[Extracted question from page 1] No questions found. [End of extracted question from page 1]

[Extracted question from page 2] (a) Let k be a constant. Express $\lim_{h\to 0} \frac{e^{-kh-h^2}-1}{h}$ in terms of k. (b) Let $f(x) = xe^{-x^2}$. Using the result of (a), find $f'(x)$ from first principles. [End of extracted question from page 2]

[Extracted question from page 3] Let n be an integer greater than 1. Define (x a)^n = $nPk = 0kx^k, whereaisanintegralconstant.Itisgiventhat2/1 = 4/3.(a)Chungchungclaimsthatnisanoddnumberand$ $0.Doyouagree?Explainyouranswer.Letnbeanintegergreaterthan1.Define(xa)^n = nPk = 0kx^k, whereaisanintegralconstant.Itisgiventhat2/1 = 4/3.(b)Let(bx6)^2n = 2nPk = 0kx^{2nk}, wherebisanintegralcon$ $nand1 = 4n1, finda, bandn.[Endofextractedquestionfrompage3]$

[Extracted question from page 4] (a) Prove that $\sin \pi x^2 + \sin 2\pi x - \sin \pi(x^2 + 2x) = 4 \sin \frac{\pi(x^2+2x)}{2} \sin \frac{\pi x^2}{2} \sin \pi x$. (b) Find the least positive value of x such that $\sin \pi x^2 + \sin 2\pi x = \sin \pi(x^2 + 2x)$. [End of extracted question from page 4]

[Extracted question from page 5] No questions found. [End of extracted question from page 5]

[Extracted question from page 6] 5. Consider the curve $\Gamma : y = e^{-2x}$, where $x \geq 0$. Let P be a moving point on Γ with h as its x-coordinate, where $h > 0$. Denote the tangent to Γ at P by L and the area of the region bounded by Γ , L and the y-axis by A .(a) Prove that $A = e^{-2h}\left(e^{2h} - 2h^2 - 2h - 1\right)/2$.(b) If $h = \ln(t + 1)$, where t is the time, find the rate of change of A when $t = 2$. [End of extracted question from page 6]

[Extracted question from page 7] (a) Find $\int \frac{x+\sin x}{1+\cos x} dx$. (b) At any point (x, y) on the curve Γ, the slope of the tangent to Γ is $1 + x - \cos x + \frac{\sin x}{1+\cos x}$. Given that Γ passes through the point $\left(\frac{\pi}{2}, 2 + \pi\right)$, does Γ pass through the point $\left(\frac{3\pi}{4}, \frac{3\pi}{4} + 2\tan\left(\frac{3\pi}{8}\right) + \frac{\pi}{4}\right)$. [End of extracted question from page 7]

[Extracted question from page 8] No questions found. [End of extracted question from page 8]

[Extracted question from page 9] (a) Let A, B be two square matrices of the same order. If AB = BA , show by math- ematical induction that for any positive integer n , (A + B)^n = $\sum_{r=0}^{n} C_r^n$ A^{n-r}B^r , where A^0 and B^0 are by definition the identity matrix I .

(b) Let A = $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ where $\theta \in \mathbb{R}$. It is given that A^n = $\begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$ for all positive integers n . Using the result of (a), prove that $\sum_{r=0}^{n} C_r^n$ cos(n - 2r) $\theta = 2^n \cos^n \theta$ and $\sum_{r=0}^{n} C_r^n$ sin(n - 2r) $\theta = 0$. (7 marks) [End of extracted question from page 9]

[Extracted question from page 10] Consider the system of linear equations in real variables x, y and z (E): $\begin{cases} x - 3y + 2z = 0 \\ (a-1)x + y + 2z = 0 \\ 2x + ay - 2z = 0 \end{cases}$ where $a \in \mathbb{R}$. It is given that (E) has infinitely many solutions when $a = \alpha$ or $a = \beta$, where $\alpha < \beta$.

Find α and β.

Solve (E) when $a = \alpha$.

Consider the system of linear equations in real variables x, y and z (F): $\begin{cases} x - 3y + 2z = 2 \\ (c-1)x + y + 2z = 3b + 5 \\ 2x + cy - 2z = 2b + 1 \end{cases}$ where $b, c \in \mathbb{R}$.

Assume that $c \neq \alpha$ and $c \neq \beta$. Express x in terms of b and c.

Assume that $c = \beta$. Find b so that (F) is consistent. Hence, solve (F).

Assume that $c = \beta$ and (F) is consistent. Find h and k such that all the solutions (x, y, z) of (F) satisfy $hx + ky - 2z = 7$. [End of extracted question from page 10]

[Extracted question from page 11] (a) Prove that $A^n = \begin{pmatrix} F_{n-1} & F_n \\ F_n & F_{n+1} \end{pmatrix}$ for all positive integers n .

(b) (i)ii Prove that $F_{n+m-1} = F_n F_m + F_{n-1} F_{m-1}$ for all positive integers m and n . (ii)i Prove that $F_{n-1} F_{n+1} - F2_n = (-1)^n$ for all positive integers n .

(c) (i)ii Let α and β be real roots of $\lambda^2 - \lambda - 1 = 0$, where $\alpha > \beta$. Prove that $A \begin{pmatrix} 1 \\ \alpha \end{pmatrix} = \mu_1 \begin{pmatrix} 1 \\ \alpha \end{pmatrix}$ for some $\mu_1 \in \mathbb{R}$ and $A \begin{pmatrix} 1 \\ \beta \end{pmatrix} = \mu_2 \begin{pmatrix} 1 \\ \beta \end{pmatrix}$ for some $\mu_2 \in \mathbb{R}$. (ii)i Hence, using (a), prove that $F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$ for all non-negative integers n . [End of extracted question from page 11]

[Extracted question from page 12] No questions found. [End of extracted question from page 12]