PENDULUM AND PROJECTILE MOTION

SPH3U Unit 1 Lab

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LaTeX document code

Contents

1 Pendulum Motion

The purpose of the pendulum motion lab is to determine the acceleration due to gravity. A pendulum with a string length of 0.33m, attached with a 1.5cm in diameter steel ball, is dropped from a 5° from the right without external forces. We recorded the time it takes for 10 oscillations for a total of 3 times for maximum accuracy.

1.1 Raw data

Length of pendulum wire: $L = 33\text{cm} = 0.33\text{m}$

Initial angular displacement: $\theta = 5°$ to the right

Mass of pendulum bob: $M =?$

Bob diameter: $d = 1.5\text{cm} = 0.015\text{m}$

1.2 Actual time trials

We did a total of 3 trials, each measuring the time for 10 pendulum oscillations to occur:

$$t_1 = 11.66\text{s}$$

$$t_2 = 11.85\text{s}$$

$$t_3 = 11.65\text{s}$$

The average actual time for 10 oscillations is:

$$\begin{aligned} t_{10avg} &= \frac{11.66\text{s} + 11.85\text{s} + 11.65\text{s}}{3} \\ &= \frac{35.16\text{s}}{3} \\ t_{10avg} &= 11.72\text{s} \end{aligned}$$

So, the average actual time for 1 oscillations is:

$$t_{avg} = \frac{11.72\text{s}}{10}$$
$$\boxed{t_{avg} = 1.172\text{s}}$$

1.3 Theoretical time

We can calculate the theoretical time for each oscillation under perfect conditions using the simple harmonic motion equation:

$$\begin{aligned}
t_{the} &= 2\pi \times \sqrt{\frac{L}{g}} \\
&= 2\pi \times \sqrt{\frac{0.33\text{m}}{9.81\text{m/s}^2}} \\
&= 2\pi \times \sqrt{0.034\text{s}^2} \\
&= 2\pi \times 0.183\text{s} \\
\boxed{t_{the} = 1.15\text{s}}
\end{aligned}$$

1.4 Error sources

Our actual time measurements are quite accurate. There is a very small difference between our actual time and the theoretical time:

$$\begin{aligned}
\Delta &= 1.172\text{s} - 1.15\text{s} \\
&= 0.022\text{s}
\end{aligned}$$

This marginal time difference can be caused by errors like:

- Imprecise measurements: EXPLAIN
- Gross errors: EXPLAIN

2 Projectile motion

The purpose of the projectile motion lab is to determine the properties of a projectile through displacement graphs, velocity graphs, and a variety of data. We placed a steel ball at the top of the ramp, and captured the trajectory of the steel ball using a slow motion camera.

2.1 Raw data

After reviewing the slow motion video, we can compile the following data points:

$\Delta\vec{d}_x$ (m [$\rightarrow$])	$\Delta\vec{d}_y$ (m [$\uparrow$])	t (s)
0.000m	0.158m	0.000s
0.015m	0.146m	0.030s
0.030m	0.128m	0.060s
0.045m	0.105m	0.090s
0.060m	0.068m	0.120s
0.075m	0.023m	0.150s
0.083m	0.000m	0.165s

Note: We measured everything using the steel ball's center.

Note: The steel ball is tracked using a 1.5cm grid plane. These values are obtained by scaling the values obtained on the grid plane.

2.2 Calculations

All calculations are based on the table of values.

2.2.1 Total Displacement (x-axis)

The total displacement in the x-axis:

$$\begin{aligned}\vec{\Delta d_x} &= \vec{\Delta d_{xf}} - \vec{\Delta d_{xi}} \\ &= 0.083\text{m } [\rightarrow] - 0.0\text{m } [\rightarrow] \\ \Aboxed{\vec{\Delta d_x} &= 0.083\text{m } [\rightarrow]}\end{aligned}$$

2.2.2 Total Displacement (y-axis)

The total displacement in the y-axis

$$\begin{aligned}\vec{\Delta d_y} &= \vec{\Delta d_{yf}} - \vec{\Delta d_{yi}} \\ &= 0.0\text{m } [\uparrow] - 0.158\text{m } [\uparrow] \\ &= -0.158\text{m } [\uparrow] \\ \Aboxed{\vec{\Delta d_y} &= 0.158\text{m } [\downarrow]}\end{aligned}$$

2.2.3 Initial Velocity & Final Velocity (x-axis)

According to the laws of projectile motion, $\vec{v_{ix}} = \vec{v_{fx}}$.

$$\begin{aligned}\vec{\Delta d_x} &= \frac{\vec{v_{ix}} + \vec{v_{fx}}}{2} \times \Delta t \\ &= \frac{2\vec{v_x}}{2} \times \Delta t \\ &= \vec{v_x} \times \Delta t \\ 0.083\text{m } [\rightarrow] &= \vec{v_x} \times 0.17\text{s} \\ \vec{v_x} &= \frac{0.083\text{m } [\rightarrow]}{0.17\text{s}} \\ \vec{v_x} &= 0.49\text{m/s } [\rightarrow] \\ \Aboxed{\vec{v_{ix}} = \vec{v_{fx}} &= 0.49\text{m/s } [\rightarrow]}\end{aligned}$$

2.2.4 Initial Velocity (y-axis)

Since we did not push the steel ball downwards, gravity is the only force that affects it. Hence, when the steel ball is slid down from the ramp, then:

$$\boxed{\vec{v_{iy}} = 0\text{m/s } [\emptyset]}$$

2.2.5 Final Velocity (y-axis)

The final velocity in the y-axis:

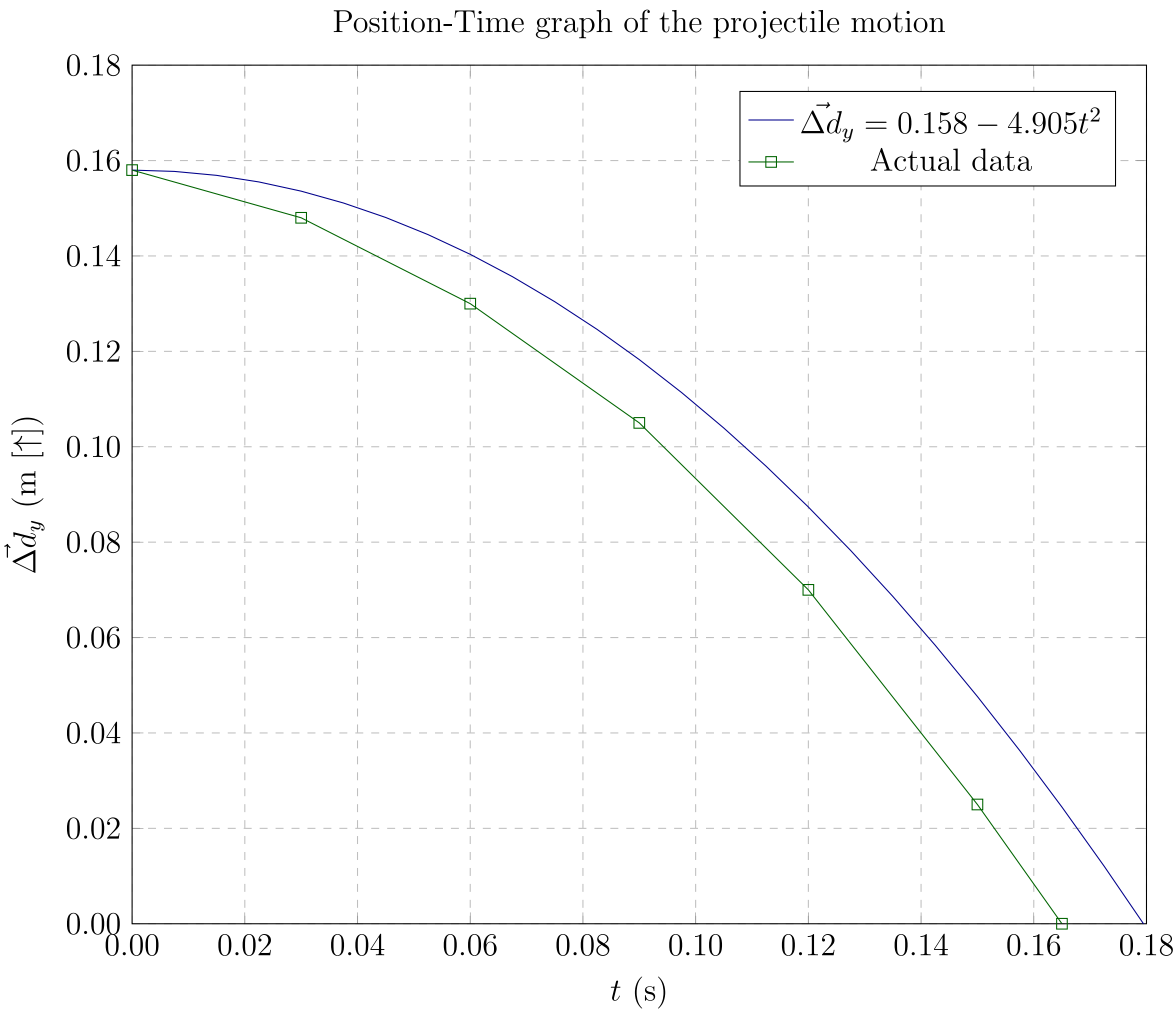
$$\begin{aligned}
\vec{\Delta d_y} &= \Delta t \left(\vec{v}_{fy} - \frac{1}{2}\vec{a}_y \Delta t \right) \\
0.158\text{m } [\downarrow] &= 0.17\text{s} \left(\vec{v}_{fy} - \frac{1}{2}\left(9.81\text{m/s}^2 \ [\downarrow]\right) 0.17\text{s} \right) \\
\frac{0.158\text{m } [\downarrow]}{0.17\text{s}} &= \vec{v}_{fy} - \frac{1}{2} \times 9.81\text{m/s}^2 \ [\downarrow] \times 0.17\text{s} \\
0.93\text{m/s } [\downarrow] &= \vec{v}_{fy} - 0.83\text{m/s } [\downarrow] \\
&\boxed{\vec{v}_{fy} = 1.76\text{m/s } [\downarrow]}
\end{aligned}$$

2.2.6 All calculations

Here are all the calculated values:

$$\begin{aligned}
\vec{\Delta d_x} &= 0.083\text{m } [\rightarrow] \\
\vec{\Delta d_y} &= 0.158\text{m } [\downarrow] \\
\vec{v_{ix}} &= 0.49\text{m/s } [\rightarrow] \\
\vec{v_{fx}} &= 0.49\text{m/s } [\rightarrow] \\
\vec{v_{iy}} &= 0\text{m/s } [\emptyset] \\
\vec{v_{fy}} &= 1.76\text{m/s } [\downarrow]
\end{aligned}$$

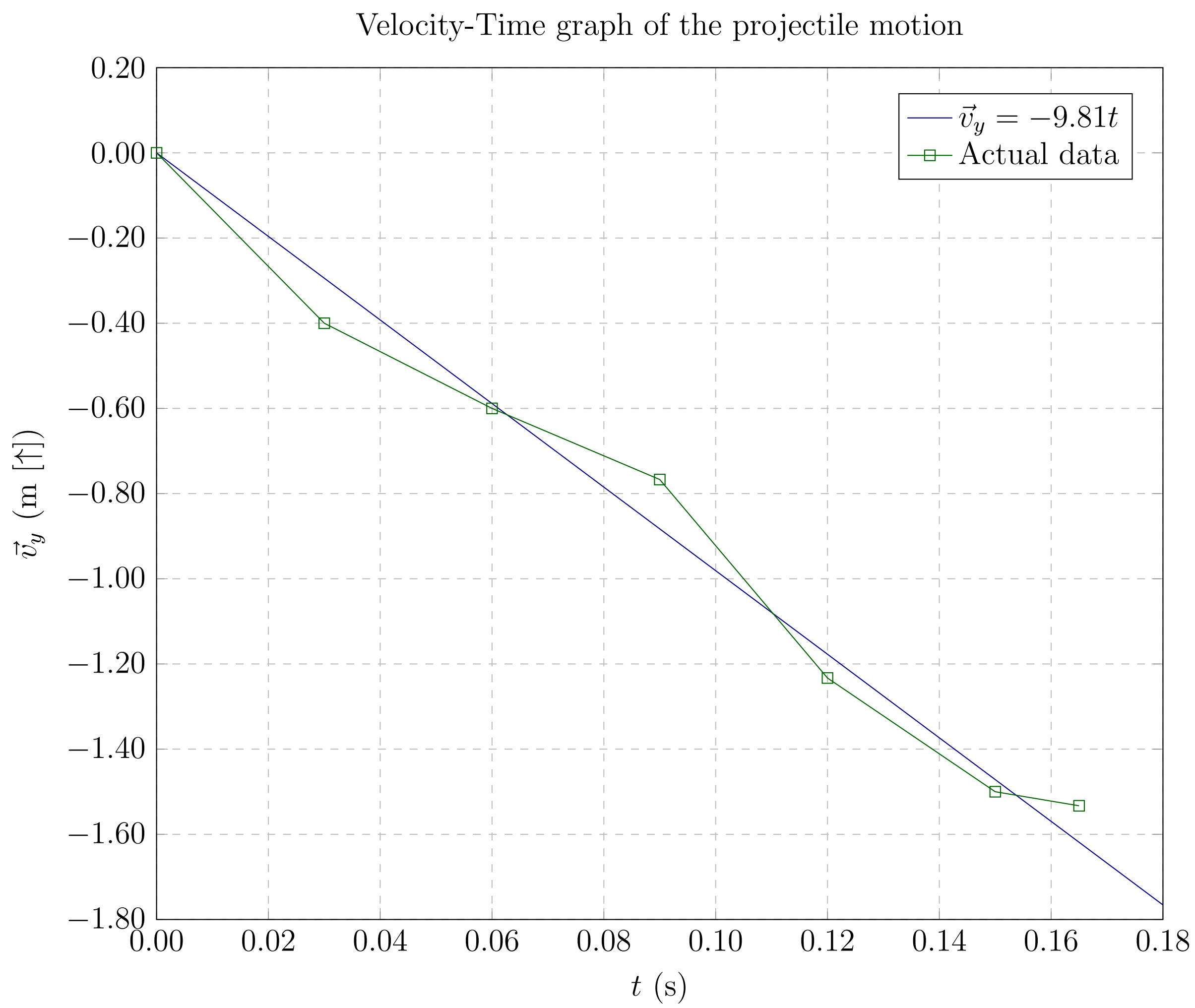
2.3 Position-Time graph



2.4 Velocity-Time graph

We calculate the velocity by calculating the slope of each interval in the position time graph:

$$
\begin{aligned}
m_0 &= 0 \\
m_1 = \frac{y_1 - y_0}{x_1 - x_0} &= \frac{0.146 - 0.158}{0.03 - 0} = \frac{-0.012}{0.03} = -0.4 \\
m_2 = \frac{y_2 - y_1}{x_2 - x_1} &= \frac{0.128 - 0.146}{0.06 - 0.03} = \frac{-0.018}{0.03} = -0.6 \\
m_3 = \frac{y_3 - y_2}{x_3 - x_2} &= \frac{0.105 - 0.128}{0.09 - 0.06} = \frac{-0.023}{0.03} \approx -0.767 \\
m_4 = \frac{y_4 - y_3}{x_4 - x_3} &= \frac{0.068 - 0.105}{0.12 - 0.09} = \frac{-0.037}{0.03} \approx -1.233 \\
m_5 = \frac{y_5 - y_4}{x_5 - x_4} &= \frac{0.023 - 0.068}{0.15 - 0.12} = \frac{-0.045}{0.03} = -1.5 \\
m_6 = \frac{y_6 - y_5}{x_6 - x_5} &= \frac{0 - 0.023}{0.165 - 0.15} = \frac{-0.023}{0.015} \approx -1.533
\end{aligned}
$$


Velocity-Time graph of the projectile motion
$\vec{v}_y = -9.81t$
Actual data
0.20
0.00
−0.20
−0.40
−0.60
−0.80
−1.00
−1.20
−1.40
−1.60
−1.80
$\vec{v}_y$ (m [↑])
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
t (s)

3 Licensing