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ICE
EIE 412

ASSIGNMENT

1. Derive the transfer function of the electro-mechanical system

solution

~~Considering the mechanical side,~~

$$\phi \propto I_f$$

$$\phi = K_1 I_f \quad \text{—} \quad (1)$$

$$T \propto \phi I_a$$

$$T = K_2 \phi I_a \quad \text{—} \quad (2)$$

Substitute equa (1) into equa (2)

$$T = K_1 K_2 I_a I_f$$

$K_1$ and $K_2$ are constants. Since, $I_f$ is from a DC source, it will also be a constant

$$T = K_T I_a \quad \text{—} \quad (3)$$

Considering the electrical side,

$$E_a = E_b + I_a R_a + L_a \frac{dI_a}{dt} \quad \text{—} \quad (4)$$

Taking the laplace transform of equa 4

$$E_{a(s)} - E_{b(s)} = I_{a(s)} R_a + s L_a I_{a(s)}$$

$$E_{a(s)} - E_{b(s)} = I_{a(s)}[R_a + sL_a]$$

Making $I_{a(s)}$ the subject of formula

$$I_{a(s)} = \frac{E_{a(s)} - E_{b(s)}}{R_a + sL_a} \quad — (5)$$

Considering the mechanical side,

$$\text{Torque, } T_m = J\ddot{\theta} + B\dot{\theta} \quad — (6)$$

$$K_T I_a = J\ddot{\theta} + B\dot{\theta} \quad — (7)$$

Taking the laplace transform of equa (7)

$$K_T I_{a(s)} = Js^2\theta_{(s)} + Bs\theta_{(s)}$$

$$T_{m(s)} = \theta_{(s)}[Js^2 + Bs] \quad — (8)$$

$$K_T I_{a(s)} = \theta_{(s)}[Js^2 + Bs] \quad — (9)$$

Substitute equa (5) in equa (9)

$$K_T\left[\frac{E_{a(s)} - E_{b(s)}}{R_a + sL_a}\right] = \theta_{(s)}[Js^2 + Bs]$$

$$\theta_{(s)}[Js^2 + Bs][R_a + sL_a] = K_T[E_{a(s)} - E_{b(s)}]$$

$E_b \propto$ rate of change of displacement

$$E_b \propto \frac{d\theta}{dt}$$

$$E_b = K_1\theta_{(s)}$$

Taking its laplace transform,

$$E_{b(s)} = K_b\, s\theta_{(s)}$$

$$\theta_{(s)}[Js^2 + sB][R_a + sL_a] = K_T[E_{a(s)} - sK_b\theta_{(s)}]$$

$$\theta_{(s)}[Js^2 + Bs][R_a + sL_a] = K_T E_{a(s)} - sK_b K_T \theta_{(s)}$$

$$\theta_{(s)}[Js^2 + Bs][R_a + sL_a] + sK_b K_T \theta_{(s)} = K_T E_{a(s)}$$

$$\theta_{(s)}\Big[[Js^2 + Bs][R_a + sL_a] + sK_b K_T\Big] = K_T E_{a(s)}$$

$$\theta_{(s)} = \frac{K_T E_{a(s)}}{[Js^2 + Bs][R_a + sL_a] + sK_b K_T}$$

The Transfer Function is :-

$$T.F = \frac{\theta_{(s)}}{E_{a(s)}} = \frac{K_T}{(Js^2 + Bs)(R_a + sL_a) + sK_b K_T}$$

2. Obtain the complete state variable representation of the translational system with 2 mass variables.

solution

For mass 1,

$$M_1 \ddot{y}_1 = U_1 - B_1(\dot{y}_1 - \dot{y}_2) - K_1(y_1 - y_2) \quad — ①$$

For mass 2,

$$M_2 \ddot{y}_2 = U_2 + B_1(\dot{y}_1 - \dot{y}_2) + K_1(y_1 - y_2) - K_2 y_2 - B_2\dot{y}_2 \quad — ②$$

From equa ①,

$$\ddot{y}_1 = \frac{U_1}{m_1} - \frac{B_1\dot{y}_1}{M_1} + \frac{B_1\dot{y}_2}{m_1} - \frac{K_1 y_1}{m_1} + \frac{K_1 y_2}{m_1}$$

From equa (2)

$$\ddot{y}_2 = \frac{U_2}{m_2} + \frac{B_1(\dot{y}_1 - \dot{y}_2)}{m_2} + \frac{k_1(y_1 - y_2)}{M_2}$$

$$- \frac{B_2\dot{y}_2}{m_2} - \frac{k_2 y_2}{m_2}$$

The state Variables are $y_1, y_2, \dot{y}_1, \dot{y}_2$. They are selected as state variables as they relate to energy storage. i.e. Velocity and height

NOTE:

$$\dot{x} = Ax(t) + Bu(t)$$

$x_1 = y_1 \qquad \dot{x}_1 = \dot{y}_1 = x_2$

$x_2 = \dot{y}_1 \qquad \dot{x}_2 = \ddot{y}_1$

$x_3 = y_2 \qquad \dot{x}_3 = \dot{y}_2 = x_4$

$x_4 = \dot{y}_2 \qquad \dot{x}_4 = \ddot{y}_2$

$$\cancel{\ddot{y}_1 = \frac{U_1}{m_1}}$$

$$\dot{x}_2 = \frac{U_1}{m_1} - \frac{B_1 x_2}{m_1} + \frac{B_2 x_4}{m_1} - \frac{k_1 x_1}{m_1} + \frac{k_1 x_3}{m_1}$$

$$\dot{x}_2 = \frac{-K_1 x_1}{m_1} - \frac{B_1 x_2}{m_1} + \frac{K_1 x_3}{m_1} + \frac{B_1 x_4}{m_1} + \frac{U_1}{m_1}$$

$$\dot{x}_1 = x_2$$

$$\ddot{y}_2 = \frac{U_2}{m_2} + \frac{B_1\dot{y}_1}{m_2} - \frac{B_1\dot{y}_2}{m_2} + \frac{k_1 y_1}{m_2} - \frac{K_1 y_2}{m_2} - \frac{K_2 y_2}{m_2}$$

$$- \frac{B_2\dot{y}_2}{m_2}$$

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$\dot{x}_4 = \ddot{y}_2$

$\frac{U_2}{m_2} + \frac{B_1 x_2}{m_2}$

$\frac{k_2 x_3}{m_2} - \frac{B_2 x}{m_2}$

$= \frac{U_2}{m_2} + \frac{B_1 x_2}{m_2}$

$\left[\frac{K_1}{m_2} + \frac{k_2}{m_2}\right]$

Recall

utting in ma

$= \begin{bmatrix} 0 \\ -k_1/m_1 \\ 0 \\ k_1/m_2 \end{bmatrix}$

$+ \begin{bmatrix} 0 \\ 1/m_1 \\ 0 \\ 0 \end{bmatrix} U_1 +$

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Recall $\dot{x}_4 = \ddot{y}_2$

$$\dot{x}_4 = \frac{U_2}{m_2} + \frac{B_1 x_2}{m_2} - \frac{B_1 x_4}{m_2} + \frac{K_1 x_1}{m_2} - \frac{K_1 x_3}{m_2} - \frac{K_2 x_3}{m_2} - \frac{B_2 x_4}{m_2}$$

$$\dot{x}_4 = \frac{U_2}{m_2} + \frac{B_1 x_2}{m_2} + \frac{K_1 x_1}{m_2} - \left[\frac{B_1}{m_2} + \frac{B_2}{m_2}\right] x_4 - \left[\frac{K_1}{m_2} + \frac{K_2}{m_2}\right] x_3$$

Recall $\dot{x}_3 = x_4$

Putting in matrix form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ x_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K_1/m_1 & -B_1/m_1 & K_1/m_1 & B_1/m_1 \\ 0 & 0 & 0 & 1 \\ K_1/m_2 & B_1/m_2 & -[K_1/m_2 + K_2/m_2] & -[B_1/m_2 + B_2/m_2] \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m_1 \\ 0 \\ 0 \end{bmatrix} U_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_2 \end{bmatrix} U_2$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

where $A =$

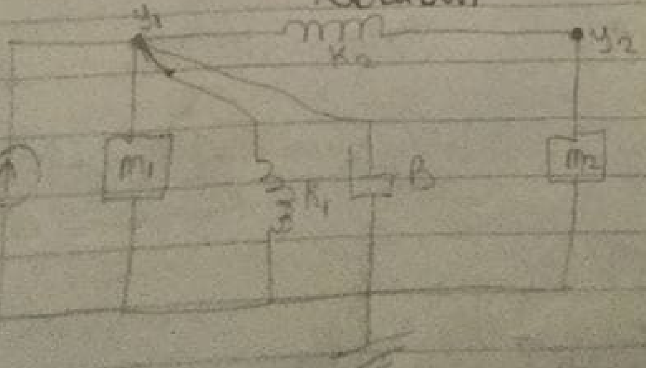
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_1/m_1 & -B_1/m_1 & k_1/m_1 & B_1/m_1 \\ 0 & 0 & 0 & 1 \\ k_1/m_2 & B_1/m_2 & -\left[k_1/m_2 + k_2/m_2\right] & -\left[B_1/m_2 + B_1/m_2\right] \end{bmatrix}$$

$$BU = \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \\ 0 & 0 \\ 0 & 1/m_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$Cx = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

3. Derive the transfer function of the translational mechanical system having 2 mass variables

solution




At Node $y_1$,
$f(t) = m_1 \ddot{y}_1 +$

Takin

both sides,

$F(s) = m_1 s^2$
$K_2[Y_1(s)$

$= Y_1(s)\,[m s^2$

At Node $y_2$,
$0 =$

Taking the

$m_2 s^2 Y_2(s)$

$K_2 Y_1(s) =$

$Y_1(s) =$

Substitute

$F(s) = \left[\frac{m_2}{}\right.$

$- K_2$

$F(s) = Y_2$

At Node $y_1$,

$$f(t) = m_1 \ddot{y}_1 + K_1 y_1 + B\dot{y} + K_2(y_1 - y_2)$$

Taking the Laplace transform of both sides,

$$F(s) = m_1 s^2 Y_1(s) + K_1 Y_1(s) + Bs Y_{1(s)} + K_2[Y_{1(s)} - Y_{2(s)}]$$

$$= Y_{1(s)}[ms^2 + Bs + K_1 + K_2] - K_2 Y_{2(s)} \quad -① $$

At Node $y_2$,

$$0 = M_2 \ddot{y}_2 + K_2(y_2 - y_1)$$

Taking the Laplace transform of both sides

$$m_2 s^2 Y_{2(s)} + K_2 Y_{2(s)} - K_2 Y_{1(s)} = 0$$

$$K_2 Y_{1(s)} = M_2 s^2 Y_{(s)} + K_2 Y_{2(s)}$$

$$Y_{1(s)} = \left[\frac{M_2 s^2 + K_2}{K_2}\right] Y_{2(s)} \quad - ②$$

Substitute equa ② into equa ①

$$F(s) = \left[\frac{M_2 s^2 + K_2}{K_2}\right] Y_{2(s)}\left[ms^2 + Bs + K_1 + K_2\right] - K_2 Y_{2(s)}$$

$$F(s) = Y_{2(s)}\left[\frac{M_2 s^2 + K_2}{K_2}(ms^2 + Bs + K_1 + K_2)\right] - K_2$$

(2) $$\frac{Y_{2(s)}}{F_{(s)}} = \frac{K_2}{M_2 s^2 + K_2 (m s^2 + B s + K_1 + K_2) - (K_2^2)}$$

$\therefore$ The transfer Function

$$\frac{Y_{2(s)}}{F_{(s)}} = T \cdot F$$

$$T \cdot F = \frac{K_2}{M_2 s^2 + K_2 (m s^2 + B s + K_1 + K_2) - K_2^2}$$