Maximum likelihood estimate
Normal distribution

$\underline{\theta_1}$ & $\underline{\theta_2}$

$\uparrow$ $\qquad$ $\downarrow$

μ $\qquad$ σ^2

$x_1, x_2, x_3, \ldots, x_n$ as n iids

$$f(\vec{X} \mid \vec{\theta}) = \prod_{i=1}^{n} f(x_i \mid \theta_1, \theta_2)$$

$$L(\theta) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

$$\log L(\theta) = \sum_{i=1}^{n} \log\left(\frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}\right)$$

$$\log \frac{1}{x} = -\log(x) \longrightarrow (1)$$

$$= \sum_{i=1}^{n} \left[-\log\left(\sqrt{2\pi\theta_2}\right) - \frac{1}{2\theta_2}(x_i - \theta_1)^2 \right]$$

$$\left[\because \; \log a^x = x \log a \; \text{—(2)}, \quad \log_e e = 1 \; \text{—(3)} \right]$$

$$LL(\theta) = -\frac{n}{2}\log(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i - \theta_1)^2$$

$$= -\frac{n}{2}\log(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}\left[x_i^2 + \theta_1^2 - 2x_i\theta_1\right]$$

$$= -\frac{n}{2}\log(2\pi\theta_2) - \frac{1}{2\theta_2}\left[\sum_{i=1}^{n} x_i^2 + n\theta_1^2 - 2\theta_1\sum_{i=1}^{n} x_i\right]$$

$$\frac{\partial LL(\theta)}{\partial \theta_1} = 0 \quad \Bigg| \quad \frac{\partial LL(\theta)}{\partial \theta_2} = 0$$

$$\frac{\partial LL(\theta)}{\partial \theta_1} = 0$$

$$0 + \frac{\cancel{2} n \theta_1}{\cancel{2} \theta_2} - \frac{\cancel{2} \sum_{i=1}^{n} x_i}{\cancel{2} \theta_2} = 0$$

if $\theta_2 \neq 0$

$$\boxed{\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n}} \therefore \boxed{\hat{\mu}}$$

$$\frac{\partial LL(\theta)}{\partial \theta_2} = 0$$

$$-\frac{n}{\theta_2} + \frac{1}{\theta_2^3} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

$$\hat{\theta}_{2\,MEE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2}$$

$$\hat{\theta}_{2\,MLE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2}$$