Likelihood: examples with real data

Let us consider an $\vec{x} = (2, 2.5, 3)$

Let us say we want to compute the likelihood of $\vec{X}$ if we assume in one case

(a) $\mu = 2$, $\sigma^2 = 1$

(b) $\mu = 4$, $\sigma^2 = 2$

$$\mathcal{L}(\mu = 2, \sigma^2 \mid \vec{x}) \equiv P(\vec{X} \mid \mu, \sigma)$$

(a) $\Rightarrow$ $f(x=2) \times f(x=2.5) \times f(x=3)$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi 1}} e^{-\frac{(x_i - 2)^2}{2 \times 1}}$$

$$= \frac{1}{\sqrt{2\pi 1}} e^{\frac{(2-2)^2}{2}} \times \frac{1}{\sqrt{2\pi 1}} e^{\frac{(2.5-2)^2}{2 \cdot 1}} \times \frac{1}{\sqrt{2\pi 1}} e^{\frac{(3-2)^2}{2 \cdot 1}}$$

$$\Rightarrow L(2, 1 \mid x = 2, 2.5, 3) = 0.39 \times 0.35 \times 0.24$$

$$= 0.0376$$

Likelihood when model is $N(4,2)$:

$$= \frac{1}{\sqrt{2\pi 2}} e^{-\frac{(2-4)^2}{2 \cdot 2}} \times \frac{1}{\sqrt{2\pi 2}} e^{\frac{(+2.5-4)^2}{2 \times 2}} \times \frac{1}{\sqrt{2\pi 2}} e^{\frac{(3-4)^2}{2 \times 2}}$$

$$= \boxed{0.00336}$$

2 Questions for audience:

Can you tell the optimal value of μ & σ^2 using these datasets?

$$\frac{\partial(\ln(L(\mu, \sigma^2 \mid x)))}{\partial \mu} = 0$$

$$\frac{\partial}{\partial \sigma^2} \left(\ln \left(L \left(\mu \mid \sigma^2 \mid \vec{x} \right) \right) = 0$$