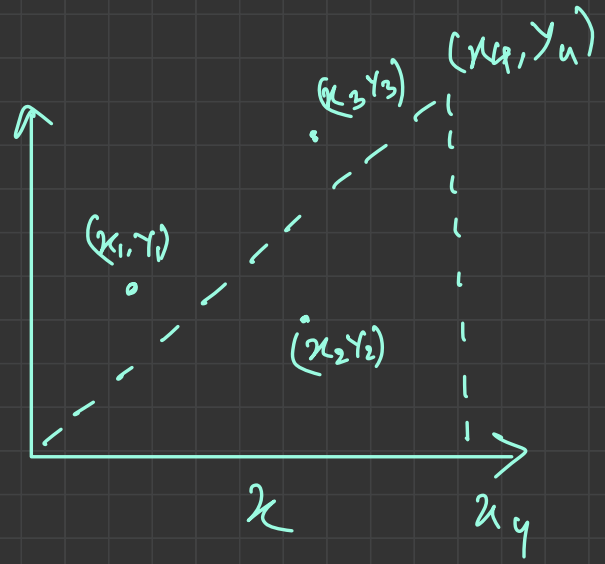
Regression Analysis

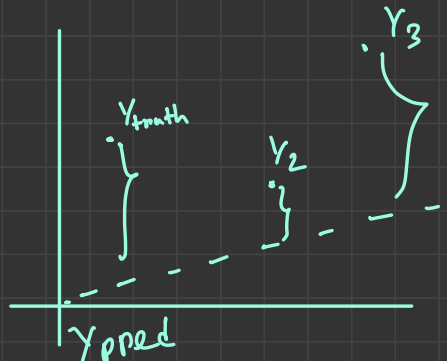
X	Y
x_1	y_1
x_2	y_2
x_3	y_3
x_4	?



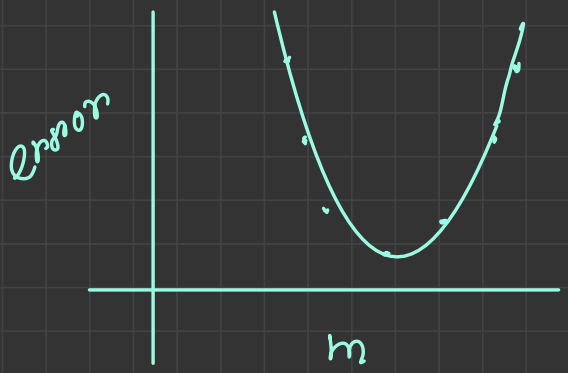
$$Y = mx + c$$

$$\Rightarrow Y = mx \quad [\text{if } c = 0]$$

if we peek a random m value



$$\text{error} = \sum_{i=1}^{3} \left(y_{i\,truth} - y_{i\,pred} \right)^2$$



$\frac{d}{dx}(\text{error})$

$$\text{error} = (Y_{i\text{truth}} - Y_{i\text{pred}})^2$$

$$= (y - (mx+b))^2$$

$$= (y - mx - b)^2$$

Partial derivative,

$$\frac{\partial}{\partial b}(\text{error}) = 2(y - mx - b) \cdot (-1) \cdot (b^{(-1)})$$

$$= -2(y - mx - b)$$

$$= 2(b + mx - y)$$

$$\frac{\partial}{\partial m}(err) = (y - mx - b)^2$$

$$= 2(y - mx - b) \cdot (-x)$$

$g_b \downarrow$

$$\frac{\partial}{\partial b}(err) = \frac{1}{2N} \cancel{2} \sum_{i=1}^{N} (b + mx - y)$$

$$= \frac{1}{N} \sum_{i=1}^{N} (b + mx - y)$$

$g_m \downarrow$

$$\frac{\partial}{\partial m}(err) = \frac{1}{2N} \cancel{2} \sum_{i=1}^{N} (Y_i - mx_i - b_i) \times (-x_i)$$

$$= N \sum_{i=1}^{N} (-x_i)(Y_i - mx_i - b)$$

#

x	Y
1	9
2	22
3	28
4	38
5	?

$$Y = mx + b$$

$$\text{error} = \frac{1}{N}\sum_{i=1}^{N}\left(Y_{\text{truth}} - mx - b\right)^2$$

Random

$\hookrightarrow m = 6,\ b = 2$

$$\text{error} = \frac{1}{4}\left[\left\{9-(6\times 1)-2\right\}^2 + \left\{22-(6\times 2)-2\right\}^2 + \left\{28-6\times 3-2\right\}^2 + \left\{38-6\ 4-2\right\}^2\right]$$

$$= 68.25$$

$$g_b = \frac{1}{4}\left[\left\{2+(6\times 1)-9\right\} + \left\{2+(6\times 2)-22\right\} + \left\{2+(6\times 3)-28\right\} + \left\{(2+(6\times 4)-38)\right\}\right]$$

$$= -7.25$$

$$g_m = \frac{1}{4}\left[(-1)\{9-(6\times 1)-2\} + (-2)\{22-(6\times 2)-2\} + (-3)\{28-(6\times 3)-2\} + (-4)\{38-(6\times 4)-2\}\right]$$

$$= -22.25$$

$$m_{new} = m_{old} - l_r g_m$$

$$= 6 - 0.1(-22.25)$$

$$= 8.225$$

↳ learning rate to minimize the value

$$b_{new} = b_{old} - l_r g_b$$

$$= 2 - 0.1(-7.25)$$

$$= 2.725$$