CSE427

Thursday, March 7, 2024 4:52 AM

#Gradient descent:-

$$\sum_{i=1}^{n} \left\{Y_i - (m_1 A_i + m_2 B_i + c)\right\}^2 = R$$

A	B	Y
1	4	3
2	4	5
3	3	4

$$\therefore \frac{\partial R}{\partial m_1} = 2\left\{Y_1 - (m_1 A_1 + m_2 B_1 + c)\right\} \times (-A_1) + \cdots$$

$$\therefore \frac{\partial R}{\partial m_2} = 2\left\{Y_1 - (m_1 A_1 + m_2 B_1 + c)\right\} \times (-B_1) + \cdots$$

$$\therefore \frac{\partial R}{\partial c} = 2\left\{Y_1 - (m_1 A_1 + m_2 B_1 + c)\right\} \times (-1) + \cdots$$

To start with, lets take, $m_1 = 1$

$m_2 = 1$

$c = 0$

$$\therefore \frac{\partial R}{\partial m_1} = 2\left\{3 - (1\times1 + 1\times4 + 0)\right\}\times(-1) + 2\left\{5 - (1\times2 + 1\times4 + 0)\right\}\times(-2) + 2\left\{4 - (1\times3 + 1\times3 + 0)\right\}\times(-3)$$

$$= 4 + 4 + 12 = 20$$

$$\therefore \frac{\partial R}{\partial m_2} = 2\left\{3 - (1\times1 + 1\times4 + 0)\right\}\times(-4) + 2\left\{5 - (1\times2 + 1\times4 + 0)\right\}\times(-4) + 2\left\{4 - (1\times3 + 1\times3 + 0)\right\}\times(-3)$$

$$= 16 + 8 + 12 = 36$$

$$\therefore \frac{\partial R}{\partial c} = 2\left\{3 - (1\times1 + 1\times4 + 0)\right\}\times(-1) + 2\left\{5 - (1\times2 + 1\times4 + 0)\right\}\times(-1) + 2\left\{4 - (1\times3 + 1\times3 + 0)\right\}\times(-1)$$

$$= 4 + 2 + 4 = 10$$

Now, the updated values, $m_1 = 1 - 0.1 \times 20 = -1$

$m_2 = 1 - 0.1 \times 36 = -2.6$

$c = 0 - 0.1 \times 10 = -1$

Now calculate again using these values

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Predicted values   [36066.13 34163.25 66512.21]
Real values        [46205. 39343. 61111.]
Trained W          9514.4
Trained b          23697.41
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Predicted values   [36335.86 34445.01 66589.52]
Real values        [46205. 39343. 61111.]
Trained W          9454.27
Trained b          24045.32
```

Decision Tree (GINI Impurity):-

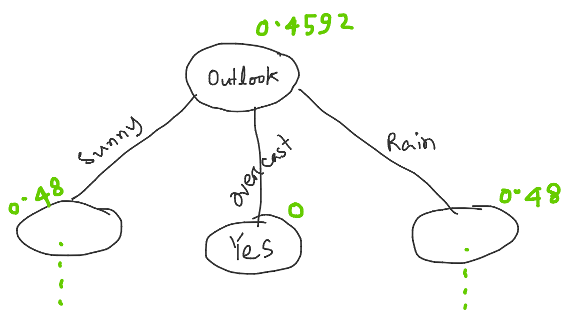
Information Gain, $\boxed{IG = 1 - \sum p_i^2} = GINI(D)$

Total Gain due to feature A, $GINI_A(D) = \frac{|D_1|}{|D|} \times GINI(D_1) + \frac{|D_2|}{|D|} \times GINI(D_2)$

GINI reduction, $\Delta GINI(A) = GINI(D) - GINI_A(D)$

$\boxed{\text{Ex:-}}$

Day	Outlook	Temparature	Humidity	Wind	Play Tennis
Day1	Sunny	Hot	High	Weak	No
Day2	Sunny	Hot	High	Strong	No
Day3	Overcast	Hot	High	Weak	Yes
Day4	Rain	Mild	High	Weak	Yes
Day5	Rain	Cool	Normal	Weak	Yes
Day6	Rain	Cool	Normal	Strong	No
Day7	Overcast	Cool	Normal	Strong	Yes
Day8	Sunny	Mild	High	Weak	No
Day9	Sunny	Cool	Normal	Weak	Yes
Day10	Rain	Mild	Normal	Weak	Yes
Day11	Sunny	Mild	Normal	Strong	Yes
Day12	Overcast	Mild	High	Strong	Yes
Day13	Overcast	Hot	Normal	Weak	Yes
Day14	Rain	Mild	High	Strong	No



Step 1: Initial GINI

Step 2:- Calculate GINI for each feature

Step 3:- Take the weighted sum

Step 4:- Calculate the reduction

Step 5:- Best feature = Highest reduction

Initial GINI $= 1 - \left(\left(\frac{9}{14}\right)^2 + \left(\frac{5}{14}\right)^2\right)$

$= \frac{45}{98} \approx 0·4592$

For outlook,

$GINI(sunny) = 1 - \left\{\left(\frac{2}{5}\right)^2 + \left(\frac{3}{5}\right)^2\right\} = \frac{12}{25} = 0·48$

$GINI(overcast) = 1 - \left\{\left(\frac{4}{4}\right)^2 + \left(\frac{0}{4}\right)^2\right\} = 0$

$GINI(Rain) = 1 - \left\{\left(\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)^2\right\} = 0·48$

$Reduction = 0·4592 - \left\{\frac{5}{14} \times 0·48 + \frac{4}{14} \times 0 + \frac{5}{14} \times 0·48\right\}$

$= \frac{509}{4375} \approx 0·1163$

For Temperature,

$GINI(Hot) = 1 - \left\{\left(\frac{2}{4}\right)^2 + \left(\frac{2}{4}\right)^2\right\} = 0·5$

$GINI(Mild) = 1 - \left\{\left(\frac{4}{6}\right)^2 + \left(\frac{2}{6}\right)^2\right\} = \frac{4}{9} \approx 0·44$

$GINI(Cool) = 1 - \left\{\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right\} = 0·375$

$Reduction = 0·4592 - \left\{\frac{4}{14} \times 0·5 + \frac{6}{14} \times 0·44 + \frac{4}{14} \times 0·375\right\}$

$= \frac{361}{17500} \approx 0·0206$

For Humidity,

$GINI(High) = 1 - \left\{\left(\frac{3}{7}\right)^2 + \left(\frac{4}{7}\right)^2\right\} = \frac{24}{49} \approx 0·4898$

$GINI(Normal) = 1 - \left\{\left(\frac{6}{7}\right)^2 + \left(\frac{1}{7}\right)^2\right\} = \frac{12}{49} \approx 0·2449$

$\therefore Reduction = 0·4592 - \left\{\frac{7}{14} \times 0·4898 + \frac{7}{14} \times 0·2449\right\}$

$= 0·09185$

For wind,

Step 5: Best feature ...

Step 6:- Make the decision node with the best feature and take the decisions

Step 7: Go to step 1 for every non-leaf node. Leaf node has 0 GINI Impurity.

... 0.0183

For wind,

$GINI(weak) = 1 - \left\{\left(\frac{6}{8}\right)^2 + \left(\frac{2}{8}\right)^2\right\} = 0.375$

$GINI(Strong) = 1 - \left\{\left(\frac{3}{6}\right)^2 + \left(\frac{3}{6}\right)^2\right\} = 0.5$

$\therefore Reduction = 0.4592 - \left\{\frac{8}{14} \times 0.375 + \frac{6}{14} \times 0.5\right\}$

$= \frac{134}{4375} \approx 0.0306$

Least square linear regression model :-

Find, X, Y, XY, X^2

Also, their sum (Σ).

$$\therefore m = \frac{n \cdot \Sigma xY - \Sigma x \cdot \Sigma y}{n \cdot \Sigma x^2 - (\Sigma x)^2}$$

$$\therefore c = \frac{\Sigma Y - m \cdot \Sigma X}{n}$$

Naive Bayes :-

$$P(Y \mid A \cap B \cap C) = \frac{P(A \cap B \cap C \mid Y) \times P(Y)}{P(A \cap B \cap C)}$$

$$= \frac{P(A \mid Y)\ P(B \mid Y)\ P(C \mid Y) \cdot P(Y)}{P(A \cap B \cap C)}$$

Conditional Independence

For continuous values:- First take the mean (μ) and standard deviation (σ)

$P(Y \mid A \cap B \cap C) = \ldots$ (Like before)

$$P(A \mid Y) \longrightarrow P(\underbrace{x}_{\text{continous value}} \mid \underbrace{c}_{\text{Yes/No of a variable}}) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

$$\mu = \frac{\sum x_i}{n}$$

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{n-1}$$

Calculate μ and σ for Yes and No separately

PCA (Principal Component Analysis):-

x	y
4	11
8	4
13	5
7	14

Here, $\bar{x} = 8$

$\bar{y} = 8.5$

$x-\bar{x}$	$y-\bar{y}$
-4	2.5
0	-4.5
5	-3.5
-1	5.5

Covariance matrix :-

$$
\begin{array}{c|cc}
 & x & y \\
\hline
x & & \\
y & &
\end{array}
\begin{bmatrix} Cov(x,x) & cov(x,y) \\ cov(y,x) & cov(y,y) \end{bmatrix}
$$

$$
= \begin{bmatrix} \dfrac{(-4)^2 + 0 + 5^2 + (-1)^2}{3} & \dfrac{(2.5\times-4) + 0 + (5\times-3.5) + (-1\times5.5)}{3} \\ & \dfrac{(2.5)^2 + (-4.5)^2 + (-3.5)^2 + (5.5)^2}{3} \end{bmatrix}
$$

$$
= \begin{bmatrix} 14 & -11 \\ -11 & 23 \end{bmatrix}
$$

Now finding the eigenvalues and eigenvectors $\rightarrow$

$$
\begin{bmatrix} 14-\lambda & -11 \\ -11 & 23-\lambda \end{bmatrix} \Rightarrow (14-\lambda)(23-\lambda) - (-11\times-11)
$$

$$\Rightarrow 322 - 14\lambda - 23\lambda + \lambda^2 - 121$$

$$\Rightarrow \boxed{\lambda^2 - 37\lambda + 201} = 0$$

$$\therefore \lambda = \frac{-(-37) \pm \sqrt{(-37)^2 - 4\times1\times201}}{2}$$

$$= 30.385 ,\quad 6.615$$

For $\lambda = 30.385$,

$$
\begin{bmatrix} 14-30.385 & -11 \\ -11 & 23-30.385 \end{bmatrix} = \begin{bmatrix} -16.385 & -11 \\ -11 & -7.385 \end{bmatrix}
$$

$$\begin{bmatrix} -11 & 23-30.385 \end{bmatrix} = \begin{bmatrix} -11 & -7.385 \end{bmatrix}$$

$\therefore -16.385\, u_1 - 11 u_2 = 0$

$\Rightarrow -16.385\, u_1 = 11 u_2 \Rightarrow \dfrac{u_1}{11} = \dfrac{u_2}{-16.385} = \boxed{t = 1}$

$\therefore u_1 = 11, \quad u_2 = -16.385$

$\therefore$ eigenevector $= \begin{bmatrix} 11 / \sqrt{11^2 + (-16.385)^2} \\ -16.385 / \quad " \end{bmatrix}$

$= \begin{bmatrix} 0.557\cancel{4} \\ -0.8303 \end{bmatrix}$

For, $\lambda = 6.615$

$$\begin{bmatrix} 14-6.615 & -11 \\ -11 & 23-6.615 \end{bmatrix} = \begin{bmatrix} 7.385 & -11 \\ -11 & 16.385 \end{bmatrix}$$

$\therefore 7.385\, u_1 - 11 u_2 = 0 \Rightarrow 7.385\, u_1 = 11 u_2$

$\Rightarrow \dfrac{u_1}{11} = \dfrac{u_2}{7.385} = \boxed{t = 1}$

$\therefore u_1 = 11, \quad u_2 = 7.385$

$\therefore e_2 = \begin{bmatrix} 11 / \sqrt{11^2 + 7.385^2} \\ 7.385 / \quad " \end{bmatrix}$

$= \begin{bmatrix} 0.8302 \\ 0.5574 \end{bmatrix}$

$\therefore \lambda_1 = 30.385$

$e_1 = \begin{bmatrix} 0.5574 \\ -0.8303 \end{bmatrix}$

$\therefore \lambda_2 = 6.615$

$\therefore e_2 = \begin{bmatrix} 0.8302 \\ 0.5574 \end{bmatrix}$

Adjusted

$x-\bar{x}$	$y-\bar{y}$
-4	2.5
0	-4.5
5	-3.5
-1	5.5

$\therefore$ New data = Adjusted × e_1

$$\begin{bmatrix} -4 & 2.5 \\ 0 & -4.5 \\ 5 & -3.5 \\ -1 & 5.5 \end{bmatrix} \times \begin{bmatrix} 0.5574 \\ -0.8303 \end{bmatrix}$$

$$= \begin{bmatrix} -4.31 \\ 3.74 \\ 5.69 \\ -5.12 \end{bmatrix}$$

— o — x — o —

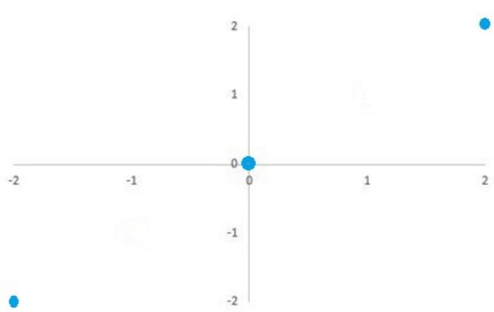


Figure 1: Dataset of the Points A(0,0), B(2, 2), C(-2, -2).

x	y
0	0
2	2
-2	-2

$x-\bar{x}$	$y-\bar{y}$
0	0
2	2
-2	-2

$$\begin{bmatrix} Cov(x,x) & cov(x,y) \\ cov(y,x) & cov(y,y) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0+4+4}{2} & \frac{0+4+4}{2} \\ & \frac{0+4+4}{2} \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 4-\lambda & 4 \\ 4 & 4-\lambda \end{bmatrix} = (4-\lambda)(4-\lambda) - 16$$

$$= \cancel{16} - 4\lambda - 4\lambda + \lambda^2 - \cancel{16} = \lambda^2 - 8\lambda$$

$$= \lambda(\lambda - 8) \Rightarrow 0$$

$$\therefore \lambda = 0, 8$$

For, $\lambda = 8 \Rightarrow \begin{bmatrix} 4-8 & 4 \\ 4 & 4-8 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ 4 & -4 \end{bmatrix}$

$$\therefore -4u_1 + 4u_2 = 0$$

$$\Rightarrow 4u_1 = 4u_2$$

$$\therefore \frac{u_1}{4} = \frac{u_2}{4} = 1$$

$$\therefore u_1 = 4, \quad u_2 = 4$$

$$e_1 = \begin{bmatrix} 4 / \sqrt{4^2 + 4^2} \\ 4 / \text{''} \end{bmatrix} = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}$$

For, $\lambda = 0$, $\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} \Rightarrow$

$$4u_1 + 4u_2 = 0$$

$$\Rightarrow 4u_1 = -4u_2$$

$$\Rightarrow \frac{u_1}{-4} = \frac{u_2}{4} = 1$$

$$\therefore u_1 = -4, \quad u_2 = 4$$

$$\therefore e_2 = \begin{bmatrix} -4 / \sqrt{4^2+4^2} \\ 4 / \text{''} \end{bmatrix} = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix}$$

$\therefore$ New data = Adjusted $\times$ $(\lambda = 8 \Rightarrow e_1)$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix} = \begin{bmatrix} 0 \\ 2.282 \\ -2.282 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 2 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2.282 & 0 \\ -2.282 & 0 \end{bmatrix}$$

T

$$\Rightarrow \underset{2\times 2}{\begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 0 & 2.282 & -2.282 \\ 0 & 0 & 0 \end{bmatrix}}$$

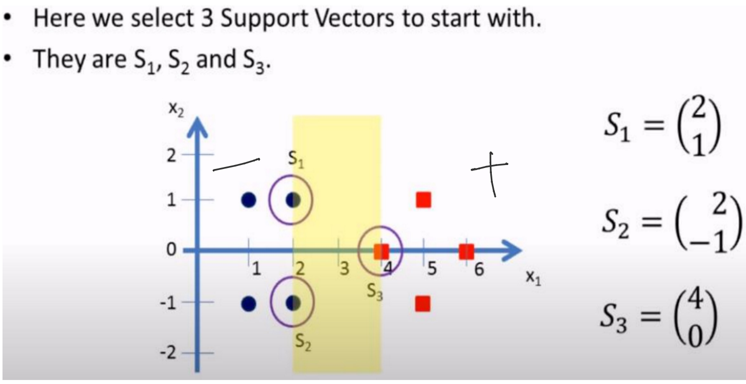
$$= \begin{bmatrix} 0 & 1.61 & -1.61 \\ 0 & 1.61 & -1.61 \end{bmatrix}^{T} + \text{mean}$$

SVM (Support Vector Machine) :-

Hard constraint = $y\left(w^T \cdot x + b\right) \geq 1$

Soft constraint $= \frac{1}{2}\|w\|^2 + C$ } Allows some misclassification and penalize

Ex:-



$$\tilde{S}_1 = \begin{pmatrix}2\\1\\1\end{pmatrix} \qquad \tilde{S}_2 = \begin{pmatrix}2\\-1\\1\end{pmatrix} \qquad \tilde{S}_3 = \begin{pmatrix}4\\0\\1\end{pmatrix}$$

$$\alpha_1 \tilde{S}_1 \tilde{S}_1 + \alpha_2 \tilde{S}_2 \tilde{S}_1 + \alpha_3 \tilde{S}_3 \tilde{S}_1 = -1$$

$$\alpha_1 \tilde{S}_1 \tilde{S}_2 + \alpha_2 \tilde{S}_2 \tilde{S}_2 + \alpha_3 \tilde{S}_3 \tilde{S}_2 = -1$$

$$\alpha_1 \tilde{S}_1 \tilde{S}_3 + \alpha_2 \tilde{S}_2 \tilde{S}_3 + \alpha_3 \tilde{S}_3 \tilde{S}_3 = 1$$

$$\alpha_1 \begin{pmatrix}2\\1\\1\end{pmatrix}\begin{pmatrix}2\\1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}2\\-1\\1\end{pmatrix}\begin{pmatrix}2\\1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}4\\0\\1\end{pmatrix}\begin{pmatrix}2\\1\\1\end{pmatrix} = -1$$

$$\Rightarrow \boxed{6\alpha_1 + 4\alpha_2 + 9\alpha_3 = -1}$$

$$\alpha_1 \begin{pmatrix}2\\1\\1\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}2\\-1\\1\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}4\\0\\1\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix} = -1$$

$$\Rightarrow \boxed{4\alpha_1 + 6\alpha_2 + 9\alpha_3 = -1}$$

$$\alpha_1 \begin{pmatrix}2\\1\\1\end{pmatrix}\begin{pmatrix}4\\0\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}2\\-1\\1\end{pmatrix}\begin{pmatrix}4\\0\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}4\\0\\1\end{pmatrix}\begin{pmatrix}4\\0\\1\end{pmatrix} = 1$$

$$\Rightarrow \boxed{9\alpha_1 + 9\alpha_2 + 17\alpha_3 = 1}$$

$$\begin{array}{ccc} \alpha_1 & \alpha_2 & \alpha_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 6 & 4 & 9 & -1 \\ 4 & 6 & 9 & -1 \\ 9 & 9 & 17 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 4 & 6 & 9 & -1 \\ 9 & 9 & 17 & 1 \end{array}\right] \quad R_1' = \frac{R_1 - R_2}{2}$$

$$\left[\begin{array}{ccc|c} 4 & 6 & 9 & -1 \\ 9 & 9 & 17 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 4 & 6 & 9 & -1 \\ 9 & 9 & 17 & 1 \end{array}\right]$$

$$\leftarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 10 & 9 & -1 \\ 0 & 1 & 1 & \frac{1}{4} \end{array}\right]$$

$10\alpha_2 + 9\alpha_3 = -1$

$\alpha_2 + \alpha_3 = \frac{1}{4}$

$\therefore \alpha_2 = \left(\frac{1}{4} - \alpha_3\right)$

$\therefore 10\left(\frac{1}{4} - \alpha_3\right) + 9\alpha_3 = -1$

$\Rightarrow 2.5 - 10\alpha_3 + 9\alpha_3 = -1 \Rightarrow -\alpha_3 = -1 - 2.5 \quad \therefore \boxed{\alpha_3 = 3.5}$

$\therefore \alpha_2 = \frac{1}{4} - 3.5 = -3.25$

$\therefore \alpha_1 - \alpha_2 = 0 \Rightarrow \boxed{\alpha_1 = \alpha_2 = -3.25}$

$\therefore \tilde{w} = \alpha_1 \tilde{s}_1 + \alpha_2 \tilde{s}_2 + \alpha_3 \tilde{s}_3$

$= -3.25 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + (-3.25)\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + 3.5 \times \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -6.5 \\ -3.25 \\ -3.25 \end{pmatrix} + \begin{pmatrix} -6.5 \\ 3.25 \\ -3.25 \end{pmatrix} + \begin{pmatrix} 14 \\ 0 \\ 3.5 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$

$\therefore w = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad b = -3$

$\therefore \quad w^T \cdot x + b = 0 \Rightarrow \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + (-3) = 0$

$\Rightarrow x - 3 = 0 \Rightarrow x = 3$

$s_1 = \begin{pmatrix}3\\1\end{pmatrix}$ then $\tilde{s}_1 = \begin{pmatrix}3\\1\\1\end{pmatrix} \longrightarrow +$

$s_2 = \begin{pmatrix}4\\0\end{pmatrix}$ then $\tilde{s}_2 = \begin{pmatrix}4\\0\\1\end{pmatrix} \longrightarrow -$

$s_3 = \begin{pmatrix}5\\1\end{pmatrix}$ then $\tilde{s}_3 = \begin{pmatrix}5\\1\\1\end{pmatrix} \longrightarrow +$

$$\alpha_1 \begin{pmatrix}3\\1\\1\end{pmatrix}\begin{pmatrix}3\\1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}4\\0\\1\end{pmatrix}\begin{pmatrix}3\\1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}5\\1\\1\end{pmatrix}\begin{pmatrix}3\\1\\1\end{pmatrix} = 1$$

$$\Rightarrow \boxed{11\alpha_1 + 13\alpha_2 + 17\alpha_3 = 1}$$

$$\Rightarrow \alpha_1 \begin{pmatrix}3\\1\\1\end{pmatrix}\begin{pmatrix}4\\0\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}4\\0\\1\end{pmatrix}\begin{pmatrix}4\\0\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}5\\1\\1\end{pmatrix}\begin{pmatrix}4\\0\\1\end{pmatrix} = -1$$

$$\Rightarrow \boxed{13\alpha_1 + 17\alpha_2 + 21\alpha_3 = -1}$$

$$\Rightarrow \alpha_1 \begin{pmatrix}3\\1\\1\end{pmatrix}\begin{pmatrix}5\\1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}4\\0\\1\end{pmatrix}\begin{pmatrix}5\\1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}5\\1\\1\end{pmatrix}\begin{pmatrix}5\\1\\1\end{pmatrix} = 1$$

$$\Rightarrow \boxed{17\alpha_1 + 21\alpha_2 + 27\alpha_3 = 1}$$

$$\begin{array}{c} \begin{array}{ccc} \alpha_1 & \alpha_2 & \alpha_3 \end{array} \\ \left[\begin{array}{ccc|c} 11 & 13 & 17 & 1 \\ 13 & 17 & 21 & -1 \\ 17 & 21 & 27 & 1 \end{array}\right] \end{array} \longrightarrow \left[\begin{array}{ccc|c} 0 & 13 & 6 & -21 \\ 1 & 2 & 4 & 2 \\ 0 & -2 & -1 & 3 \end{array}\right]$$

$13\alpha_2 + 6\alpha_3 = -21$

$13\alpha_2 + 6(-2\alpha_2 - 3) = -21$

$13\alpha_2 - 12\alpha_2 - 18 = -21$

$\alpha_2 = -21 + 18 = -3$

$-2\alpha_2 - \alpha_3 = 3$

$\therefore \alpha_3 = -2\alpha_2 - 3$

$= -2 \times (-3) - 3$

$= 6 - 3 = 3$

$\alpha_1 + \alpha_3 = 2$

$\Rightarrow \alpha_1 = 2 - \alpha_3 = 2 - 3 = -1$

$$\tilde{w} = \alpha_1 \begin{pmatrix}3\\1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}4\\0\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}5\\1\\1\end{pmatrix}$$

$$= -1 \begin{pmatrix}3\\1\\1\end{pmatrix} + (-3) \begin{pmatrix}4\\0\\1\end{pmatrix} + 3 \begin{pmatrix}5\\1\\1\end{pmatrix}$$

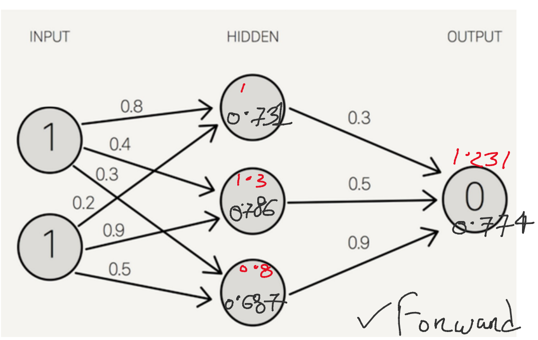
$$= \begin{pmatrix}-3\\-1\\-1\end{pmatrix} + \begin{pmatrix}-12\\0\\-3\end{pmatrix} + \begin{pmatrix}15\\3\\3\end{pmatrix} = \begin{pmatrix}0\\2\\-1\end{pmatrix}$$

$\therefore\ w = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ $\qquad b = -1$

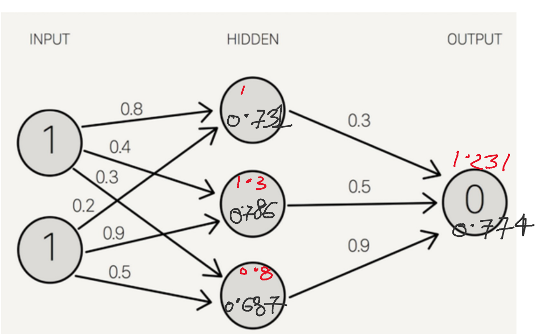
$\therefore\ w^T x + b \Rightarrow \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + (-1) = 0$

$\Rightarrow 2y - 1 = 0 \Rightarrow y = \frac{1}{2}$

ANN (Artificial Neural Network) :-



✓ Forward Propagation done



$$\Delta \text{ output sum} = \underbrace{0{\cdot}774 \times (1 - 0{\cdot}774)}_{\text{Derivative}} \times \underbrace{(0 - 0{\cdot}774)}_{\text{Error}}$$

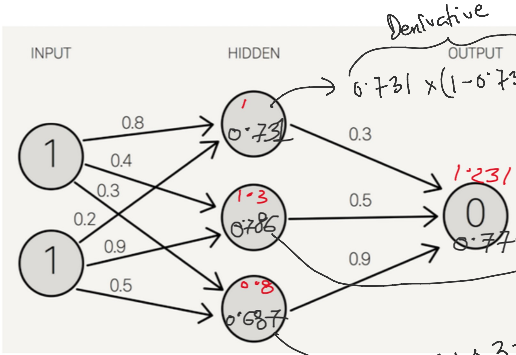
$$= -0{\cdot}1354$$

$$\therefore\ \Delta W_{h \to o} = \frac{-0{\cdot}1354}{[0{\cdot}731,\ 0{\cdot}786,\ 0{\cdot}687]} = [-0{\cdot}185,\ -0{\cdot}172,\ -0{\cdot}197]$$

✓ New weights = $[0{\cdot}115,\ 0{\cdot}328,\ 0{\cdot}703]$

Derivative, Error/Δ

$$0{\cdot}731 \times (1-0{\cdot}731) \times \frac{-0{\cdot}1354}{0{\cdot}3} = -0{\cdot}0887$$

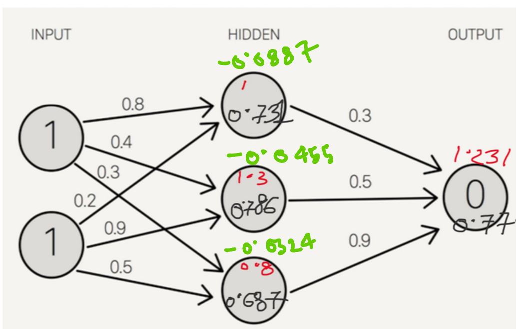


$$0{\cdot}786 \times (1-0{\cdot}786) \times \frac{-0{\cdot}1354}{0{\cdot}5}$$

$$= -0{\cdot}0455$$

$-0{\cdot}0324$

∴



✓ New weights

$$0{\cdot}8 + \frac{-0{\cdot}0887}{1} = 0{\cdot}7113$$

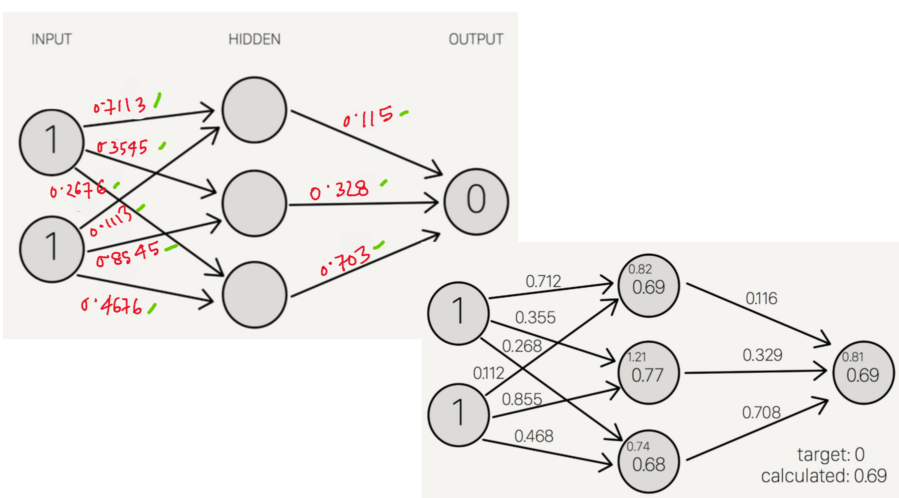
$$0{\cdot}4 + \frac{-0{\cdot}0455}{1} = 0{\cdot}3545$$

$$0{\cdot}3 + \frac{-0{\cdot}0324}{1} = 0{\cdot}2676$$

$$0{\cdot}2 + \frac{-0{\cdot}0887}{1} = 0{\cdot}1113$$

$$0{\cdot}9 + \frac{-0{\cdot}0455}{1} = 0{\cdot}8545$$

$$0{\cdot}5 + \frac{-0{\cdot}0324}{1} = 0{\cdot}4676$$



X				wh			bh			hidden_layer_input			hidden_layer_activations			wout	bout	output	y	E
1	0	1	0	0.42	0.88	0.55	0.46	0.72	0.08	1·48	1·78	1·1	0·815	0·856	0·75	0.30	0.69	0·789	1	0·21
1	0	1	1	0.10	0.73	0.68				2·4			0·917			0.25			1	
0	1	0	1	0.60	0.18	0.47										0.23			0	
				0.92	0.11	0.52														

0·42 + 0·6 + 0·46   ·88 + ·18 + ·72   ·55 + ·47 + 0·68

$.42 + .6 + .92 + .46$

$0.815 \times 0.3 + 0.856 \times 0.25 + 0.75 \times 0.23$
$+ 0.69$

X				wh			bh			hidden_layer_input			hidden_layer_activations			wout	bout	output	y	E
1	0	1	0	0.42	0.88	0.55	0.46	0.72	0.08	1.48	1.78	1.10	0.81	0.86	0.75	0.30	0.69	0.79	1	0.21
1	0	1	1	0.10	0.73	0.68				2.40	1.89	1.61	0.92	0.87	0.83	0.25		0.80	1	0.20
0	1	0	1	0.60	0.18	0.47				1.48	1.56	1.27	0.81	0.83	0.78	0.23		0.79	0	-0.79
				0.92	0.11	0.52														

Step 1 delta - output sum = $\boxed{\text{Output} \times (1 - \text{output}) \times E}$

$0.79 \times (1 - 0.79) \times 0.21 = \begin{bmatrix} 0.035 \\ 0.032 \\ -0.131 \end{bmatrix} = \begin{bmatrix} 0.04 \\ 0.03 \\ -0.13 \end{bmatrix}$

Step 2 w_out = w_out + delta_w_out

$= \text{w\_out} + \left[\left(\text{Hidden layer activation} \right)^T \times \left(\text{delta output sum} \right) \times \text{learning rate} \right]$

Step 3:- $\text{bias}_{out} = \text{bias}_{out} + \left[\text{Sum} \left(\text{delta\_output sum} \right) \times \text{Learning Rate} \right]$

$(0.04 + 0.03 - 0.13) \times \text{Learning Rate}$

Step4 Delta hidden layer

Elemen wise multiply

$= \boxed{\text{Activation\_hidden} \times (1 - \text{Activation hidden})} \times \underbrace{\boxed{\text{Error}}}_{\text{delta\_output sum} \times (\text{W\_out})^T}$

$\begin{bmatrix} 0.04 \\ 0.03 \\ -0.13 \end{bmatrix} \times \begin{bmatrix} 0.3 & 0.25 & 0.23 \end{bmatrix}$

$\boxed{\text{Step 5}}$ $W_h = W_h + \left[(X)^T \cdot \left(\begin{array}{c} \text{delta} \\ \text{hidden} \\ \text{layer} \end{array} \right) \times \text{Learning Rate} \right]$

$\boxed{\text{Step 6}}$ $\text{bias}_{in} = \text{bias}_{in} + \left[\text{Sum} \left(\begin{array}{c} \text{delta hidden} \\ \text{layer} \end{array} \right) \times LR \right]$

CNN (Convolutional Neural Network):-

For convolution:-

$(m - k + s) / s$

For pooling:-

$\dfrac{(m-k)}{s} + 1$

Number of filters (for each of the convolution operations) = 8
Filter size = 2 x 2 (for each of the convolution operations)
stride=1
padding='valid'

Pool size = 2 x 2 (for each of the pooling operations)
stride=2
padding='valid'