Non-Equilibrium Transport Through Two Defect Bonds in a Free-Fermion Chain

Model

Consider a one-dimensional chain of L spinless fermions with nearest-neighbour hopping amplitude J, except for *two* consecutive defect bonds of reduced strength J_{imp}. Label the defect bonds between sites $j_0 \leftrightarrow j_0 + 1$ and $j_0 + 1 \leftrightarrow j_0 + 2$. The single-particle Hamiltonian is

$$h_{ij} = \frac{J_j}{2} \left(\delta_{i,j+1} + \delta_{i+1,j}\right), \quad J_j = \begin{cases} J_{\text{imp}}, & j = j_0 \text{ or } j = j_0 + 1, \\ J, & \text{otherwise.} \end{cases}$$

Initial State and Quench

At $t = 0$, the left half (sites $0, \ldots, j_0$) is prepared in a zero-temperature Fermi sea at chemical potential μ_L, and the remainder (sites $j_0 + 1, \ldots, L - 1$) in a Fermi sea at μ_R with $\mu_L > \mu_R$. The many-body initial state Ψ_0 factorizes into two equilibrium vacua.

Correlation-Matrix Formalism

Define the single-particle correlator

$$C_{ij}(t) = \left\langle \Psi_0 \middle| c_i^\dagger(t)\, c_j(t) \middle| \Psi_0 \right\rangle, \quad c_j(t) = e^{iHt} c_j\, e^{-iHt}.$$

If $\{\phi_n, \varepsilon_n\}$ diagonalize h, then

$$C(t) = U(t)\, C(0)\, U^\dagger(t), \quad U(t) = \phi\, e^{-i\varepsilon t}\, \phi^\dagger,$$

and $C(0)$ is block-diagonal with fillings prescribed by μ_L and μ_R.

Real and Imaginary Parts

- $\Re\, C_{ii}(t)$ gives site occupations $\langle n_i \rangle$.
- $\Im\, C_{ij}(t)$ is antisymmetric ($\Im\, C_{ij} = -\Im\, C_{ji}$) and encodes the particle current from j into i:

$$j_{j \to i}(t) \;\propto\; \Im\, C_{ij}(t).$$

Defect-Bond Current

The two weakened links carry currents

$$j_{j_0 \to j_0+1}(t) = 2 J_{\text{imp}}\, \Im\, C_{j_0,\, j_0+1}(t),$$