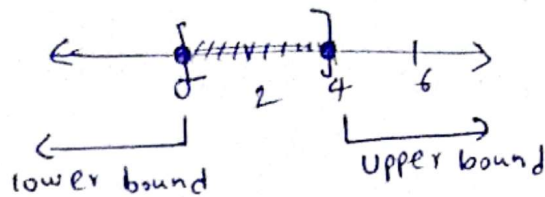


The interval $[0, 4]$ has a maximum of 4 and minimum of 0



- * Smallest upper bound (4) called the Supremum
- * Greatest/highest lower bound (0) called the infimum
- * $(0, 4)$ and $[0, 4]$ supremum and infimum are same to same. $(0, 4)$ has no max and min.

Examples:

$$T = \{q \in \mathbb{Q} : 0 \leq q \leq \sqrt{2}\}$$

$$\min(T) = 0$$

$\max(T)$ = does not exist because $\sqrt{2}$ is not a rational number.

$$A = \left\{ \frac{1}{n^2} : n \in \mathbb{N} \text{ and } n \geq 3 \right\} \quad A = \left\{ \dots, \frac{1}{25}, \frac{1}{16}, \frac{1}{9} \right\}$$

$$A(\max) = 1/9$$

$$A(\min) = 0$$

$$A(\text{supremum}) = 1/9$$

$$A(\text{infimum}) = 0$$

$$\lim_{n \rightarrow \infty} \frac{1}{n^2} = 0$$

$$D = \{x \in \mathbb{R} \text{ and } x^2 < 10\}$$

$$-\sqrt{10} < x < \sqrt{10}$$

$$D(\max) = \text{Does not exist}$$

$$D(\min) = \text{Does not exist}$$

$$D(\text{supreme}) = \sqrt{10}$$

$$D(\text{infimum}) = -\sqrt{10}$$

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, \dots\} \quad \mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, 3, \dots\}$$

$\mathbb{N}(\max) = \text{Does not exist}$

$\mathbb{Z}(\max) = \text{Does not exist.}$

$\mathbb{N}(\min) = 1$

$\mathbb{Z}(\min) = \text{Does not exist.}$

$\mathbb{N}(\text{supremum}) = \text{Does not exist}$

$\mathbb{Z}(\text{supremum}) = \text{Does not exist.}$

$\mathbb{N}(\text{infimum}) = 1$

$\mathbb{Z}(\text{infimum}) = \text{Does not exist.}$

$$\mathbb{Q} = \left\{ -\frac{5}{6}, -1, 0, \frac{1}{2}, \frac{3}{4}, 2 \right\}$$

$\mathbb{Q}(\max) = \text{does not exist.}$

$\mathbb{Q}(\min) = \text{does not exist.}$

$\mathbb{Q}(\text{supremum}) = \text{does not exist.}$

$\mathbb{Q}(\text{infimum}) = \text{does not exist.}$

$$|n| = \begin{cases} n & \text{if } n \geq 0, \\ -n & \text{if } n < 0, \end{cases}$$

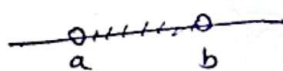
Standard notation for intervals



closed interval $[a, b]$

(endpoint included)

$$[a, b] = \{n \in \mathbb{R} : a \leq n \leq b\}$$



open interval (a, b)

endpoint excluded

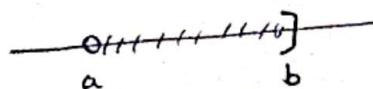
$$(a, b) = \{n : a < n < b\}$$



Half open interval

$[a, b)$

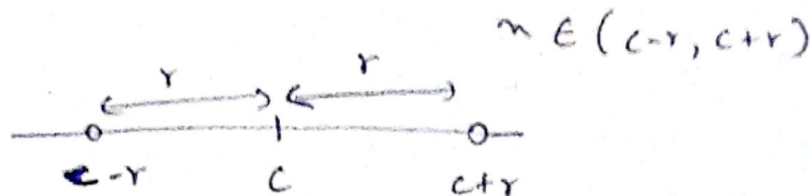
$$[a, b) = \{n : a \leq n < b\}$$



Half open interval $(a, b]$

$$(a, b] = \{n : a < n \leq b\}$$

• $|m-c| < r \Rightarrow$ ~~$m \in (c-r, c+r)$~~
 $-r < m-c < r \Rightarrow c-r < m < c+r$



~~r as radius~~ r as radius (շրջան)
 C as center (տեղանկյուն)

$(a, b) = (c-r, c+r)$

$c = \frac{a+b}{2}$

$r = \frac{b-a}{2}$

(radius)

• example (center)

① what is the midpoint and radius $[7, 13]$

② $c = \frac{7+13}{2} = 10$ $r = \frac{13-7}{2} = 3$

② $S = \{n : |\frac{1}{2}n - 3| > 4\}$

$|\frac{1}{2}n - 3| > 4$ օգտագործելով $|\frac{1}{2}n - 3| \leq 4$ օգտագործելով
 դրանք առաջին և երկրորդ թվերը համարժեցում են $\frac{1}{2}n - 3 \leq 4$ և $\frac{1}{2}n - 3 \geq -4$

~~$\frac{1}{2}n - 3 \leq 4$~~ $-4 \leq \frac{1}{2}n - 3 \leq 4$

$-1 \leq \frac{1}{2}n \leq 7$

$-2 \leq n \leq 14$

• The set S is the complement consisting of all numbers n not in $[-2, 14]$. we can describe S as a union of two intervals.

$S = (-\infty, -2) \cup (14, \infty)$

example:

Find the roots and sketch the graph of

$$f(x) = x^3 - 2x$$

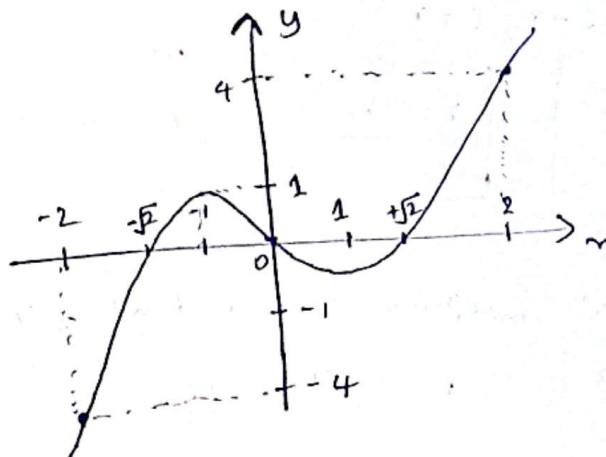
~~$$= x(x^2 - 2)$$~~

Q. (y=0 at x=0)

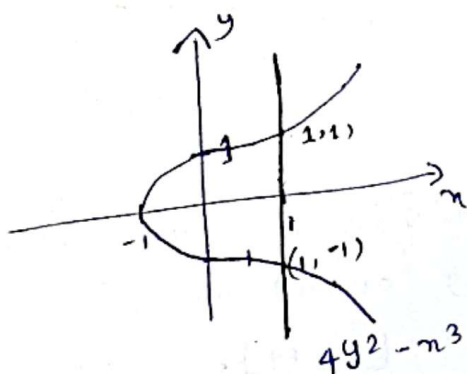
$$f(x) = x(x^2 - 2)$$

The roots of $f(x)$ are $x=0$ and $x=\pm\sqrt{2}$. To sketch the graph, we plot the roots and a few values listed in table. and join them by curve figure.

x	$x^3 - 2x$
-2	-4
-1	1
0	0
1	-1
2	4



① Vertical line test: If you can draw a vertical line (parallel to the y-axis) anywhere on the graph and it intersects the graph at more than one point, then the graph is not a function.



This graph fails the vertical line test.

So it is not the graph of a function.

why? because in a function every input x can have only one output y .

~~That is function~~

② What is function?

Special kind of relationship between two set.

Each input has exactly one output.

~~more~~
• one input has more than one output it is not a function (vertical line test)

$$f(m) = m + 2$$

if $m = 3$

$$f(3) = 3 + 2$$

$$f(3) = 5$$

one output

• function

$$x^2 + y^2 = 4$$

if $x = 0$

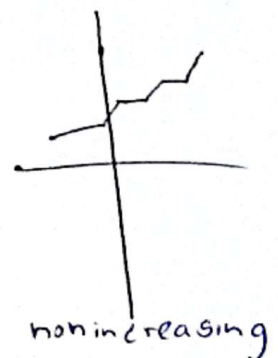
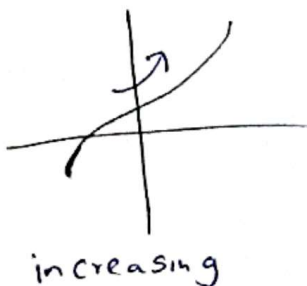
$$y = \pm 2$$

two output

not a function

Increasing and Decreasing function.

- | | | |
|------------------|----------------------|-------------|
| Ⓐ Increasing | $f(m_1) < f(m_2)$ | $m_1 > m_2$ |
| Ⓑ Nondecreasing | $f(m_1) \leq f(m_2)$ | $m_1 > m_2$ |
| Ⓒ Decreasing | $f(m_1) > f(m_2)$ | $m_1 > m_2$ |
| Ⓓ Non Increasing | $f(m_1) \geq f(m_2)$ | $m_1 > m_2$ |



(Monotonic)

we say that $f(x)$ is Monotonic if it is either increasing or decreasing