CS340: Assignment 2

October 31, 2022

Time: 5 days **Maximum Marks**: 140

Question 1. (10+20 marks) For a finite automata $\mathcal{F} = (Q, q_0, \Sigma, \delta, F)$ and input x, define a *configuration* of $\mathcal{F}$ to be the pair $\langle q, y\rangle$ where $q \in Q$, y is a suffix of x, and automata $\mathcal{F}$ on input $x = zy$, ends up in state q after reading z. Note that if $\mathcal{F}$ is an NFA, then there may be more than one state in which $\mathcal{F}$ ends in after reading z. An *accepting configuration sequence* is a string $\langle q_0, y_0\rangle\langle q_1, y_1\rangle \cdots \langle q_m, \rangle$ where (1) each $\langle q_i, y_i\rangle$ is a configuration, (2) $\mathcal{F}$ moves from configuration $\langle q_i, y_i\rangle$ to $\langle q_{i+1}, y_{i+1}\rangle$ in one step, (3) $q_m \in F$, and (4) $y_0 = x$.

Prove that

- Input x is accepted by $\mathcal{F}$ iff there exists an accepting configuration sequence that starts with $\langle q_0, x\rangle$.
- The set of all accepting configuration sequences of $\mathcal{F}$ is computable but is not a CFL.

Question 2. (20 marks) A *2-PDA* is a pushdown automata with two stacks. In a transition, the automata can push/pop both stacks independently. Prove that any computable set can be accepted by a 2-PDA.

Question 3. (20 marks) A *counter TM* is a Turing machine with an input tape that is read-only (input is initially written on the tape and cannot be changed during computation) and a finite number of counters. In one move of the TM, a counter either remains unchanged, or is incremented by one, or is decremented by one. Prove that every computable set can be accepted by a counter TM.

Question 4. (20 marks) Design a Turing Machine (draw the state diagram) to multiply two numbers represented in unary alphabet, that is, 1 is represented as 1, 2 as 11, 3 as 111, ... The input alphabet is $\Sigma = \{1, \times, =\}$. Input has the form $1^n \times 1^m =$, which denotes multiplication of number n with m. The TM should write the result of the multiplication, 1^{nm}, as output immediately after the $=$ sign on the tape.

Question 5. (20 marks) Design a TM (draw the state diagram) that accepts following set of strings:

$$L = \{ww \mid w \in \{a, b\}^*\}.$$

Question 6. (10 marks) Prove that the following set is not computable:

$$L = \{(p, q) \mid \text{there exists a string } x \text{ accepted by both TM } M_p \text{ and TM } M_q\}.$$