Assignment - 1.

Q1

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$$

(a) $(2A)^T$

$$\rightarrow \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix} \Longrightarrow \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 6 & 8 \end{bmatrix}$$

(b) $(A-B)^T$

Not possible.

(c) $(3B^T - A)^T$

$$\begin{bmatrix} 3 & 0 \\ 6 & 3 \\ 9 & 6 \end{bmatrix}^T - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ -2 & 2 & 2 \end{bmatrix}$$

(d) $(-A)^T . E$

$$\begin{bmatrix} -1 & -2 & -3 \\ -2 & -1 & -4 \end{bmatrix}^T \cdot \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 3+4 & -2+8 \\ 6+2 & -4+4 \\ 9+4 & -6+16 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 6 \\ 8 & 0 \\ 13 & 10 \end{bmatrix}$$

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(e) $(C + 2D^T + E)^T$

Not possible.

Q2 $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$

AB

$\Rightarrow \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2-12 & -1+16 \\ 6-6 & -3+8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -10 & 15 \\ 0 & 5 \end{bmatrix}$

BA

$\Rightarrow \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2-3 & 8-2 \\ -3+12 & -12+8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -1 & 6 \\ 9 & -4 \end{bmatrix}$

$AB \neq BA$.

Q3. Orthogonal set.

↳ Dot product any 2 vectors in the set is 0.

Orthonormal set

↳ Orthogonal set with all unit vectors.

$V_1 = (-2, 0, 10)$ $\quad V_2 = (0, 1, 0)$. $\quad V_3 = (2, 0, 4)$.

) $V_1 \cdot V_2 = -40 + 40 = 0$.

$\therefore$ Orthogonal but not orthonormal.

(b) $V_1 = \dfrac{-2}{\sqrt{2^2+10^2}}, \dfrac{0}{\sqrt{2^2+10^2}}, \dfrac{10}{\sqrt{2^2+10^2}}$

$= \left(\dfrac{-2}{\sqrt{104}}, 0, \dfrac{10}{\sqrt{104}} \right)$

$V_2 = (0, 1, 0)$

$V_3 = \left(\dfrac{2}{\sqrt{20}}, \dfrac{0}{\sqrt{20}}, \dfrac{4}{\sqrt{20}} \right)$

Q4 Rank of matrix.

$\hookrightarrow$ Maximum number of linearly independent vectors in a matrix is equal to the number of non-zero rows in its row echelon matrix.

Thus, we find no. of non-zero rows in the row echelon form of matrix.

Let $A = xy^T$ $\quad$ | Given $\Rightarrow$ $x \in R^m$, $y \in R^n$.

~~Let $x \in R$~~

~~We know that $x \in R^n$.~~

~~Thus,~~ r

Then A is $m \times n$ matrix

Let $u \in R^m$, $v \in R^m$

Then $Au = xy^Tu$

| Thus rank(A) = 1 |

$\Rightarrow Au = (u \cdot y)x$

||ly $AuV^T = (u\,y)(xV^T)$

$= (u \cdot y)(v \cdot x)$

$AuV^T \in R^m R^m$

[=]

Q5. $X = [x_1, x_2, \dots x_n] \in R^{m \times n}$ where $x_i \in R^m$ for all i and $Y^T = [y^1, y^2 \dots y^n] \in R^{p \times n}$ where $y^i \in R^p$ for all i. Show that

$$XY = \sum_{i=1}^{n} (x_i)(y^i)^T.$$

A5 $x_i \in R^m$ and $X \in R^{m \times n}$.

Similarly $Y^T \in R^{p \times n}$ & $y^i \in R^p$

~~Thus we can say that~~ ~~and~~

Thus, $(y^i)^T \in (n \times p)$.

$(x^i) \in (m \times n)$

Therefore $XY^T = [x_1, x_2 \dots x_n] \cdot [y^{1T}, y^{2T} \dots y^{nT}]$

$$= \sum_{i=1}^{n} x_i (y^i)^T.$$

Q6. $X \in R^{m \times n}$. Show that $X^T X$ is symmetric and +ve semi-definite. When is it positive definite?

A6 ~~$\langle x, AA^T x \rangle = \langle A^T x, A^T x \rangle = \|A^T x\|^2 \geq 0$.~~

~~Thus, $A^T A$.~~

For matrix to be +ve definite, $x^T M x \geq 0$.

$$(x^T X^T X x) = X^T x X x.$$
$$= X^T X x^T x$$
$$= R^0.$$

$\Rightarrow$ ~~$x^T X^T X x =$~~

$\Rightarrow$ ~~$(X^T (x^T)^T (x X^T)$~~

$\Rightarrow$ ~~$(X^T x \cdot x X^T)$~~ $\Rightarrow$ ~~$(X^T x^2 X^T)$~~

$\Rightarrow$ ~~$x^2 X X^T$~~

Therefore it cannot be strictly +ve definite.

It can be +ve semi-definite $\Rightarrow \langle x, X^T X x \rangle \geq 0$.

$\langle x, X^T X x \rangle = \langle X^T x, X^T x \rangle$

$= \|X^T x\|^2$ which is ≥ 0.

Hence proved.

Q7 $g(x,y) = e^x + e^y + e^{-2xy} - \log_e(-x^2 y)$.

Compute $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$

A7. $\frac{\partial g}{\partial x} = e^x + 0 + (e^{-2xy})(-2y) - \frac{1}{(-x^2 y)}(-2xy)$.

$$\boxed{= e^x - 2y e^{-2xy} - \frac{2}{x}}.$$

$\frac{\partial g}{\partial y} = 0 + e^y + (e^{-2xy})(-2x) - \frac{1}{(-x^2 y)}(-x^2)$

$$\boxed{= e^y - 2x e^{-2xy} - \frac{1}{y}}.$$

Q8 $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 3 & 2 & 5 \end{pmatrix}$.

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