Maximum likelihood estimate

(Paramater estimation)

1. Normal
2. uniform
3. Poisson
4. Binomial

} Random variable

Normal : mean, variance

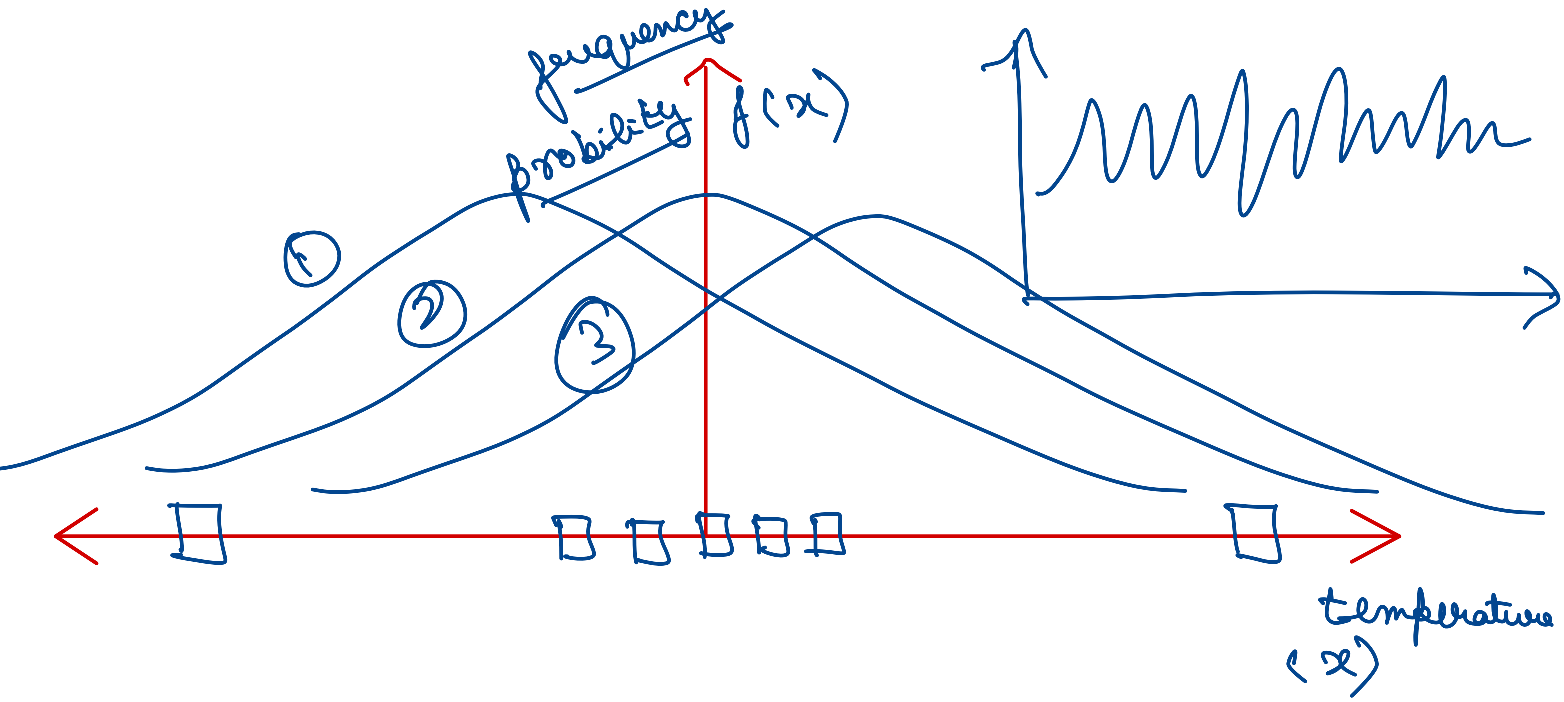
$$(\mu, \quad \sigma^2)$$

parameters

Bernoulli : 1 or 0

$$p \qquad 1-p$$

univariate parametric distr.



$$Y = m X + b$$

$\vec{\theta} = (m, b)$ Linear regression

$\vec{\theta} = (\mu, \sigma)$ Normal dis^n

$\vec{\theta} = (a, b)$ uniform

$\vec{\theta} = \lambda$ Poisson

1. Maximum Likelihood estimate
2. Maximum A Posteriori (MAP)

$\{x_1, x_2, x_3, \ldots, x_n\}$: Temperature : iids

identical

independent

Baye's Rule:

$$P(A, B) = P(A \mid B) \times P(B) \quad \longrightarrow (1)$$

$$= P(B \mid A) \times P(A) \quad \longrightarrow (2)$$

$$P(A \mid B) \times P(B) = P(B \mid A) \times P(A)$$

$$\Rightarrow \boxed{P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}} \quad \longrightarrow (3)$$

"Baye's rule"

$$P(\vec{\theta} \mid \vec{x}) = \frac{P(\vec{x} \mid \vec{\theta}) \times P(\vec{\theta})}{P(\vec{x})} \quad \longrightarrow (4)$$

Posteriori likelihood evidence prior

"Baye's rule"

$$P(\vec{\theta} \mid \vec{X}) = \frac{P(\vec{X} \mid \vec{\theta}) \times P(\vec{\theta})}{P(\vec{X})}$$ (4)

Posteriori — likelihood — evidence — prior

Intuition : likelihood

⇒ $x_1, x_2, x_3, \ldots x_n$: independent & identically distributed.

⇒ Either all of them have same pdf if X is continuous or pmf if X: discrete.

⇒ **Independence:**

$$f(\vec{X} \mid \theta) = f(\vec{X}) = f(X_1 = x_1, X_2 = x_2, \ldots X_n = x_n)$$

$$= p(X = x_1) \cdot p(X = x_2) \cdot p(X = x_3) \cdot p(X = x_4) \ldots p(X = x_n)$$

$$f(\vec{X}) = \prod_{i=1}^{n} p(X = x_i)$$

$$= \prod_{i=1}^{n} f(X = x_i)$$

⇒ Temperature : follows normal distribⁿ

(1) $\mathcal{N}(\mu, \sigma^2)$

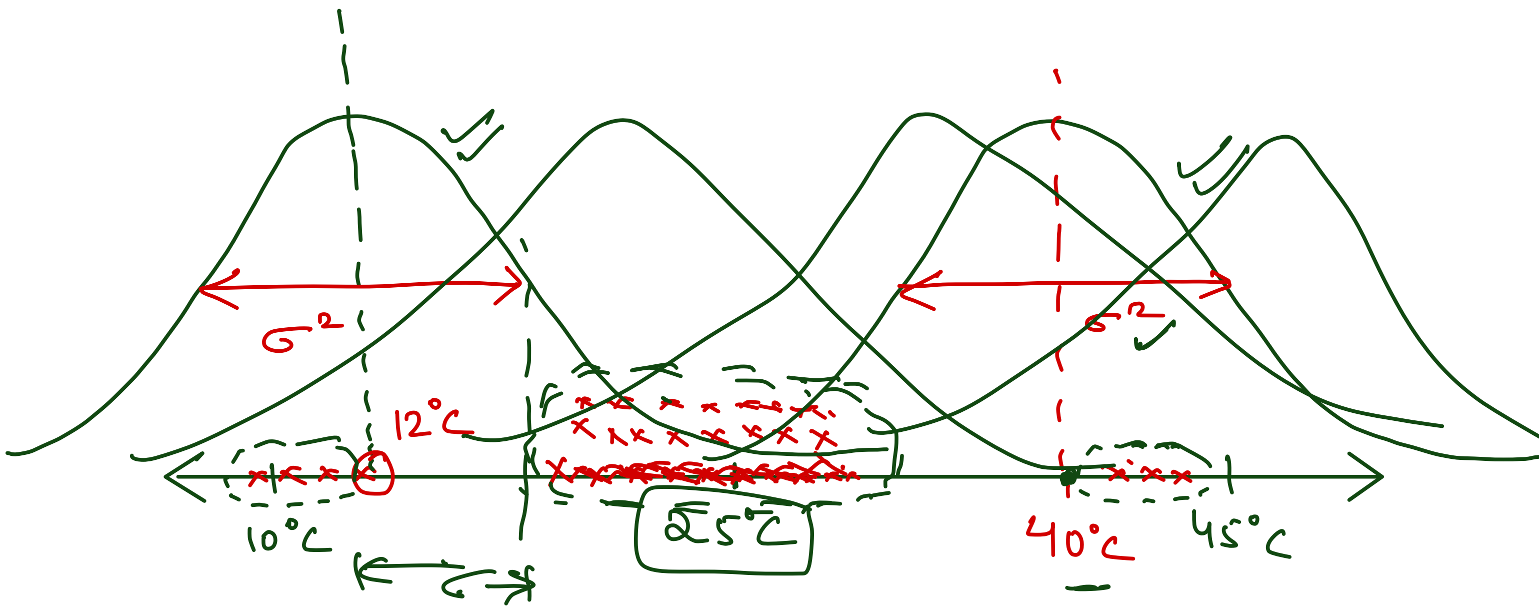
$\vec{\theta} = \mu, \sigma^2$: determine the exact dsⁿ that X is following

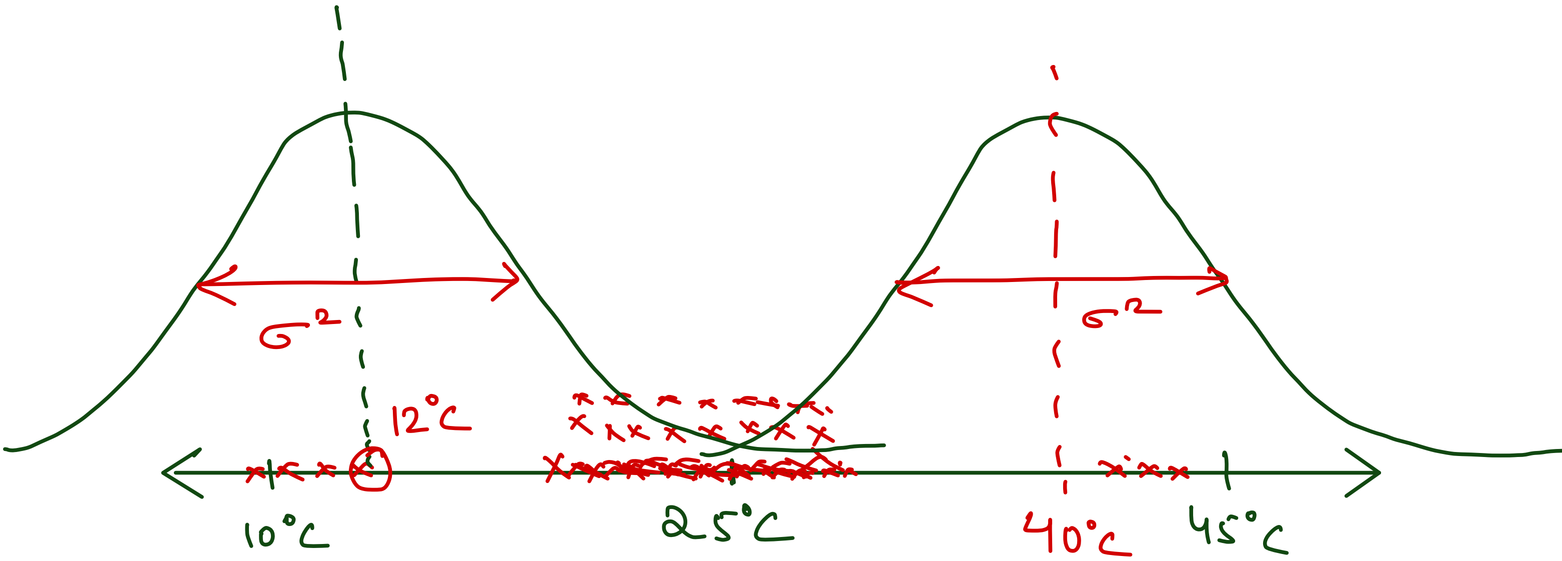
Intuition behind likelihood

$$p(\vec{x} \mid \vec{\theta}) = p(x = x_1, x = x_2, x = x_3 \mid \mu, \sigma^2)$$

$$\Rightarrow p(\vec{x} \mid \vec{\theta}) = \underbrace{p(x=x_1) \cdot p(x=x_2) \cdots p(x=x_n)}_{\text{Conditional } \mu \,\&\, \sigma^2}$$

Maximum: $\quad \arg\max \; p(\vec{x} \mid \vec{\theta})$





$$\operatorname{argmax} p(\vec{X} \mid \mu, \sigma^2) = \operatorname{argmax} p(x_1 \mid \vec{\theta}) \times p(x_2 \mid \vec{\theta}) \times p(x_3 \mid \vec{\theta}) \ldots$$

$$y = f(\vec{x})$$

$$\left\{ \frac{\partial y}{\partial x} = 0 \quad \begin{array}{l} : \text{Stationary} \\ : \text{second order} \end{array} \right\}$$

log of likelihoods

$$\operatorname{argmax} \left\{ \log \left[p(\vec{x} \mid \vec{\theta}) \right] \right\} = \ln \left\{ \prod_{i=1}^{n} p(x_1) \, p(x_2) \ldots p(x_n) \right\}$$