① [L1] 21/08/2025

Calculus (DS 1103)

Lecture - Mr. P.G.P. Kumara

(credit 02)

∴ ~~Complex~~ Natural number ($\mathbb{N}$)

- No negatives → -1 are not natural number
- ~~Some times 0 include natural number~~ no zero
- No fraction / decimals → 2.5, $\frac{1}{2}$ are not natural number

$$\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$$

∴ Integer number ($\mathbb{Z}$)

- No fractions or decimals → 2.5 or $7/3$ not integer number
- Can be negative, zero, positive)

$$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, 3, \dots\}$$

∴ Rational number ($\mathbb{Q}$)

$$Q = \{ p/q \mid p, q \in \mathbb{Z}, q \neq 0 \}$$

- Numerator p and q are integers)
- Denominator can not be zero
- Rational number can be positive, nagative, zero)

$$\left.\begin{array}{l} \frac{1}{2} = 0.5 \\ -3/4 = -0.75 \\ 0.333\dots = 1/3 \end{array}\right\} \text{Rational number}$$

✓

$$\left.\begin{array}{l} \pi\ (3.14159\dots \text{ no pattern}) \\ \sqrt{2}\ (1.4142135\dots \text{ no pattern}) \\ e\ (2.71828\dots \end{array}\right\} \text{not Rational number}$$

✗

∴ Real number ($\mathbb{R}$)

any number that can be placed on the number line

examples of real numbers. | not real

$5, -2, 0, 3/4, -7.5, \pi, \sqrt{2}$ | $2i, 3+4i$

↑ real numbers | ↑ not real numbers

∴ Complex number ($\mathbb{C}$)

has two parts

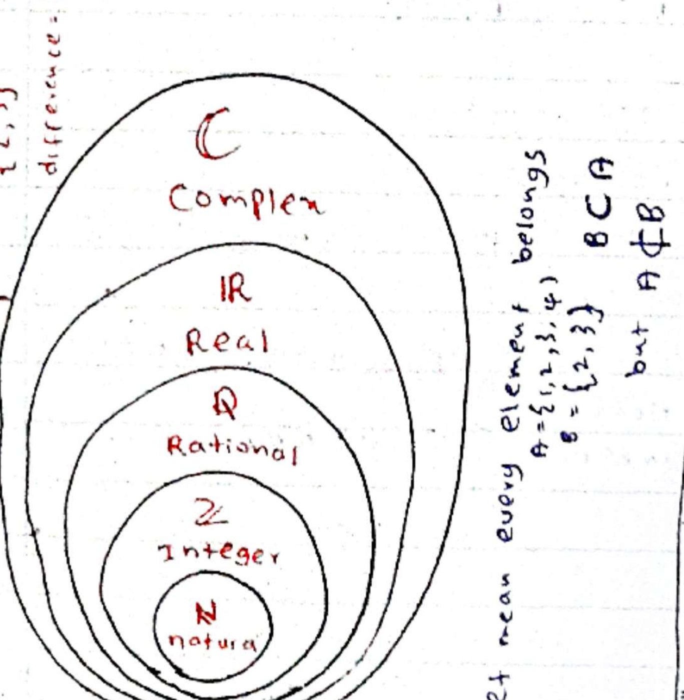
$z = a + bi$ (a: real part, bi: imaginary part)

$i^2 = -1$

complex number can also be shown on the argand plane (like graph)

- Horizontal axis - real part
- vertical axis - imaginary part

∩ (intersection) $A=\{1,2\}$, $B=\{2,3\}$ ∴ $A \cap B = \{2\}$	U (union) $A=\{1,2\}$, $B=\{2,3\}$, $A \cup B = \{1,2,3\}$
/ difference $A=\{1,2,3\}$, $B=\{2,3\}$, difference = 1	⊆ subset or equal $A=\{1,2,3\}$, $B=\{1,2,3\}$, $A \subseteq B$, $B \subseteq A$



• subset mean every element belongs

$A=\{1,2,3,4\}$, $B=\{2,3\}$: $B \subset A$ but $A \not\subset B$

{ } curly brackets: set written inside brackets.	∈ belongs to, ∉ does not belong to
, comas: elements separate	⊂ subset, ⊄ not subset
∅ or { } empty: A set with no elements	⊆ means one set is either a subset or equal to another
∩ (intersection): the common elements of two set	U (union): combined all elements of two set (without repetition)
Ellipsis (dot, dot): set continues in pattern	\ (difference): elements in one set but not in another

∴ Field Axioms:

(Real numbers and complex number must follow.)

A set $\mathbb{R}$ has more than one element is said to be a field under two composition of Addition and multipication defined in it if the following properties are ~~sta~~ satisfied for all $a, b, c \in \mathbb{R}$

Name	Addition	multiplication
Closure	$a, b \in \mathbb{R}$ $a+b \in \mathbb{R}$	$a, b \in \mathbb{R} \rightarrow ab \in \mathbb{R}$
Associativity	$(a+b)+c = a+(b+c)$	$(ab)c = a(bc)$
Identity	$a+0 = 0+a = a$	$a1 = 1a = a$
Inverse	$a+(-a) = (-a)+a = 0$	$aa^{-1} = a^{-1}a = 1$ if $a \neq 0$
Commutativity	$a+b = b+a$	$ab = ba$
Distributivity	$a(b+c) = ab+ac$	

- Order Axioms

"positive" and "negative" numbers in a consistenht way.

- complex number Not an ordere field because you cannot consistently say whether i is positive or negative
- Rational numbers, Real numbers ordered field

~~Generally, the order relation does not~~
If it has a relation < (less than that ~~these~~ satisfies these conditions. (all conditions)

Reflexivity	$a \leq a$
Antisymmetry	$a \leq b$ am $b \leq a \Rightarrow a = b$
Transitivity	$a \leq b$ and $b \leq c \Rightarrow a \leq c$
Trichotomy	Either $a < b$ or $a = b$ or $a > b$
	$a \leq b \Rightarrow a + c \leq b + c$; $a \leq b$ and $c \geq 0 \Rightarrow ac \leq bc$

Summery

$\mathbb{N} \rightarrow$ no field , no order field

$\mathbb{Z} \rightarrow$ no field , no order field

$\mathbb{Q} \rightarrow$ no complete field , order field

$\mathbb{R} \rightarrow$ field , Order field

$\mathbb{C} \rightarrow$ field , no order field