"Baye's rule"

$$P(\vec{\theta} \mid \vec{x}) = \frac{P(\vec{x} \mid \vec{\theta}) \times P(\vec{\theta})}{P(\vec{x})} \quad (4)$$

Posteriori, likelihood, evidence, prior

## Intuition behind likelihood

$x_1, x_2, x_3, \ldots, x_n$

X : random variable : temperature

Independent & identically distributed ( i i ds)

(1) pdf : same (continous) ✓

pmf : discrete ( discrete )

$$p(X_1 = x_1, X_2 = x_2, \ldots X_n = x_n) = p(X_1 = x_1) \times p(X_2 = x_2) \times \ldots p(X_3 = x_n)$$

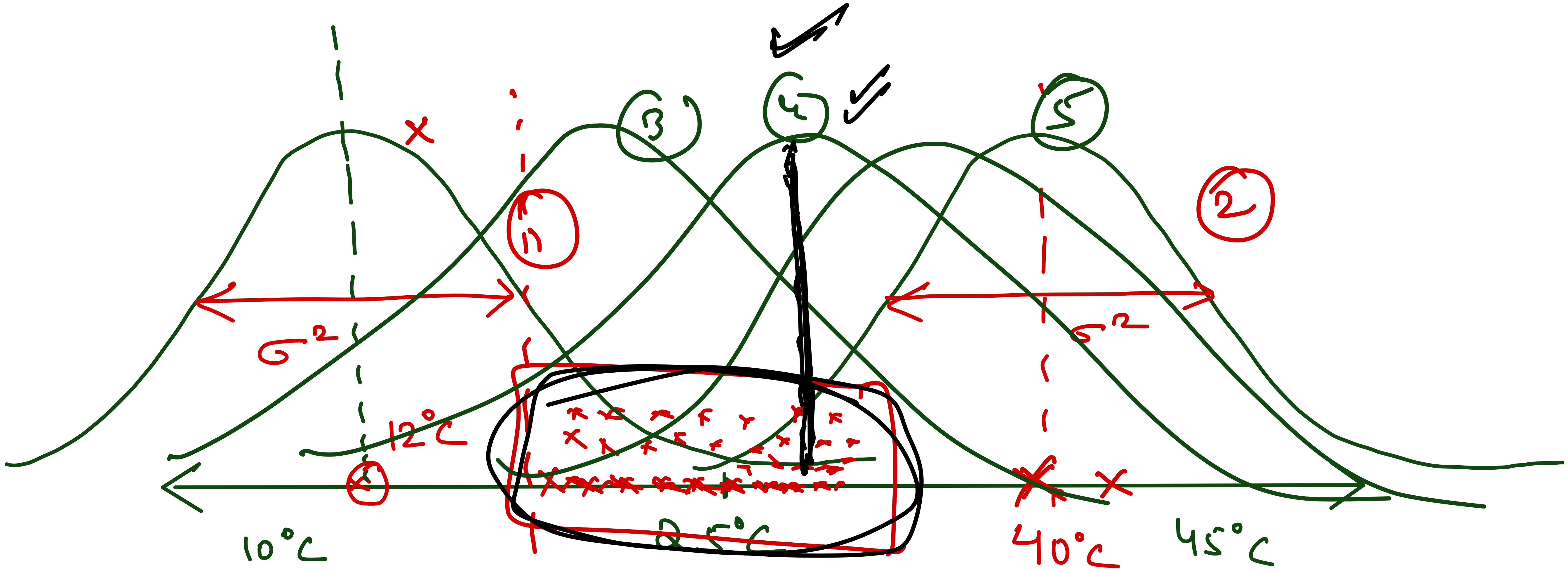
random variable ($X_1$), value R.V. ($x_1$)

$$\vec{X} = (x_1, x_2, x_3 \ldots x_n)$$

$$p(X_1 = x_1, X_2 = x_2, X_3 = x_3 \ldots) = \prod_{i=1}^{n} p(x_i)$$

$$p(\vec{X} \mid \vec{\theta}) = \prod_{i=1}^{n} p(x_i \mid \vec{\theta})$$

① Temperature : $\mathcal{N}(\mu, \sigma^2)$



$$p(\vec{x} \mid \theta) \equiv p(\vec{x} \mid \mu)$$

$$\mu = \mu_1 \quad (12^\circ C)$$

$$\mu = \mu_2 \quad (25^\circ C)$$

$$\mu = \mu_3 \quad (40^\circ C)$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\boxed{\sigma^2}}}$$

$$f(x = 25^\circ \mid \underline{\mu} = 25^\circ C) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(25-25)^2}{2\sigma^2}}$$

conditional on

$$= \frac{1}{\sqrt{2\pi\sigma}}$$

$$\boxed{f(x = 25^\circ \mid \mu = 12^\circ C)} = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(25-12)^2}{2\sigma^2}}$$

$$f(x_1 = 25^\circ, x_2 = 30^\circ, x_3 = 40^\circ, x_4 = 27^\circ \ldots x_n)$$

$$= f(x_1) \times f(x_2) \times f(x_3) \ldots f(x_n)$$

conditional on $\mu$ & $\sigma^2$

$p(\vec{X} \mid \vec{\theta})$ choose the value of $\theta$

such that arg max $p(\vec{X} \mid \vec{\theta}) = \text{argmax} \prod_{i=1}^{n} f(x_i \mid \theta)$

maximum likelihood estimate

$$\left\{ \begin{array}{l} y = f(\vec{X}) \\ \dfrac{\partial y}{\partial x_i} = 0 \quad : \text{stable points} \\ \dfrac{\partial^2 y}{\partial^2 x_i} \quad \text{"Hessian"} \end{array} \right\}$$