PH419: Physics of Biological Systems
Assignment 3

1. **Nernst relation and electrophoretic flux (Concept: Advection-Diffusion)**

   Many of the molecules floating in water carry net electric charge. For example, when ordinary salt dissolves, the individual sodium and chlorine atoms separate, resulting in negative chloride ions and positive sodium ions. Any electric field E present in the solution will then exert forces on the individual ions.

   Suppose we have a uniform density solution of charged particles, each of charge q, in a region with electric field E.

   (a) What is the drift velocity of a charged particle, given that ξ is the viscous friction coefficient?

   (b) What is the electrophoretic flux of ions induced by the field E? Assume that the constant concentration in this case is denoted by c.

   Now suppose that the density of ions is *not* uniform, $c = c(x)$. The concentration is assumed to vary only along the direction of the electric field, for simplicity.

   (c) Using Fick's Law and Einstein relation, write down the expression of for the *driven* electrophoretic flux in this case.

   (d) What electric field, and hence potential difference would be needed to get zero net flux, canceling out the diffusive tendency to erase nonuniformity? This is the condition for equilibrium. Assume a planar geometry, with the concentrations at the top and bottom surfaces given by c_{top} and c_{bot}. The separation between the top and bottom surfaces is d.

   (e) Consider a singly charged ion like Na^+, such that $q = e$. Assume the concentration jump is given by $c_{bot}/c_{top} = 10$. Working at room temperature, calculate the potential difference that is required to maintain this concentration difference at equilibrium. This gives an estimate of the potential differences across the membranes of living cells.

2. **Chemoreceptor clustering (Concept: Diffusion and signaling)**

   There is strong evidence that chemoreceptors in E. coli tend to cluster near one pole as shown in the figure (Kentner and Sourjik (2006)). One hypothesis about the role of such clustering is that it might increase the ability of a bacterium to better detect molecules in its environment. Determine if this is the most efficient strategy for counting (absorbing) molecules of chemoattractant. Approximate E. coli as a sphere $a = 1\mu m$ in radius and neglect its motion. Then compare the diffusive current to $N = 1000$ receptors (absorbing patches of radius $s = 1nm$)

scattered over the surface of the cell with the diffusive current to the same receptors incorporated into a single patch with the same total area. Make use of the result that the diffusive current onto a sphere of radius a with N absorbing patches of radius s spread uniformly over its surface is

$$I = \frac{4\pi D a c_\infty}{1 + \frac{\pi a}{N s}}$$

where D is the diffusion constant of the molecules, while c_∞ is their concentration far from the cell.

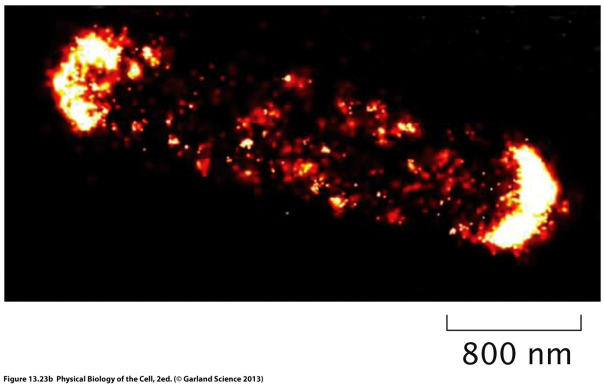

Figure 1: Clustering of *E. Coli* chemotaxis receptors at the poles, Greenfield *et. al.* PLoS Biol. 2009

3. **Mean number of visits for a reflecting sphere (Concept: Capture processes (Simulation))**

   Write a code to simulate a diffusion process in three dimensions. Assume that the diffusion occurs in the region between two concentric spheres of radius a and c, with $c > a$. A random walker starts at radius b, with $a < b < c$, and performs a random walk in three dimensions with constant step length l $(= a/100)$. The spherical boundary at $r = a$ is reflecting, such that a walk reverses direction when it reaches $r = a$. The spherical boundary at $r = c$ is absorbing, so a given walk terminates when it reaches $r = c$. By considering an ensemble of such walks, determine the average number of times $\langle n \rangle$ the random walker visits the reflecting sphere at $r = a$ before being absorbed at $r = c$. Plot this $\langle n \rangle$ as a function of the starting radius b. How does this compare to the mean field estimate obtained in class ($\langle n \rangle = \frac{a(c-b)}{c(b-a)}$)? (Assume a=1 and c=10 for the simulation.)

4. **Mean capture times in 3D**

Consider the three dimensional capture problems studied in class,

(i) Capture between two absorbing spheres at $r = a$ and $r = c$ for a diffusing molecule starting at some radius $a < r < c$.

(ii) Capture at the absorbing sphere for a diffusing molecule between an absorbing sphere at $r = a$ and a reflecting sphere at $r = c$. The molecule starts at some radius $a < r < c$.

Determine the expression for the capture times $W(r)$ as a function of the starting position in the two cases.

In the first case (two absorbing boundaries), at which initial radius is the mean capture time maximal? How does this compare to the results for the capture time in the one-dimensional diffusion process?