

Design of a structured product

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1 Introduction, Assumptions and relevant market data

In this assignment the design of two structured products for an underlying stock will be discussed. Namely the underlying stock is Advanced Micro Devices Inc. , which is also known as AMD. AMD is an American multinational semiconductor company, which produces Computer processors and related technologies for business and consumer markets. For many investors AMD is a really interesting company, as it is beside of Intel (Intel Corp. v. Advanced Micro Devices) the biggest producer for computer processors with even growing partial of the market. Additionally the recent release of the new graphics processing units (GPUs) is very promising as these represent serious competition to Nvidia graphics cards, which dominated the market in the last years.

Following structured products will be introduced:

- Product 1: Partially principal protected note
- Product 2: Airbag note

As underlying Finance instruments we will basically consider three different assets:

- the risk-free bank account with an fixed interest rate r
- the stock S (in this case AMD)
- Call and Put options on the stock S

As starting date $t = 0$ Friday the 20st November is choosen. The maturity date of the products is Friday 21st January 2022. More precisely the structured products, which will be discussed later, are build on the data presented below, accessed at Friday 20st November at 4:45 pm (MEZ), which is one hour and a quarter after the market opening at that day. The stock price at this moment was $S_0 = 85.35$ USD. The data is shown in the tables at the end of this section. A small extract of the data is moreover shown in a corresponding screenshot in the appendix. As the tables already contain the full data, the screenshot only shows the stock price and the first few call options.

To get a fixed interest rate for the whole time period, the daily treasury yield rate of the US treasure bond at the starting date $t = 0$ is picked, which we denote by $r = 0.11\%$. In the appendix also an associated table is shown, which is published at [1] an official website of the United States Government.

The reason of choice is that the base interest rate of the United States, the United States Fed Funds Rate, is given by a target range of $0 - 0.25\%$. Moreover the daily treasury yield curve rates are given for different time periods exactly. Since the time period of our products is around one year and two months the rate for an investment over one year is picked. As the investment is even longer than one year, that is a rather cautious estimate for the interest rate.

LastTradeDate	Strike	LastPrice	Bid	Ask	Volume	ImpliedVolatility
2020-11-18 3:10PM EST	5.0	78.48	79.95	81.5	232	145.31%
2020-11-19 11:49AM EST	10.0	74.86	75.2	76.4	1	110.25%
2020-11-18 1:28PM EST	15.0	69.01	70.4	71.1	5	86.82%
2020-11-09 10:15AM EST	18.0	68.35	67.5	68.7	22	88.04%
2020-11-05 9:50AM EST	20.0	65.5	65.6	66.5	1	80.96%
2020-11-17 1:44PM EST	23.0	61.0	62.55	63.55	4	73.44%
2020-11-18 3:36PM EST	25.0	59.0	60.45	61.55	1	67.97%
2020-11-09 1:24PM EST	28.0	56.0	57.7	58.85	1	66.99%
2020-11-18 10:32AM EST	30.0	54.0	55.8	56.95	3	64.70%
2020-10-13 1:53PM EST	32.0	54.99	50.35	51.25	1	0.00%
2020-11-19 3:19PM EST	35.0	51.5	51.3	52.0	5	59.45%
2020-11-20 10:02AM EST	37.0	49.5	49.4	50.3	2	58.35%
2020-11-19 3:18PM EST	40.0	46.95	46.9	47.4	35	56.45%
2020-11-13 1:09PM EST	42.0	41.83	45.2	45.75	7	56.07%
2020-11-16 12:14PM EST	45.0	40.8	42.45	43.5	14	55.18%
2020-11-17 12:40PM EST	47.0	39.65	41.0	41.65	7	54.47%
2020-11-20 10:10AM EST	50.0	39.0	38.7	39.2	6	53.71%
2020-11-16 10:47AM EST	52.5	35.04	36.75	37.25	15	52.95%
2020-11-19 3:36PM EST	55.0	35.0	34.9	35.95	11	53.76%
2020-11-18 3:20PM EST	57.5	33.26	33.1	34.5	5	53.92%
2020-11-20 10:16AM EST	60.0	31.7	31.35	32.4	10	52.55%
2020-11-19 2:22PM EST	62.5	29.1	29.25	31.15	1	52.05%
2020-11-20 9:30AM EST	65.0	28.38	28.1	28.5	33	50.53%
2020-11-16 11:54AM EST	67.5	25.5	26.45	27.8	1	51.45%
2020-11-20 10:09AM EST	70.0	25.55	25.2	25.9	3	50.73%
2020-11-17 11:13AM EST	72.5	23.0	23.7	24.25	5	50.75%
2020-11-20 10:07AM EST	75.0	22.77	22.6	22.95	4	50.02%
2020-11-19 2:10PM EST	77.5	22.0	21.45	21.85	7	50.21%
2020-11-20 9:39AM EST	80.0	20.35	20.25	20.7	8	50.07%
2020-11-20 10:17AM EST	82.5	19.0	19.1	19.5	12	50.35%
2020-11-20 10:05AM EST	85.0	18.45	18.15	18.5	7	50.39%
2020-11-20 9:58AM EST	87.5	17.45	17.1	17.5	3	50.29%
2020-11-20 10:12AM EST	90.0	16.29	16.15	16.55	15	50.19%
2020-11-20 10:06AM EST	92.5	15.6	15.2	16.1	1	50.10%
2020-11-20 10:05AM EST	95.0	14.8	14.4	15.4	9	50.29%
2020-11-20 10:19AM EST	97.5	14.1	13.7	14.0	1	49.96%
2020-11-20 10:05AM EST	100.0	13.0	12.95	13.3	10	50.06%
2020-11-20 10:12AM EST	105.0	11.6	11.5	11.8	5	49.67%
2020-11-20 10:17AM EST	110.0	10.45	10.3	10.65	7	49.85%
2020-11-20 10:05AM EST	115.0	9.5	9.2	9.55	2	49.82%
2020-11-20 10:07AM EST	120.0	8.37	8.3	8.55	8	49.73%
2020-11-20 10:05AM EST	125.0	7.7	7.45	7.7	2	49.79%
2020-11-19 3:57PM EST	130.0	6.95	6.6	6.95	457	49.87%
2020-11-20 10:05AM EST	135.0	6.2	6.0	6.25	6	49.87%
2020-11-20 10:05AM EST	140.0	5.6	5.4	5.6	23	49.77%

Table 1: European Call Options AMD (data extracted from [2] at 20/11/2020 16:45:35)

LastTradeDate	Strike	LastPrice	Bid	Ask	Volume	ImpliedVolatility
2020-11-20 10:12AM EST	5.0	0.04	0.03	0.04	2	103.13%
2020-11-16 2:39PM EST	10.0	0.07	0.06	0.17	5	88.87%
2020-11-16 2:10PM EST	15.0	0.12	0.1	0.22	36	75.20%
2020-11-12 11:11AM EST	18.0	0.15	0.1	0.26	7	68.56%
2020-11-17 2:49PM EST	20.0	0.22	0.15	0.3	33	66.21%
2020-11-04 9:30AM EST	23.0	0.35	0.25	0.36	3	63.14%
2020-11-19 1:38PM EST	25.0	0.34	0.25	0.46	1	60.89%
2020-11-10 10:33AM EST	28.0	0.54	0.35	0.63	3	58.94%
2020-11-09 10:25AM EST	30.0	0.66	0.5	0.75	12	58.30%
2020-11-09 3:49PM EST	32.0	0.85	0.5	0.86	584	55.96%
2020-11-18 2:57PM EST	35.0	0.95	0.75	1.08	15	54.79%
2020-11-05 11:50AM EST	37.0	1.41	0.76	1.2	2	52.59%
2020-11-19 3:08PM EST	40.0	1.42	1.35	1.5	152	53.05%
2020-11-19 2:43PM EST	42.0	1.74	1.61	1.81	37	52.73%
2020-11-19 3:26PM EST	45.0	2.24	2.01	2.2	155	51.54%
2020-11-19 3:13PM EST	47.0	2.59	2.3	2.61	13	51.16%
2020-11-20 9:30AM EST	50.0	2.95	2.6	3.1	43	50.92%
2020-11-19 2:43PM EST	52.5	3.7	3.5	3.65	23	50.01%
2020-11-19 3:44PM EST	55.0	4.2	4.15	4.3	23	50.13%
2020-11-19 2:43PM EST	57.5	4.9	4.85	5.0	13	49.81%
2020-11-19 2:43PM EST	60.0	5.8	5.55	5.75	17	49.43%
2020-11-19 2:43PM EST	62.5	6.5	6.4	6.6	14	49.22%
2020-11-19 2:43PM EST	65.0	7.6	7.35	7.5	38	48.95%
2020-11-19 2:43PM EST	67.5	8.6	8.3	8.5	12	48.82%
2020-11-19 3:55PM EST	70.0	9.25	9.35	9.5	47	48.46%
2020-11-20 9:45AM EST	72.5	10.5	10.45	10.65	100	48.41%
2020-11-20 10:26AM EST	75.0	11.8	11.65	11.8	3	48.15%
2020-11-19 2:43PM EST	77.5	13.25	12.9	13.15	10	48.32%
2020-11-19 3:25PM EST	80.0	14.36	14.3	14.45	17	48.16%
2020-11-20 10:26AM EST	82.5	15.7	15.6	15.85	2	48.11%
2020-11-20 10:15AM EST	85.0	17.25	17.05	17.35	26	48.19%
2020-11-17 2:07PM EST	87.5	18.8	18.55	19.55	2	50.07%
2020-11-17 2:10PM EST	90.0	20.4	20.1	20.7	3	48.87%
2020-11-20 10:13AM EST	92.5	22.0	21.7	22.05	2	48.11%
2020-11-20 10:13AM EST	95.0	23.65	23.35	23.65	1	47.91%
2020-11-12 3:01PM EST	97.5	25.35	24.0	26.3	1	50.43%
2020-11-17 3:06PM EST	100.0	28.07	26.7	27.65	2	49.30%
2020-11-13 2:49PM EST	105.0	33.4	29.85	31.35	3	49.46%
2020-11-10 3:29PM EST	110.0	39.2	34.1	34.5	4	47.69%
2020-10-28 9:18AM EST	115.0	45.85	38.0	38.35	1	47.47%
2020-11-06 2:25PM EST	120.0	43.0	42.0	42.65	10	48.21%
2020-10-09 11:29AM EST	125.0	51.85	44.65	48.0	21	51.79%
2020-11-04 10:51AM EST	130.0	55.12	50.3	51.15	10	48.55%
2020-11-02 2:45PM EST	135.0	64.1	54.5	55.55	1	48.82%
2020-11-17 3:06PM EST	140.0	60.73	59.0	59.55	2	47.47%

Table 2: European Put Options AMD (data extracted from [2] at 20/11/2020 16:45:35)

2 Product 1: Partially principal protected note (PPPN)

2.1 Descriptive part: What is a PPPN and why is it interesting?

A partially principal protected note (PPPN) is an investment option over an underlying stock, in our case AMD. Many people are initially skeptical when it comes to investing in stocks. The reason for this is that the opportunities of profit come with risks. A PPPN is an investment option, where the risk is partially absorbed and additionally there can even be profit when the stock falls under a specific level. For instance imagine investing 850 USD in a PPPN with fixed maturity date T over an underlying stock with initial stock price $S_0 = 85$ USD.

If one would buy the stock directly this would mean buying ten stocks. If at maturity the stock is up to a value of 110 USD one would have a profit of $1100 \text{ USD} - 850 \text{ USD} = 250 \text{ USD}$. But in the case of $S_T = 60$ USD one would lose the same amount of money. The PPPN is preventing exactly that: One cannot lose more than 10% of the initial investment. But additionally there is profit if the stock rises above a certain strike and if the stock falls below another fixed strike. The following plot explains what exactly is meant for the example situation above ($S_0 = 85$ USD and an initial investment of 850 USD with strikes $K_1 = 60$ USD and $K_2 = 110$ USD):

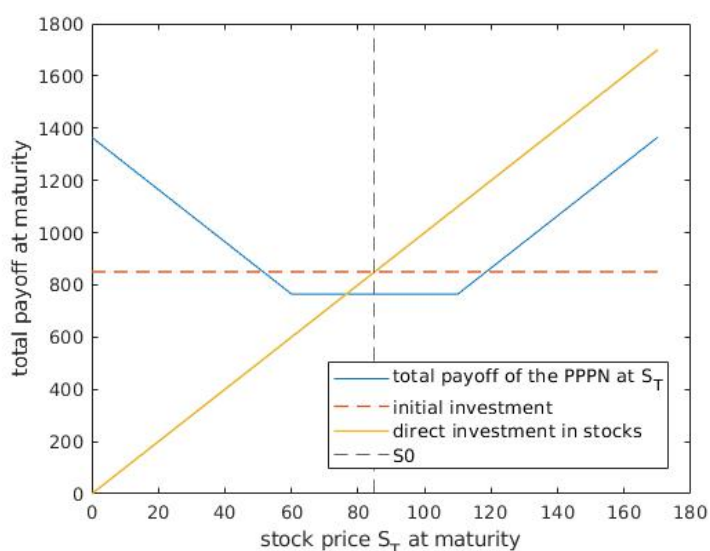


Figure 1: payoff for an PPPN

The yellow line represents the direct investment in the stock, where the blue line represents the total payoff of the PPPN. One can see that there is less risk but one has to pay with a smaller profit in the case of a rising stock and a possible loss of up to 10% if the stock stays close to S_0 . Therefore one can even get profit in the case that the stock goes down.

2.2 Concrete product

The concrete product which can be offered has the following specifications:

- Since the underlying stock is AMD, consider $S_0 = 85.35$ USD.
- The investment can be any multiple of $N = 85.35$ USD which is the stock price S_0 .
- The maturity date is 21st January 2022 so the duration is $T = 14$ months.
- As lower strike $K_1 = 40$ USD is picked.
- As higher strike $K_2 = 130$ USD is chosen.

Then the total payoff can be visualized in the same way as in the descriptive part:

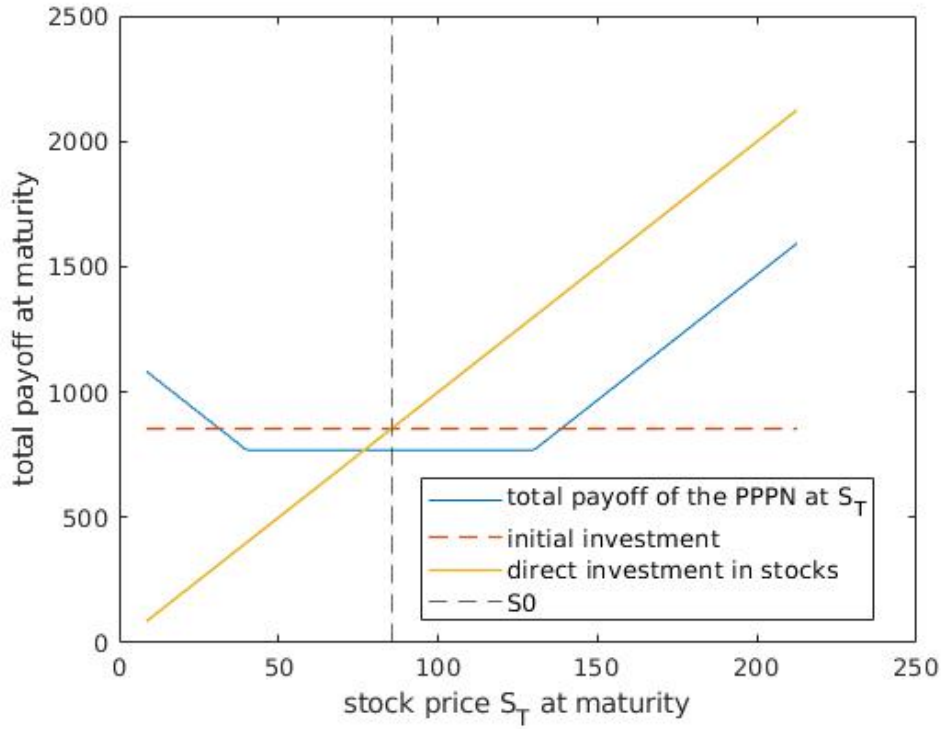


Figure 2: payoff for the concrete PPPN offer

If the stock goes up 100% till the maturity day one would end up with 117.52 USD for an investment of $N = 85.35$ USD . And no matter how the stock moves in the next 14 months one can never end up with less than 76.5 USD.

2.3 Technical Part: How does a PPPN work?

A PPPN is the combination of three different assets:

- A risk free asset with interest rate r .
- Call options on the underlying stock.
- Put options on the underlying stock.

First the part of the investment has to be determined, which has to be invested in a risk free way. Therefore 90% of the initial investment have to get discounted by the interest rate r over the time period T . In the present case consider an investment in the US Treasury Bond of

$$N_{\text{fixed}} = 0.9N \cdot \exp(-rT) = 0.9 \exp(-0.0011 \frac{14}{12})N = 0.89885N .$$

It remains to invest

$$N_{\text{options}} \leq N - N_{\text{fixed}} = 0.10115N .$$

The Bank is now buying $\frac{N}{S_0}$ European Put options for the considered maturity date with strike $K_1 = 40$ USD and the same amount of European Call options for the considered maturity date with strike $K_2 = 130$ USD. To look up the prices of these one has to look up the tables of relevant market data stated above. There can be found for the put option

LastTradeDate	Strike	LastPrice	Bid	Ask	Volume	ImpliedVolatility
2020-11-19 3:08PM EST	40.0	1.42	1.35	1.5	152	53.05%

and for the Call option

LastTradeDate	Strike	LastPrice	Bid	Ask	Volume	ImpliedVolatility
2020-11-19 3:57PM EST	130.0	6.95	6.6	6.95	457	49.87%

Since N is chosen as a multiple of the initial stock price S_0 , the ratio $a = \frac{N}{S_0}$ is a natural number, which is the amount of each call and put options the bank has to buy. With the ask prices of the options the bank can calculate how much money it need for doing so.

$$\begin{aligned} c_{\text{put,ask}} &= 1.5 \text{ USD} \approx 0.017575S_0 \\ c_{\text{cal,ask}} &= 6.95 \text{ USD} \approx 0.081429S_0 \\ N_{\text{options}} &= \frac{N}{S_0} 0.017575S_0 + \frac{N}{S_0} 0.081429S_0 \approx 0.099004N \end{aligned}$$

The remaining amount ($\approx 0.002146N$) is then a small profit for the bank for selling the option. Let us now have a look at the payoff of those three assets at maturity:

- The risk free asset will end up at $0.9 \cdot N$.
- Each of the a Put options generates a payoff at maturity of $\text{payoff}_{\text{put}} = (K_1 - S_T)^+$.
- Each of the a Call options generates a payoff at maturity of $\text{payoff}_{\text{call}} = (S_T - K_2)^+$.

Thus the total payoff is given by

$$\begin{aligned}\text{payoff}_{\text{total}} &= 0.9 \cdot N + a \cdot \text{payoff}_{\text{put}} + a \cdot \text{payoff}_{\text{call}} \\ &= 0.9N + a \cdot ((K_1 - S_T)^+ + (S_T - K_2)^+) \\ &= \begin{cases} 0.9N + \frac{N}{S_0}(K_1 - S_T) & \text{if } S_T < K_1 \\ 0.9N & \text{if } K_1 \leq S_T \leq K_2 \\ 0.9N + \frac{N}{S_0}(S_T - K_2) & \text{if } S_T > K_2 \end{cases}\end{aligned}$$

and the bank just delivers to the investor the payoff in every case. For an investment of

$$N = 10 \cdot S_0 = 853.5 \text{ USD}$$

this would give the bank a profit of around 1.84 USD by having nearly no risk following the common assumptions. Additional margin can be achieved if one is able to invest the risk free asset with a slightly higher interest rate or if its possible to get the desired options a little bit under the ask value.

3 Product 2: Airbag note (AN)

3.1 Descriptive part: What is a AN and why is it interesting?

Another structured product which tries to handle the risk while investing into the stock market is the so-called Airbag note (AN). The main difference to the PPPN is that the risk handling is done a little bit different. Imagine for example the stock AMD. For a PPPN we saw that the investor can only make profit from a falling stock in quite extreme scenarios. Since the company did quite well in the last time one would not expect the stock to fall below 80% of its value at time $t = 0$. How can now an investor get a downside protection only up to a certain level when investing in stocks, for instance those 80% of the current stock price. The answer is an Airbag note. If the stock falls but ends up above the so-called airbag level the investor is protected and just gets his initial investment back. If the stocks price S_T at maturity is below that level, then no complete downside protection is provided but the investor still gets a little bit more then when he would have invested in the stock directly. But that is anyway the case which the investor is almost sure that it won't happen. More precisely the total payoff at maturity is then

$$\text{payoff} = \frac{N}{lS_0} \cdot S_T$$

where lS_0 is the airbag level, N is again the initial investment, S_0 the initial stock price and S_T the stock price at maturity. Since $0 < l < 1$ (in our case $l = 0.8$) this is more than the payoff at maturity which a direct investment in the stock would give. In the case of the rising stock the investor is happy, because he gets a partial participation in the stock performance. This means that in this case the payoff at maturity is given by

$$\text{payoff} = N + p \cdot \frac{N}{S_0} (S_T - S_0)$$

with a fixed participation rate p . Of course the participation rate p will be lower than one, which means in that case the investor will get a little bit less as if he had invested directly in a stock - that is the trade off in reducing the risk. But in contrast to the PPPN there is no scenario where the stock goes up and the investor will not also make profit from the airbag note. As before we can plot the payoff in dependence of S_T :

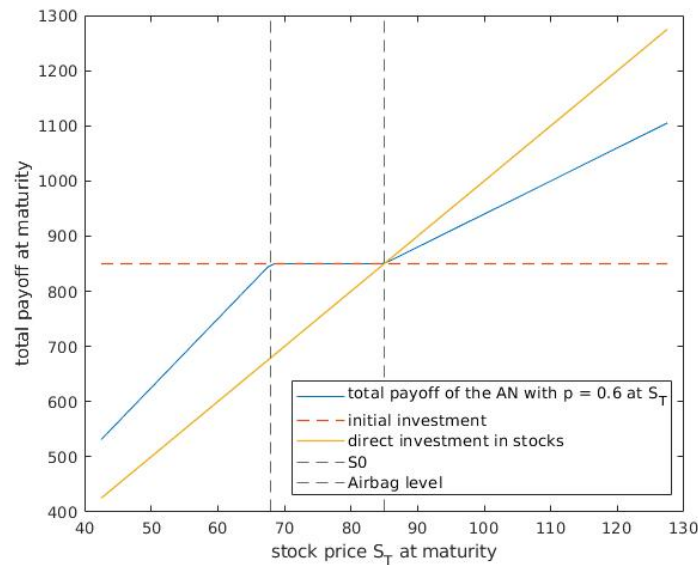


Figure 3: payoff an example AN with $p = 0.6$

Again the yellow line represents the direct investment in the underlying stock and the dashed verticle lines are representing the initial stock price S_0 and the airbag level $A = 0.8 \cdot S_0$.

3.2 Concrete Product

The concrete product which can be offered has the following specifications:

- The investment can be any multiple of $N = 1707$ USD. If the bank is capable of buying and selling partial options, for instance if it is expected, that the product is sold often and the bank can handle multiple customers together, also less artificial numbers for N are possible.
- Since the underlying stock is AMD, consider $S_0 = 85.35$ USD.
- The investment can be any multiple of $N = 85.35$ USD which is the stock price S_0 .
- The maturity date is 21st January 2022 so the duration $T = 14$ months.
- As Airbag level $A = l \cdot S_0 = 0.8 \cdot S_0 = 68.28$ USD is picked.
- The participation rate which can be offered is $p = 55\%$.

Then the total payoff can be visualized in the same way as in the descriptive part:

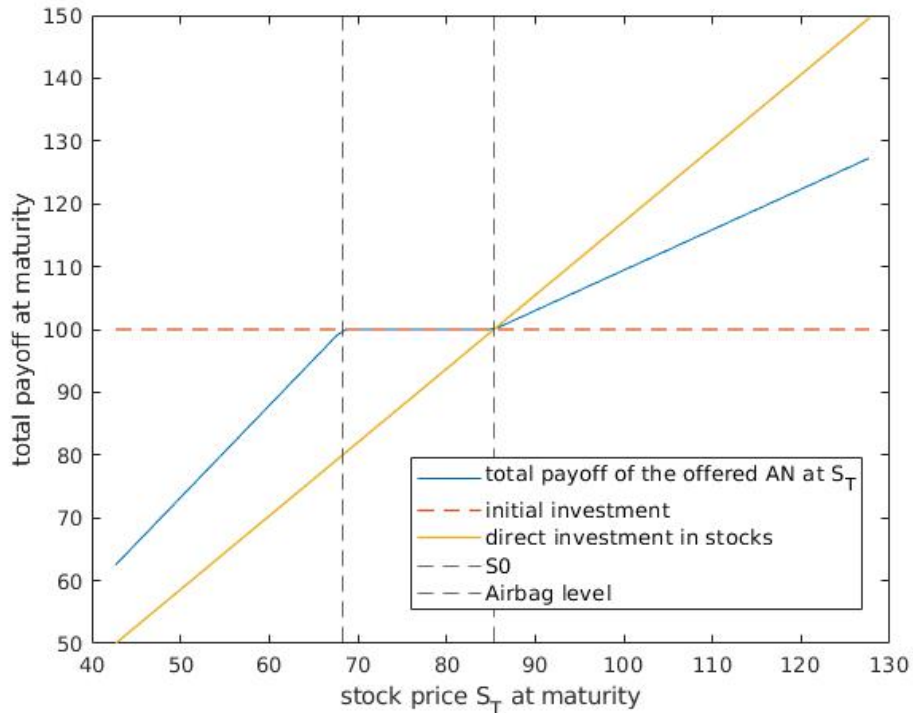


Figure 4: payoff for the concrete AN offer

If the stock goes up 100% till the maturity day one would end up with 2645.85 USD for an investment of $N = 1707$ USD . That is a gain of 55%. If the stock price ends up to be somewhere between S_0 and the airbag level (in our case $0.8 \cdot S_0$) then a possible investor just obtains his investment. Note, that below that airbag level no complete downside protection is guaranteed, but a possible investor would nevertheless end up higher than in the case of an direct investment in the underlying stock.

3.3 Technical Part: How does a AN work?

An Airbag Note is again the combination of the three assets, we already saw before:

- A risk free asset with interest rate r .
- Call options on the underlying stock.
- Put options on the underlying stock.

A big difference in contrast to the PPPN is that the Put options will not get bought by the bank. Quite the reverse! To design the AN the Put options have to be sold by the bank, in other words: the bank goes short on the put options. So how does an AN work?

Again first it has to be determined the part of the investment, which has to be invested in the risk free asset. Therefore we are now discounting the full initial investment by the interest rate r over the time period T . In our case consider again an investment in the US Treasury Bond of

$$N_{\text{fixed}} = N \cdot \exp(-rT) = \exp(-0.0011 \frac{14}{12})N$$

In the case of $N = 1707$ USD this is corresponding to 1704.81 USD. Now the bank goes short $\frac{N}{A}$ put options with strike A . Considering the chosen participation rate that is just enough to buy (together with the remaining few dollars which are not invested in the risk free asset) exactly $\frac{pN}{S_0}$ Call options with strike S_0 . N is chosen in way to obtain natural numbers for $\frac{N}{A}$ and $\frac{pN}{S_0}$. That is also the reason why the concrete number might look artificial. To look up the prices of these options we have to look up again the tables of relevant market data.

We can find for the put option

LastTradeDate	Strike	LastPrice	Bid	Ask	Volume	ImpliedVolatility
2020-11-19 2:43PM EST	67.5	8.6	8.3	8.5	12	48.82%

and for the Call option

LastTradeDate	Strike	LastPrice	Bid	Ask	Volume	ImpliedVolatility
2020-11-20 10:05AM EST	85.0	18.45	18.15	18.5	7	50.39%

Note that the strike of the put option is chosen a little bit lower than $S_0 * 0.8 = 68.28$ USD. This will make sure that the portfolio always ends up a little bit higher then the actual agreed payoff at maturity. In the same reasoning the strike of the Call is chosen just a little bit below S_0 . The reason why we have to consider those is that the strikes are not offered in a continuous way at the market, but in discrete steps. This is discussed more precisely in the discussion about the profit of the bank at the end of this subsection.

First check that the bank is capable of buying and selling the options without making loss: Since the bank goes short on put options and buys call options, the bid price of the put option and the ask price of the call option has to be considered. More precisely it is:

$$c_{\text{put,bid}} = 8.3 \text{ USD}$$

$$c_{\text{call,ask}} = 18.5 \text{ USD}$$

Thus it remains after selling the put options, getting the few remaining dollars which are not invested in the risk free asset and buying the call options:

$$\begin{aligned}
 N_{\text{remaining}} &= N - N_{\text{fixed}} + \frac{N}{A} c_{\text{put,bid}} - \frac{pN}{S_0} c_{\text{call,ask}} \\
 &= N - N_{\text{fixed}} + \frac{N}{lS_0} c_{\text{put,bid}} - \frac{pN}{S_0} c_{\text{call,ask}} \\
 &= 0.0036N
 \end{aligned}$$

The calculation is done with Matlab to reduce rounding errors and can be found in the attached scripts (AN.m). This remaining money is one part of the margin. After setting up the portfolio the bank is capable of delivering the agreed payoff at the maturity date in every scenario:

1. In the case the stock ends up between A and S_0 both the bought call options and the sold put options expire without any profit or loss. Then the bank just takes N from the risk free asset, which is now there since $\exp(-rT)N$ got invested there for the time period T .
2. If the stock ends up higher the bank has still N from the risk free asset and gets additionally $\frac{pN}{S_0}(S_T - K_{\text{call}})$ from the call options payoff at maturity.
3. If the stock falls below the airbag level A at maturity, the bank will use a part of the money from the risk free asset to payout the sold put options. The payoff for the customer will then be the remaining money which is $N - \frac{N}{A}(K_{\text{put}} - S_T)$.

We can write the total payoff in the following way, using that $K_{\text{put}} > A$ and $K_{\text{call}} < S_0$:

$$\begin{aligned}
 \text{payoff}_{\text{total}} &= N - \frac{N}{A}(K_{\text{put}} - S_T)^+ + \frac{pN}{S_0}(S_T - K_{\text{call}})^+ \\
 &\geq N - \frac{N}{A}(A - S_T)^+ + \frac{pN}{S_0}(S_T - S_0)^+ \\
 &= \begin{cases} N - \frac{N}{A}(A - S_T) & \text{if } S_T < A \\ N & \text{if } A \leq S_T \leq S_0 \\ N + \frac{pN}{S_0}(S_T - S_0) & \text{if } S_T > S_0 \end{cases} \\
 &= \begin{cases} \frac{N}{A}S_T & \text{if } S_T < A \\ N & \text{if } A \leq S_T \leq S_0 \\ N + \frac{pN}{S_0}(S_T - S_0) & \text{if } S_T > S_0 \end{cases}
 \end{aligned}$$

Thus the bank can deliver the agreed payoff in every scenario. Lastly lets have a look at the additional margin which the bank gets because of the choice of K_{put} and K_{call} . Therefore we can look at the difference of the payoffs:

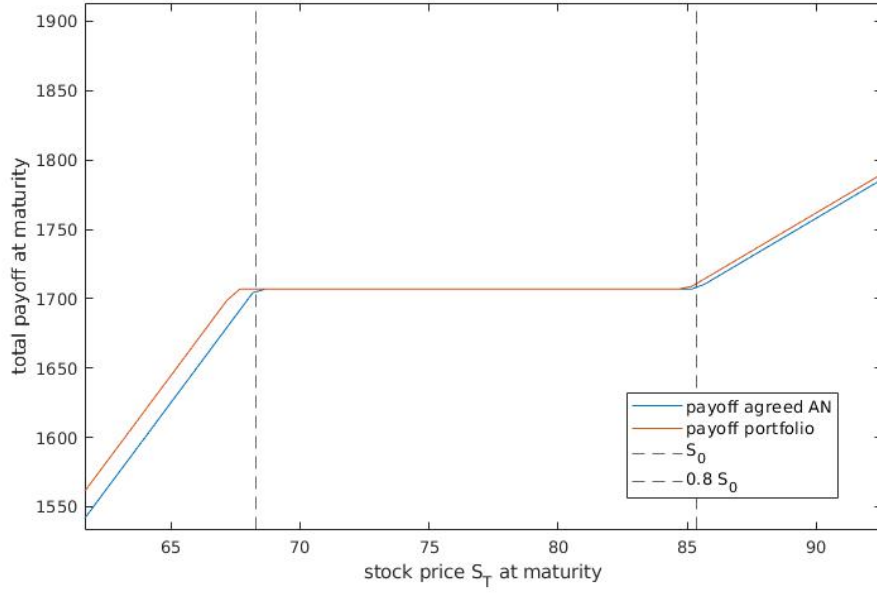


Figure 5: Difference of actual payoff and agreed payoff

There is nearly no difference in the case of $A \leq S_T \leq S_0$. Only at the borders there might be some differences, which are smaller than in the other two cases. Thus these differences are not discussed in a precise way, but we can give an interval, where the margin will end up in.

If the stock is higher than S_0 the difference is from a certain point on equal to $0.0023N$. If the stock is lower than K_{put} the difference is $0.0114N$. Together with the margin obtained at time $t = 0$ the following margin can be derived:

$$\begin{cases} \text{margin} = 0.0036N + 0.0114N & \text{if } S_T \leq K_{\text{put}} \\ 0.0036N \leq \text{margin} \leq 0.0036N + 0.0114N & \text{if } K_{\text{put}} < S_T < A \\ \text{margin} = 0.0036N & \text{if } A \leq S_T \leq K_{\text{call}} \\ 0.0036N \leq \text{margin} \leq 0.0036N + 0.0023N & \text{if } K_{\text{call}} < S_T < S_0 \\ \text{margin} = 0.0036N + 0.0114N & \text{if } S_T \geq S_0 \end{cases}$$

In the concrete case of an investment of $N = 1707$ USD this is corresponding to:

$$\begin{cases} \text{margin} = 25.6 \text{ USD} & \text{if } S_T \leq K_{\text{put}} \\ 6.15 \text{ USD} \leq \text{margin} \leq 25.6 \text{ USD} & \text{if } K_{\text{put}} < S_T < A \\ \text{margin} = 6.15 \text{ USD} & \text{if } A \leq S_T \leq K_{\text{call}} \\ 6.15 \text{ USD} \leq \text{margin} \leq 10.07 \text{ USD} & \text{if } K_{\text{call}} < S_T < S_0 \\ \text{margin} = 10.07 \text{ USD} & \text{if } S_T \geq S_0 \end{cases}$$

Additional margin can again be achieved if the bank is able to invest the risk free asset with a slightly higher interest rate, if it is possible to get the desired call options a little bit under the ask value or if the put options can be sold a little bit higher the bid value.

4 Appendix

4.1 Screenshot of the market data

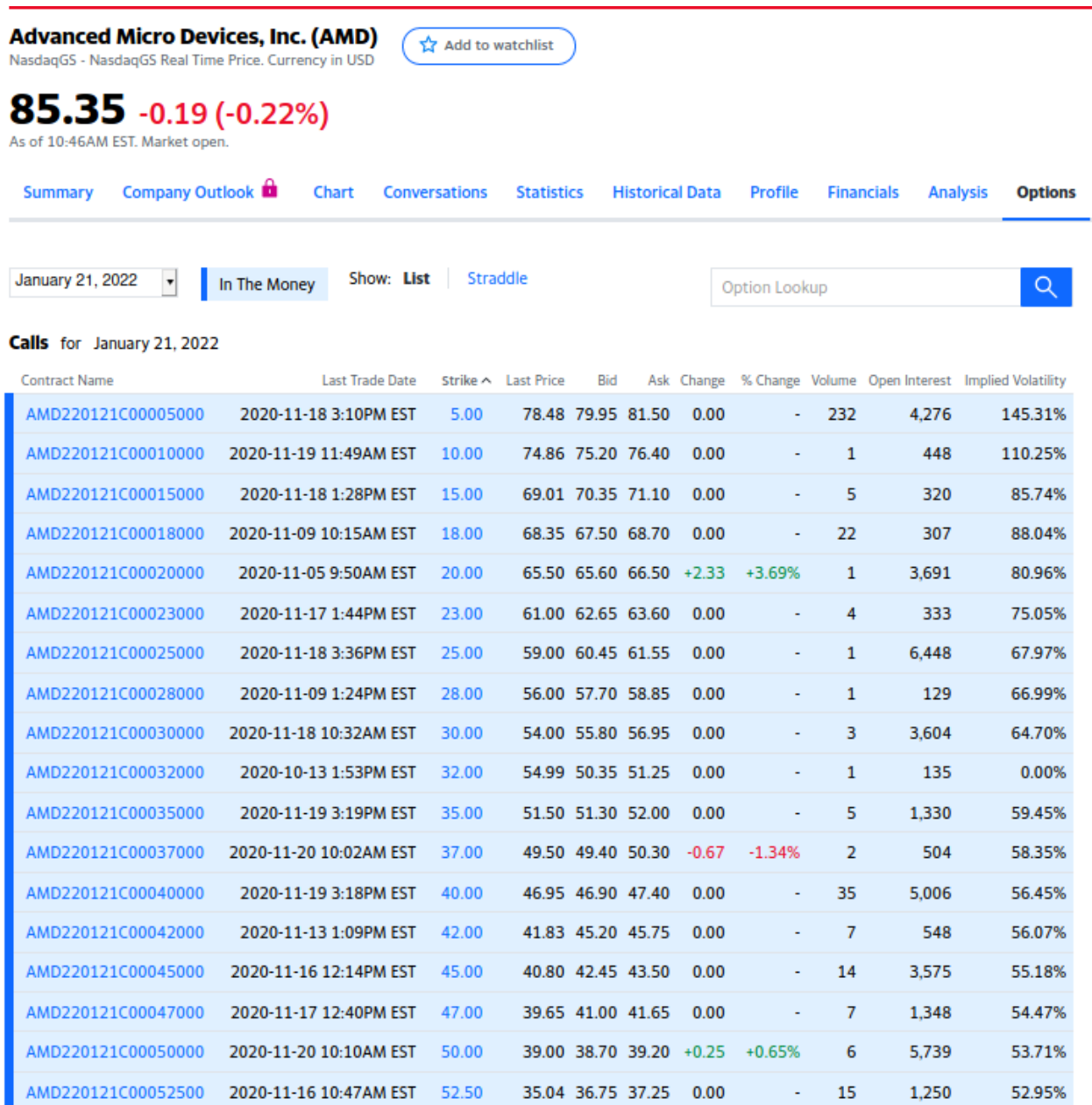


Figure 6: Small extract of the relevant market data from [2]

The data got extracted directly from the website with a small python script which is also included in the attached .zip folder.

4.2 Daily Treasury Yield Curve Rates

Date	1 Mo	2 Mo	3 Mo	6 Mo	1 Yr	2 Yr	3 Yr	5 Yr	7 Yr	10 Yr	20 Yr	30 Yr
11/02/20	0.09	0.09	0.09	0.11	0.13	0.16	0.20	0.38	0.63	0.87	1.41	1.63
11/03/20	0.09	0.10	0.10	0.12	0.14	0.17	0.21	0.39	0.65	0.90	1.44	1.66
11/04/20	0.08	0.09	0.10	0.10	0.12	0.14	0.18	0.33	0.55	0.78	1.33	1.55
11/05/20	0.09	0.10	0.10	0.10	0.12	0.14	0.18	0.33	0.56	0.79	1.32	1.54
11/06/20	0.10	0.10	0.10	0.11	0.12	0.16	0.21	0.36	0.59	0.83	1.37	1.60
11/09/20	0.10	0.10	0.11	0.11	0.12	0.17	0.25	0.44	0.70	0.96	1.51	1.73
11/10/20	0.09	0.09	0.10	0.11	0.12	0.19	0.26	0.46	0.72	0.98	1.53	1.75
11/12/20	0.10	0.11	0.10	0.10	0.13	0.17	0.23	0.40	0.64	0.88	1.42	1.64
11/13/20	0.10	0.09	0.09	0.10	0.12	0.17	0.23	0.41	0.65	0.89	1.43	1.65
11/16/20	0.09	0.10	0.09	0.12	0.12	0.19	0.24	0.41	0.66	0.91	1.44	1.66
11/17/20	0.08	0.08	0.09	0.10	0.12	0.18	0.22	0.39	0.63	0.87	1.40	1.62
11/18/20	0.07	0.09	0.09	0.10	0.11	0.16	0.22	0.40	0.64	0.88	1.42	1.62
11/19/20	0.08	0.08	0.07	0.10	0.11	0.18	0.22	0.39	0.63	0.86	1.38	1.58
11/20/20	0.09	0.09	0.07	0.10	0.11	0.16	0.21	0.38	0.62	0.83	1.33	1.53

Figure 7: Daily Treasury Yield Curve Rates from [1]

References

- [1] Daily treasury yield curve rates - the united states government. <https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield>. Accessed: 2020-11-20 - 16:45:35.
- [2] yahoo! finance - advanced micro devices, inc.(amd) - options. <https://finance.yahoo.com/quote/AMD/options?p=AMD&date=1642723200>. Accessed: 2020-11-20 - 16:45:35.