Maximum Likelihood Estimate

Examples

MLE choose or estimate

$\theta \quad : \quad \hat{\theta} = \underset{\theta}{\operatorname{argmax}}\ p(\vec{X} \mid \vec{\theta})$

$= \underset{\theta}{\operatorname{argmax}}\ \mathcal{L}(\theta)$

$$\left.\begin{array}{l} \ln\left[p(\vec{X} \mid \vec{\theta})\right] \\ : \ \ln\left[\mathcal{L}(\theta)\right] \end{array}\right\} \text{mathematical convenience}$$

$\vec{X} = [\,X_1, X_2, X_3, \ldots X_n\,]$

iids

$$L(\theta) = p(\vec{X} \mid \vec{\theta}) = \prod_{i=1}^{n} p(X_i \mid \vec{\theta})$$

$$\log_e(L(\theta)] = \sum_{i=1}^{n} \ln p(X_i \mid \vec{\theta})$$

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\ln\left[p(x_1|\vec{\theta}) \times p(x_2|\vec{\theta}) \dots p(x_n|\vec{\theta})\right]$$
$$= \ln p(x_1|\vec{\theta}) + \ln p(x_2|\vec{\theta}) + \dots + \ln p(x_n|\vec{\theta})$$

$\Rightarrow$

$$f(x|\theta) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

2 simple examples

discrete

Bernoulli dis^n

continuous

(normal)

$$\begin{Bmatrix} 1 & : & p \\ 0 & : & 1-p \end{Bmatrix}$$

$H \equiv 1 \quad : \quad p = 1/2$

$T \equiv 0 \quad : \quad 1-p = 1/2$

$\boxed{p}$: parameter

$[x_1, x_2, x_3, \ldots, x_n]$

$x_i \in 0$ or 1

✓ $1-p : x_i = 0$ if zero precipitation

$p : x_i = 1$ if non-zero precipitation

$$f(X=x) = p^x (1-p)^{1-x}$$

$$f(X=0) = p^0 (1-p)^{1-0} = (1-p)$$

$$f(X=1) = p$$

$$f(x_1, x_2, x_3, \ldots x_n) = \prod_{i=1}^{n} p^{x_i} (1-p)^{1-x_i}$$

$$f(\vec{X} \mid \theta = p]$$

$$p = ??$$

$$\mathcal{L}(\theta) = \prod_{i=1}^{n} p^{x_i} (1-p)^{1-x_i}$$

$$\ln[\mathcal{L}(\theta)] = \sum_{i=1}^{n} \log\left[p^{x_i} (1-p)^{1-x_i}\right]$$

$$= \sum_{i=1}^{n} x_i \log p + \sum_{i=1}^{n} (1-x_i) \log(1-p)]$$

$$\boxed{\sum_{i=1}^{n} x_i = Y}$$

$$\log \mathcal{L}(\theta) = Y \log p + [n - Y \log(1-p)]$$

$$\frac{\partial LL(\theta)}{\partial \theta} = 0$$

$$\frac{\partial LL(p)}{\partial p} = 0$$

$$\boxed{\hat{p} = \frac{Y}{n} = \frac{\sum_{i=1}^{n} x_i}{n}}$$

$$\vec{X} = [1, 0, 1, 1, 0] \to 3$$

$$P(\text{Rainfall}) = \frac{3}{5}$$

$$\hat{p} = \frac{\sum_{i=1}^{n} x_i}{(5 = n)}$$