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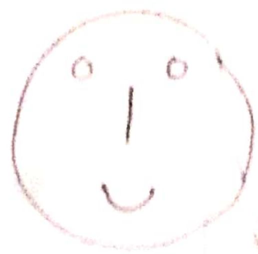

Ans to the ques no :- 01

a

$$A = \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

b

Here, $A = \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$

$$\Rightarrow A^{(1)} = \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{bmatrix} \qquad \begin{matrix} r_2' = r_2 - 3r_1 \\ r_3' = r_3 - 4r_1 \end{matrix}$$

$$\therefore F^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

And, $A^{(2)} = \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -\frac{1}{16} \end{bmatrix}$ $\quad r_3' = r_3 - \frac{19}{16} r_2$

$$\therefore F^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{19}{16} & 1 \end{bmatrix}$$

$\boxed{c}$

We know, $L = \left(F^{(1)}\right)^{-1} \cdot \left(F^{(2)}\right)^{-1}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{19}{16} & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{19}{16} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{19}{16} & 1 \end{bmatrix}$$

$\boxed{d}$

Here, $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{19}{16} & 1 \end{bmatrix}$

$$U = A^{(2)} = \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -\frac{1}{16} \end{bmatrix}$$

We know, $L \cdot Y = b$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{19}{16} & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

$\therefore y_1 = 10$

$\therefore 3y_1 + y_2 = 6$

$\Rightarrow 3 \times 10 + y_2 = 6$

$\Rightarrow y_2 = -24$

$\therefore 4y_1 + \frac{19}{16} y_2 + y_3 = 9$

$\Rightarrow 4 \times 10 + \frac{19}{16} \times (-24) + y_3 = 9$

$\therefore y_3 = -2.5$

And, $U \cdot X = Y$

$$\Rightarrow \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -\frac{1}{16} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -24 \\ -2.5 \end{bmatrix}$$

$\therefore -\frac{1}{16} x_3 = -2.5$

$\Rightarrow x_3 = 40$

$\therefore -16x_2 - 5x_3 = -24$

$\Rightarrow -16x_2 - 5 \times 40 = -24$

$\therefore x_2 = -11$

$\therefore x_1 + 6x_2 + 2x_3 = 10$

$\Rightarrow x_1 + 6 \times (-11) + 2 \times 40 = 10$

$\therefore x_1 = -4$

$\therefore x_1 = -4, \quad x_2 = -11, \quad x_3 = 40$

—o——x——o—

Ans to the ques no:-2

(a)

$$A = \begin{bmatrix} 0 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix} \begin{matrix} r_1 \\ \\ \\ \end{matrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 4 \\ 2 & 2 & 3 \end{bmatrix}$$

$r_2 = r_2 - \frac{7}{4} r_1$ $\begin{bmatrix} 4 & 5 & 2 \\ 7 & 0 & 3 \\ 4 & 2 & 1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 4 & 5 & 2 \\ 0 & -\frac{35}{4} & \end{bmatrix}$

~~b~~

Ans to the ques no:- 2

a

$$A = \begin{bmatrix} 0 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

b

Yes, matrix A has pivoting problem.

Making the lower triangle of A to 0 →

$$A = \begin{bmatrix} 0 & 6 & 2 \\ 0 & -\frac{7}{4} & -\frac{1}{2} \\ 4 & 5 & 2 \end{bmatrix} \qquad r_2' = r_2 - \frac{3}{4} r_3$$

So, we can see, to make the lower triangle 0, we must shift the rows. So, matrix ~~has~~ A has ~~pori~~ pivoting problem.

$\boxed{C}$

The Augmented matrix,

$$Aug(A) = \begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \end{array}$$

$$Aug(A) = \left[\begin{array}{ccc|c} 0 & 6 & 2 & 10 \\ 3 & 2 & 1 & 6 \\ 4 & 5 & 2 & 9 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 0 & 6 & 2 & 10 \\ 0 & -\frac{7}{4} & -\frac{1}{2} & -\frac{3}{4} \\ 4 & 5 & 2 & 9 \end{array}\right] \quad r_2' = r_2 - \frac{3}{4} r_3$$

$$= \left[\begin{array}{ccc|c} 4 & 5 & 2 & 9 \\ 0 & -\frac{7}{4} & -\frac{1}{2} & -\frac{3}{4} \\ 0 & 0 & \frac{2}{7} & \frac{52}{7} \end{array}\right] \quad \begin{array}{l} r_1 \leftrightarrow r_3 \\ r_3'' = r_3' + \frac{24}{7} r_2 \end{array}$$

In this point, all the values of the lower triangle is 0. So, the remaining matrix is a upper triangle matrix U.

$$\therefore U = \begin{bmatrix} 4 & 5 & 2 \\ 0 & -\frac{7}{4} & -\frac{1}{2} \\ 0 & 0 & \frac{2}{7} \end{bmatrix}$$

(Ans)

d

The achieved Augmented matrix in upper triangle form ⇒

$$\left[\begin{array}{ccc|c} 4 & 5 & 2 & 9 \\ 0 & -\frac{7}{4} & -\frac{1}{2} & -\frac{3}{4} \\ 0 & 0 & \frac{2}{7} & \frac{52}{7} \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 4 & 5 & 2 & 9 \\ 0 & 1 & \frac{2}{7} & \frac{3}{7} \\ 0 & 0 & 1 & 26 \end{array}\right] \qquad r_2' = \frac{r_2}{\frac{7}{4}}, \quad r_3' = \frac{r_3}{\frac{2}{7}}$$

$$\cancel{= 4 \quad 4 \quad \frac{12}{7} \quad \frac{60}{7}} \quad \cancel{0} \qquad \cancel{r_1' = r_1 - r_2} \quad \cancel{r_2' = r_2 - r_3}$$

$$= \left[\begin{array}{ccc|c} 4 & 0 & \frac{4}{7} & \frac{48}{7} \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 26 \end{array}\right] \qquad r_1' = r_1 - 5r_2, \quad r_2' = r_2 - \frac{2}{7} r_3$$

$$r_1' = \frac{r_1 - \frac{4}{7} r_3}{4}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 26 \end{array}\right]$$

Ans:- $x_1 = -2$

$x_2 = -7$

$x_3 = 26$