

First set the separation between the wells equal to 0 by setting $a=-1$, $b=c=0$ and $d=1$ with $V_0=-10$. Do the values you observe for the first 2 eigenvalues correspond to what you obtained studying the finite well?

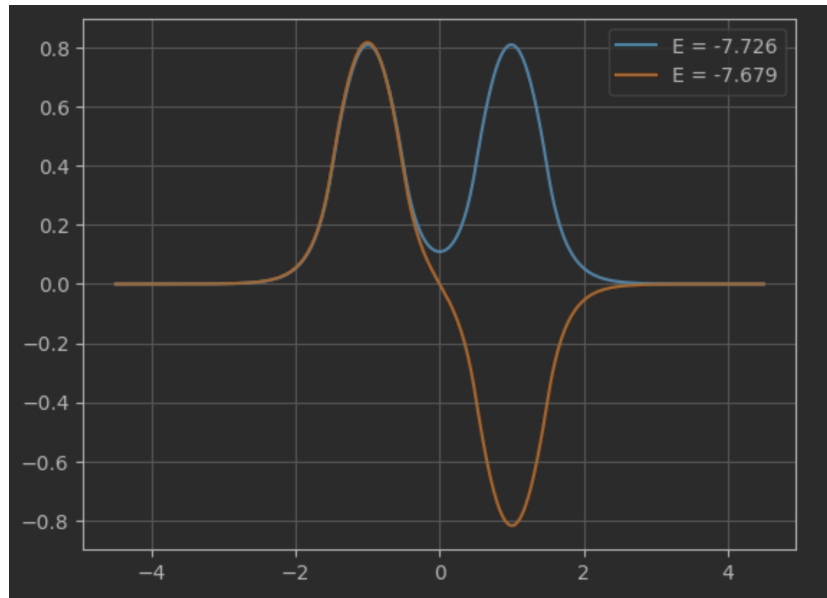
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-9.1792636  
-6.7752639  
-3.0467048  
0.5384127  
1.2251747  
2.2165245  
4.3354712  
5.6710366  
8.3572009  
10.8037609
```

Referring to the Jupyter notebook from Assignment 2, the first two energies are -9.1802599 and -6.7790593 , which agree roughly with the first two energies here.

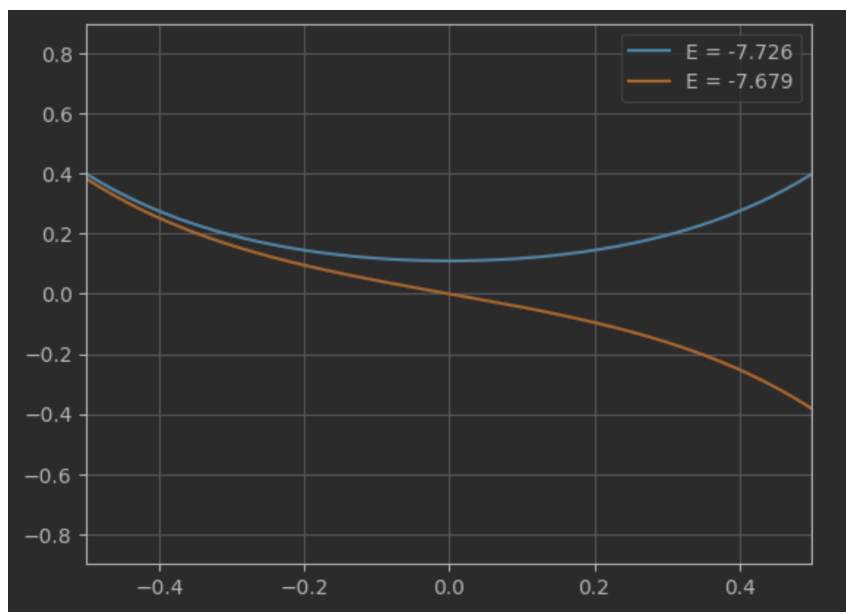
The energy levels are calculated using different methods, so it's not surprising that they are slightly different

Next consider the intermediate case where the width of the two wells is comparable to their separation. Set $a=-1.5$, $b=-0.5$, $c=0.5$ and $d=1.5$ with $V_0=-10$. What is the functional form of the wavefunction in the interval -0.5 to 0.5 for the ground state and the first excited state?

The ground state and first excited state:

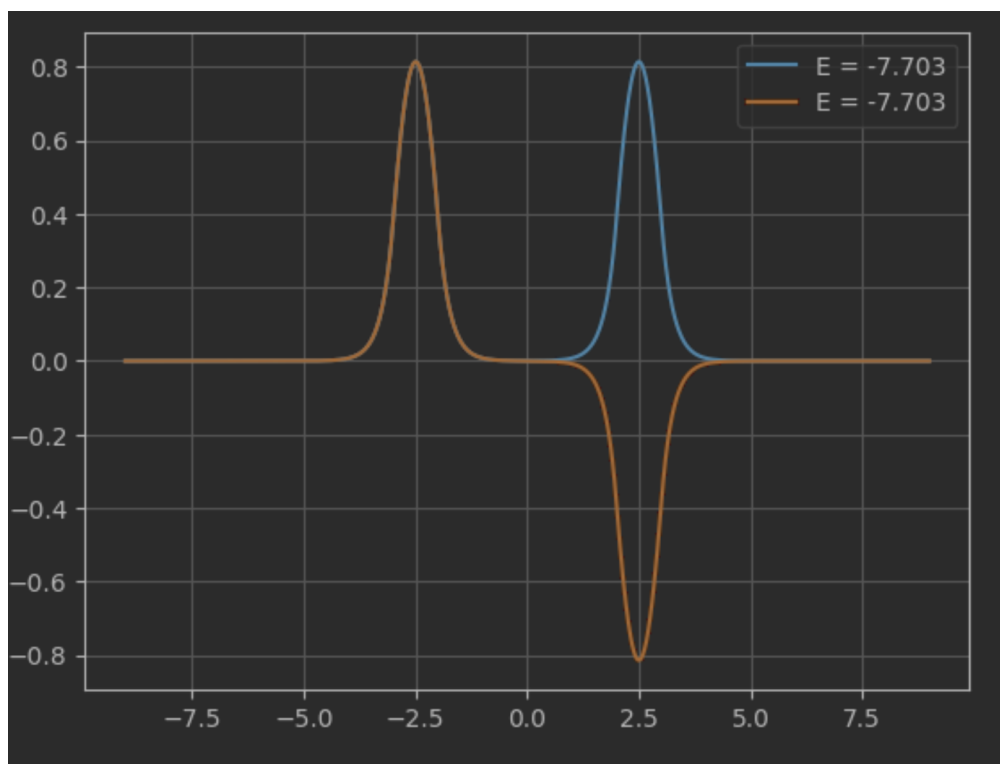


On the interval $-\frac{1}{2} < x < \frac{1}{2}$, the ground state resembles a parabola, while the first excited state is reminiscent of a cubic function.

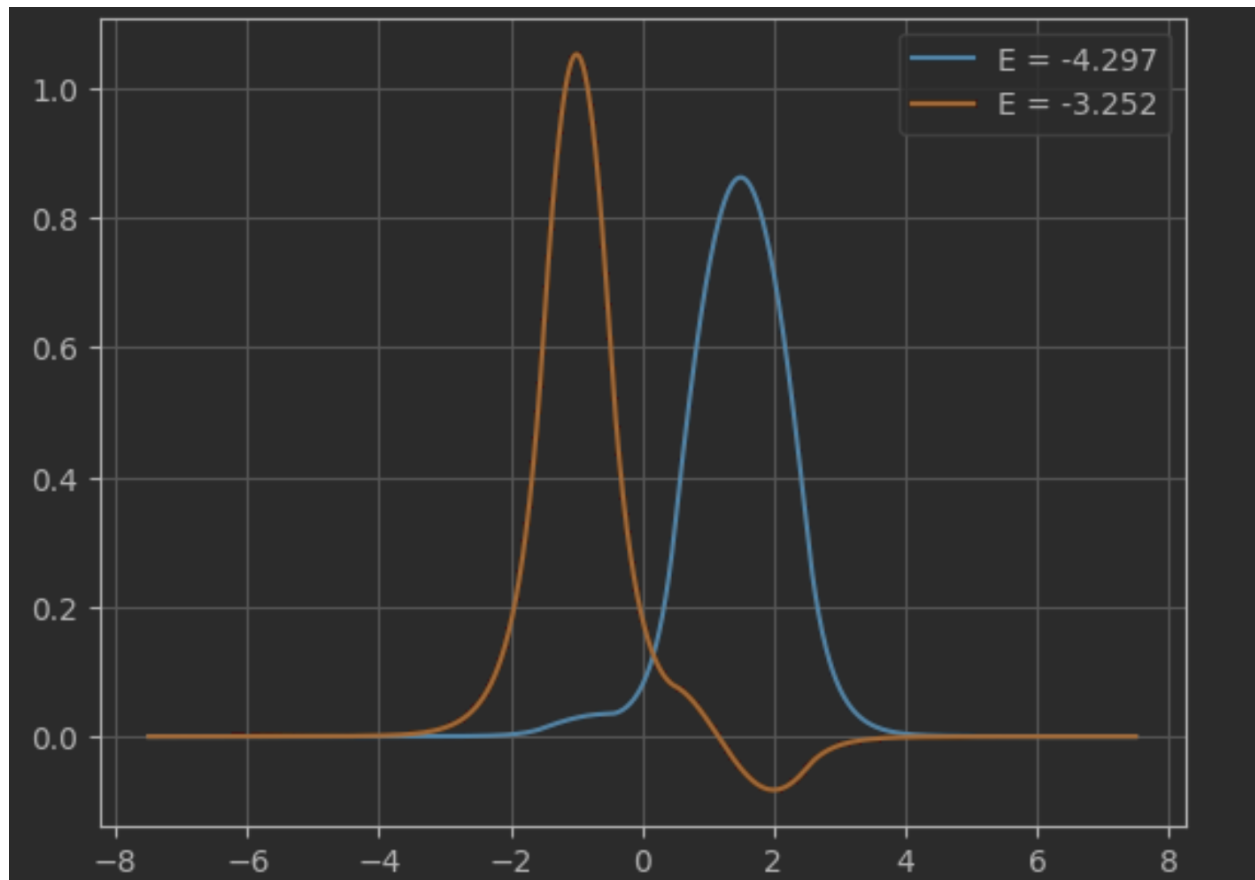


Next let us analyze the case where there is a significant distance between the two wells by setting $a=-3$, $b=-2$, $c=2$ and $d=3$ with $V_0=-10$. What do you observe when you plot the first 2 wave-functions? Explain.

The first two wave states are degenerate. This is because that wavefunctions are essentially the same, except in the first excited state, the negative x region has been flipped to make an odd function. The wavefunctions come in pairs, odd and even, that represent the same state.



Now let's try something new! Modify the potential so that the two wells have different widths. Choose $a=-1.5$, $b=-0.5$, $c=0.5$ and $d=2.5$ but this time with $V_0=-5$. Where is the wave-function mostly likely to be located for the ground state and the first excited state? Explain.



The particle is most likely to be found in the larger well for the ground state. For the second excited state, the particle is more likely to be found in the smaller well.

To explain this, consider the energy levels permitted by an infinite well:

$$E_n = n^2 \pi^2 \hbar^2 / 2ma^2$$

The energy level E_n is inversely proportional to the width of the well. When the well is smaller, the particle has higher energy. Therefore, it makes sense that the first excited state is more confined to a smaller well because this would imply the state represented by the wave function has a higher energy.