Assignment 3 - Probability

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1) Question 1

- $P(\text{black} \vee \text{tabby}) = P(\text{black}) + P(\text{tabby}) - P(\text{black} \wedge \text{tabby})$

$$= \frac{12}{50} + \frac{11}{50} + 0 = \frac{23}{50}$$

- $P(\text{white}) = \frac{5}{50}$

- $P(!\text{calico}) = 1 - P(\text{calico}) = 1 - \frac{15}{50} = \frac{35}{50}$

2) Question 2 : Inference with JPP

- $P(\text{calico} \wedge \text{male}) = 0.12$

- $P(\text{black} \wedge \text{male}) \vee (\text{white} \wedge \text{female}) = P(\text{black} \wedge \text{male}) + P(\text{white} \wedge \text{female})$

$$= 0.1 + 0.08 = 0.18$$

- $P(\text{male} \vee \text{calico}) = P(\text{male}) + P(\text{calico}) - P(\text{male} \wedge \text{calico})$

$$= (0.1 + 0.12 + 0.08 + 0.02 + 0.12) + (0.12 + 0.02)$$

$$= 0.1 + 0.12 - 0.12$$

$$= 0.1$$

- $P(\text{female}) = (0.14 + 0.02 + 0.16 + 0.08 + 0.18)$

$$= 0.42$$

- $P((\text{tabby} \vee \text{white}) \wedge \text{female}) = P(\text{tabby} \wedge \text{female}) + P(\text{white} \wedge \text{female})$

$$= 0.16 + 0.08$$

$$= 0.24$$

- $P(\text{gray} \wedge !\text{male}) = P(\text{gray} \wedge \text{female}) = 0.02$

★ $!\text{male} = \text{female}$

3) Question 3: Conditional Probability

- $P(\text{male} \mid \text{gray} \vee \text{white})$

$$\frac{P(\text{male} \wedge (\text{gray} \vee \text{white}))}{P(\text{gray} \vee \text{white})} = \frac{P(\text{male} \wedge \text{gray}) + P(\text{male} \wedge \text{white}) - P(\text{gray} \wedge \text{white})}{P(\text{gray} \vee \text{white})}$$

$$= \frac{0.12 + 0.02 - 0.02}{0.12} = \frac{0.14}{0.12} = 1.1667$$

- $P(\text{female} \mid !\text{black})$

$$\frac{P(\text{female} \wedge !\text{black})}{P(!\text{black})} = \frac{P(\text{fmale}) - P(\text{fmale} \wedge \text{black})}{1 - P(\text{black})}$$

$$= \frac{0.14 + 0.02 + 0.08 + 0.18 - 0.14}{1 - 0.1}$$

$$= \frac{0.28}{0.9} = 0.3111$$

- $P(\text{gray} \mid \text{female}) = \frac{P(\text{gray} \wedge \text{female})}{P(\text{female})}$

$$= \frac{0.02}{0.14 + 0.02 + 0.08 + 0.18}$$

$$= \frac{0.02}{0.42} = 0.0476$$

Question 4: Bayes' Rule

Given:

$$P(\text{friendly} \mid \text{calico}) = 0.2$$
$$P(\text{friendly} \mid !\text{calico}) = 0.4$$
$$P(\text{calico}) = 0.3$$

By bayes rule

$$P(\text{calico} \mid \text{friendly}) = \frac{P(\text{friendly} \mid \text{calico}) * P(\text{calico})}{P(\text{friendly})}$$

$$= \frac{P(\text{friendly} \mid \text{calico}) * P(\text{calico})}{P[(\text{friendly} \mid \text{calico}) * P(\text{calico})] + P(\text{friendly} \mid !\text{calico} * P(!\text{calico}))}$$

$$= \frac{(0.2 * 0.3)}{[(0.2 * 0.3) + (0.4 * 0.7)]}$$

$$= \frac{0.06}{0.06 + 0.28}$$

$$= \frac{0.06}{0.34}$$

$$\approx 0.1765$$

∴ The probability of seeing a friendly cat who is a calico is 17.65% or 0.1765