MAT 215

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Section : 02

Ans to the ques no:- 1

$$\mathcal{L}(\cos at) = \int_0^\infty e^{-st} \cdot \cos at \ dt$$

$$= \left[\cos at \cdot \int e^{-st} dt\right]_0^\infty - \left[\int_0^\infty \frac{d}{dx}(\cos at) \cdot \left(\int e^{-st} dt\right) dt\right]$$

$$= \left[\cos at \cdot \frac{1}{-s} \cdot e^{-st}\right]_0^\infty - \left[\int_0^\infty -a \sin at \cdot \frac{1}{-s} \cdot e^{-st} \ dt\right]$$

$$= \left[\left(\cos\infty \cdot \frac{1}{-s} \cdot e^{-\infty}\right) - \left(\cos 0 \cdot \frac{1}{-s} e^{0}\right)\right] - \frac{a}{s} \int_0^\infty e^{-st} \cdot \sin at \ dt$$

$$= 0 - \frac{1}{-s} - \frac{a}{s}\left[\left[\sin at \cdot \int e^{-st} dt\right]_0^\infty - \int_0^\infty \frac{d}{dx}(\sin at) \cdot \left(\int e^{-st} dt\right) dt\right]$$

$$= \frac{1}{s} - \frac{a}{s}\left[\left[\sin at \cdot \frac{1}{-s} \cdot e^{-st}\right]_0^\infty - \int_0^\infty a \cos at \cdot \frac{1}{-s} e^{-st} \ dt\right]$$

$$= \frac{1}{s} - \frac{a}{s}\left[\left[\left(\sin\infty \cdot \frac{1}{-s} \cdot e^{-\infty}\right) - \left(\sin 0 \cdot \frac{1}{-s} \cdot e^{0}\right)\right] + \frac{a}{s}\int_0^\infty \cos at \cdot e^{-st} dt\right]$$

$$= \frac{1}{s} - \frac{a}{s}\left[(0-0) + \frac{a}{s}\int_0^\infty e^{-st} \cdot \cos at \ dt\right]$$

$$= \frac{1}{s} - \frac{a^2}{s^2}\int_0^\infty e^{-st} \cdot \cos at \ dt$$

$$= \frac{1}{s} - \frac{a^2}{s^2} \cdot \mathcal{L}(\cos at)$$

$$\therefore \mathcal{L}(\cos at) = \frac{1}{s} - \frac{a^2}{s^2} \cdot \mathcal{L}(\cos at)$$

$$\Rightarrow \mathcal{L}(\cos at) + \frac{a^2}{s^2} \mathcal{L}(\cos at) = \frac{1}{s}$$

$$\Rightarrow \mathcal{L}(\cos at) \cdot \left(1 + \frac{a^2}{s^2}\right) = \frac{1}{s}$$

$$\Rightarrow \mathcal{L}(\cos at) = \frac{1}{s\left(1 + \frac{a^2}{s^2}\right)}$$

$$\Rightarrow \mathcal{L}(\cos at) = \frac{1}{s + \frac{a^2}{s}}$$

$$\therefore \mathcal{L}(\cos at) = \frac{s}{s^2 + a^2}$$

Now, $\mathcal{L}(\cos 16t \cdot \cos 10t)$

$$= \mathcal{L}\left[\cos\left(\frac{26t + 6t}{2}\right) \cdot \cos\left(\frac{26t - 6t}{2}\right)\right]$$

$$= \mathcal{L}\left[\frac{1}{2}\left(\cos 26t + \cos 6t\right)\right]$$

$$= \mathcal{L}\left(\frac{1}{2}\cos 26t\right) + \mathcal{L}\left(\frac{1}{2}\cos 6t\right)$$

$$= \frac{1}{2}\left[\mathcal{L}(\cos 26t) + \mathcal{L}(\cos 6t)\right]$$

$$= \frac{1}{2}\left[\frac{s}{s^2+26^2} + \frac{s}{s^2+6^2}\right]$$

$$= \frac{1}{2}\left[\frac{s}{s^2+676} + \frac{s}{s^2+36}\right]$$

$$= \frac{s}{2}\left[\frac{1}{s^2+676} + \frac{1}{s^2+36}\right]$$

—— o —— x —— o ——

Ans to the ques no:- 2

Given, $\mathcal{L}^{-1}\left[\frac{s}{(s^2+64)(s-4)}\right]$

Decomposing $\frac{s}{(s^2+64)(s-4)}$ into parctaial fraction $\Rightarrow$

$$\frac{s}{(s^2+64)(s-4)} \equiv \frac{As+B}{(s^2+64)} + \frac{C}{(s-4)}$$

$$= \frac{(As+B)(s-4)+C(s^2+64)}{(s^2+64)(s-4)}$$

So, $s = (As+B)(s-4) + C(s^2+64)$ ——— (i)

$\Rightarrow s = As^2 + Bs - 4As - 4B + Cs^2 + 64C$

$\therefore s = s^2(A+C) + s(B-4A) + (-4B+64C)$ ——— (ii)

Pluggin, $s=4$ in equation (i),

$4 = (As+B)(4-4) + C(4^2+64)$

$\Rightarrow 4 = C(16+64)$

$\therefore C = \frac{1}{20}$

Plugging, $C = \frac{1}{20}$ in equation (ii),

$s = s^2\left(A + \frac{1}{20}\right) + s(B-4A) + \left(-4B + \frac{64}{20}\right)$

Now, Equating the coefficients,

$A + \frac{1}{20} = 0$

$\therefore A = -\frac{1}{20}$

$B - 4A = 1$

$\Rightarrow B - 4\left(-\frac{1}{20}\right) = 1$

$\Rightarrow B + \frac{1}{5} = 1$

$\therefore B = \frac{4}{5}$

$$\therefore \frac{s}{(s^2+64)(s-4)} = \frac{-\frac{1}{20}s + \frac{4}{5}}{(s^2+64)} + \frac{\frac{1}{20}}{(s-4)}$$

$$= \frac{\frac{-s+16}{20}}{(s^2+64)} + \frac{1}{20(s-4)}$$

$$= \frac{-s+16}{20(s^2+64)} + \frac{1}{20(s-4)}$$

$$\therefore \mathcal{L}^{-1}\left[\frac{s}{(s^2+64)(s-4)}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{-s+16}{20(s^2+64)} + \frac{1}{20(s-4)}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{-s+16}{20(s^2+64)}\right] + \mathcal{L}^{-1}\left[\frac{1}{20(s-4)}\right]$$

$$= \frac{1}{20}\mathcal{L}^{-1}\left[\frac{-s+16}{s^2+64}\right] + \frac{1}{20}\mathcal{L}^{-1}\left[\frac{1}{s-4}\right]$$

$$= \frac{1}{20}\mathcal{L}^{-1}\left[\frac{-s}{s^2+64} + \frac{16}{s^2+64}\right] + \frac{1}{20}e^{4t}$$

$$= \frac{1}{20}\mathcal{L}^{-1}\left[\frac{-s}{s^2+64}\right] + \frac{1}{20}\mathcal{L}^{-1}\left[\frac{16}{s^2+64}\right] + \frac{1}{20}e^{4t}$$

$$= -\frac{1}{20}\mathcal{L}^{-1}\left[\frac{s}{s^2+8^2}\right] + \frac{2}{20}\mathcal{L}^{-1}\left[\frac{8}{s^2+8^2}\right] + \frac{1}{20}e^{4t}$$

$$= -\frac{1}{20}\cos 8t + \frac{1}{10}\sin 8t + \frac{1}{20}e^{4t}$$

—— o —— x —— o ——

Ans to the ques no :- 3

Given differential equation,

$$y'' - 10y' + 9y = e^{-9t}$$

where, $y(0) = 1$ and $y'(0) = 1$

Taking Laplace Transform,

$$\mathcal{L}(y'' - 10y' + 9y) = \mathcal{L}(e^{-9t})$$

$$\Rightarrow \mathcal{L}(y''(t)) - 10\mathcal{L}(y'(t)) + 9\mathcal{L}(y(t)) = \frac{1}{s-(-9)}$$

$$\Rightarrow \left[s^2 Y(s) - s\cdot y(0) - y'(0)\right] - 10\left[s\cdot Y(s) - y(0)\right] + 9Y(s) = \frac{1}{s+9}$$

$$\Rightarrow s^2 Y(s) - s\cdot 1 - \cancel{8}1 - 10sY(s) + 10\times 1 + 9Y(s) = \frac{1}{s+9}$$

$$\Rightarrow Y(s)\left[s^2 - 10s + 9\right] - s - 1 + 10 = \frac{1}{s+9}$$

$$\Rightarrow Y(s)\left[s^2 - 10s + 9\right] - s + 9 = \frac{1}{s+9}$$

$$\Rightarrow Y(s)\cdot\left[s^2 - 10s + 9\right] = \frac{1}{s+9} + s - 9$$

$$\Rightarrow Y(s) = \frac{1}{s^2 - 10s + 9}\cdot\left[\frac{1 + (s-9)(s+9)}{s+9}\right]$$

$$= \frac{1}{s^2 - 9s - s + 9}\cdot\frac{1 + s^2 - 9^2}{s+9}$$

$$= \frac{1}{s(s-9) - (s-9)}\cdot\frac{1 + s^2 - 81}{s+9}$$

$$= \frac{1}{(s-9)(s-1)}\cdot\frac{s^2 - 80}{(s+9)}$$

$$= \frac{s^2 - 80}{(s+9)(s-9)(s-1)}$$

Here, $\frac{s^2-80}{(s+9)(s-9)(s-1)} = \frac{A}{(s+9)} + \frac{B}{(s-9)} + \frac{C}{(s-1)}$

$= \frac{A(s-9)(s-1)+B(s+9)(s-1)+C(s+9)(s-9)}{(s+9)\ (s-9)\ (s-1)}$

$\therefore s^2-80 = A(s-9)(s-1)+\ B(s+9)(s-1)+C(s+9)\ (s-9)$

When, $s=9$,

$9^2-80 = A(9-9)(9-1)+B(9+9)(9-1)+C(9+9)(9-9)$

$\Rightarrow 81-80 = B\times 18\times 8$

$\therefore B = \frac{1}{144}$

When, $s=1$,

$1^2-80 = A(1-9)(1-1)+B(1+9)(1-1)+C(1+9)(1-9)$

$\Rightarrow 1-80 = C\times 10\times(-8)$

$\therefore C = \frac{79}{80}$

when, $s=-9$,

$\varepsilon.\ (-9)^2 - 80 = A(-9-9)(-9-1) + B(-9+9)(-9-1) + C(-9+9)(-9-9)$

$\Rightarrow 81 - 80 = A \times (-18) \times (-10)$

$\therefore A = \frac{1}{180}$

$$\text{So, } \frac{s^2-80}{(s+9)(s-9)(s-1)} = \frac{\frac{1}{180}}{(s+9)} + \frac{\frac{1}{144}}{(s-9)} + \frac{\frac{79}{80}}{(s-1)}$$

$$\therefore Y(s) = \frac{\frac{1}{180}}{(s+9)} + \frac{\frac{1}{144}}{(s-9)} + \frac{\frac{79}{80}}{(s-1)}$$

$$\Rightarrow y(t) = \mathcal{L}^{-1}\left[\frac{\frac{1}{180}}{(s+9)} + \frac{\frac{1}{144}}{(s-9)} + \frac{\frac{79}{80}}{(s-1)}\right]$$

$$= \frac{1}{180}\mathcal{L}^{-1}\left(\frac{1}{s+9}\right) + \frac{1}{144}\mathcal{L}^{-1}\left(\frac{1}{s-9}\right) + \frac{79}{80}\cdot\mathcal{L}^{-1}\left(\frac{1}{s-1}\right)$$

$$= \frac{1}{180}\cdot e^{-9t} + \frac{1}{144}\cdot e^{9t} + \frac{79}{80}\cdot e^{t}$$

—o———x———o———