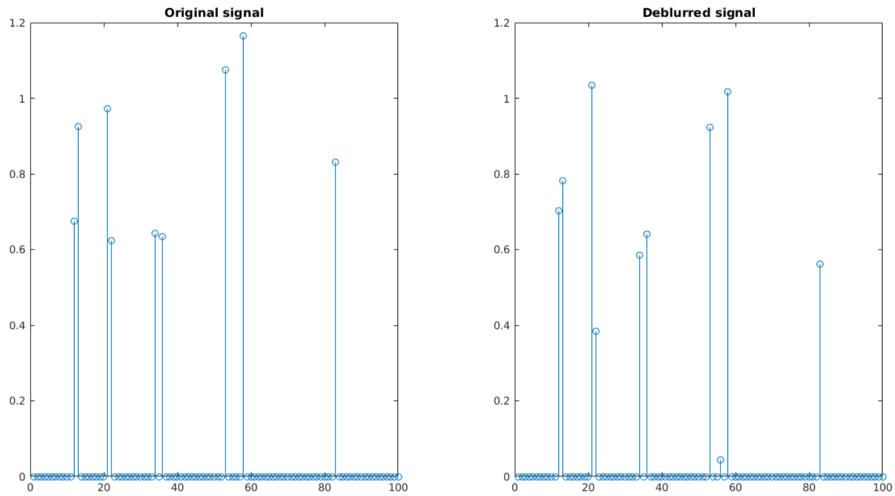
In the folder q1, run the file deblur.m to get the deblurred image.

Parameters used:

$$\lambda = 50$$
$$\alpha = max(eig(A' * A))$$
$$\sigma = 0.01 * ||x||_2$$
$$\hat{\lambda} = \lambda * (2 * \sigma^2)$$



(a) Blurred Signal



(b) Deblurred Signal

Figure 3: Figure for Q1c

Q2

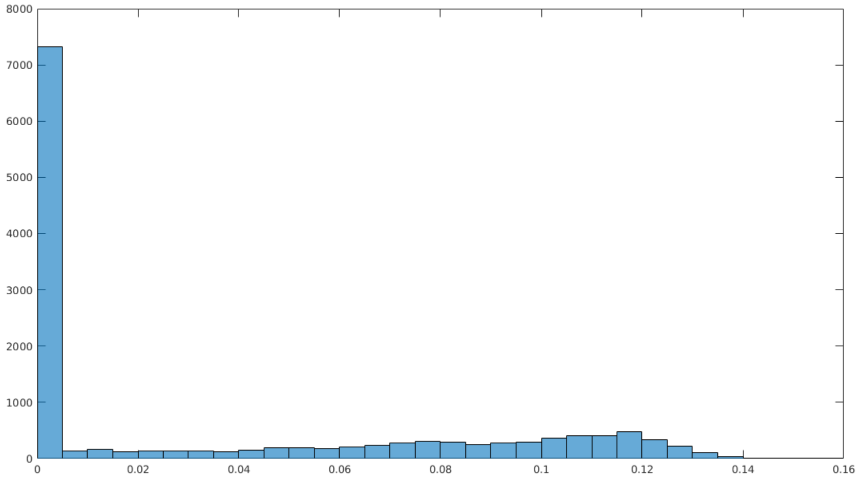
A2

In the folder q2, run the file integration.m to get the numerical solution for the case of uniform variance. Parameters chosen:

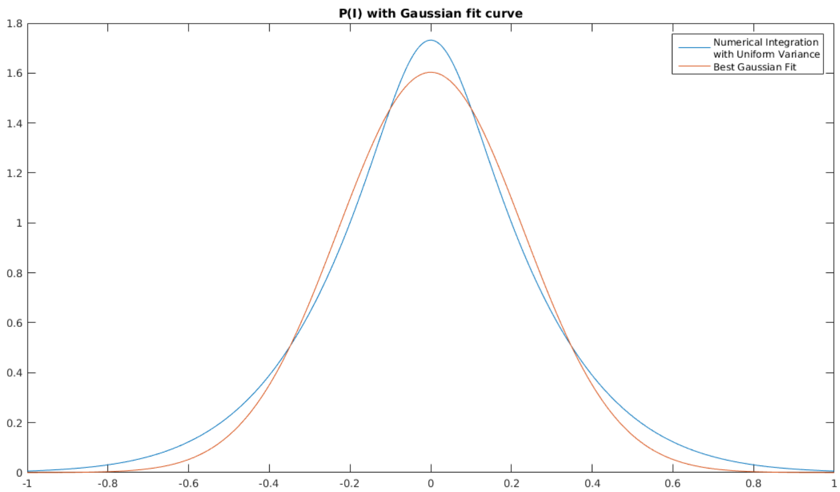
$$a = 0.005$$

$$b = 0.14$$

$$dI = 10^{-3}$$



(a) Variance distribution



(b) P(I) and Gaussian Fit

Figure 4: Figures for Q2

Q3

A3.1

Assuming the image is of size NxN, and there are M filters. From equation (6) we note that, there are 4 terms to be considered. The first term is : $\rho(f_{ik}.I_1)$. To convert it into the form of $\rho(A_{j\rightarrow}v - b_j)$ where v = vectorized image I_1, we need to construct A_j such that its rows denote the derivative filter, which on multiplication with v, will result in the derivative image. Every possible $f_{i,k}$ is represented by one row of A_j, such that $A_j \cdot v$ will give a column vector containing every term of the summation. We will have to redefine $\rho_j(\vec{x})$ as $\sum_k \rho(x_k)$.

For all other terms A_j will be constructed in the same way. Only for the third and fourth term, we will take $f_{i,k}$ for only $i \in S_1/S_2$. b_j will be calculated as $f_{i,k} \cdot I$ for second and third term. Effectively $b_j = A_j \cdot I$. For first and fourth term b_j will be 0.

Size of A_j for first and second term is $N^2M \times N^2$, and for third and fourth term it will be $S_{(1|2)}M \times N^2$.

A3.2

In equation (6) the first two terms are obtained from prior, while the last two terms are obtained from likelihood.
That is, the prior terms are:

$$\sum_{i,k} \rho(f_{i,k} \cdot I_1)$$

$$\sum_{i,k} \rho(f_{i,k} \cdot I_1 - f_{i,k} \cdot I)$$

The likelihood terms are :

$$\lambda \sum_{i \in S_1,k} \rho(f_{i,k} \cdot I - f_{i,k} \cdot I_1)$$

$$\lambda \sum_{i \in S_2,k} \rho(f_{i,k} \cdot I_1)$$

The prior used in this paper is the mixed Laplacian model, given by:

$$Pr(I) \approx \prod_{i,k} Pr(f_{i,k} \cdot I)$$

Approx sign is used, since we are assuming independence of derivative filters over space which is strictly not true.

The likelihood used in this paper is that the gradient of I_1 at all locations S_1 agree with the gradients of the input image I and similarly gradient of I_2 at all locations S_2 agree with the gradients of the input image I. That is,
At all points S_1:

$$\nabla I_1 = \nabla I$$

At all points S_2

$$\nabla I_2 = \nabla I$$

A3.3

Consider the third term :

$$\lambda \sum_{i \in S_1,k} \rho(f_{i,k} \cdot I - f_{i,k} \cdot I_1)$$

This can be reduced to

$$\lambda \sum_{i \in S_1,k} \rho(f_{i,k} \cdot I_2)$$

We have implicitly assumed that at points S_1 the gradient of I_2 is approximately zero (since the user will predominantly pick edges of I_1 and they are highly unlikely to coincide with the edges of I_2). Hence we expect this term to be even more sparse than the second term, which is

$$\sum_{i,k} \rho(f_{i,k} \cdot I_2)$$

Since we have seen Gaussian model doesn't even fit

$$\sum_{i,k} \rho(f_{i,k} \cdot I_2)$$

It is definitely not expected to fit the sparser term (which is the third term). In fact we can use a sparser model (instead of current mixed Laplacian) but we definitely cannot use Gausian likelihood.

Similar argument can be provided for the fourth term.