Charge sharpening in a $\mathbb{Z}_3$-symmetric circuit

1 Model

We consider a 1D chain of L qutrits, which evolves under brickwork unitary evolution interspersed with single-site measurements. Each time step t involves two layers of unitary operators:

$$\mathbb{U}(t) = \bigotimes_{x\,\text{even}} U_{x,x+1}(x,t) \bigotimes_{x'\,\text{odd}} U_{x',x'+1}(x',t)\ . \tag{1}$$

Here, each $U_{x,x+1}$ is a two-qutrit unitary that acts on sites x and $x+1$. In the computational basis, $U_{x,x+1}$ decomposes into 3 blocks, each block being of size 3×3:

$$U_{x,x+1} = U^{(0)} \oplus U^{(1)} \oplus U^{(2)}. \tag{2}$$

The superscript label refers to the total charge modulo 3: $U^{(0)}$ acts on the subspace spanned by $\{|00\rangle, |12\rangle, |21\rangle\}$; $U^{(1)}$ acts on the subspace spanned by $\{|01\rangle, |10\rangle, |22\rangle\}$; and $U^{(2)}$ acts on the subspace spanned by $\{|02\rangle, |20\rangle, |11\rangle\}$.

Each $U^{(j)}$ is *Haar random* (drawn independently for each x and t), so the evolution is random within each charge sector, but conserves the charge modulo 3. Following the two layers of unitary evolution, at each time step t independent single-site projective measurements are applied to the system with a rate p (each site has a probability p to be measured).

We then look at the following charge-sharpening problem: starting from an equal-superposition state $|\Psi_0\rangle = \left[\frac{1}{\sqrt{3}}\left(|0\rangle + |1\rangle + |2\rangle\right)\right]^{\otimes L}$, consider the state $|\Psi_f\rangle$ after $t = L$ time steps. Denoting by p_j the probability to measure a total charge j in $|\Psi_f\rangle$ (with $j = 0,1,2$), we can ask to what extent p_j is concentrated at a certain value. This is measured by the entropy $S = -\sum_j p_j \ln p_j$, which is further averaged over different realizations of the circuit. Conversely, assuming that $|\Psi_0\rangle$ is in a definite charge sector j, S measures how likely it is to correctly learn j from the results of the measurements performed throughout the evolution (learnability increases with decreasing S).