

Maximum likelihood estimate  
Normal distribution,

$$\begin{array}{cc} \theta_1 & \& \theta_2 \\ \uparrow & & \downarrow \\ \mu & & \sigma^2 \end{array}$$

$x_1, x_2, x_3, \dots, x_n$  as  $n$  iids

$$f(\vec{x} | \vec{\theta}) = \prod_{i=1}^n f(x_i | \theta_1, \theta_2)$$
$$L(\theta) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

$$\log L(\theta) = \sum_{i=1}^n \log \left( \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}} \right)$$

$$= \sum_{i=1}^n \left[ -\log \left( \sqrt{2\pi\theta_2} \right) - \frac{1}{2\theta_2} (x_i - \theta_1)^2 \right] \quad \text{--- (1)}$$

$$\left[ \because \log a^x = x \log a \right] \quad \text{--- (2)}$$

$$\left[ \log_e e = 1 \right] \quad \text{--- (3)}$$

$$LL(\theta) = -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^n (x_i - \theta_1)^2$$

$$= -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^n [x_i^2 + \theta_1^2 - 2x_i\theta_1]$$

$$= -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2\theta_2} \left[ \sum_{i=1}^n x_i^2 + n\theta_1^2 - 2\theta_1 \sum_{i=1}^n x_i \right]$$

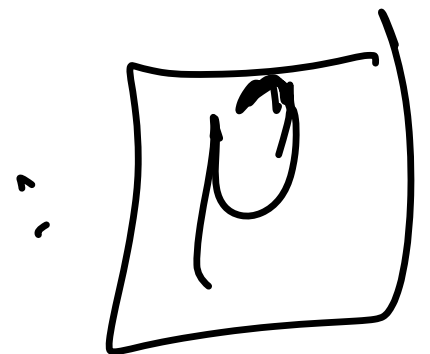
$$\frac{\partial LL(\theta)}{\partial \theta_1} = 0 \quad \left| \quad \frac{\partial LL(\theta)}{\partial \theta_2} = 0 \right.$$

$$\frac{\partial L L(\theta)}{\partial \theta_1} = 0$$

$$0 + \cancel{\frac{\sum \theta_1}{\sum \theta_2}} - \cancel{\frac{\sum_{i=1}^n x_i}{\sum \theta_2}} = 0$$

$$\text{if } \theta_2 \neq 0$$

$$\boxed{\theta_1 = \frac{\sum_{i=1}^n x_i}{n}}$$



$$\frac{\partial L L(\theta)}{\partial \theta_2} = 0$$

$$-\frac{n}{\theta_2} + \frac{1}{\theta_1^3} \sum_{i=1}^n (x_i - \theta_1)^2 = 0$$

$$\hat{\theta}_{2MLE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \theta_1)^2}$$

$$\hat{\theta}_{2MLE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2}$$