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Ans to the ques no:- 01

$\boxed{a}$

For conditional independence, $P(\text{cheat} \cap \text{pass} \mid \text{study})$ should be equal to $P(\text{cheat} \mid \text{study}) \cdot P(\text{pass} \mid \text{study})$.

Here, $$P(\text{cheat} \cap \text{pass} \mid \text{study}) = \frac{P(\text{cheat} \cap \text{pass} \cap \text{study})}{P(\text{study})}$$

$$= \frac{0.25}{0.25 + 0.15 + 0.02 + 0.03}$$

$$= \frac{5}{9} \approx 0.56$$

Now, $$P(\text{cheat} \mid \text{study}) \cdot P(\text{Pass} \mid \text{study}) = \frac{P(\text{study} \mid \text{cheat})\, P(\text{cheat})}{P(\text{study})} \times \frac{P(\text{study} \mid \text{Pass}) \cdot P(\text{pass})}{P(\text{study})}$$

$$= \frac{\cancel{0.25}\,(0.25 + 0.02)(0.25 + 0.02 + 0.1 + 0.22)}{0.25 + 0.15 + 0.02 + 0.03} \times \frac{(0.25 + 0.15)(0.25 + 0.15 + 0.1 + 0.13)}{0.25 + 0.15 + 0.02 + 0.03}$$

$$\approx 0.198$$

Ans. $P(\text{cheat} \cap \text{pass} | \text{study}) \neq P(\text{cheat} | \text{study}) \cdot P(\text{pass} | \text{study})$

Therefore, cheat and pass are not conditionally independent given study.

$\boxed{b}$

$P(\text{pass} \cup \text{cheat}) = P(\text{pass}) + P(\text{cheat}) - P(\text{pass} \cap \text{cheat})$

$= (0.25 + 0.15 + 0.10 + 0.13) + (0.25 + 0.02 + 0.10 + 0.22) - (0.25 + 0.10)$

$= 0.87$

(Ans)

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Ans to the ques no:- 2

a

$$P(\sim cold) = 0.12 + 0.04 + 0.10 + 0.07$$

$$= 0.33$$

(Ans)

b

$$P(\neg cloudy \mid \neg Rain \cap \neg cold) = \frac{P(\neg cloudy \cap \neg Rain \cap \neg cold)}{P(\neg Rain \cap \neg cold)}$$

$$= \frac{0.07}{0.04 + 0.07}$$

$$= \frac{7}{11} \approx 0.636$$

(Ans)

c

$$P(\neg Rain \mid \neg cloudy) = \frac{P(\neg Rain \cap \neg cloudy)}{P(\neg cloudy)}$$

$$= \frac{0.03 + 0.07}{0.26 + 0.03 + 0.10 + 0.07}$$

$$= \frac{5}{23} \approx 0.217$$

(Ans)

$\boxed{d}$

$$P(\neg Rain \cup cloudy) = P(\neg Rain) + P(cloudy) - P(\neg Rain \cap cloudy)$$

$$= (0.06 + 0.04 + 0.03 + 0.07) + (0.32 + 0.12 + 0.06 + 0.04)$$

$$- (0.06 + 0.04)$$

$$= 0.64$$

(Ans)

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Ans to the ques no:- 3

$\boxed{a}$

~~P(Football ∩ Left-handed) = 0.15~~

$$P(Football \mid Left\text{-}handed) = \frac{P(Football \cap Left\text{-}handed)}{P(Left\text{-}handed)}$$

$$= \frac{0.15}{0.24 + 0.15 + 0.15}$$

$$= \frac{5}{18} \approx 0.278$$

(Ans)

$\boxed{b}$

$$P(\text{Right-handed} \mid \text{Cricket}) = \frac{P(\text{Right-handed} \cap \text{Cricket})}{P(\text{Cricket})}$$

$$= \frac{0.1}{0.24 + 0.1}$$

$$= \frac{5}{17} \approx 0.294$$

(Ans)

$\boxed{c}$

$$\cancel{P(\text{Football} \cup \text{Cricket}) = P(\text{Football}) + P(\text{Cricket}) - P(\text{Football} \cap \text{Cricket})}$$

$$\cancel{= (0.15 + 0.1) + (0.24 + 0.1) - 0}$$

$$\cancel{= 0.59}$$

~~(Ans)~~

$$P(\text{Football} \cap \text{Cricket}) = P(\text{Football}) + P(\text{Cricket}) - P(\text{Football} \cup \text{Cricket})$$

$$= (0.15+0.1) + (0.24+0.1) - (0.24+0.1+0.15+0.1)$$

$$= 0$$

(Ans)

$\boxed{d}$

$P(\text{Right-handed} \cup \text{Left-handed})$

$$= P(\text{Right-handed}) + P(\text{Left-handed}) - P(\text{Right-handed} \cap \text{Left-handed})$$

$$= (0.1+0.1+0.26) + (0.24+0.15+0.15) - 0$$

$$= 1$$

(Ans)

$\boxed{e}$

Here, $P(\text{Football} \cap \text{Right-handed}) = 0.1$

And, $P(\text{Football}) \cdot P(\text{Right-handed}) = (0.15+0.1) * (0.1+0.1+0.26)$

$$= 0.115$$

As, $P(\text{Football} \cap \text{Right-handed}) \neq P(\text{Football}) \cdot P(\text{Right-handed})$

Therefore, playing football ~~does not~~ depends on being right-handed.

Ans to the ques no:- 4

Let,

Machine detects lie = D

Person tells lie = P

Here,

$P(D \mid P) = 0.96$ $\quad \therefore P(\neg D \mid P) = 1 - 0.96 = 0.04$

$P(\neg D \mid \neg P) = 0.95$ $\quad \therefore P(D \mid \neg P) = 1 - 0.95 = 0.05$

$P(P) = 0.02$ $\quad \therefore P(\neg P) = 1 - 0.02 = 0.98$

Event of machine = D

$\boxed{a}$

$$P(P \mid D) = \frac{P(D \mid P) \cdot P(P)}{P(D)} = \frac{0.96 \times 0.02}{P(D)} = \frac{0.0192}{P(D)}$$

$$P(\neg P \mid D) = \frac{P(D \mid \neg P) \cdot P(\neg P)}{P(D)} = \frac{0.05 \times 0.98}{P(D)} = \frac{0.049}{P(D)}$$

$\because \frac{0.0192}{P(D)} < \frac{0.049}{P(D)}$

So, it is more likely that the person is not a liar.

$\boxed{b}$

Here, $P(\neg P \mid D) = \frac{P(D \mid \neg P) \cdot P(\neg P)}{P(D)}$

$= \frac{0.05 \times 0.98}{P(D \cap P) + P(D \cap \neg P)}$

$= \frac{0.05 \times 0.98}{P(D \mid P) \cdot P(P) + P(D \mid \neg P) \cdot P(\neg P)}$

$= \frac{0.05 \times 0.98}{0.96 \times 0.02 + 0.05 \times 0.98}$

≈ 0.718

(Ans)

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CamScanner

Ans to the question no:- 5

Bayes theorem is complicated when the dataset is quite large and we have to find joint probability under some condition. Bayes theorem's this issue is solved by Naive Bayes by introducing conditional independence among the ~~indepe~~ features given the output variable.

For example, lets consider the dataset below:-

Outlook	Wind	Play
Sunny	False	No
Sunny	True ~~False~~	No
Rainy	False	Yes
Sunny	True	Yes

Here, outlook and wind are features and play is output variable

~~Now, if our query is $P(Sunny \cap False \mid Yes)$, that is probability of play = Yes outlook = Sunny and wind = False when Play = Yes.~~

~~Following By Bayes theorem, P(sunny ∩~~

Now, we want to find the probability of play = Yes when outlook = Sunny and wind = False, that is, $P(\text{Yes} \mid \text{sunny} \cap \text{False})$.

Applying Naive Bayes, $P(\text{Yes} \mid \text{sunny} \cap \text{False})$

$$= \frac{P(\text{sunny} \cap \text{False} \mid \text{Yes}) \cdot P(\text{Yes})}{P(\text{sunny} \cap \text{False})}$$

Here, we come across 0 probability problem because $P(\text{Sunny} \cap \text{False} \mid \text{Yes}) = 0$.

This problem is solved by Naive Bayes by introducing conditional probability.

$$\therefore P(\text{Yes} \mid \text{sunny} \cap \text{False}) = \frac{P(\text{Sunny} \mid \text{Yes}) \cdot P(\text{False} \mid \text{Yes}) \cdot P(\text{Yes})}{P(\text{sunny} \cap \text{False})}$$

$$= \frac{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{2}{4}}{\frac{1}{4}}$$

$$= \frac{1}{8} \times \frac{4}{1} = \frac{1}{2}$$

So, we get a value for playing in this

process which solves the problem of Bayes theorem.

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Ans to the ques no:- 6

[a]

P(Yes | Sunny ∩ Mild ∩ Normal ∩ windy).

$$= \frac{P(\text{Sunny} \cap \text{Mild} \cap \text{Normal} \cap \text{windy} \mid \text{Yes}) \cdot P(\text{Yes})}{P(\text{Sunny} \cap \text{Mild} \cap \text{Normal} \cap \text{windy})}$$

$$= \frac{P(\text{Sunny} \mid \text{Yes}) \cdot P(\text{mild} \mid \text{Yes}) \cdot P(\text{Normal} \mid \text{Yes}) \cdot P(\text{windy} \mid \text{Yes}) \cdot P(\text{Yes})}{P(\text{sunny} \cap \text{mild} \cap \text{Normal} \cap \text{windy})}$$

Similarly,

P(No | Sunny ∩ Mild ∩ Normal ∩ windy)

$$= \frac{P(\text{Sunny} \mid \text{No}) \cdot P(\text{mild} \mid \text{No}) \cdot P(\text{Normal} \mid \text{No}) \cdot P(\text{windy} \mid \text{No}) \cdot P(\text{No})}{P(\text{Sunny} \cap \text{Mild} \cap \text{Normal} \cap \text{windy})}$$

Now, the learning phase:-

Outlook	PlayTennis = Yes	PlayTennis = No
Overcast	$\frac{2}{5}$	$\frac{0}{3}$
Sunny	$\frac{2}{5}$	$\frac{3}{3}$
Rainy	$\frac{1}{5}$	$\frac{0}{3}$

Humidity	Yes	No
cool	$\frac{2}{5}$	$\frac{0}{3}$
mild	$\frac{3}{5}$	$\frac{2}{3}$
Hot	$\frac{0}{5}$	$\frac{1}{3}$

Temp	Yes	No
Normal	$\frac{4}{5}$	$\frac{0}{3}$
High	$\frac{1}{5}$	$\frac{3}{3}$

Wind	Yes	No
True	$\frac{3}{5}$	$\frac{1}{3}$
False	$\frac{2}{5}$	$\frac{2}{3}$

$\therefore P(Yes \mid Sunny \cap mild \cap Normal \cap windy)$

$$= \frac{\frac{2}{5} \cdot \frac{3}{5} \cdot \frac{4}{5} \cdot \frac{3}{5} \cdot \frac{5}{8}}{P(Sunny \cap mild \cap Normal \cap windy)}$$

$$= \frac{0.072}{P(Sunny \cap mild \cap Normal \cap windy)}$$

And, $P(No \mid Sunny \cap mild \cap Nonormal \cap windy)$

$$= \frac{\frac{3}{3} \cdot \frac{2}{3} \cdot \frac{0}{3} \cdot \frac{1}{3} \cdot \frac{3}{8}}{P(Sunny \cap mild \cap Normal \cap windy)}$$

$$= \frac{0}{P(Sunny \cap mild \cap Normal \cap windy)}$$

$\therefore P(\text{Yes} | \text{Sunny} \cap \text{mild} \cap \text{Normal} \cap \text{windy}) > P(\text{No} | \text{sunny} \cap \text{mild} \cap \text{Normal} \cap \text{windy})$

So, it is more likely that the player is going to play tennis.

b)

$$P(\text{Yes} | \text{Overcast} \cap \text{Hot} \cap \text{Normal} \cap \text{windy})$$

$$= \frac{P(\text{Overcast} | \text{Yes}) \cdot P(\text{Hot} | \text{Yes}) \cdot P(\text{Normal} | \text{Yes}) \cdot P(\text{windy} | \text{Yes}) \cdot P(\text{Yes})}{P(\text{Overcast} \cap \text{Hot} \cap \text{Normal} \cap \text{windy})}$$

$$= \frac{\frac{2}{5} \cdot \frac{0}{5} \cdot \frac{4}{5} \cdot \frac{3}{5} \cdot \frac{5}{8}}{P(\text{overcast} \cap \text{Hot} \cap \text{Normal} \cap \text{windy})}$$

$$= \frac{0}{P(\text{Overcast} \cap \text{Hot} \cap \text{Notmal} \cap \text{windy})}$$

Similarly,

$$P(\text{No} | \text{Overcast} \cap \text{Hot} \cap \text{Normal} \cap \text{windy})$$

$$= \frac{P(\text{Overcast} \mid \text{No}) \cdot P(\text{Hot} \mid \text{No}) \cdot P(\text{Normal} \mid \text{No}) \cdot P(\text{windy} \mid \text{No}) \cdot P(\text{No})}{P(\text{Overcast} \cap \text{Hot} \cap \text{Normal} \cap \text{windy})}$$

$$= \frac{\frac{0}{3} \cdot \frac{1}{3} \cdot \frac{0}{3} \cdot \frac{1}{3} \cdot \frac{3}{8}}{P(\text{Overcast} \cap \text{Hot} \cap \text{Normal} \cap \text{windy})}$$

$$= \frac{0}{P(\text{Overcast} \cap \text{Hot} \cap \text{Normal} \cap \text{windy})}$$

$\because P(\text{Yes} \mid \text{Overcast} \cap \text{Hot} \cap \text{Normal} \cap \text{Windy}) = P(\text{No} \mid \text{Overcast} \cap \text{Hot} \cap \text{Normal} \cap \text{windy})$

Therefore the result is inconclusive, as both are same.

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Ans to the ques no. 7

$\boxed{a}$

$$E(\text{Edible}) = -\frac{9}{16}\log_2\frac{9}{16} - \frac{7}{16}\log_2\frac{7}{16}$$

~~= 0.987~~ ~~= 9~~

$= 0.989$ (Ans)

$\boxed{b}$

For Color :-

$$E(\text{Edible} \mid \text{Color} = \text{Yellow}) = -\frac{8}{13}\log_2\frac{8}{13} - \frac{5}{13}\log_2\frac{5}{13}$$

$= 0.961$

$$E(\text{Edible} \mid \text{Color} = \text{Green}) = -\frac{1}{3}\log_2\frac{1}{3} - \frac{2}{3}\log_2\frac{2}{3}$$

$= 0.918$

$$\therefore IG(\text{Edible} \mid \text{Color}) = 0.989 - \left(\frac{13}{16} \times 0.961 + \frac{3}{16} \times 0.918\right)$$

$= 0.036$

~~IG~~ For Size:-

$$E(\text{Edible} \mid \text{Size} = \text{Small}) = -\frac{6}{8}\log_2\frac{6}{8} - \frac{2}{8}\log_2\frac{2}{8}$$

$$= 0.811$$

$$E(\text{Edible} \mid \text{Size} = \text{Large}) = -\frac{3}{8}\log_2\frac{3}{8} - \frac{5}{8}\log_2\frac{5}{8}$$

$$= 0.954$$

$$\therefore IG(\text{Edible} \mid \text{size}) = 0.989 - \left(\frac{8}{16} \times 0.811 + \frac{8}{16} \times 0.954\right)$$

$$= 0.1065$$

For Shape:-

$$E(\text{Edible} \mid \text{Shape} = \text{Round}) = -\frac{6}{12}\log_2\frac{6}{12} - \frac{6}{12}\log_2\frac{6}{12}$$

$$= 1$$

$$E(\text{Edible} \mid \text{Shape} = \text{Irregular}) = -\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4}$$

$$= 0.811$$

$$\therefore IG(\text{Edible} \mid \text{Shape}) = 0.989 - \left(\frac{12}{16} \times 1 + \frac{4}{16} \times 0.811\right)$$

$$= 0.03625$$

As the information gain of 'size' is largest, it is the better feature.

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