Convergence of probabilities obtained from Rejection Sampling Method:

We know that variable elimination is an exact inference method, so the probability of a query obtained using it will be the true probability.

Using the true probability we can now find error in the rejection sampling method and observe how it is converging.

The network that we will be using for the experiment is this one:-

```
5
4 2 3
0.5 0.5
0.5 0.5
0.5 0.5
0.5 0.5
2 1
0.5 0.5
0.5 0.5
3 1
0.5 0.5
0.5 0.5
1
0.5 0.5
5 3
0.5 0.5
0.5 0.5
```

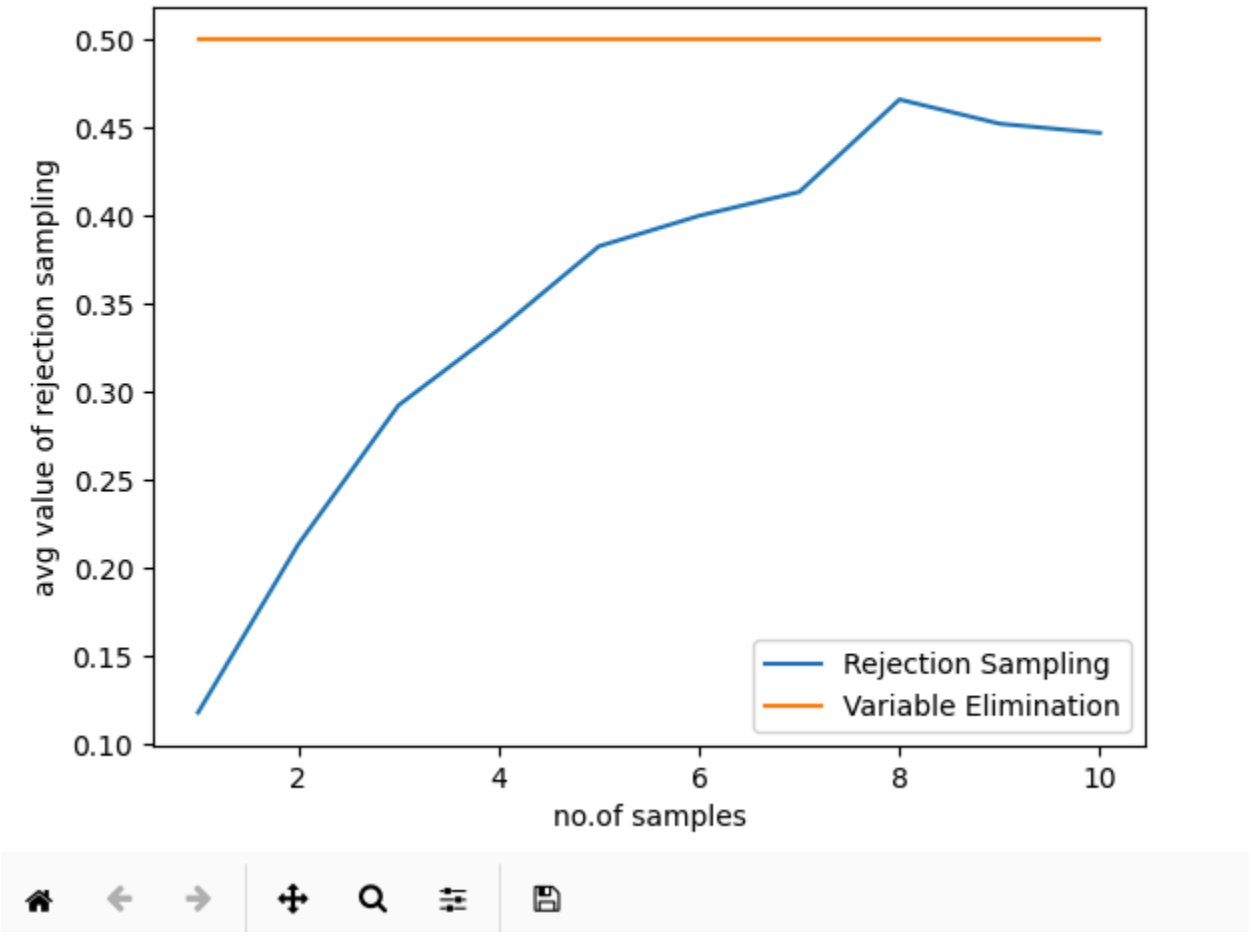
And the query will be to find the value P(4 | -2, 3)

Positive value means that the variable is True and negative value means that the variable is False.

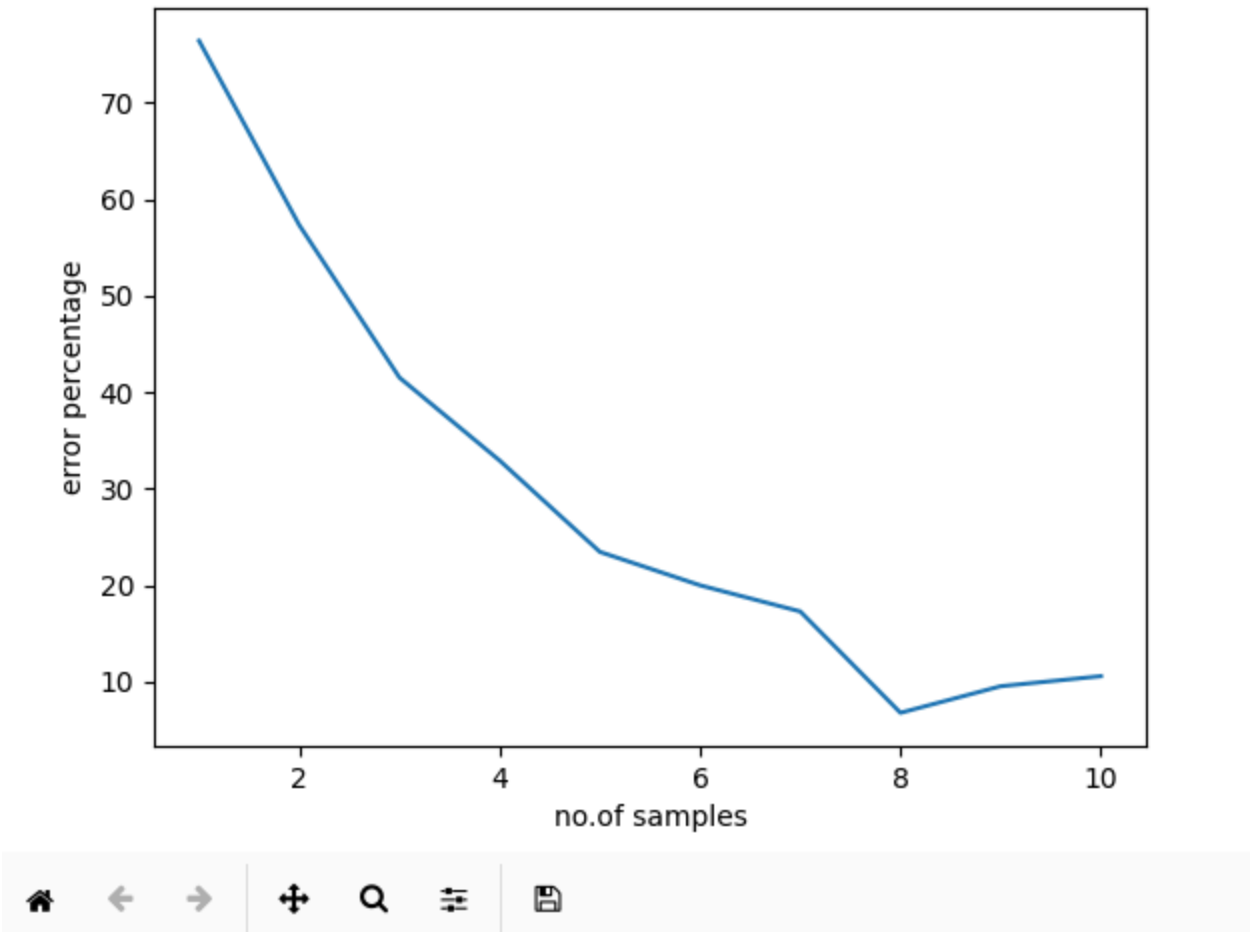
Here we need to find the probability of 4 being true given that 2 is false and 3 is true.

For this purpose we made some functions in a new file convergence_calculator.
There we are finding out the true probability once and then taking average of results obtained from rejection sampling by doing it 1000 times for each value of N (number of samples that we are taking). The value of N goes from 1 to 10 inclusive.

Now,
The convergence graph:-



The error vs no. of samples plot:-



As we see as the sample size increases the error in the estimated value is following a decreasing trend. Similarly the estimated probability obtained from rejection sampling seems to converge with the value obtained from variable elimination (true probability) as the number of samples that we are taking for rejection sampling increases.

From this we can conclude that as the number of samples are increased, the value of probability obtained from rejection sampling converges to the true probability value and our results get more accurate.

The below table has more detailed information about the values I used for plotting the graph.

No.of Samples	Truel Probability	Estimated Probability	Error Percentage
1	0..5	0.1180	76.40
2	0..5	0.2135	57.30
3	0..5	0.2925	41.50
4	0..5	0.3355	32.90
5	0..5	0.3828	23.44
6	0..5	0.4001	19.97
7	0..5	0.4137	17.26
8	0..5	0.4661	6.76
9	0..5	0.4524	9.52
10	0..5	0.4472	10.56