

Deep Learning Neural Net Basics

Outline

- Neural Networks
- *Convolutional* Neural Networks
- Variants
 - Detection
 - Segmentation
 - Siamese Networks
- Visualization of Deep Networks

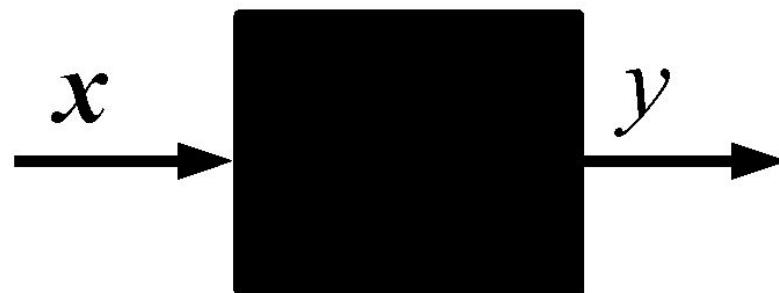
Supervised Learning

$\{(\mathbf{x}^i, y^i), i=1 \dots P\}$ training dataset

\mathbf{x}^i i-th input training example

y^i i-th target label

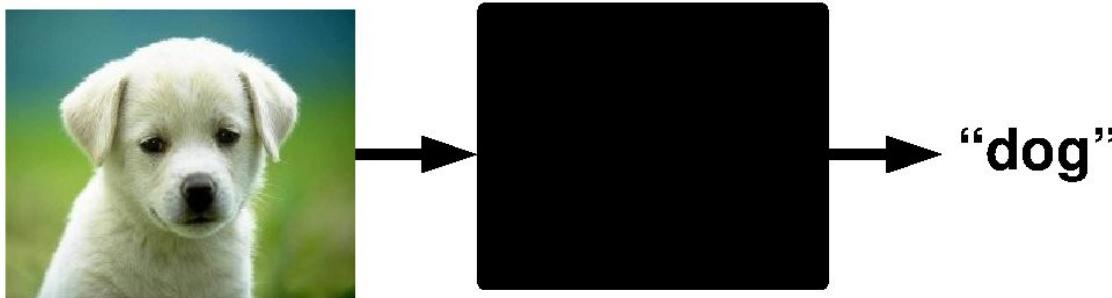
P number of training examples



Goal: predict the target label of unseen inputs.

Supervised Learning: Examples

Classification



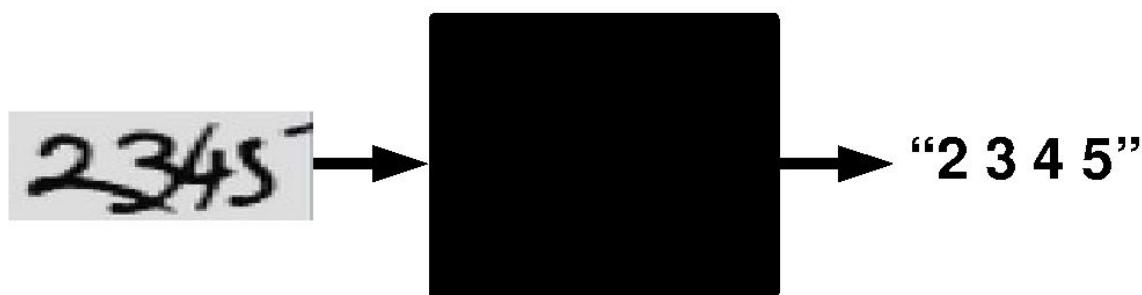
classification

Denoising



regression

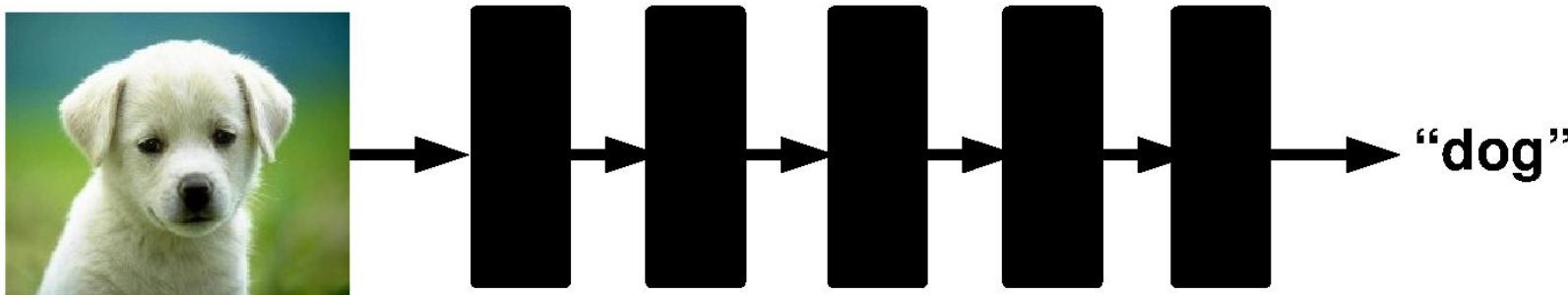
OCR



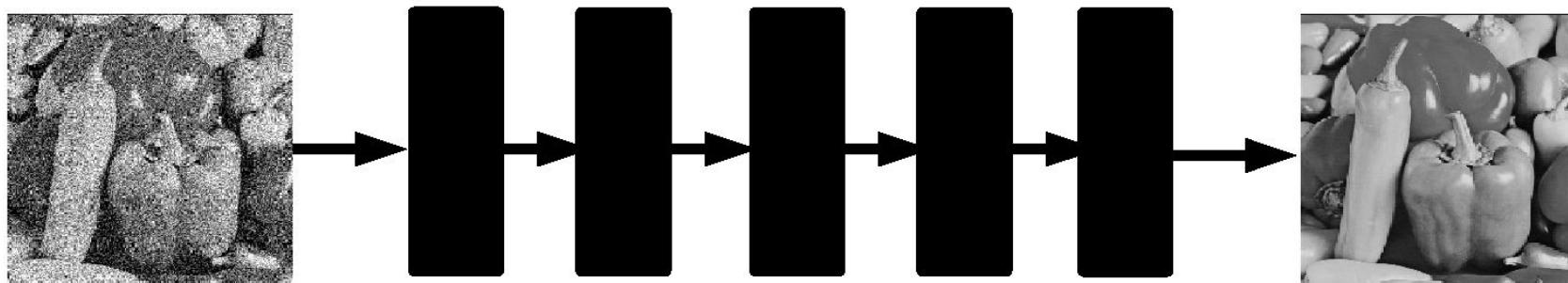
structured
prediction

Supervised Deep Learning

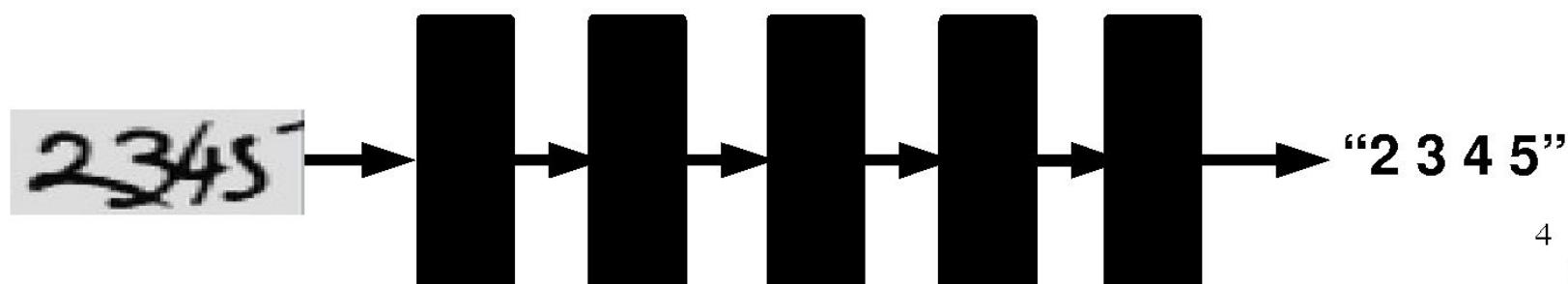
Classification



Denoising



OCR



Project 3: Scene Classification with Deep Nets

Dataset

The dataset to be used in this assignment is the 15-scene dataset, containing natural images in 15 possible scenarios like bedrooms and coasts. It was first introduced by Lazebnik et al, 2006 [1]. The images have a typical size of around 200 by 200 pixels, and serve as a good milestone for many vision tasks. A sample collection of the images can be found below:



Figure 1: Example scenes from each of the categories of the dataset.

Download the data (link at the top), unzip it and put the `data` folder in the `proj4` directory.

1 Part 1: SimpleNet

Introduction

In this project, scene recognition with deep learning, we are going to train a simple convolutional neural net from scratch. We'll be starting with some modification to the dataloader used in this project to include a few extra pre-processing steps. Subsequently, you will define your own model and optimization function. A trainer class will be provided to you, and you will be able to test out the performance of your model with this complete pipeline of classification problem.

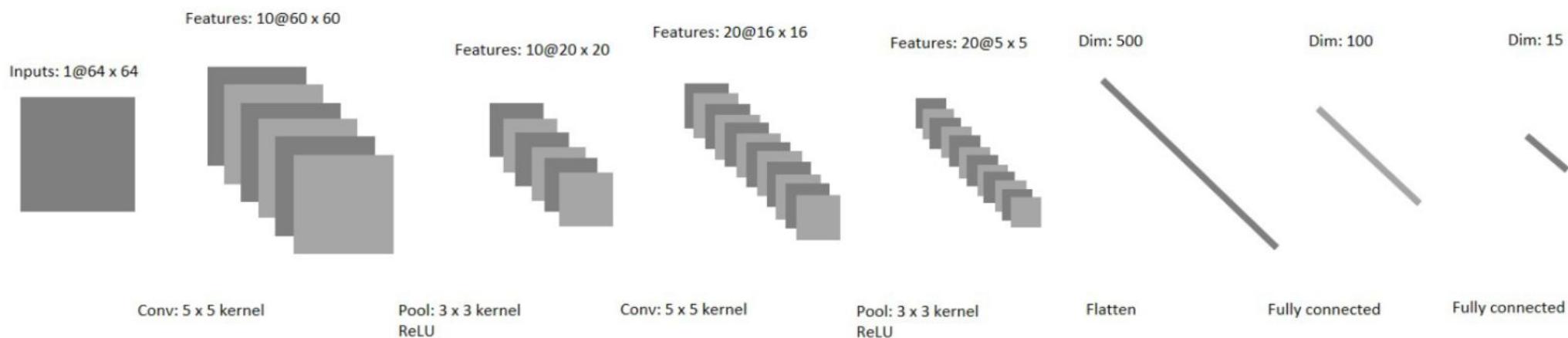


Figure 2: The base SimpleNet architecture for Part 1.

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Neural Networks

Assumptions (for the next few slides):

- The input image is vectorized (disregard the spatial layout of pixels)
- The target label is discrete (classification)

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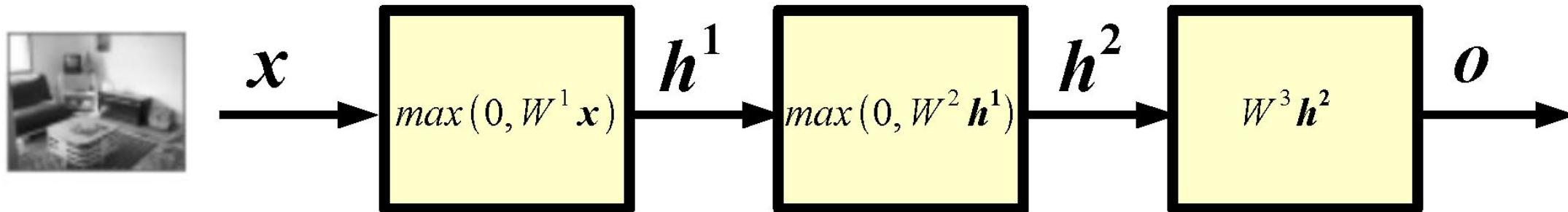
Question: what class of functions shall we consider to map the input into the output?

Answer: composition of simpler functions.

Follow-up questions: Why not a linear combination? What are the “simpler” functions? What is the interpretation?

Answer: later...

Neural Networks: example



x input

h^1 1-st layer hidden units

h^2 2-nd layer hidden units

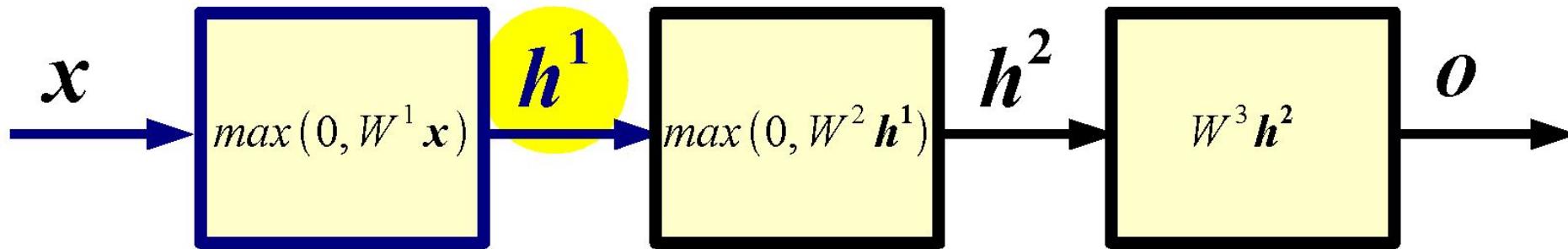
o output

Example of a 2 hidden layer neural network (or 4 layer network,
counting also input and output).

Forward Propagation

Def.: Forward propagation is the process of computing the output of the network given its input.

Forward Propagation



$$x \in R^D \quad W^1 \in R^{N_1 \times D} \quad b^1 \in R^{N_1} \quad h^1 \in R^{N_1}$$

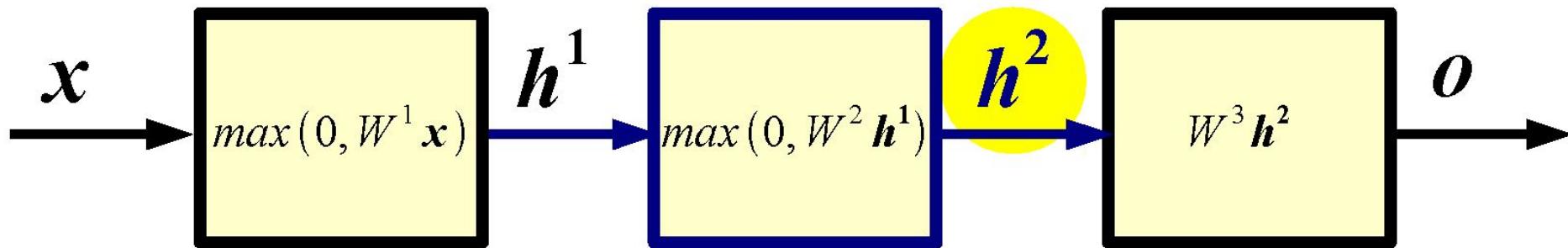
$$h^1 = \max(0, W^1 x + b^1)$$

W^1 1-st layer weight matrix or weights

b^1 1-st layer biases

The non-linearity $u = \max(0, v)$ is called **ReLU** in the DL literature. Each output hidden unit takes as input all the units at the previous layer: each such layer is called “**fully connected**”.

Forward Propagation

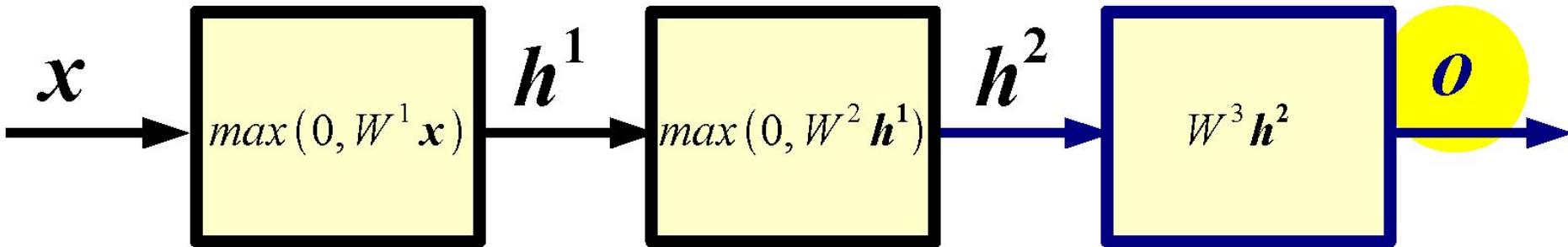


$$h^1 \in R^{N_1} \quad W^2 \in R^{N_2 \times N_1} \quad b^2 \in R^{N_2} \quad h^2 \in R^{N_2}$$

$$h^2 = \max(0, W^2 h^1 + b^2)$$

W^2 2-nd layer weight matrix or weights
 b^2 2-nd layer biases

Forward Propagation

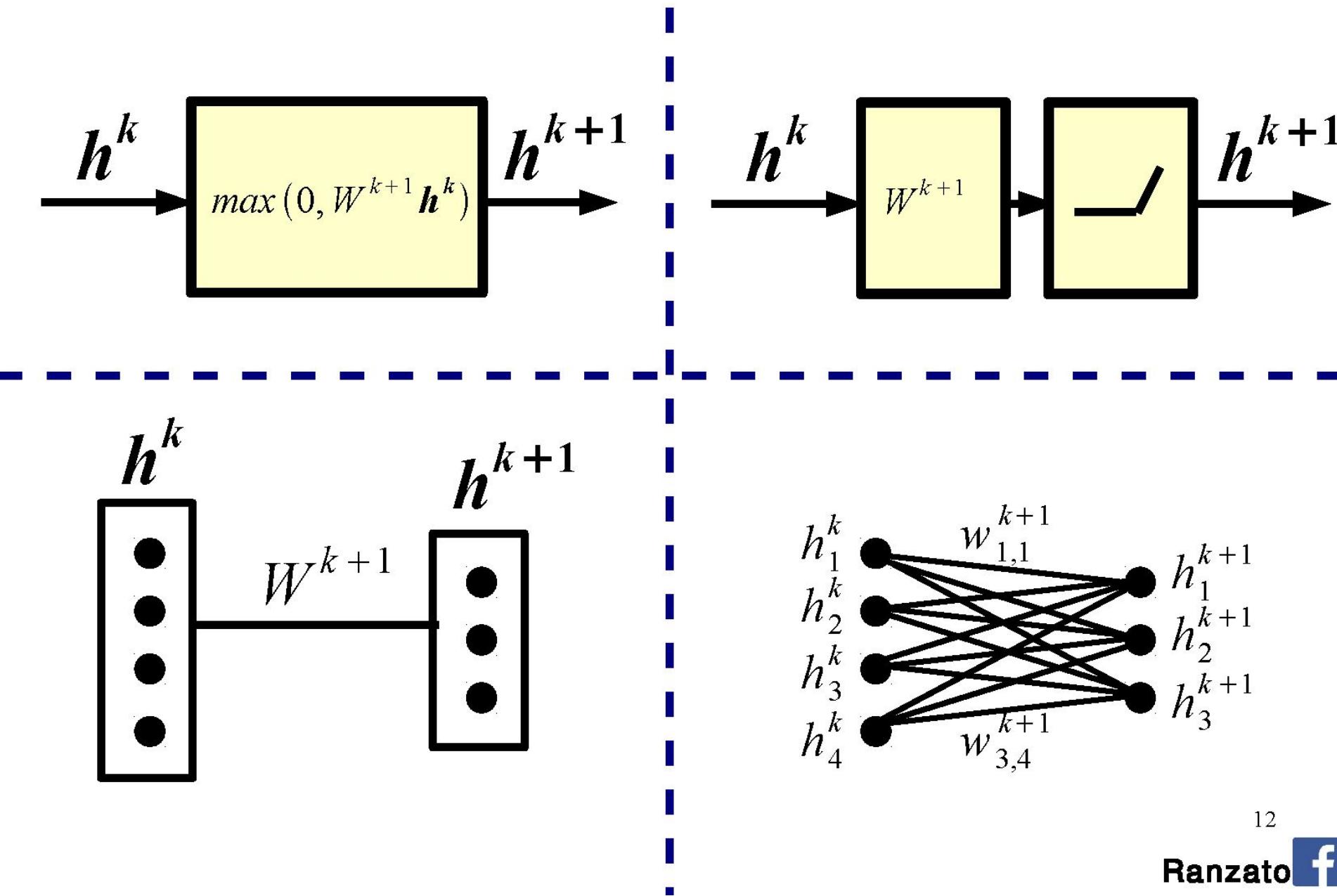


$$h^2 \in R^{N_2} \quad W^3 \in R^{N_3 \times N_2} \quad b^3 \in R^{N_3} \quad o \in R^{N_3}$$

$$o = \max(0, W^3 h^2 + b^3)$$

W^3 3-rd layer weight matrix or weights
 b^3 3-rd layer biases

Alternative Graphical Representation



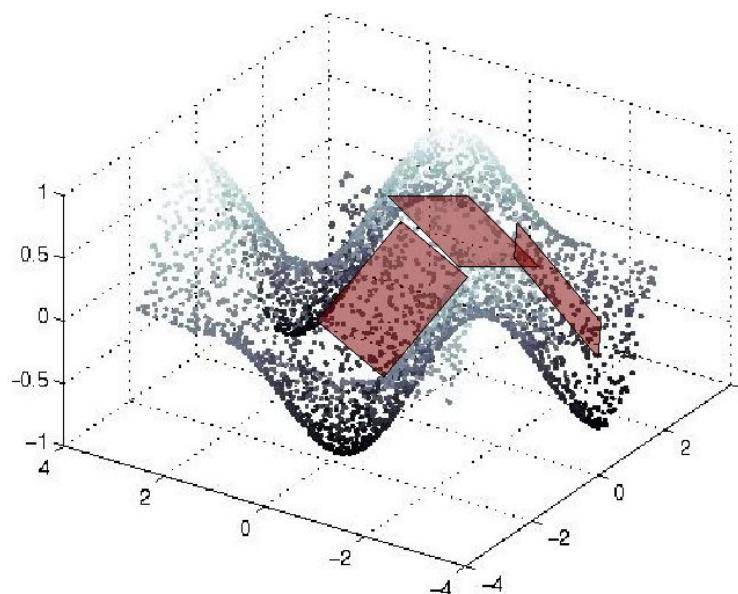
Interpretation

Question: Why can't the mapping between layers be linear?

Answer: Because composition of linear functions is a linear function.
Neural network would reduce to (1 layer) logistic regression.

Question: What do ReLU layers accomplish?

Answer: Piece-wise linear tiling: mapping is locally linear.

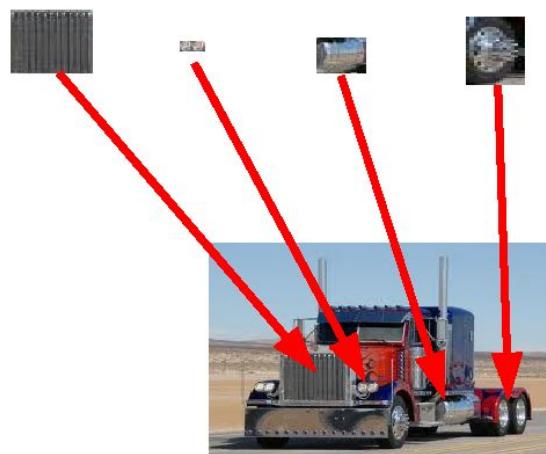


Interpretation

Question: Why do we need many layers?

Answer: When input has hierarchical structure, the use of a hierarchical architecture is potentially more efficient because intermediate computations can be re-used. DL architectures are efficient also because they use **distributed representations** which are shared across classes.

[0 0 1 0 0 0 0 1 0 0 0 1 1 0 0 1 0 ...] truck feature

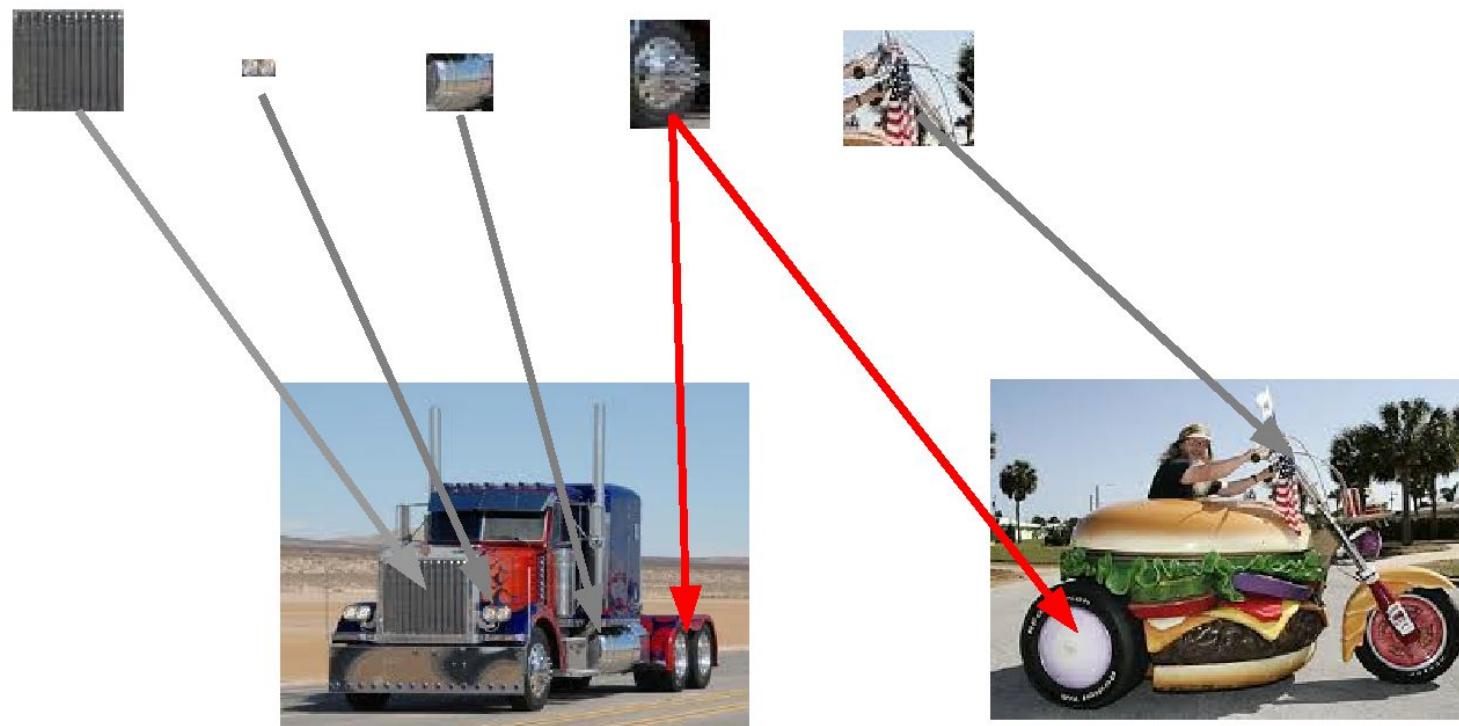


Exponentially more efficient than a
1-of-N representation (a la k-means)

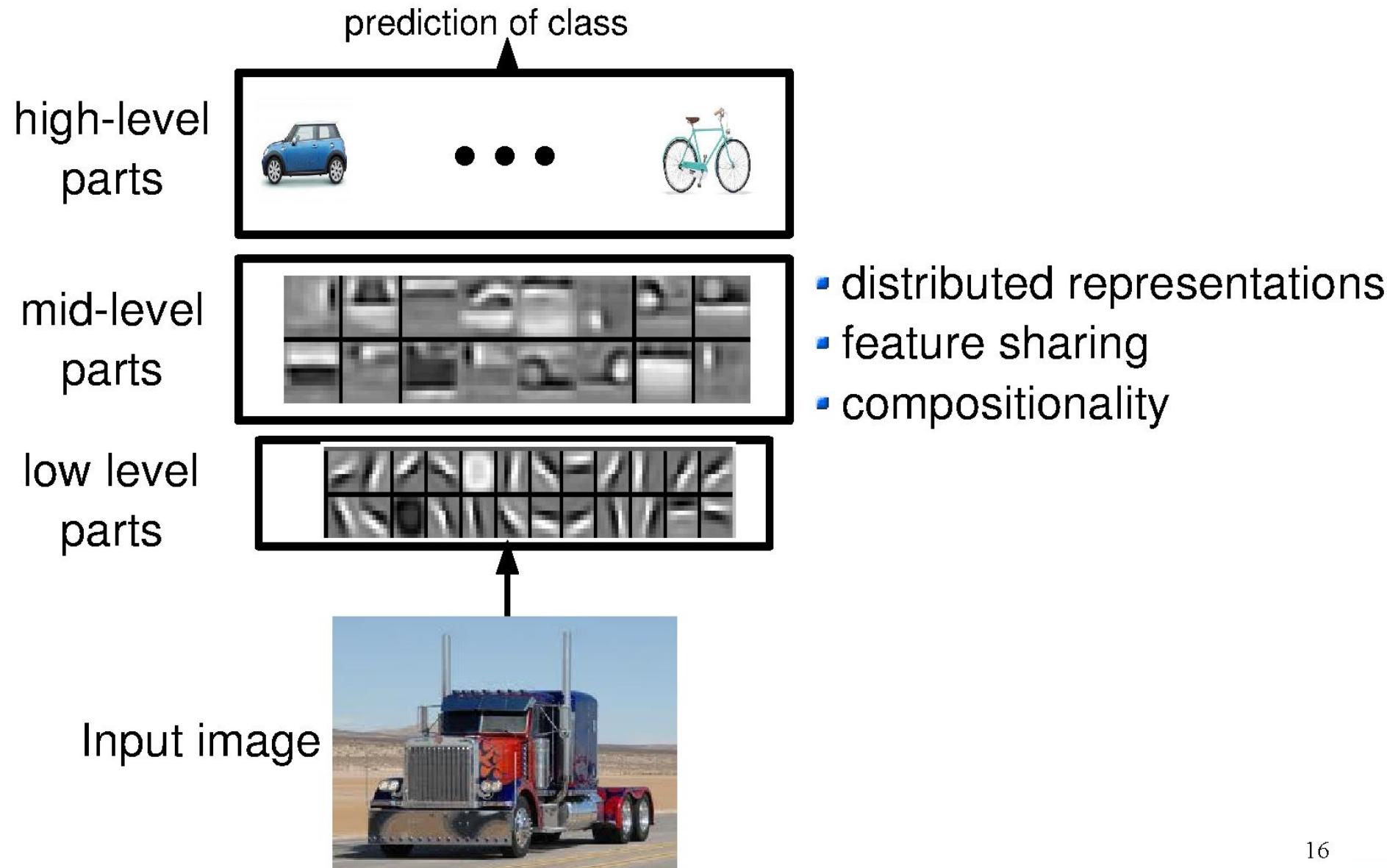
Interpretation

[1 1 0 0 0 1 0 1 0 0 0 0 1 1 0 1...] motorbike

[0 0 1 0 0 0 0 1 0 0 1 1 0 0 1 0...] truck



Interpretation



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Answer: It can be thought of as a classifier or feature detector.

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Question: How many layers? How many hidden units?

Answer: Cross-validation or hyper-parameter search methods are the answer. In general, the wider and the deeper the network the more complicated the mapping.

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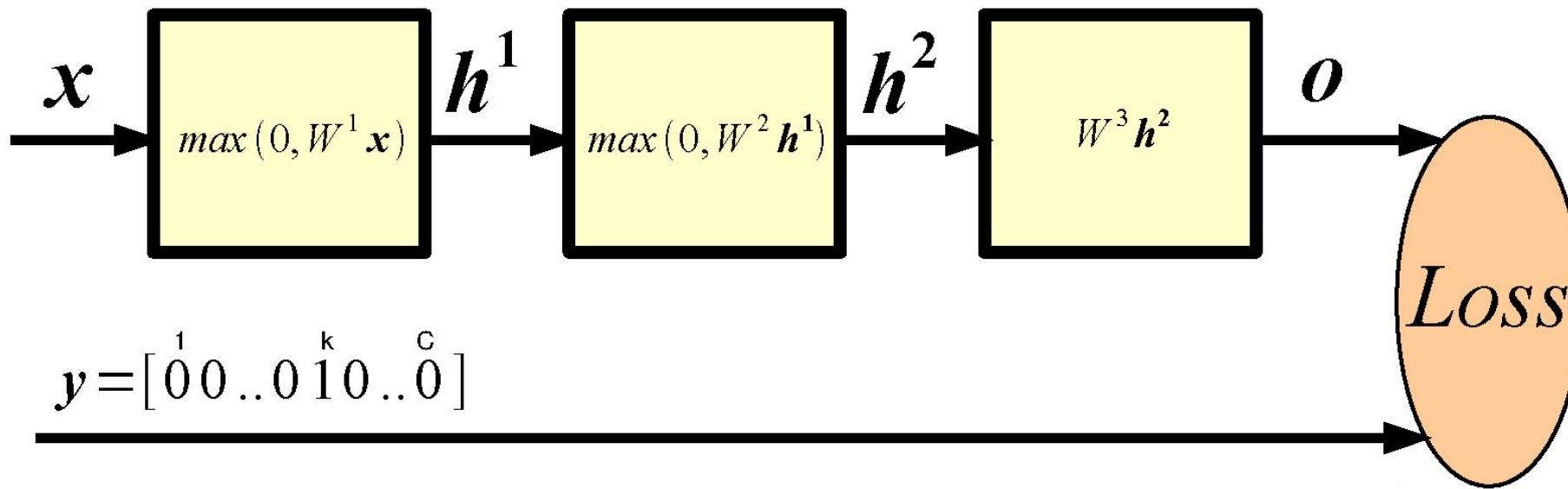
Answer: Cross-validation or hyper-parameter search methods are the answer. In general, the wider and the deeper the network the more complicated the mapping.

Question: How do I set the weight matrices?

Answer: Weight matrices and biases are learned.

First, we need to define a measure of quality of the current mapping.
Then, we need to define a procedure to adjust the parameters.

How Good is a Network?



Probability of class k given input (softmax):

$$p(c_k=1|x) = \frac{e^{o_k}}{\sum_{j=1}^C e^{o_j}}$$

(Per-sample) **Loss**; e.g., negative log-likelihood (good for classification of small number of classes):

$$L(x, y; \theta) = -\sum_j y_j \log p(c_j|x)$$

Training

Learning consists of minimizing the loss (plus some regularization term) w.r.t. parameters over the whole training set.

$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta}} \sum_{n=1}^P L(\mathbf{x}^n, y^n; \boldsymbol{\theta})$$

Training

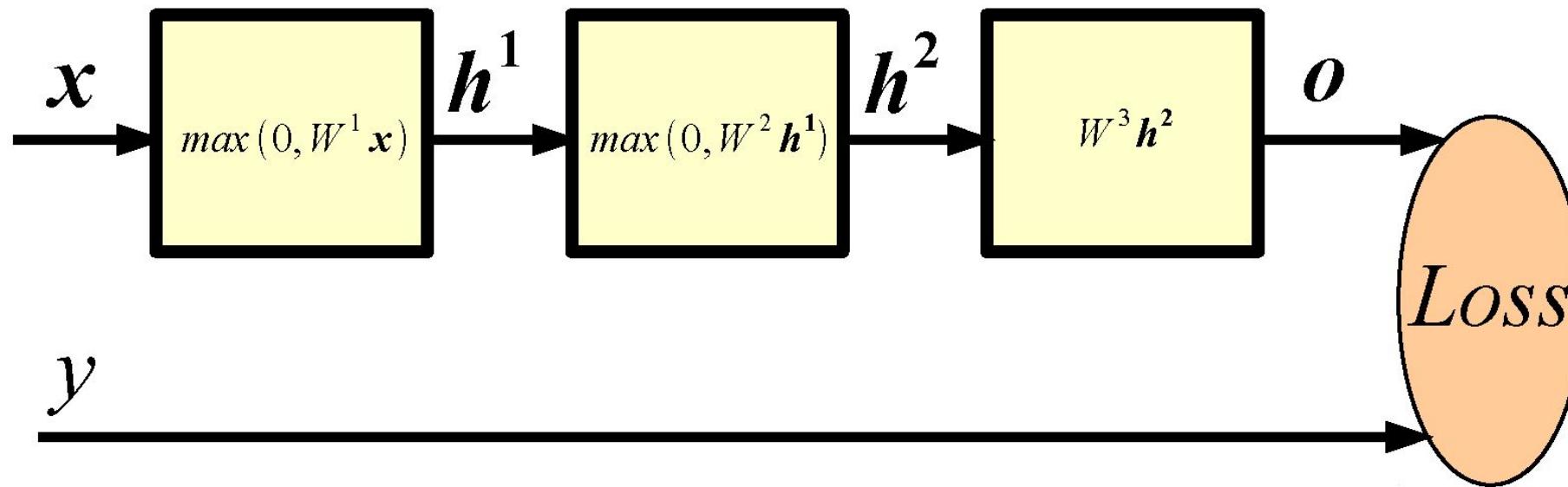
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$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta}} \sum_{n=1}^P L(\mathbf{x}^n, y^n; \boldsymbol{\theta})$$

Question: How to minimize a complicated function of the parameters?

Answer: Chain rule, a.k.a. **Backpropagation**! That is the procedure to compute gradients of the loss w.r.t. parameters in a multi-layer neural network.

Key Idea: Wiggle To Decrease Loss



Let's say we want to decrease the loss by adjusting $W_{i,j}^1$.
We could consider a very small $\epsilon = 1e-6$ and compute:

$$L(\mathbf{x}, y; \boldsymbol{\theta})$$

$$L(\mathbf{x}, y; \boldsymbol{\theta} \setminus W_{i,j}^1, W_{i,j}^1 + \epsilon)$$

Then, update:

$$W_{i,j}^1 \leftarrow W_{i,j}^1 + \epsilon \operatorname{sgn}(L(\mathbf{x}, y; \boldsymbol{\theta}) - L(\mathbf{x}, y; \boldsymbol{\theta} \setminus W_{i,j}^1, W_{i,j}^1 + \epsilon))$$

Derivative w.r.t. Input of Softmax

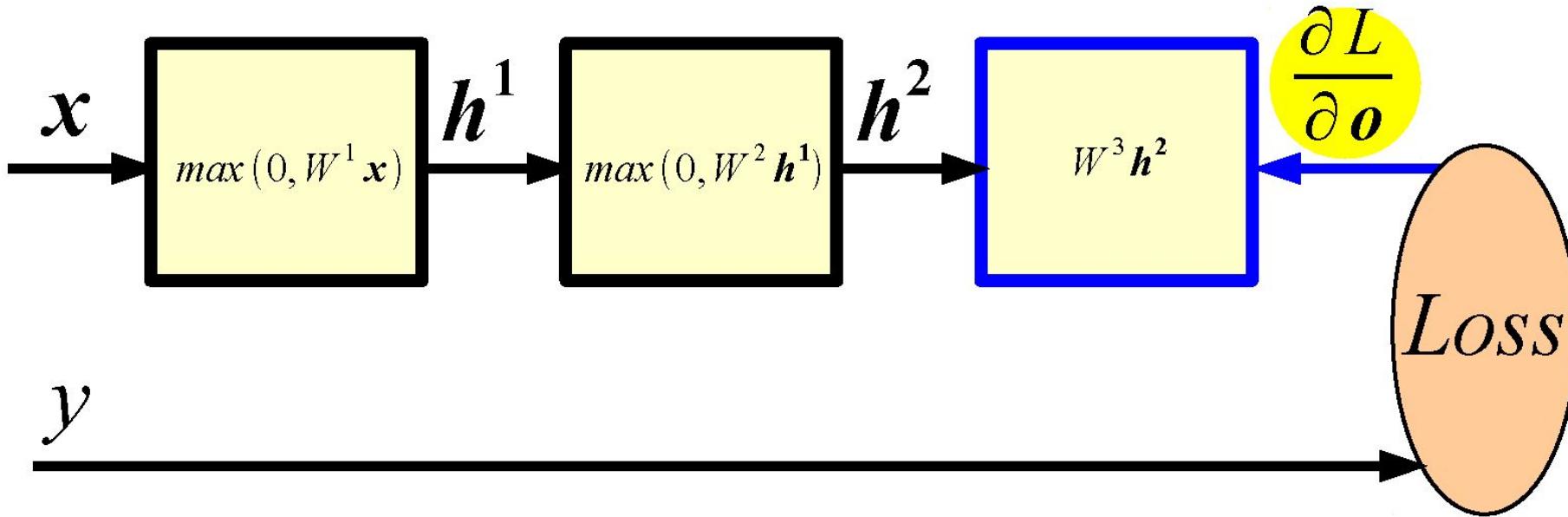
$$p(c_k=1|\mathbf{x}) = \frac{e^{o_k}}{\sum_j e^{o_j}}$$

$$L(\mathbf{x}, \mathbf{y}; \boldsymbol{\theta}) = -\sum_j y_j \log p(c_j|\mathbf{x}) \quad \mathbf{y} = [0^1 0^0 .. 0^1 0^k .. 0^c]$$

By substituting the first formula in the second, and taking the derivative w.r.t. \mathbf{o} we get:

$$\frac{\partial L}{\partial o} = p(c|\mathbf{x}) - y$$

Backward Propagation

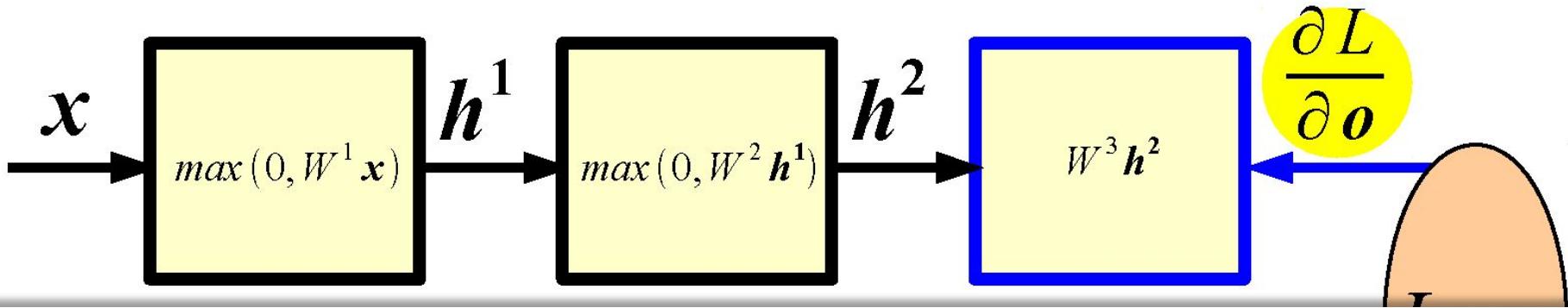


Given $\frac{\partial L}{\partial \mathbf{o}}$ and assuming we can easily compute the Jacobian of each module, we have:

$$\frac{\partial L}{\partial W^3} = \frac{\partial L}{\partial \mathbf{o}} \frac{\partial \mathbf{o}}{\partial W^3}$$

$$\frac{\partial L}{\partial h^2} = \frac{\partial L}{\partial \mathbf{o}} \frac{\partial \mathbf{o}}{\partial h^2}$$

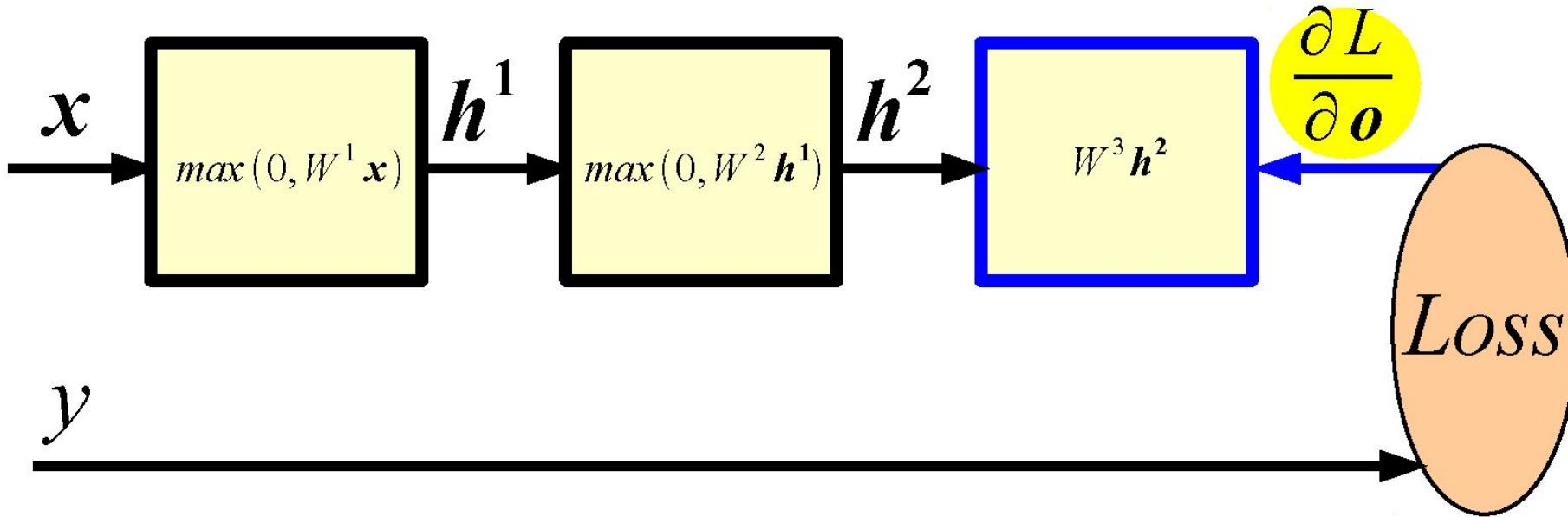
Backward Propagation



Suppose $\mathbf{f} : \mathbf{R}^n \rightarrow \mathbf{R}^m$ is a function such that each of its first-order partial derivatives exist on \mathbf{R}^n . This function takes a point $\mathbf{x} \in \mathbf{R}^n$ as input and produces the vector $\mathbf{f}(\mathbf{x}) \in \mathbf{R}^m$ as output. Then the Jacobian matrix of \mathbf{f} is defined to be an $m \times n$ matrix, denoted by \mathbf{J} , whose (i,j) th entry is $\mathbf{J}_{ij} = \frac{\partial f_i}{\partial x_j}$, or explicitly

$$\mathbf{J} = \left[\frac{\partial \mathbf{f}}{\partial x_1} \quad \dots \quad \frac{\partial \mathbf{f}}{\partial x_n} \right] = \begin{bmatrix} \nabla^T f_1 \\ \vdots \\ \nabla^T f_m \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \dots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

Backward Propagation

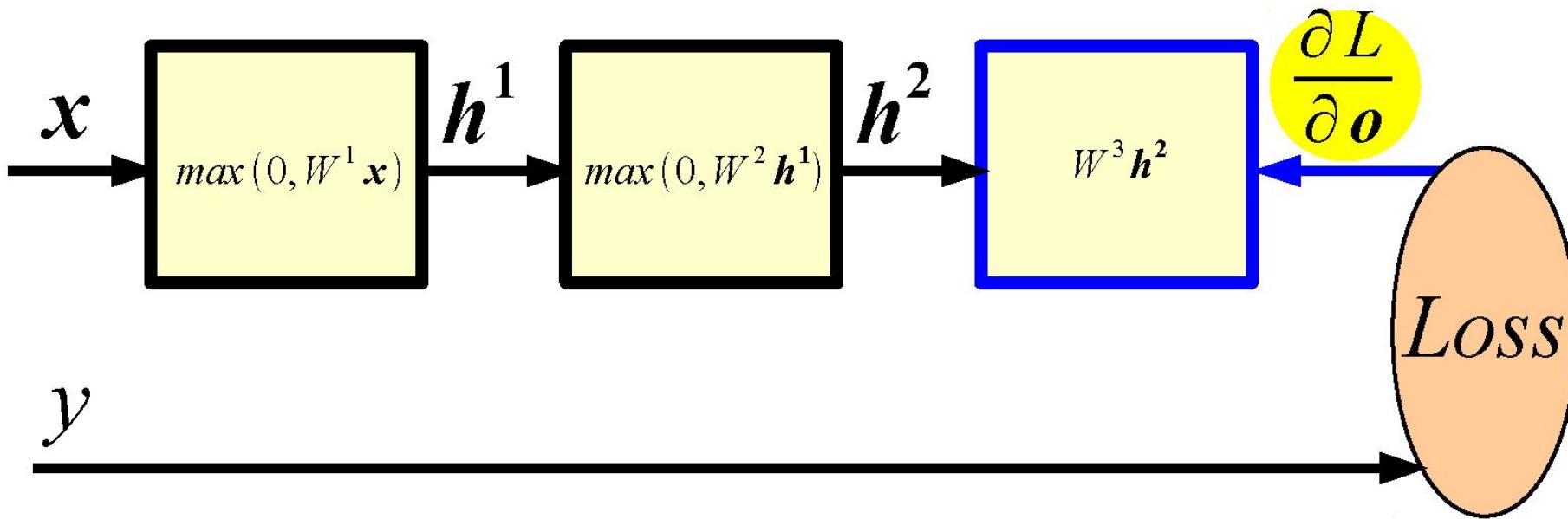


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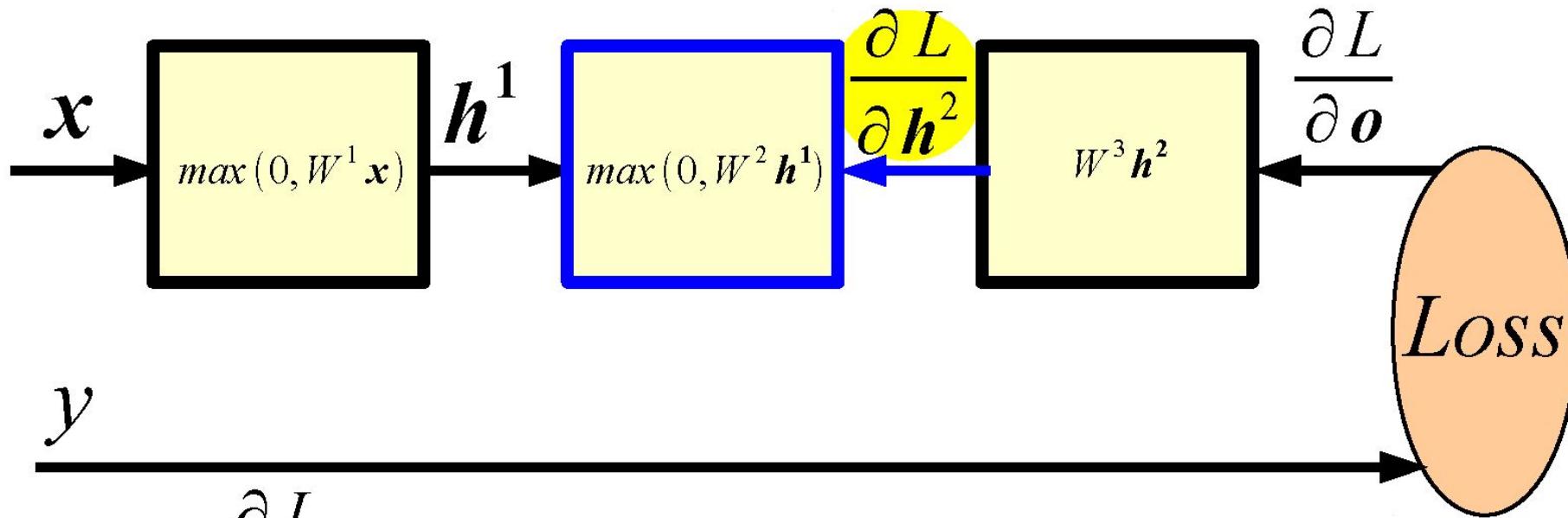
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$$\frac{\partial L}{\partial h^2} = \frac{\partial L}{\partial \mathbf{o}} \frac{\partial \mathbf{o}}{\partial h^2}$$

$$\frac{\partial L}{\partial W^3} = (p(c|x) - y) h^{2T}$$

$$\frac{\partial L}{\partial h^2} = W^{3T} (p(c|x) - y)$$

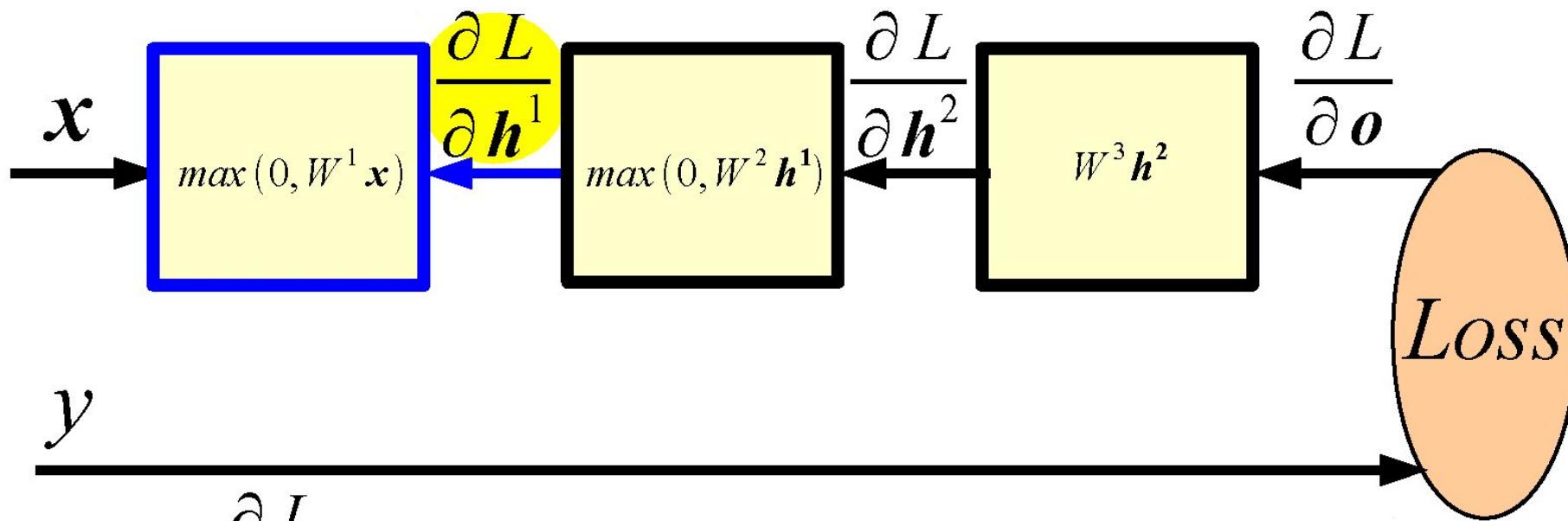
Backward Propagation



Given $\frac{\partial L}{\partial h^2}$ we can compute now:

$$\frac{\partial L}{\partial W^2} = \frac{\partial L}{\partial h^2} \frac{\partial h^2}{\partial W^2} \quad \frac{\partial L}{\partial h^1} = \frac{\partial L}{\partial h^2} \frac{\partial h^2}{\partial h^1}$$

Backward Propagation



Given $\frac{\partial L}{\partial h^1}$ we can compute now:

$$\frac{\partial L}{\partial W^1} = \frac{\partial L}{\partial h^1} \frac{\partial h^1}{\partial W^1}$$

Backward Propagation

Question: Does BPROP work with ReLU layers only?

Answer: Nope, any a.e. differentiable transformation works.

Backward Propagation

Question: Does BPROP work with ReLU layers only?

Answer: Nope, any a.e. differentiable transformation works.

Question: What's the computational cost of BPROP?

Answer: About twice FPROP (need to compute gradients w.r.t. input and parameters at every layer).

Optimization

Stochastic Gradient Descent (on mini-batches):

$$\theta \leftarrow \theta - \eta \frac{\partial L}{\partial \theta}, \eta \in (0, 1)$$

Stochastic Gradient Descent with Momentum:

$$\theta \leftarrow \theta - \eta \Delta$$

$$\Delta \leftarrow 0.9 \Delta + \frac{\partial L}{\partial \theta}$$

Note: there are many other variants...

Toy Code (Matlab): Neural Net Trainer

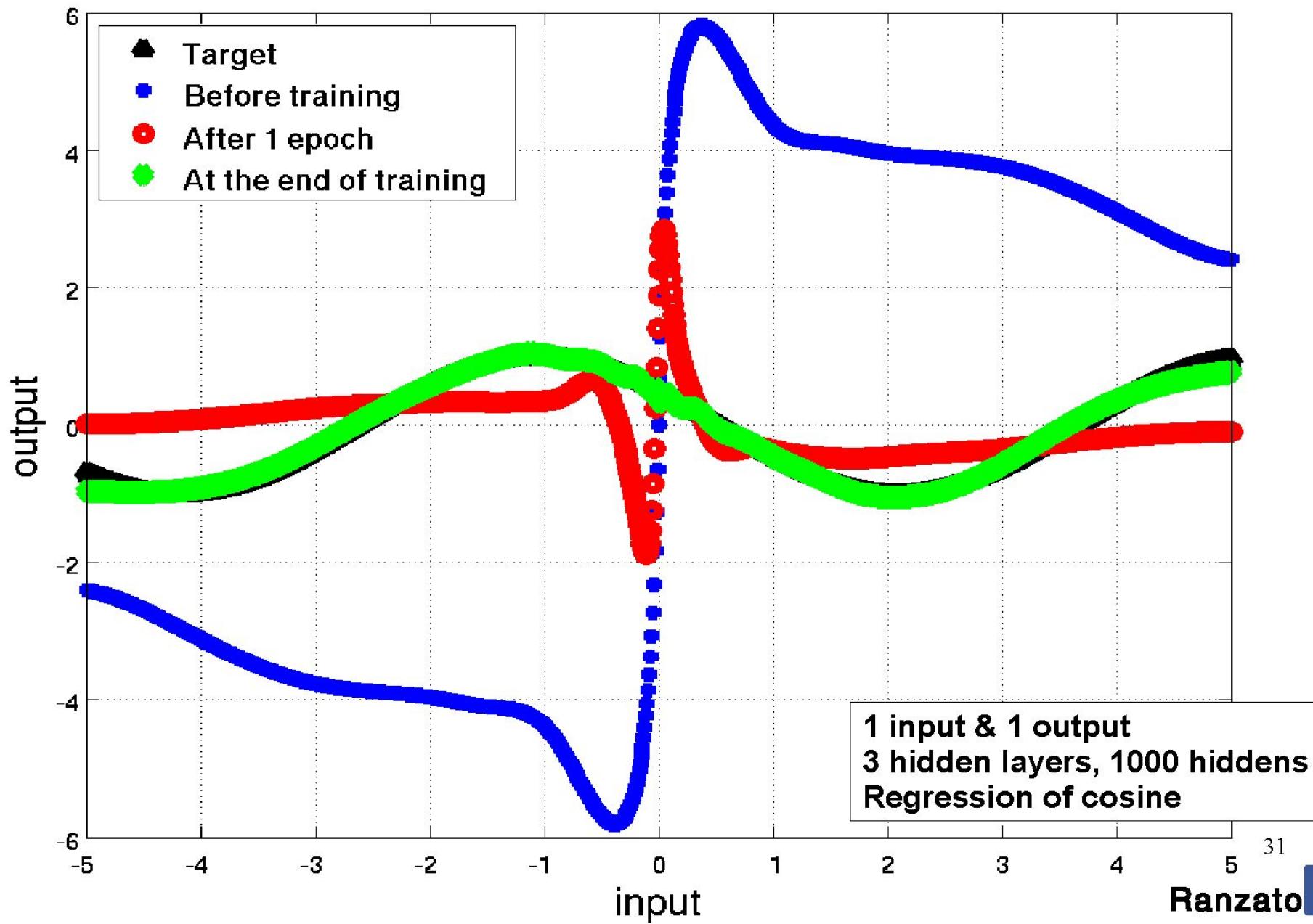
```
% F-PROP
for i = 1 : nr_layers - 1
    [h{i} jac{i}] = nonlinearity(W{i} * h{i-1} + b{i});
end
h{nr_layers-1} = W{nr_layers-1} * h{nr_layers-2} + b{nr_layers-1};
prediction = softmax(h{l-1});

% CROSS ENTROPY LOSS
loss = - sum(sum(log(prediction) .* target)) / batch_size;

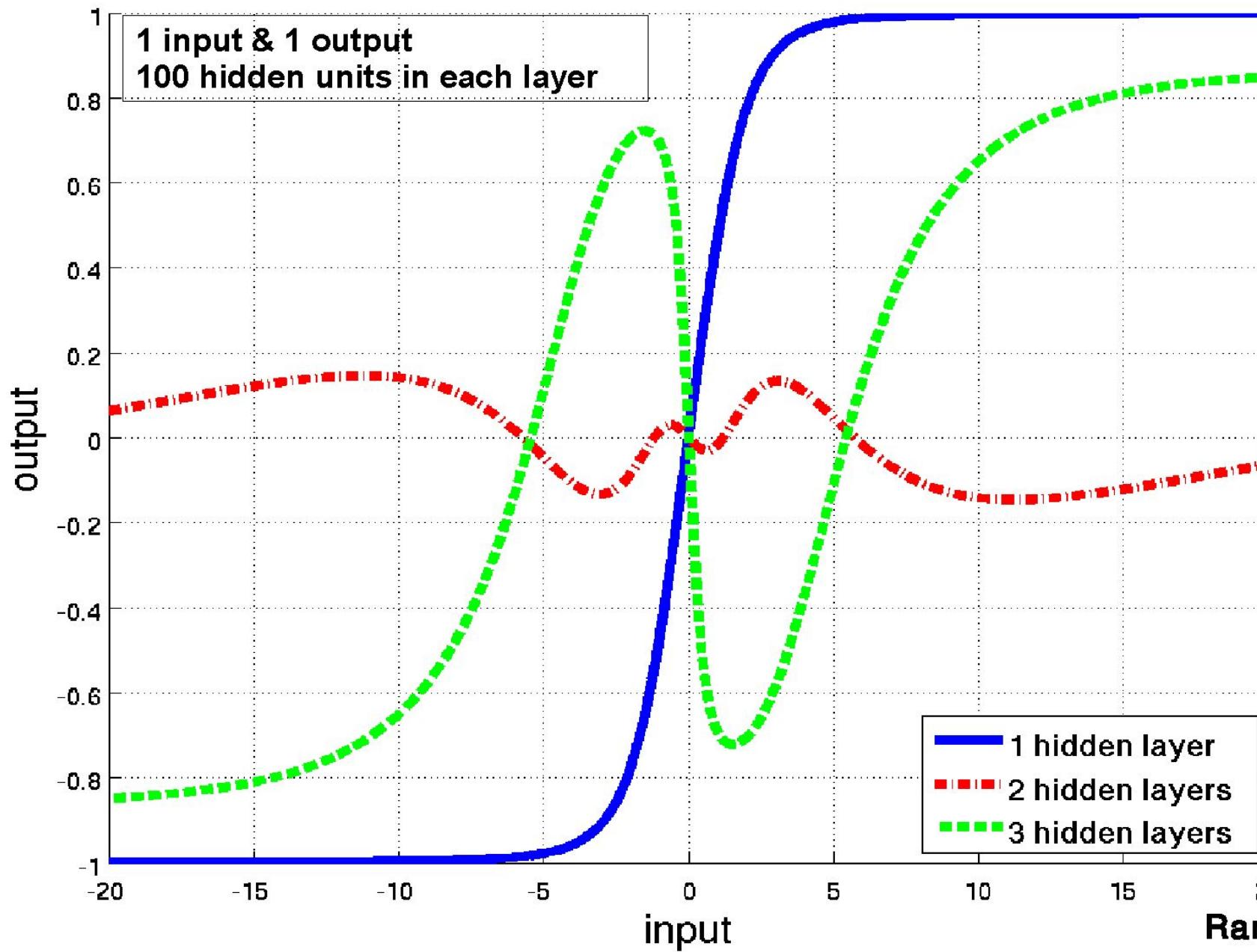
% B-PROP
dh{l-1} = prediction - target;
for i = nr_layers - 1 : -1 : 1
    Wgrad{i} = dh{i} * h{i-1}';
    bgrad{i} = sum(dh{i}, 2);
    dh{i-1} = (W{i}' * dh{i}) .* jac{i-1};
end

% UPDATE
for i = 1 : nr_layers - 1
    W{i} = W{i} - (lr / batch_size) * Wgrad{i};
    b{i} = b{i} - (lr / batch_size) * bgrad{i};
end
```

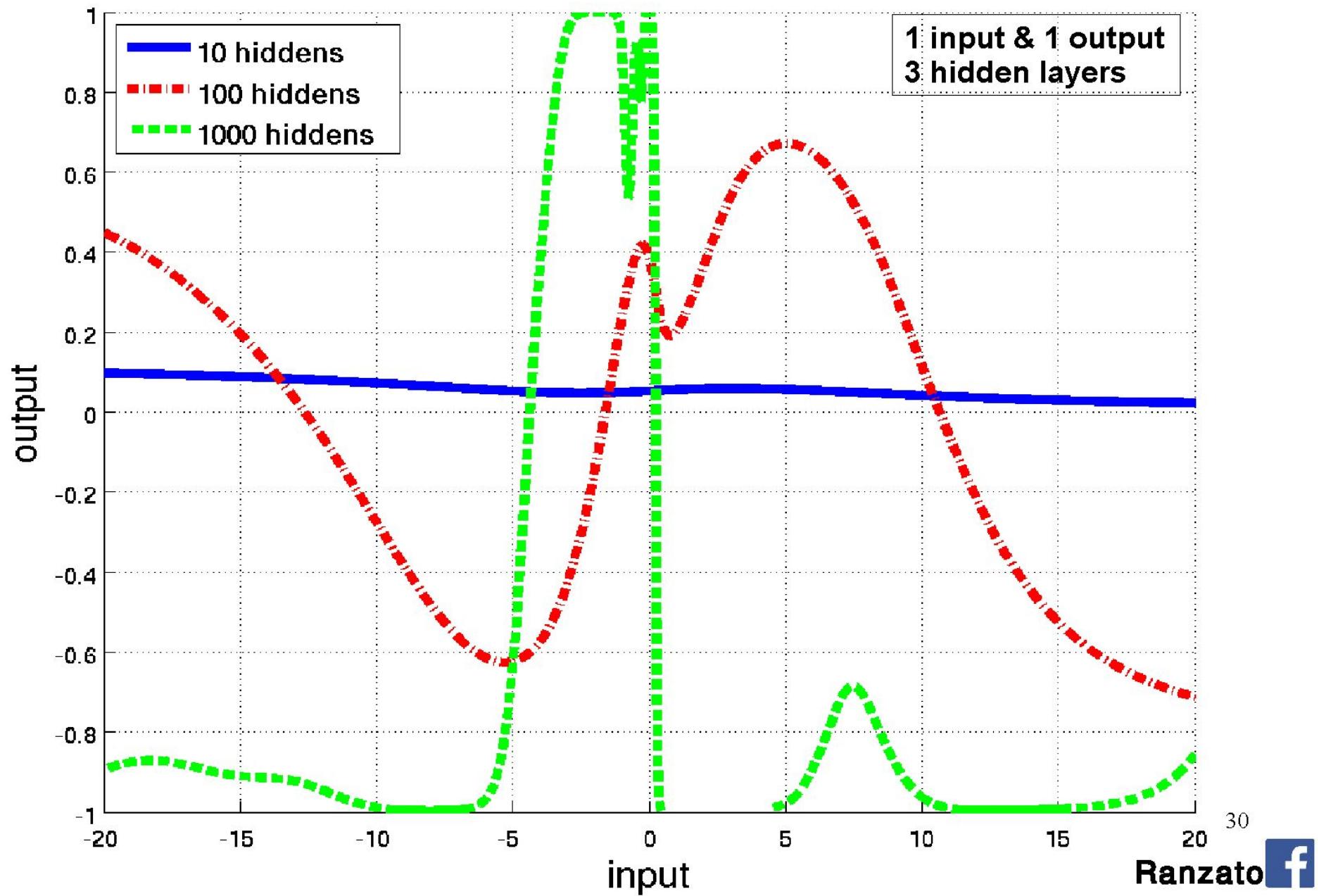
Toy Example: Synthetic Data



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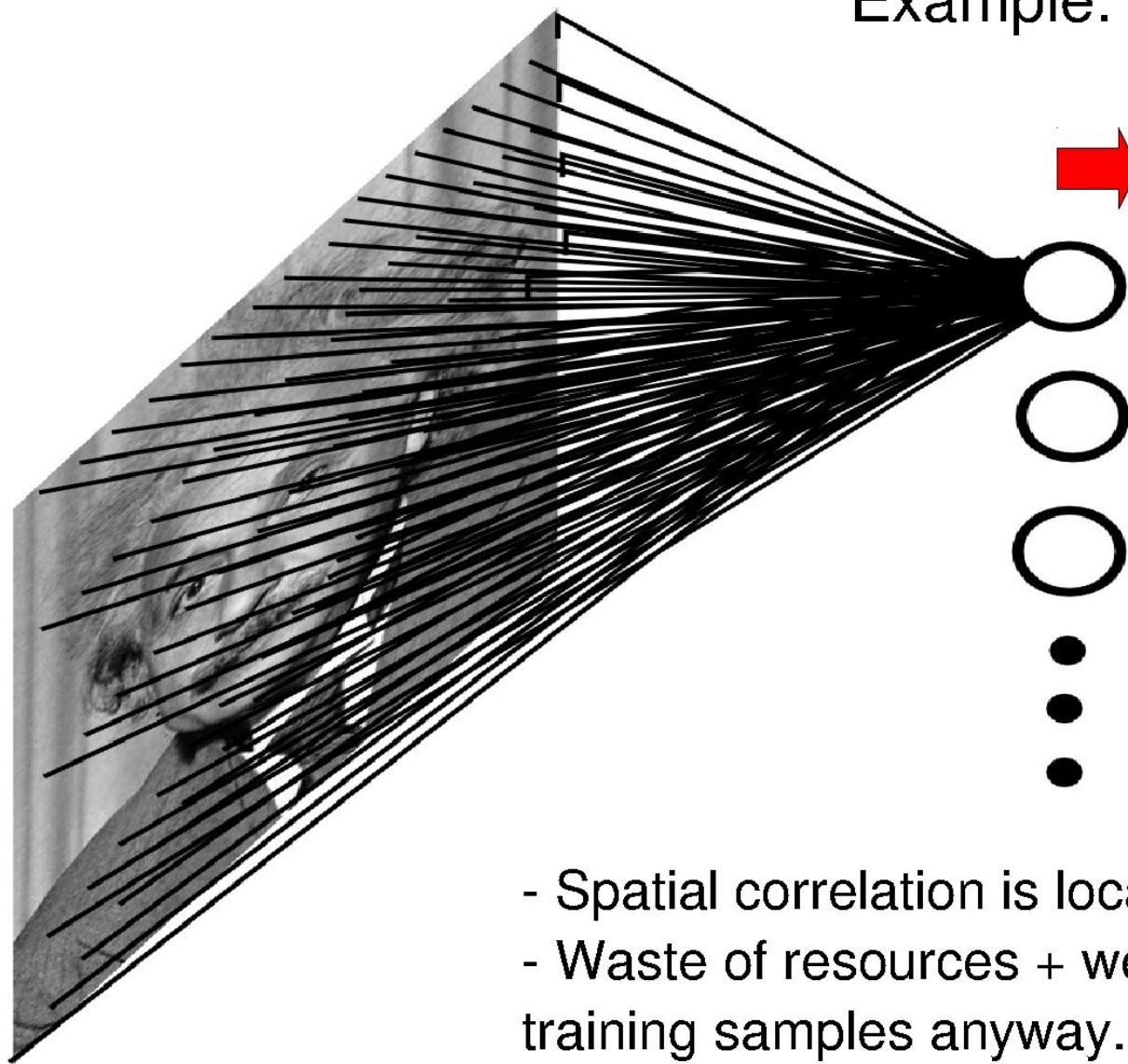
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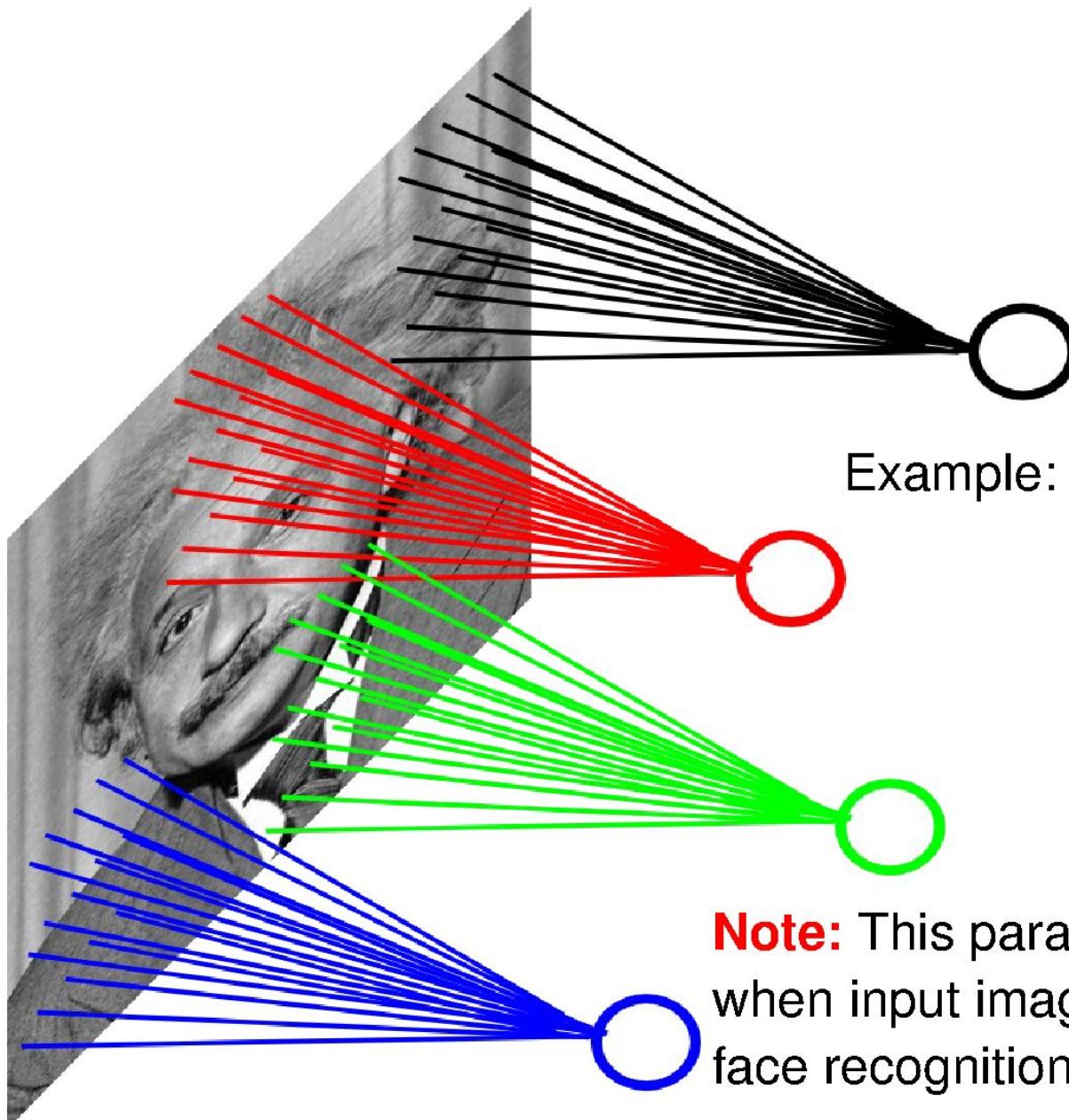
Fully Connected Layer



Example: 200x200 image
40K hidden units
→ **~2B parameters!!!**

- Spatial correlation is local
- Waste of resources + we have not enough training samples anyway..

Locally Connected Layer

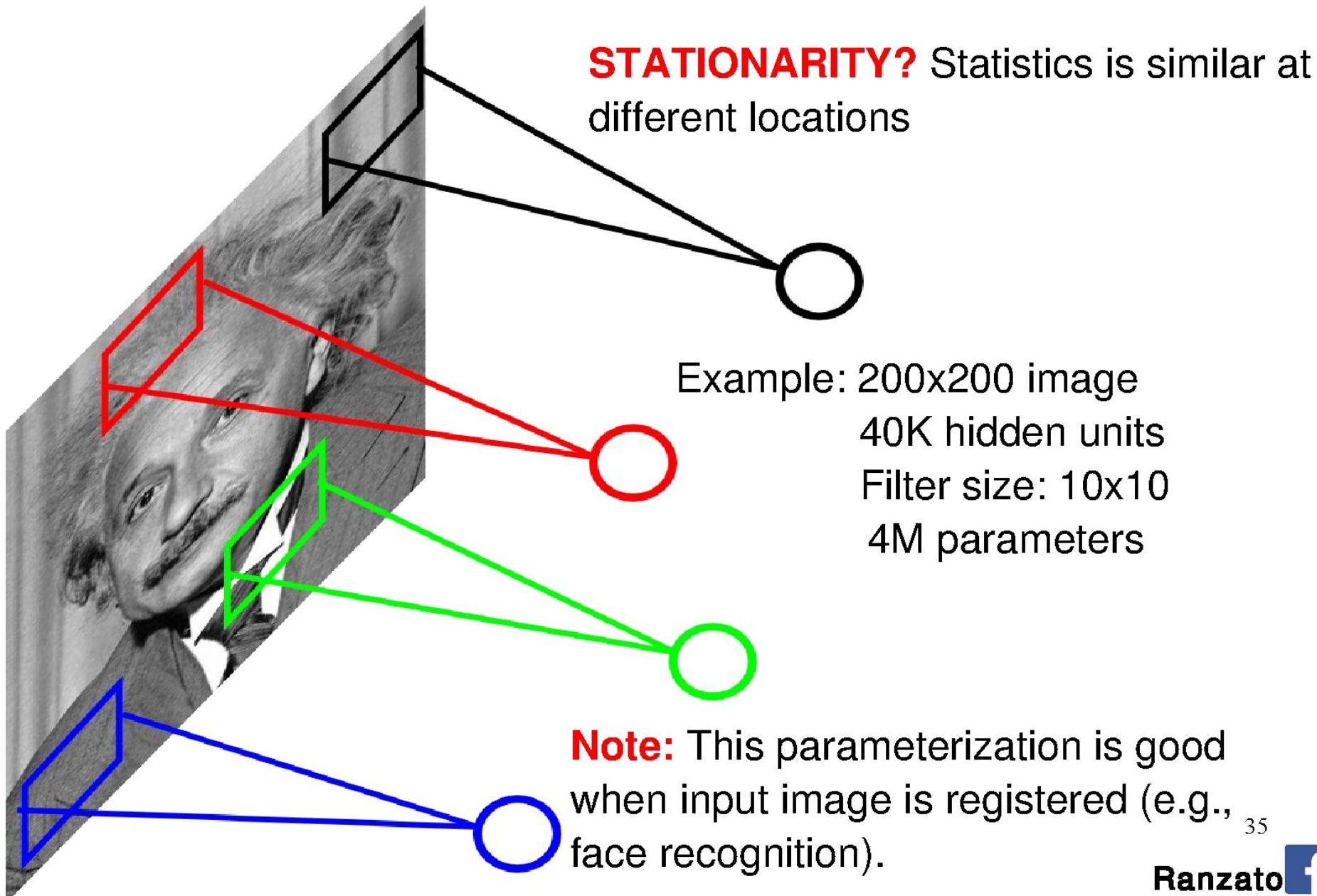


Example: 200x200 image
40K hidden units
Filter size: 10x10
4M parameters

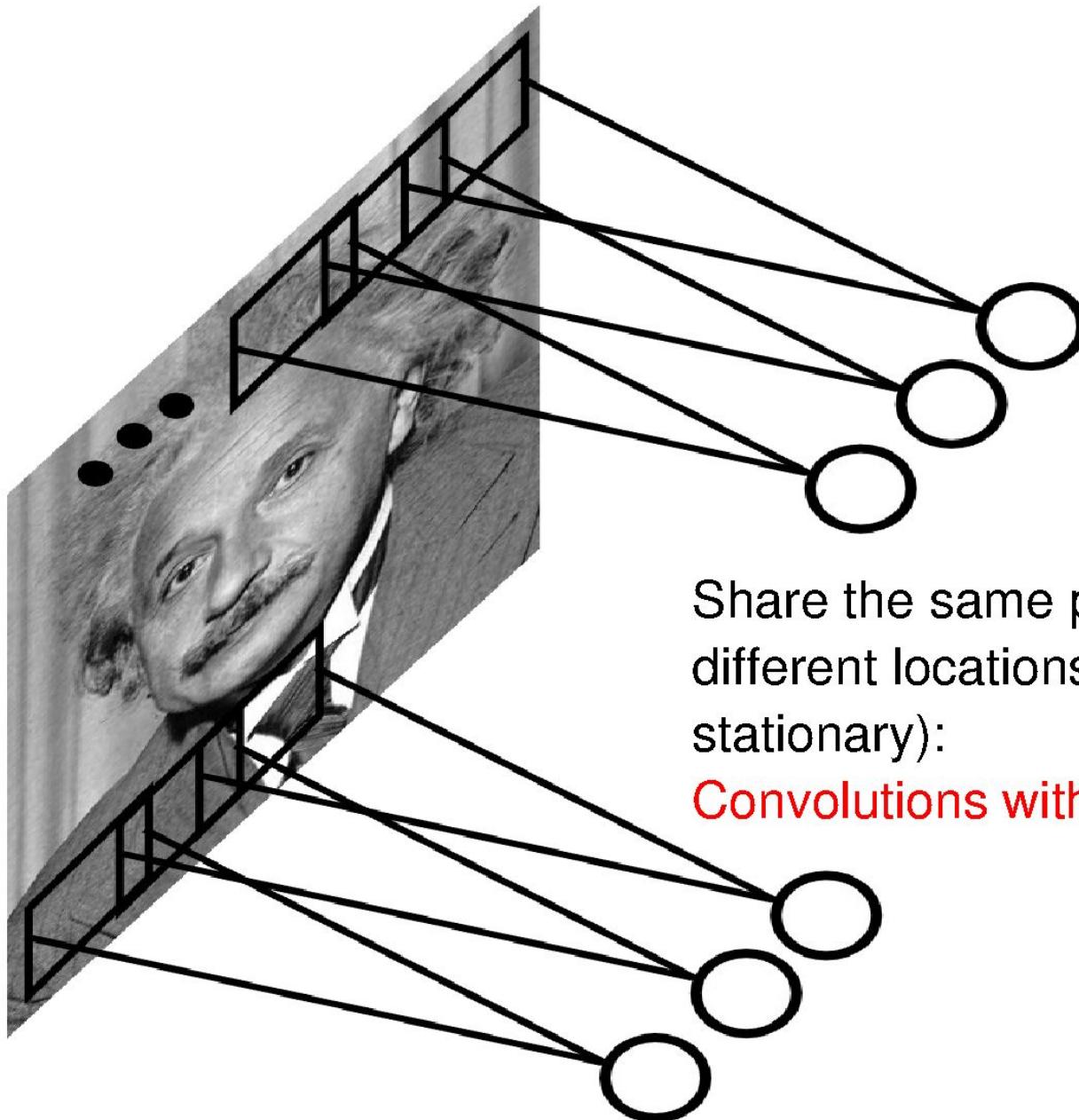
Note: This parameterization is good
when input image is registered (e.g.,
face recognition).

34

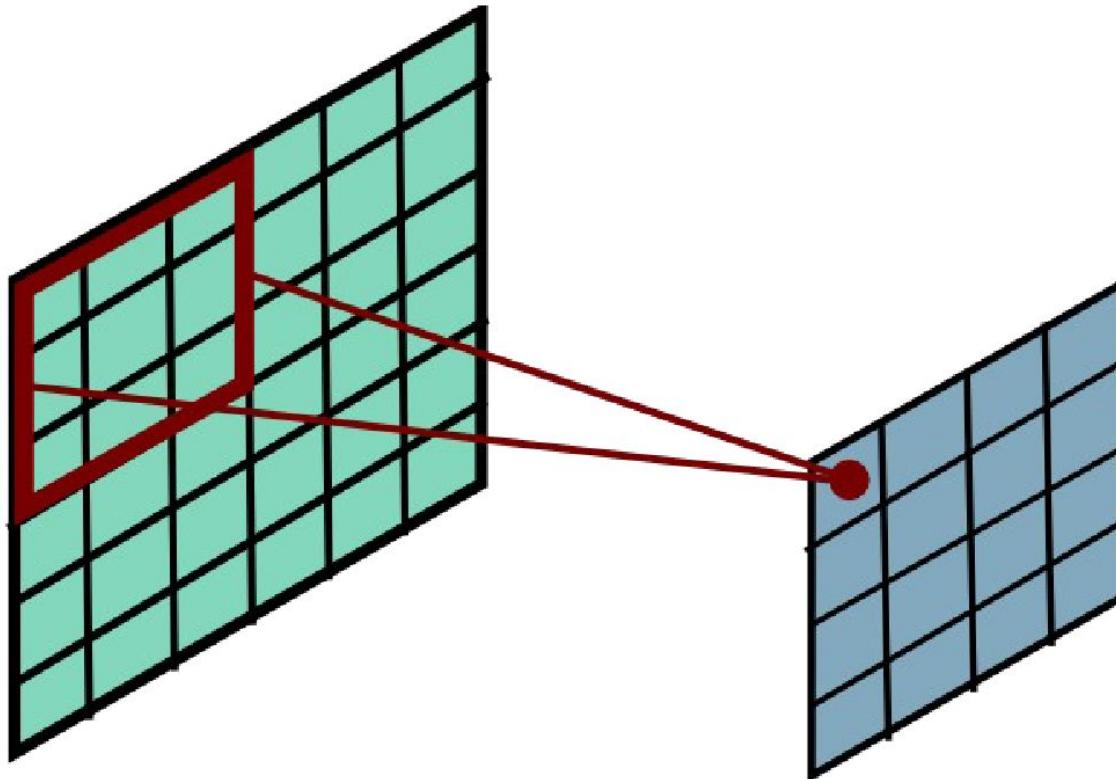
Locally Connected Layer



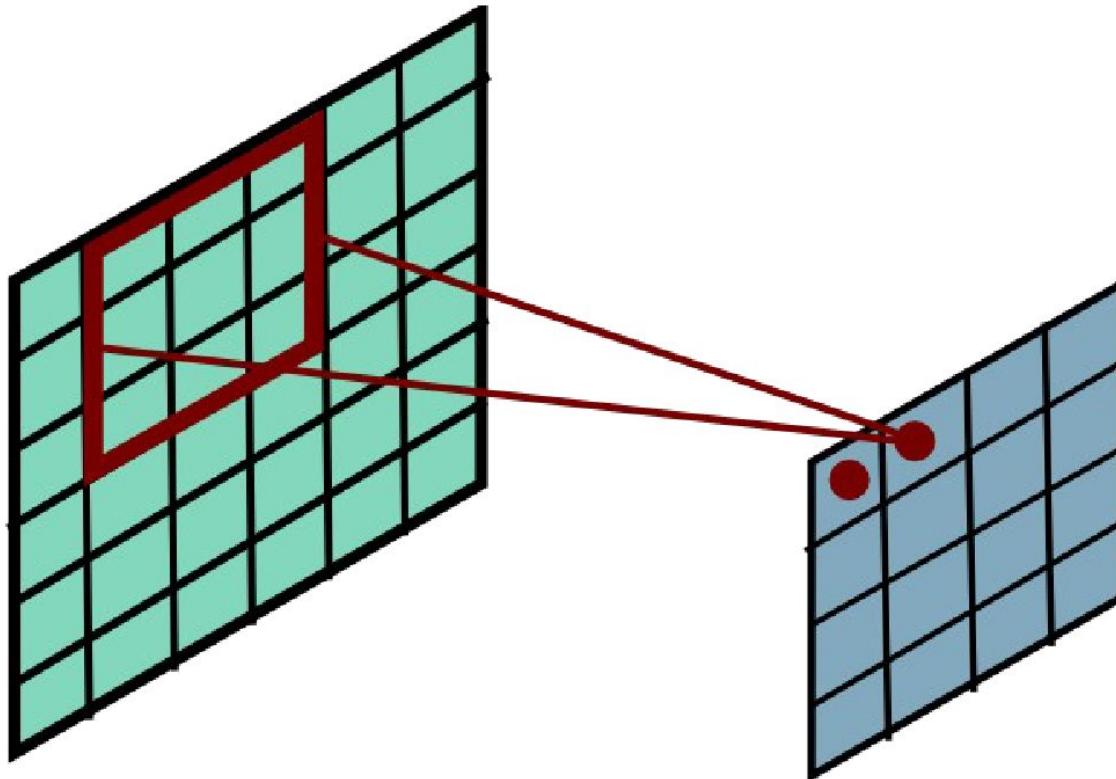
Convolutional Layer



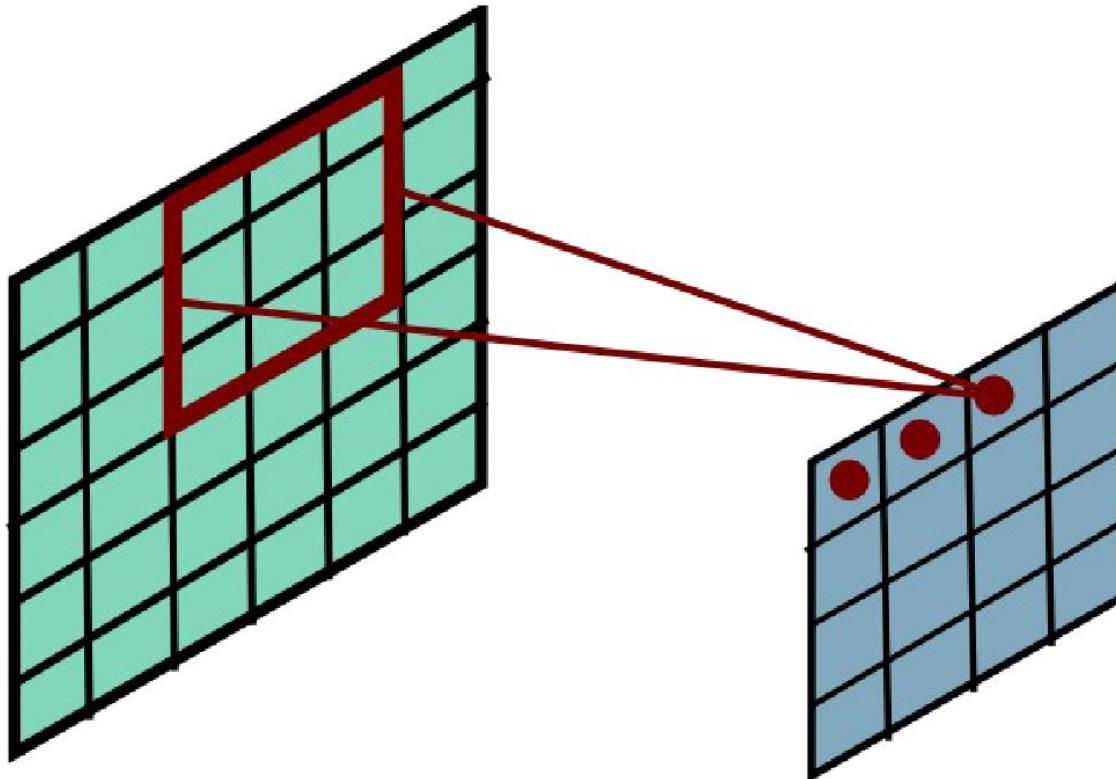
Convolutional Layer



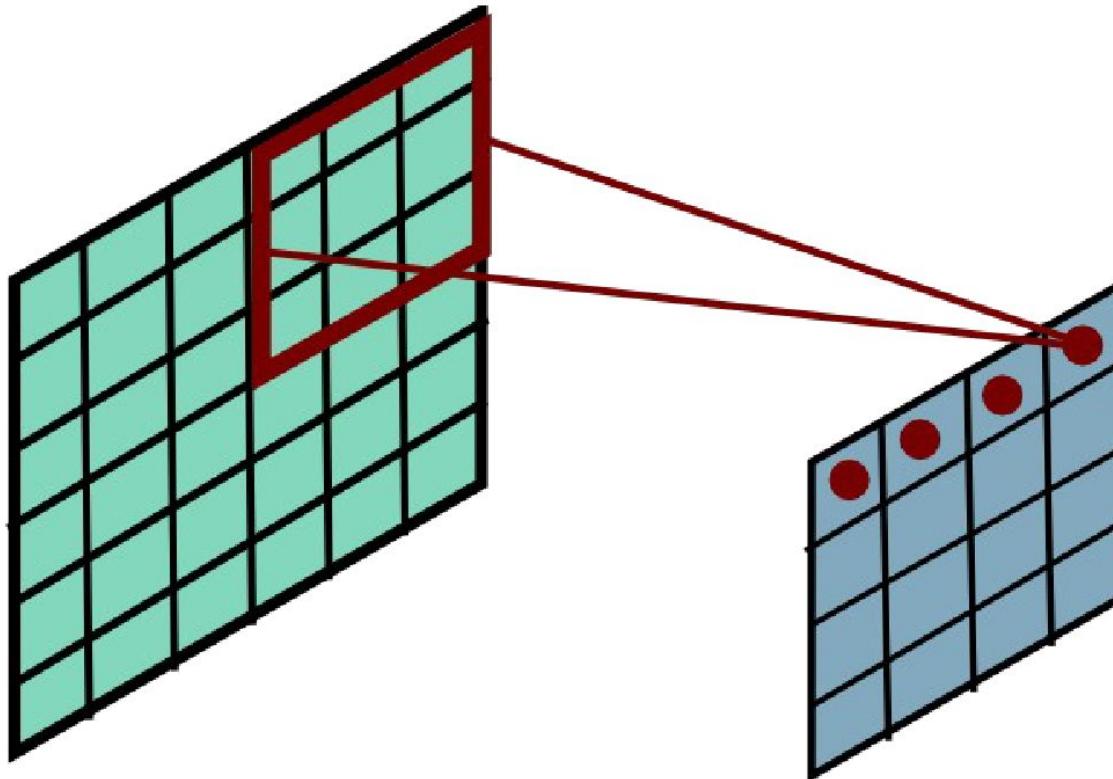
Convolutional Layer



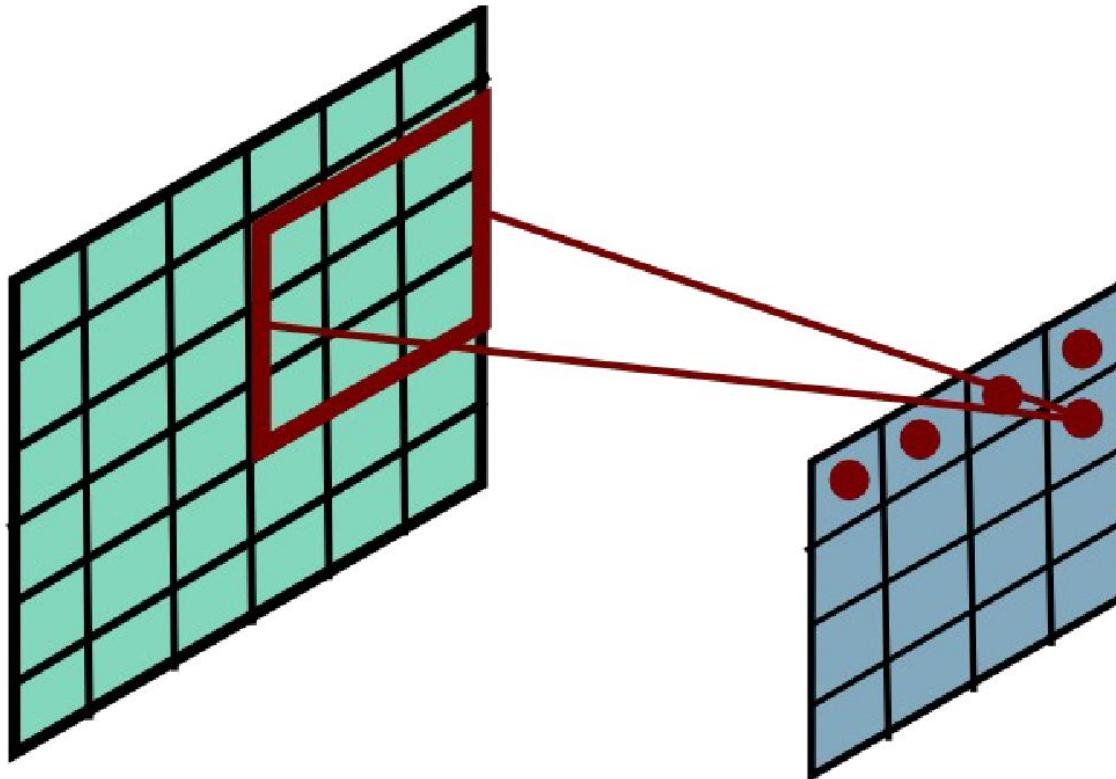
Convolutional Layer



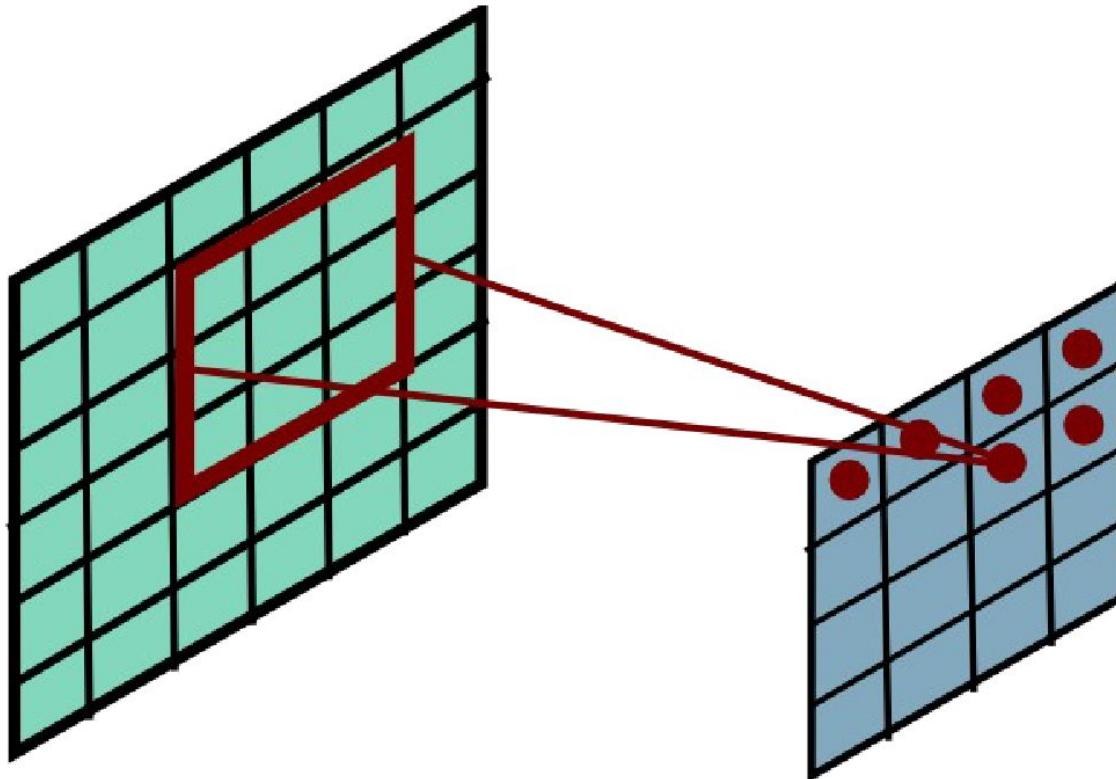
Convolutional Layer



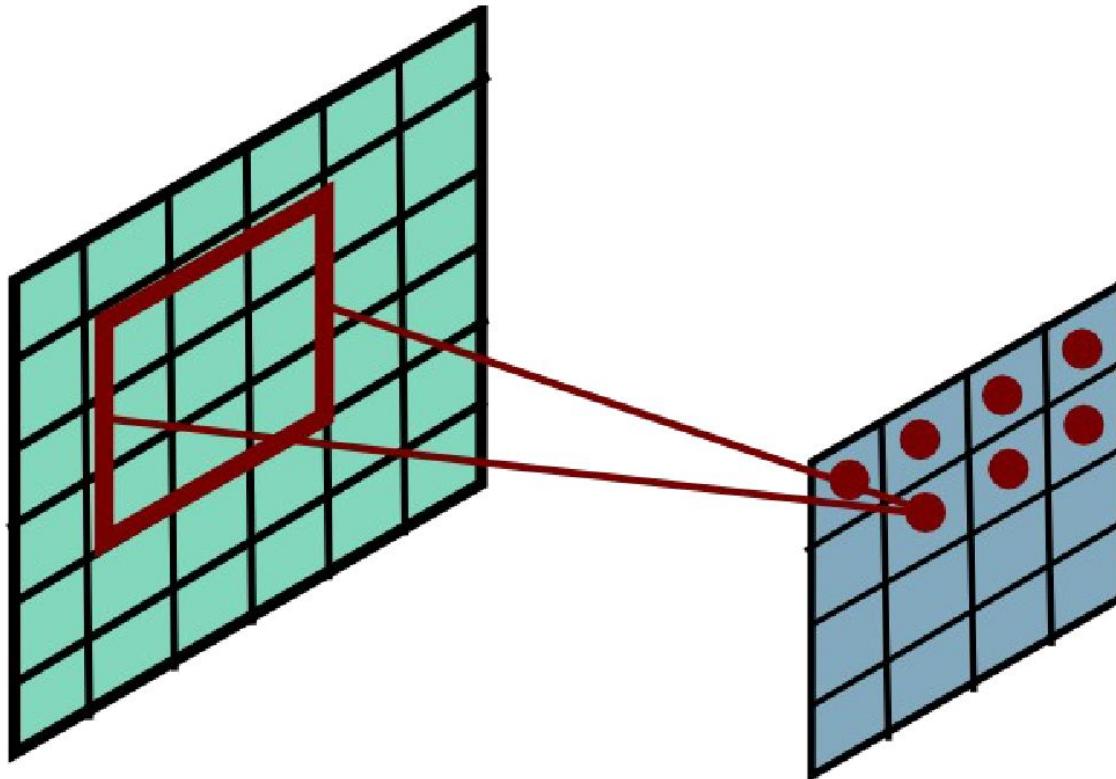
Convolutional Layer



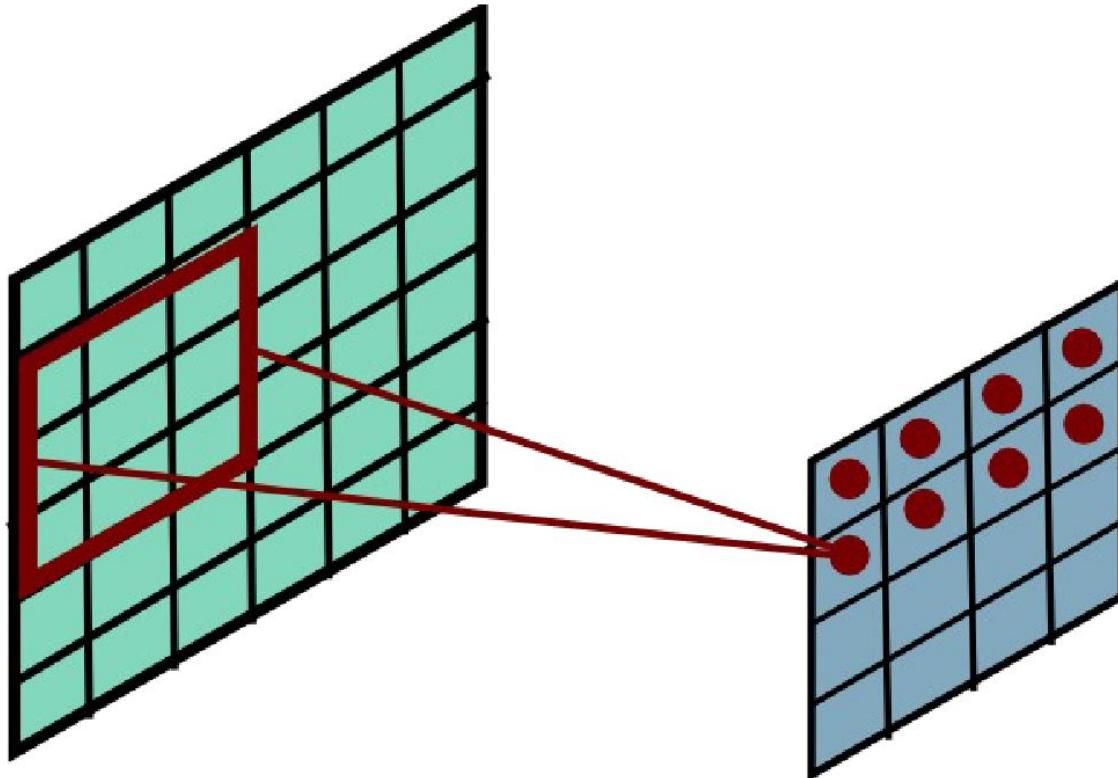
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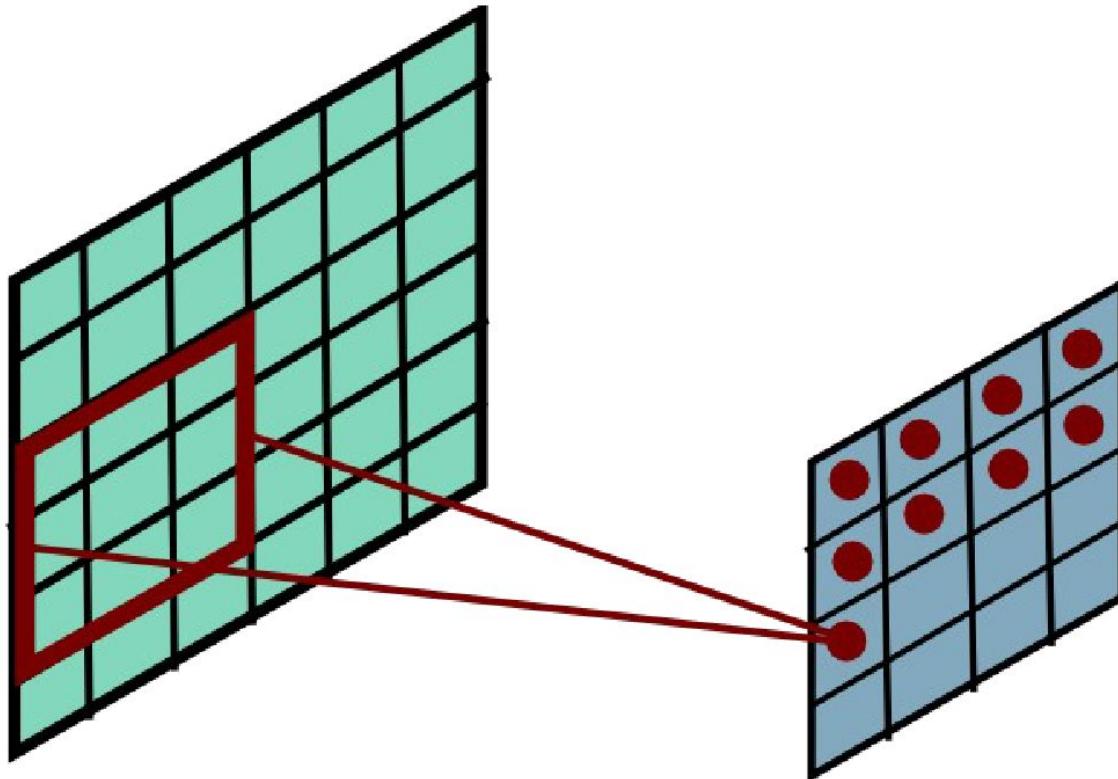
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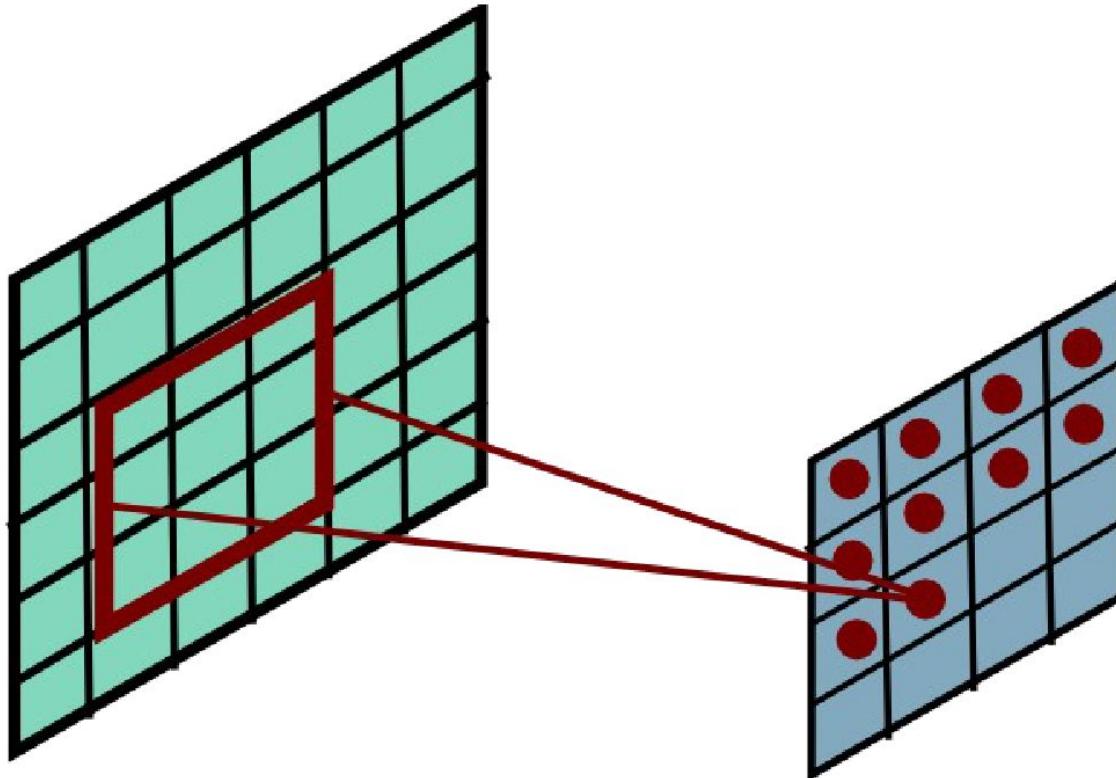
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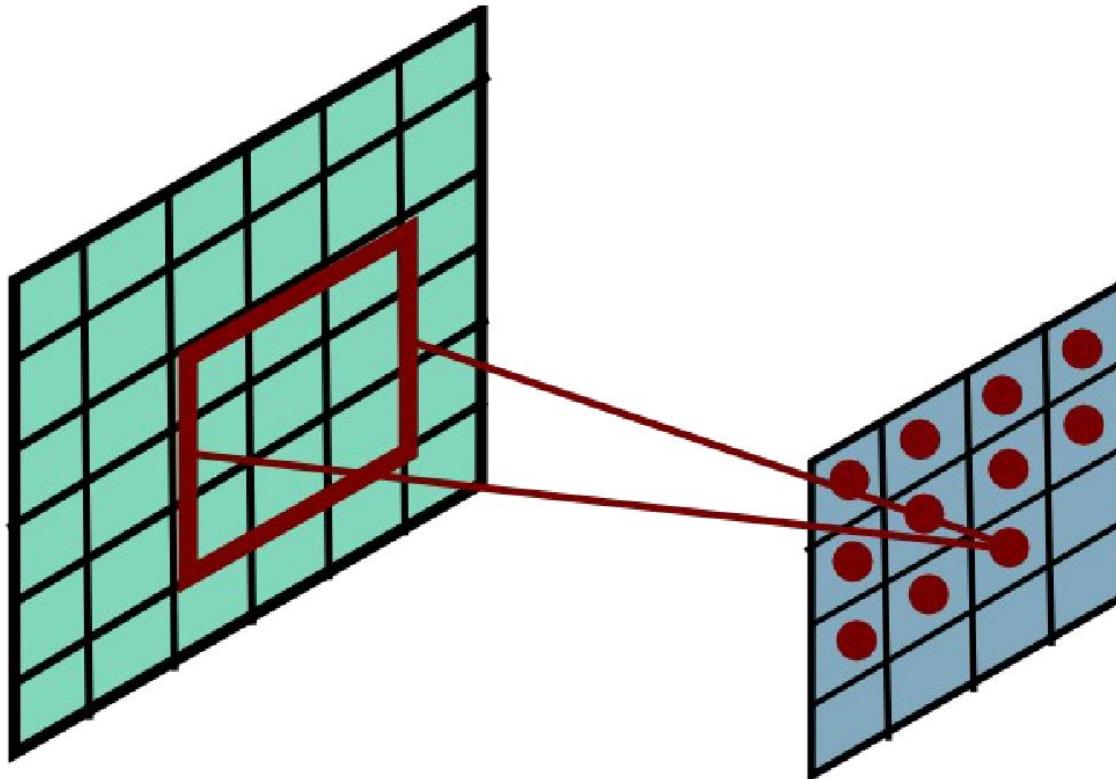
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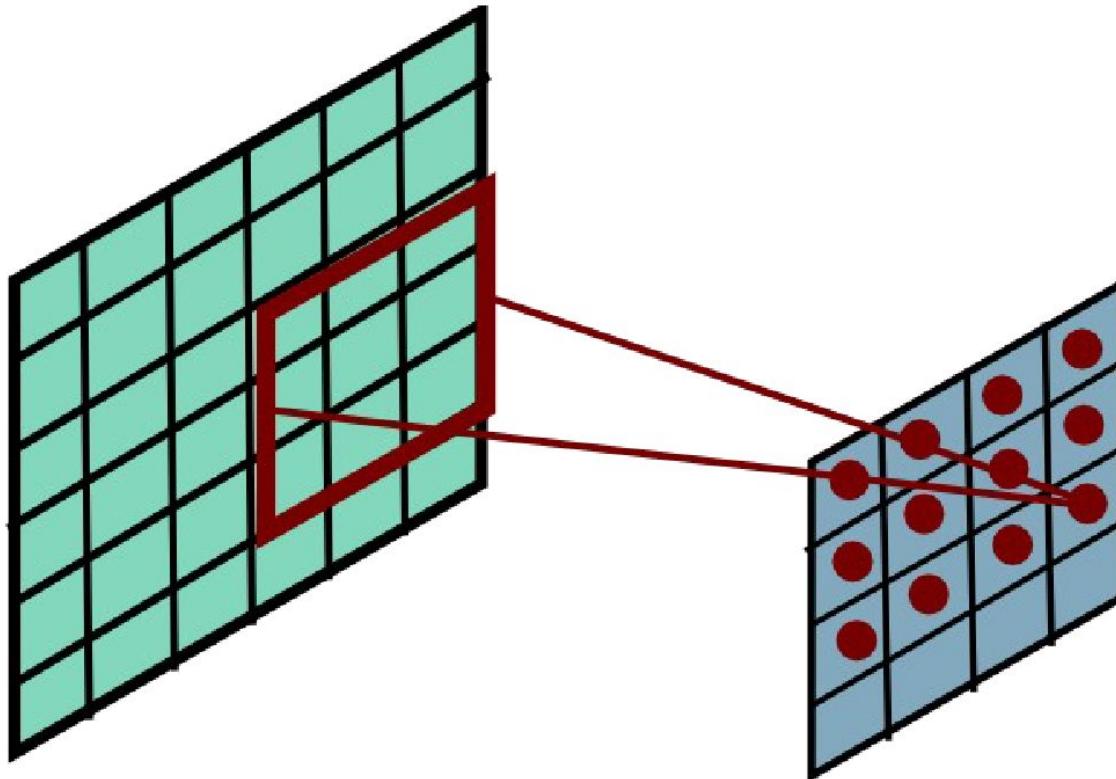
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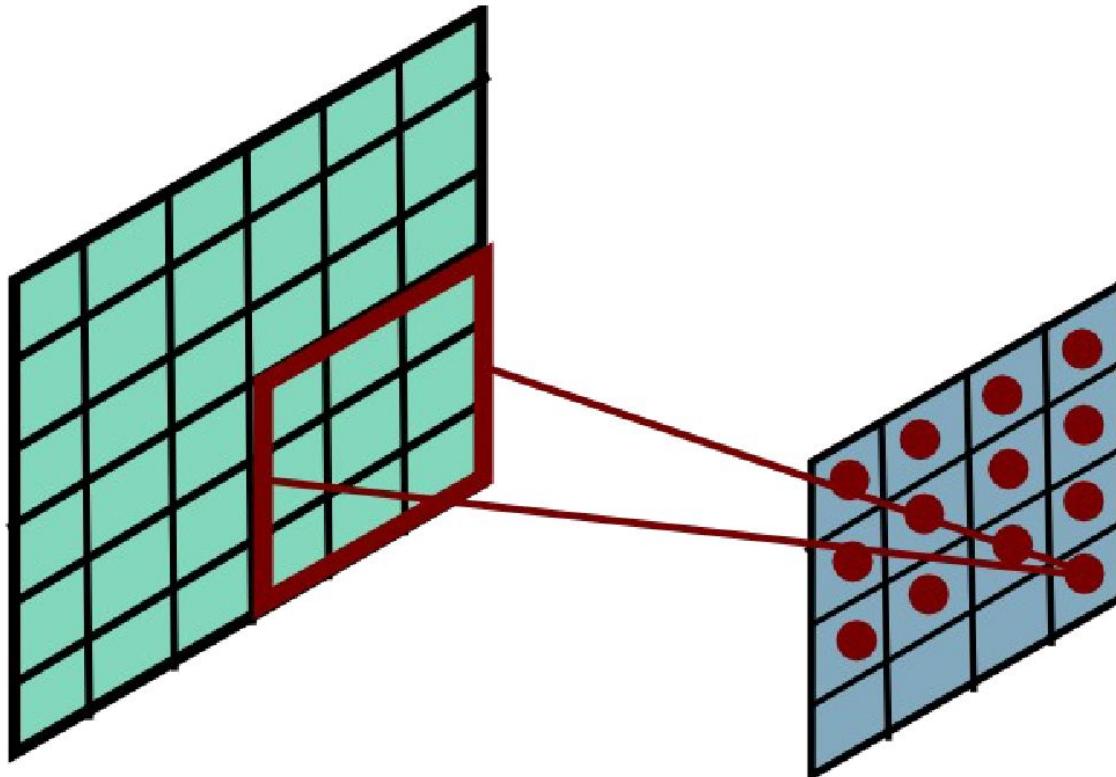
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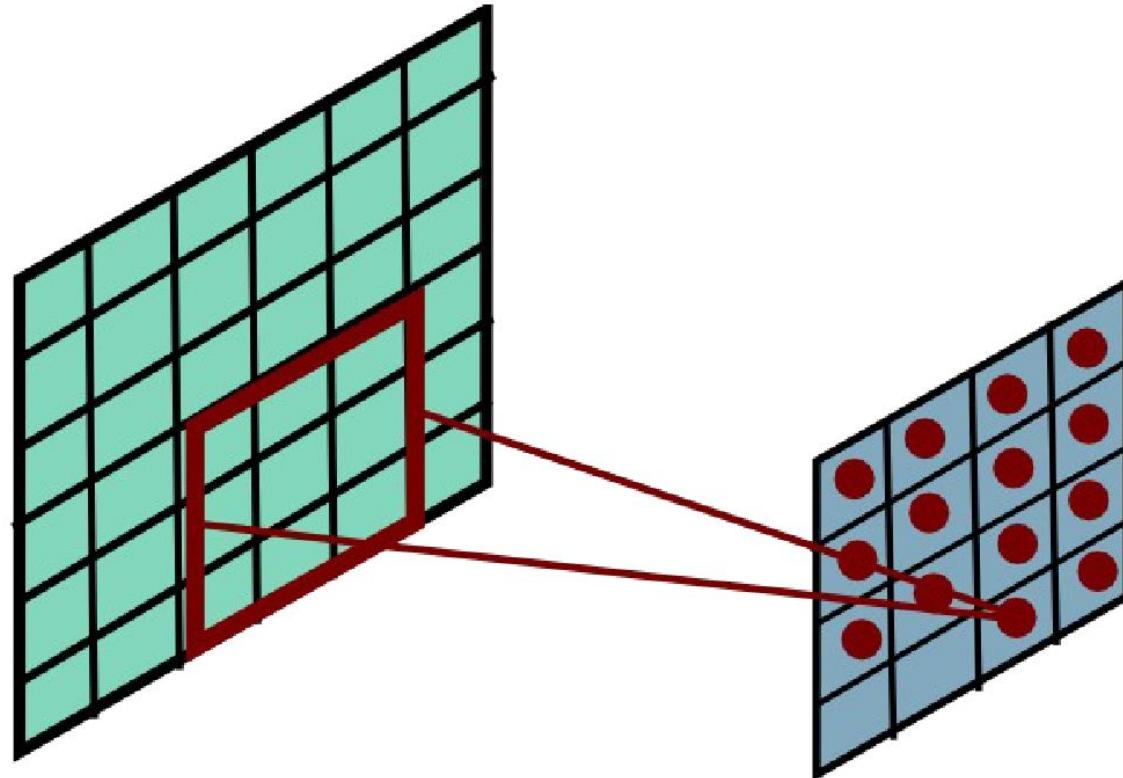
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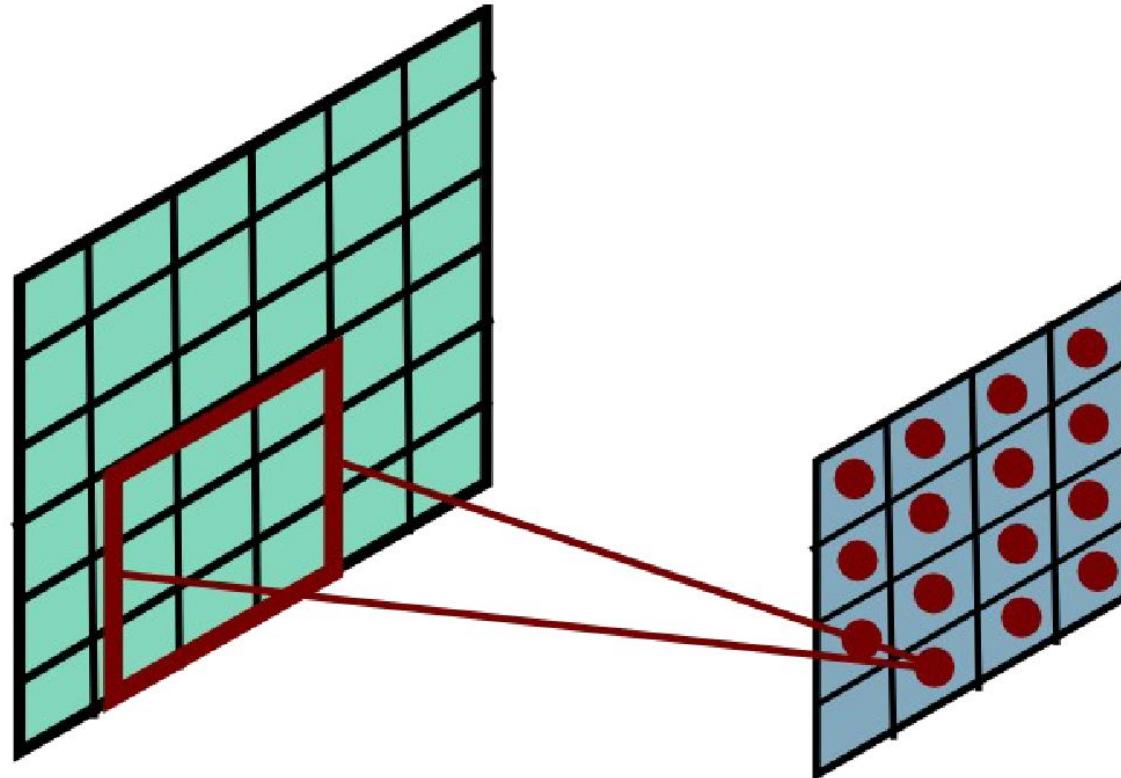
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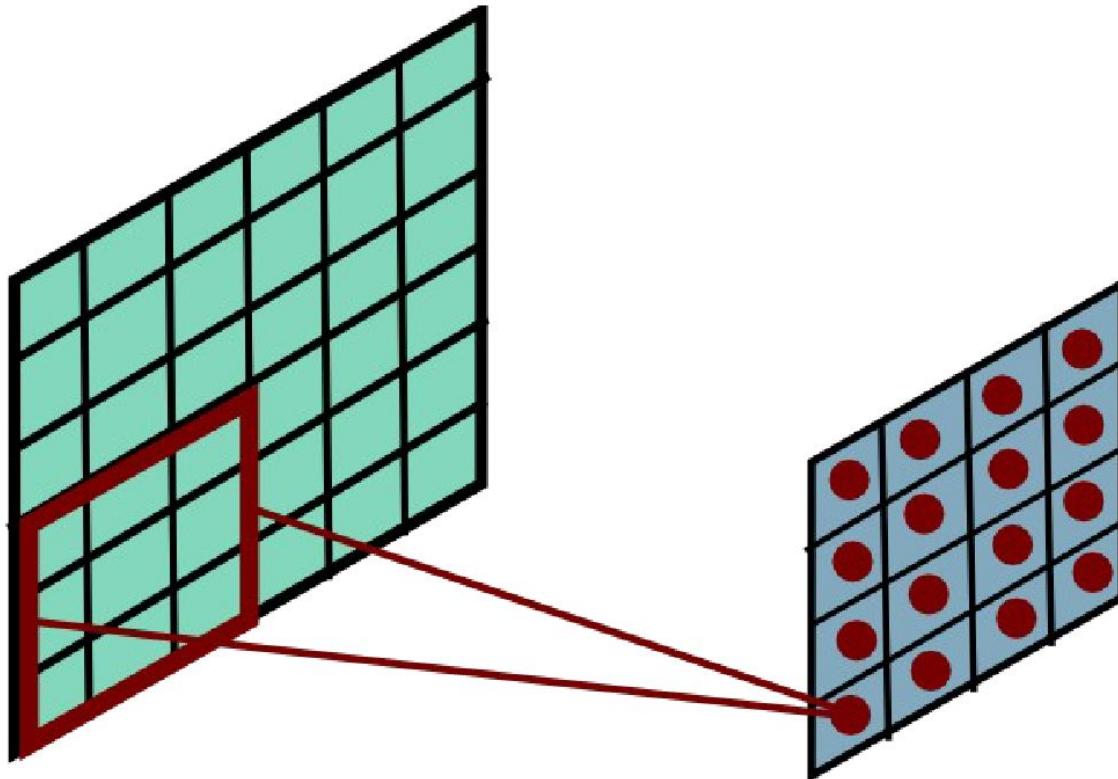
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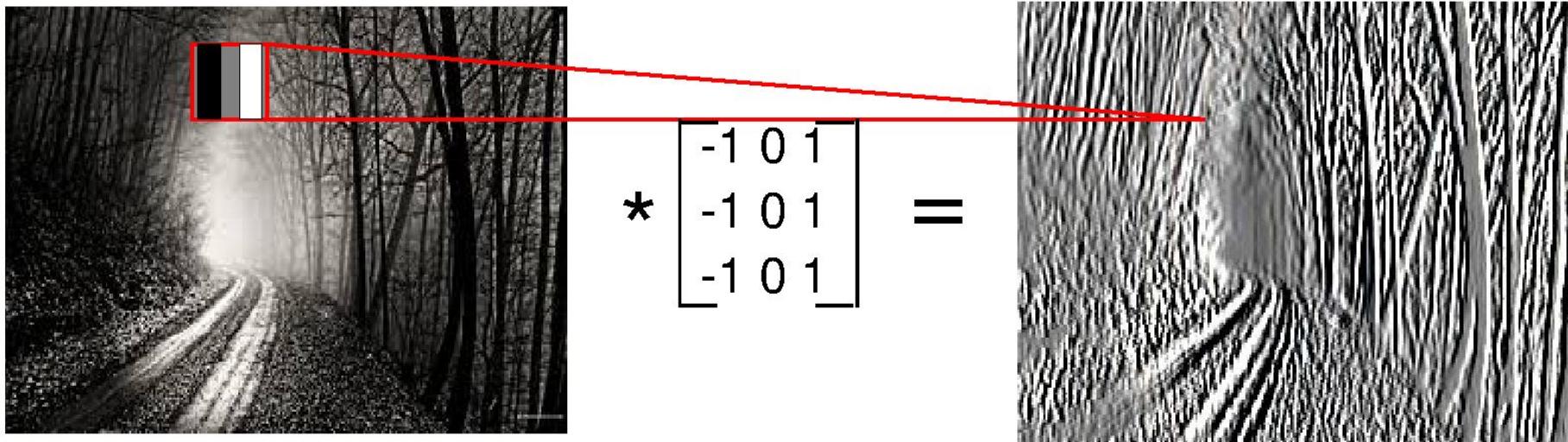
Convolutional Layer



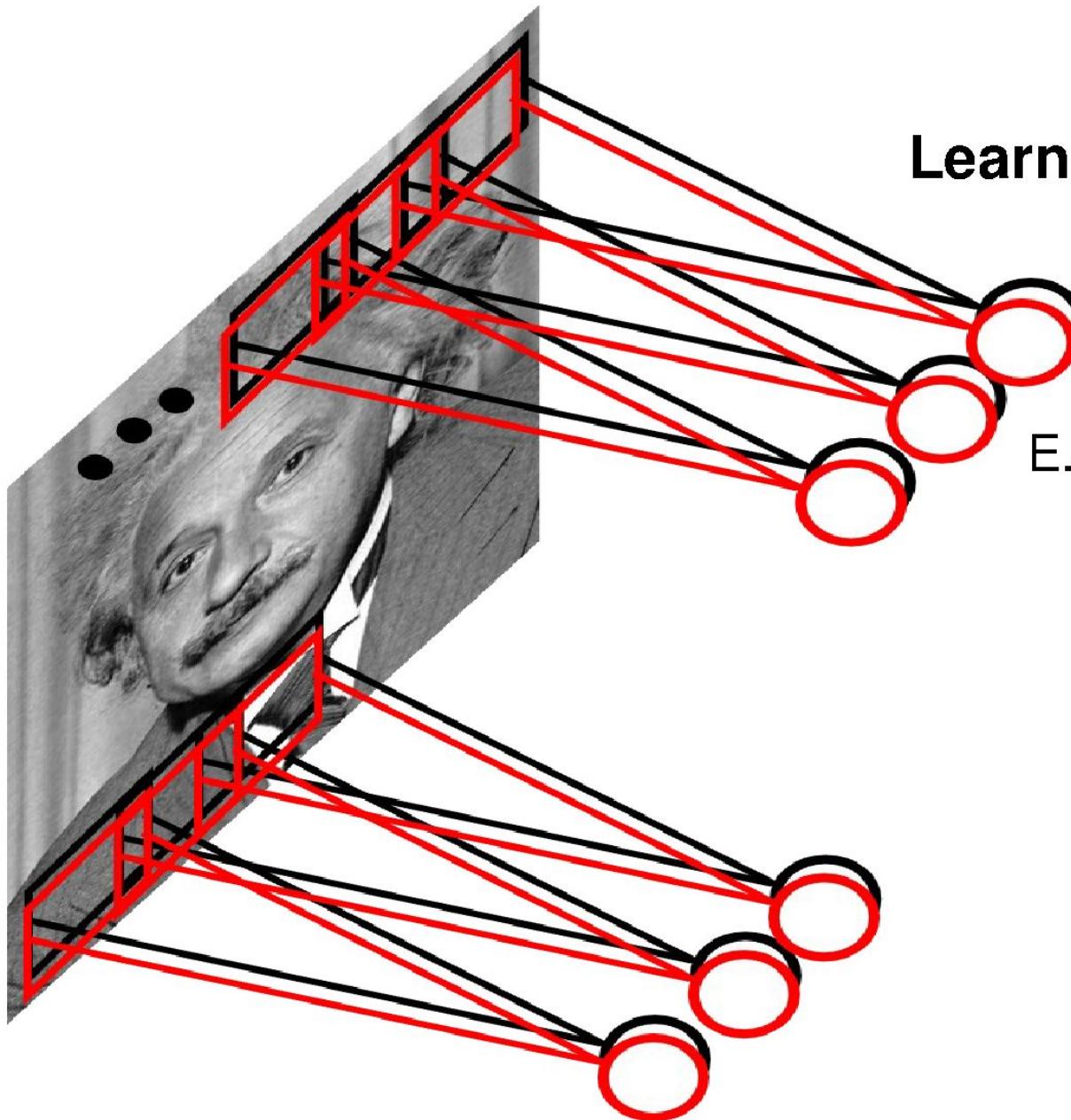
Convolutional Layer



Convolutional Layer



Convolutional Layer



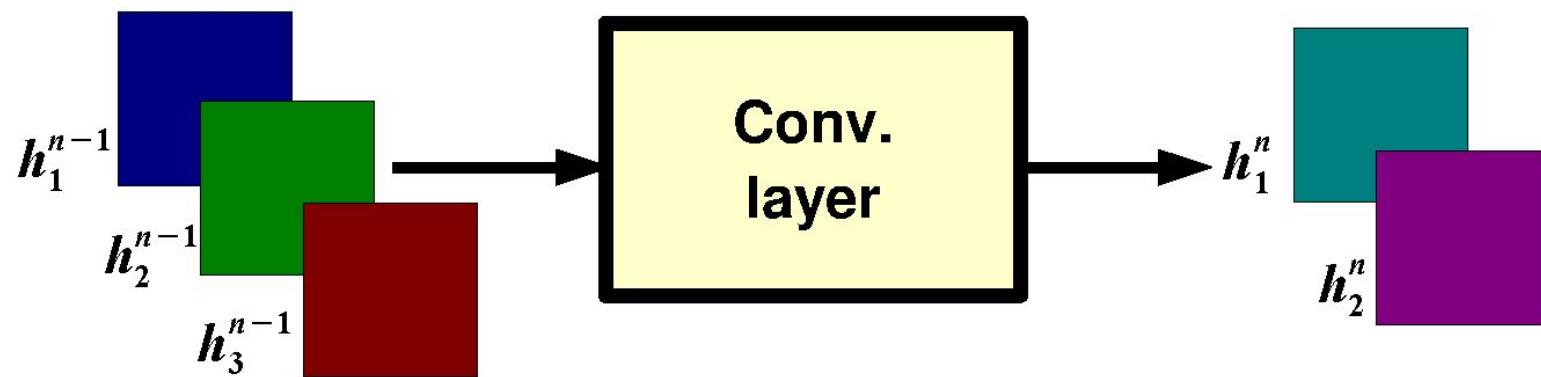
Learn multiple filters.

E.g.: 200x200 image
100 Filters
Filter size: 10x10
10K parameters

Convolutional Layer

$$h_j^n = \max \left(0, \sum_{k=1}^K h_k^{n-1} * w_{kj}^n \right)$$

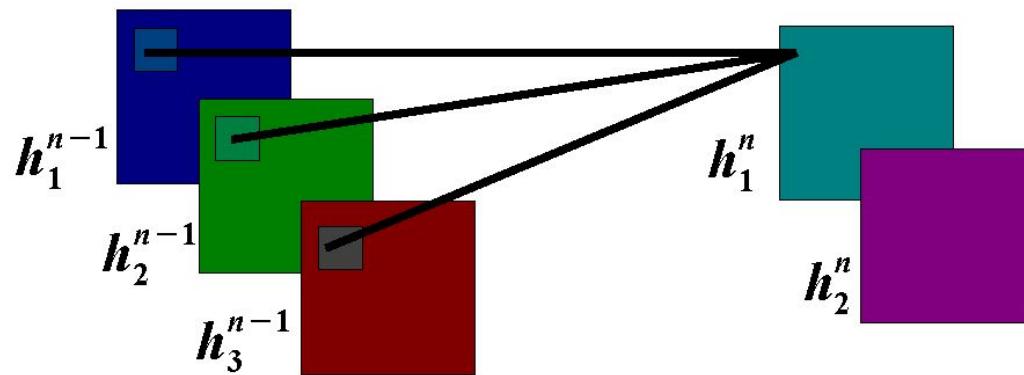
output feature map **input feature map** **kernel**



Convolutional Layer

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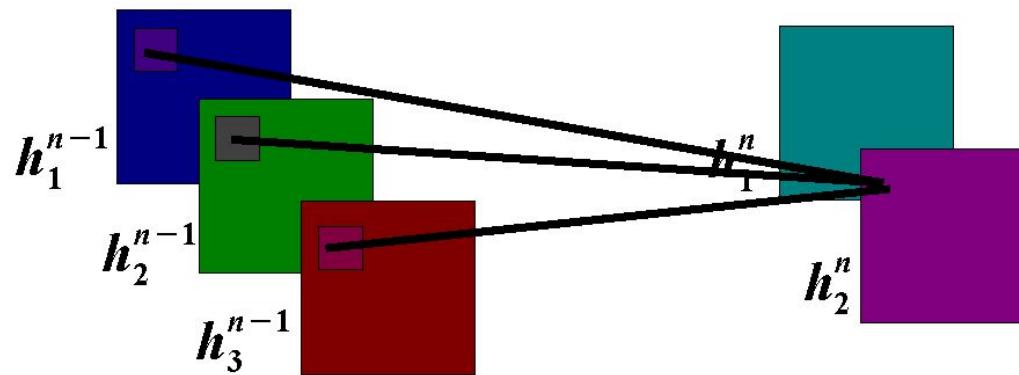
output feature map **input feature map** **kernel**



Convolutional Layer

$$h_j^n = \max \left(0, \sum_{k=1}^K h_k^{n-1} * w_{kj}^n \right)$$

output feature map **input feature map** **kernel**



Convolutional Layer

Question: What is the size of the output? What's the computational cost?

Answer: It is proportional to the number of filters and depends on the stride. If kernels have size $K \times K$, input has size $D \times D$, stride is 1, and there are M input feature maps and N output feature maps then:

- the input has size $M @ D \times D$
- the output has size $N @ (D - K + 1) \times (D - K + 1)$
- the kernels have $M \times N \times K \times K$ coefficients (which have to be learned)
- cost: $M * K * K * N * (D - K + 1) * (D - K + 1)$

Question: How many feature maps? What's the size of the filters?

Answer: Usually, there are more output feature maps than input feature maps. Convolutional layers can increase the number of hidden units by big factors (and are expensive to compute).

The size of the filters has to match the size/scale of the patterns we want to detect (task dependent).

Key Ideas

A standard neural net applied to images:

- scales quadratically with the size of the input
- does not leverage stationarity

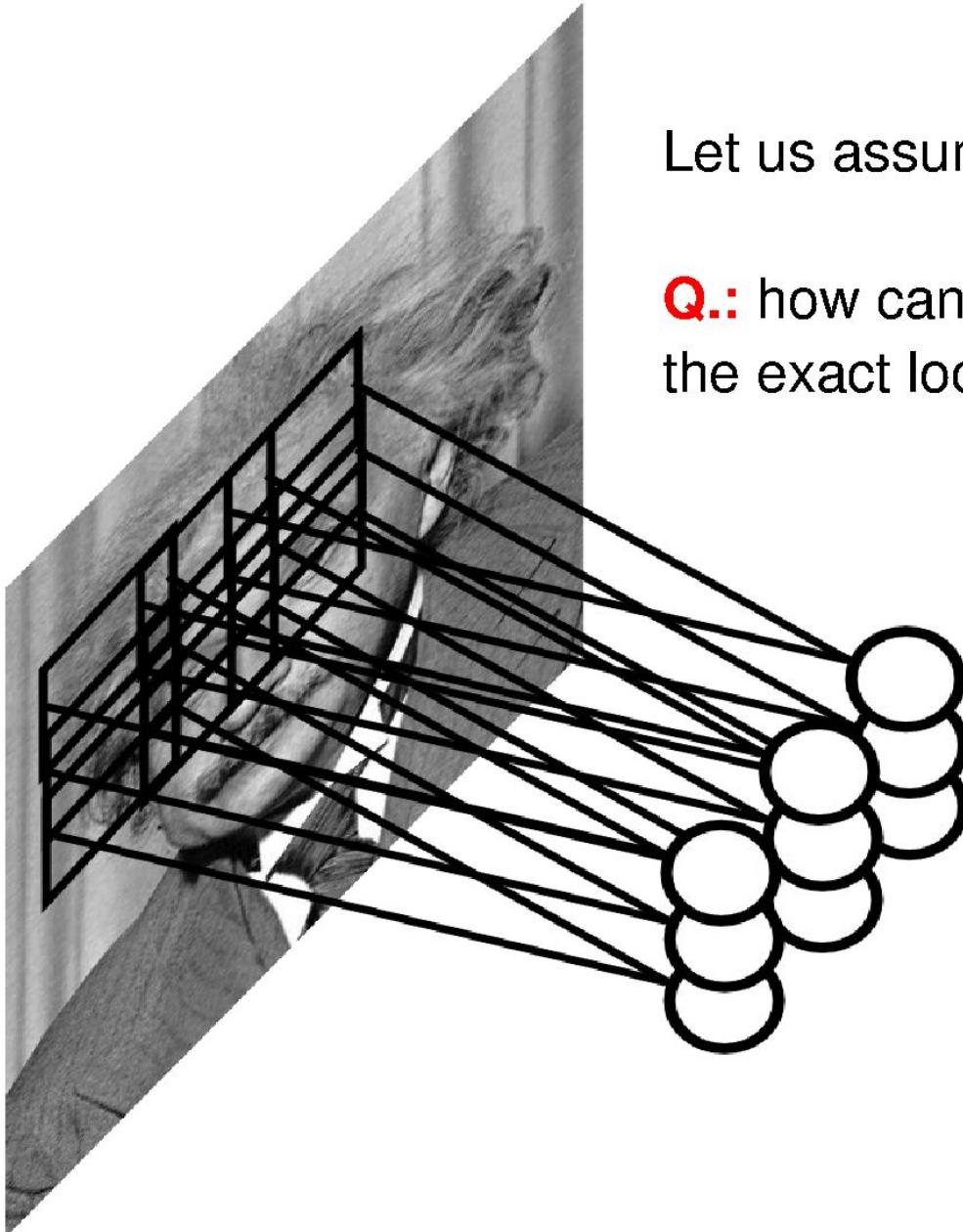
Solution:

- connect each hidden unit to a small patch of the input
- share the weight across space

This is called: **convolutional layer**.

A network with convolutional layers is called **convolutional network**.

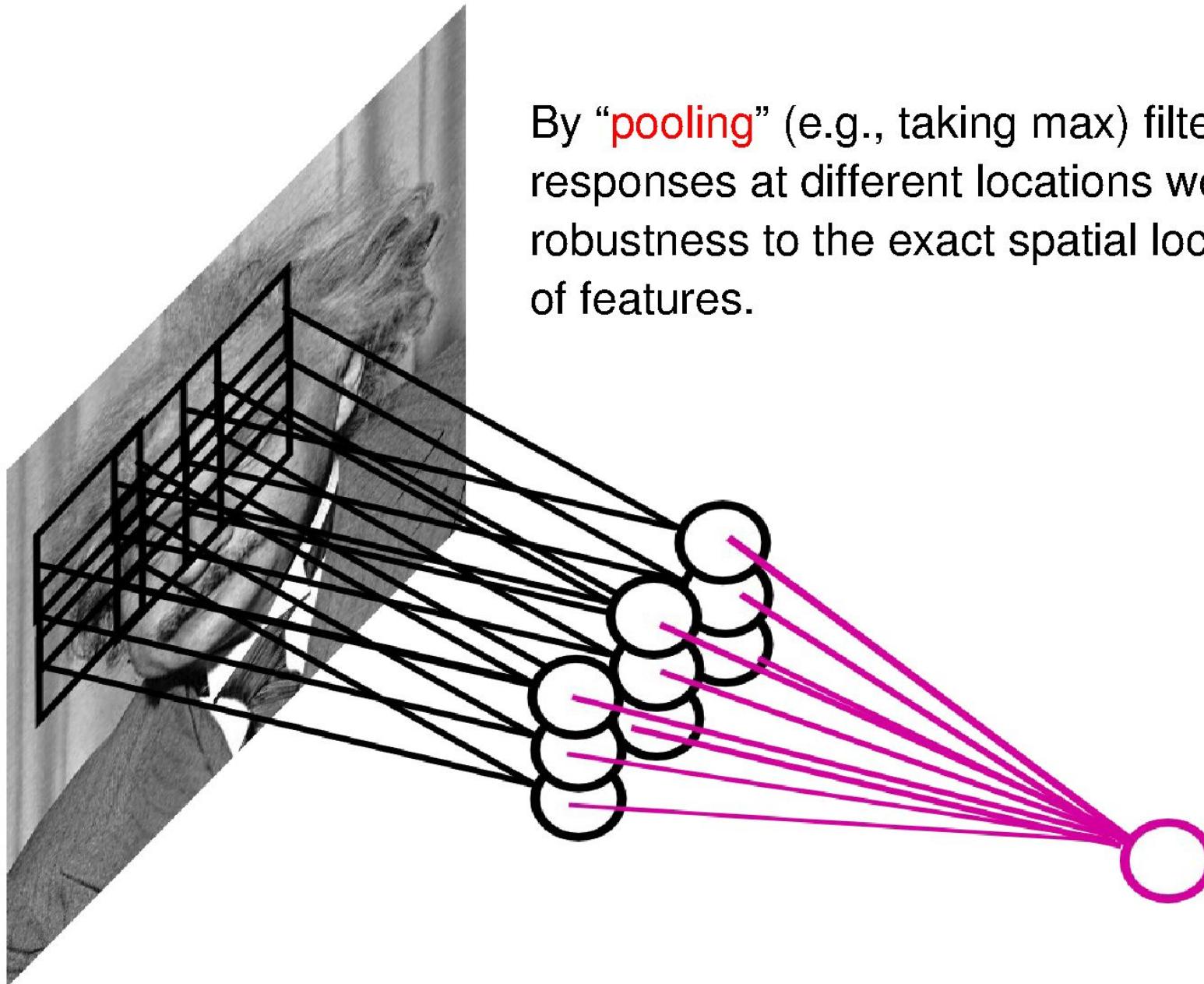
Pooling Layer



Let us assume filter is an “eye” detector.

Q.: how can we make the detection robust to the exact location of the eye?

Pooling Layer



Pooling Layer: Examples

Max-pooling:

$$h_j^n(x, y) = \max_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

Average-pooling:

$$h_j^n(x, y) = 1/K \sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

L2-pooling:

$$h_j^n(x, y) = \sqrt{\sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})^2}$$

L2-pooling over features:

$$h_j^n(x, y) = \sqrt{\sum_{k \in N(j)} h_k^{n-1}(x, y)^2}$$

Pooling Layer

Question: What is the size of the output? What's the computational cost?

Answer: The size of the output depends on the stride between the pools. For instance, if pools do not overlap and have size $K \times K$, and the input has size $D \times D$ with M input feature maps, then:

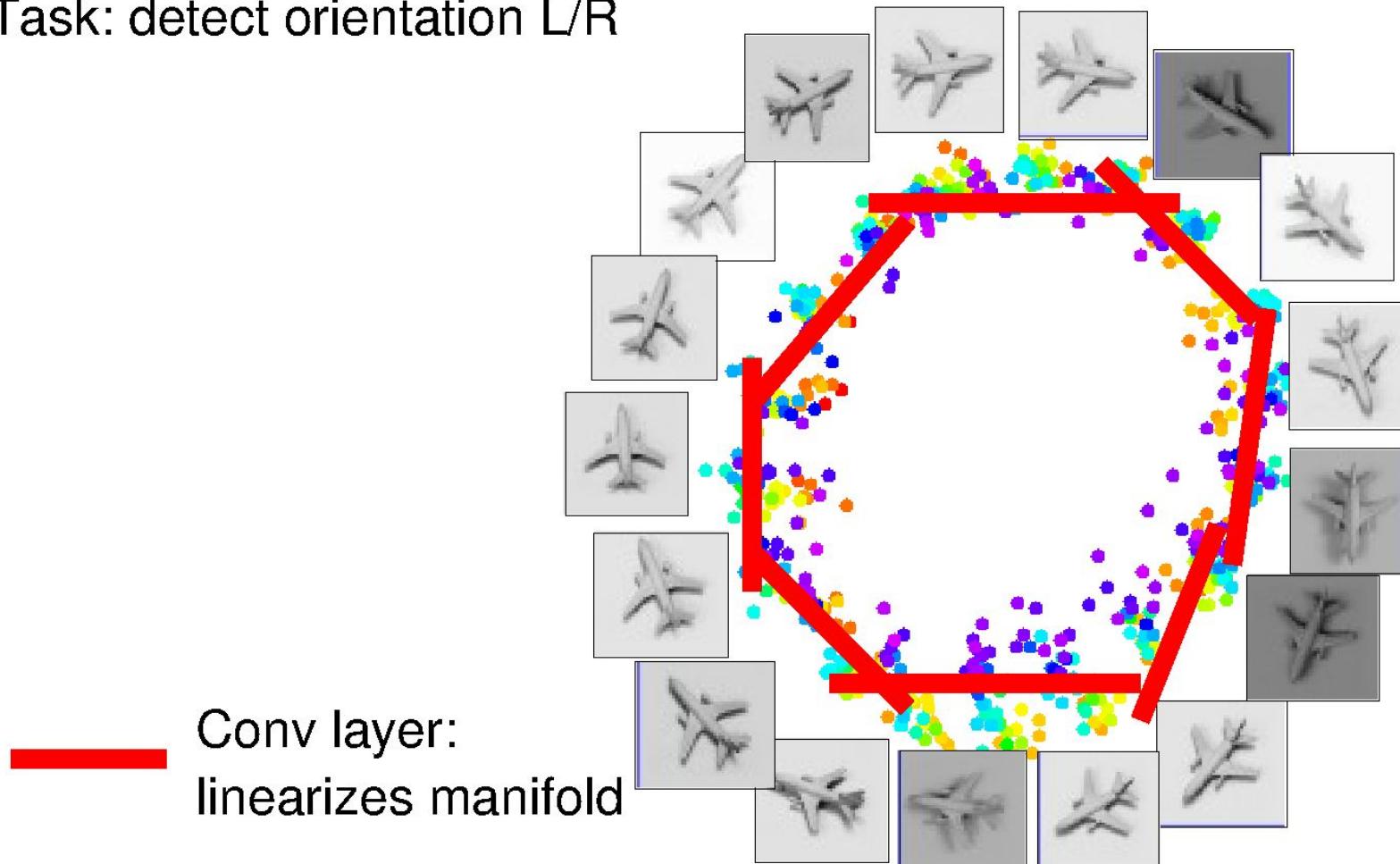
- output is $M @ (D/K) \times (D/K)$
- the computational cost is proportional to the size of the input (negligible compared to a convolutional layer)

Question: How should I set the size of the pools?

Answer: It depends on how much “invariant” or robust to distortions we want the representation to be. It is best to pool slowly (via a few stacks of conv-pooling layers).

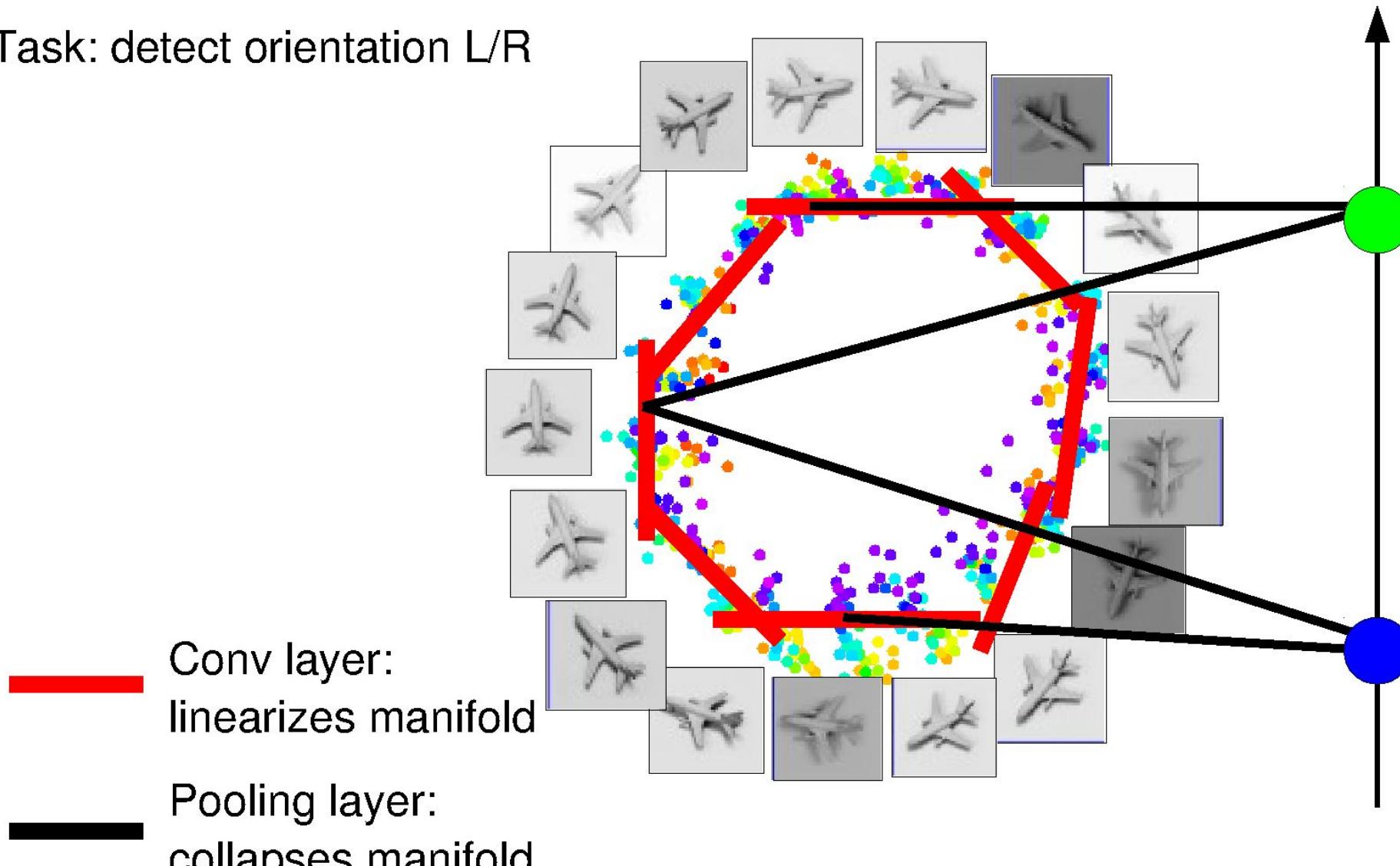
Pooling Layer: Interpretation

Task: detect orientation L/R

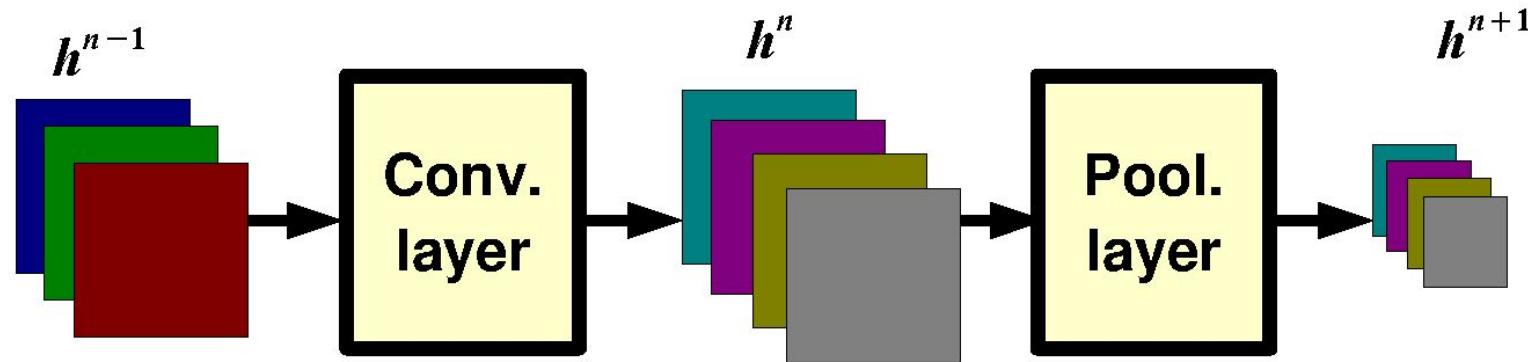


Pooling Layer: Interpretation

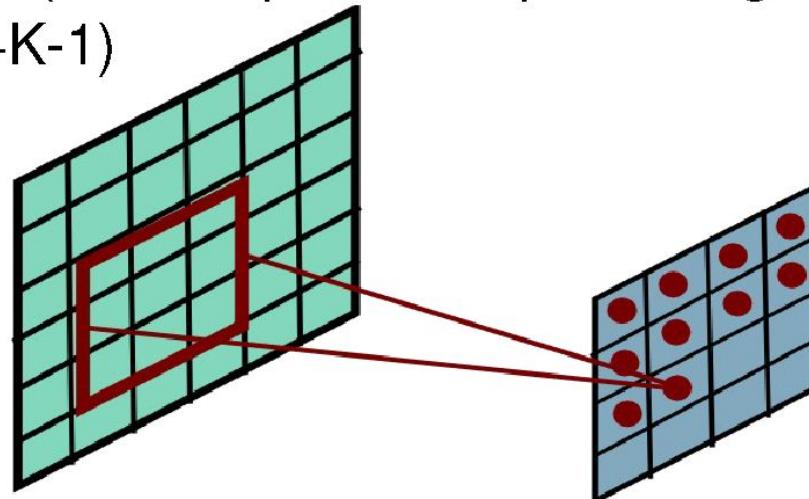
Task: detect orientation L/R



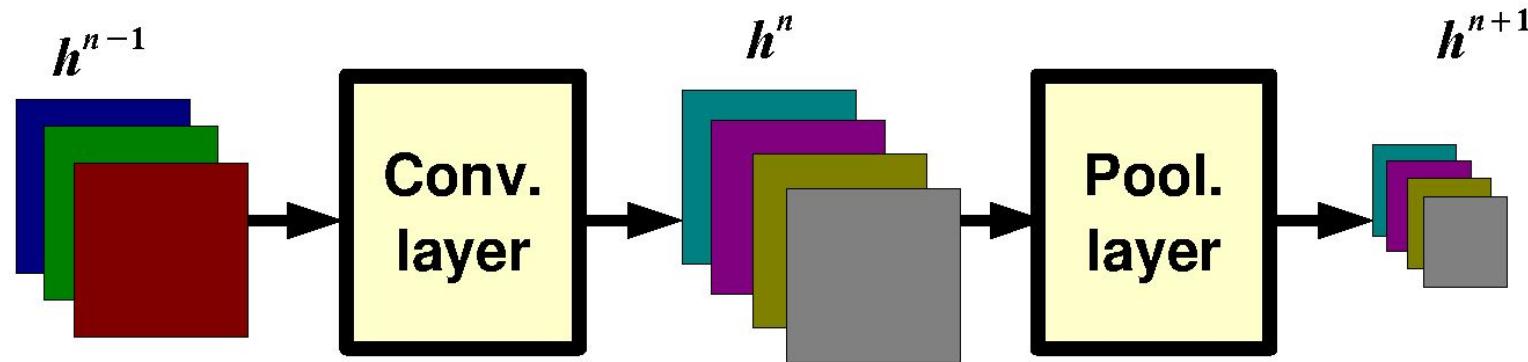
Pooling Layer: Receptive Field Size



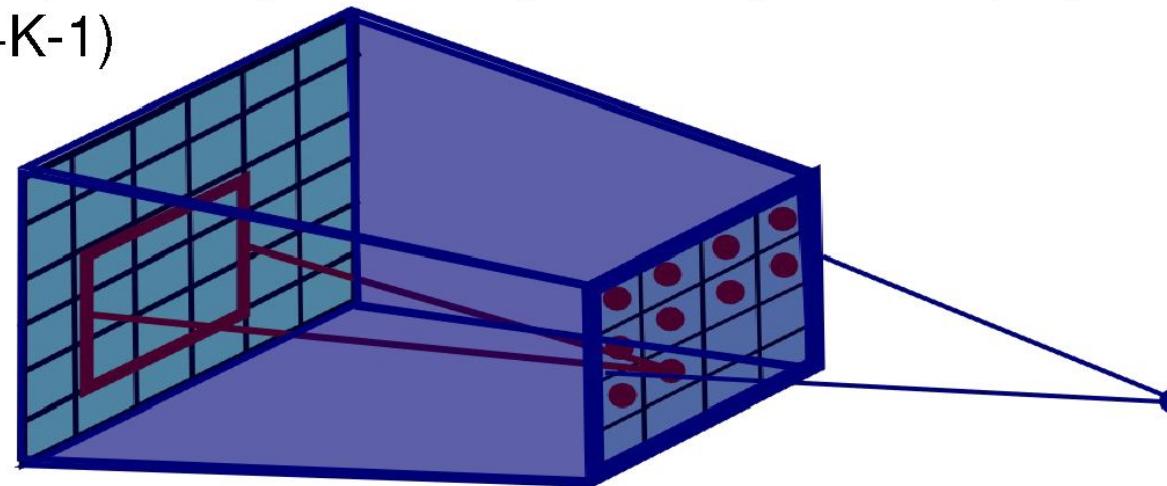
If convolutional filters have size $K \times K$ and stride 1, and pooling layer has pools of size $P \times P$, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size:
 $(P+K-1) \times (P+K-1)$



Pooling Layer: Receptive Field Size

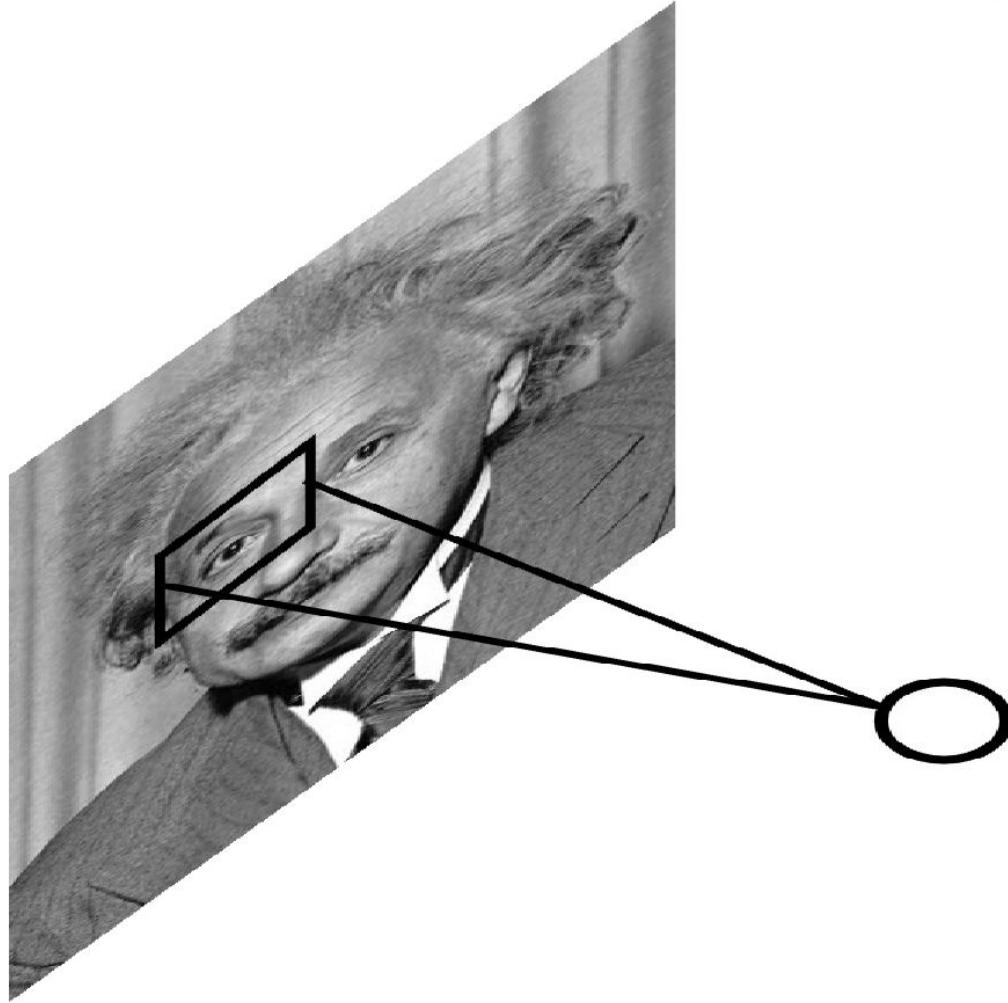


If convolutional filters have size $K \times K$ and stride 1, and pooling layer has pools of size $P \times P$, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size:
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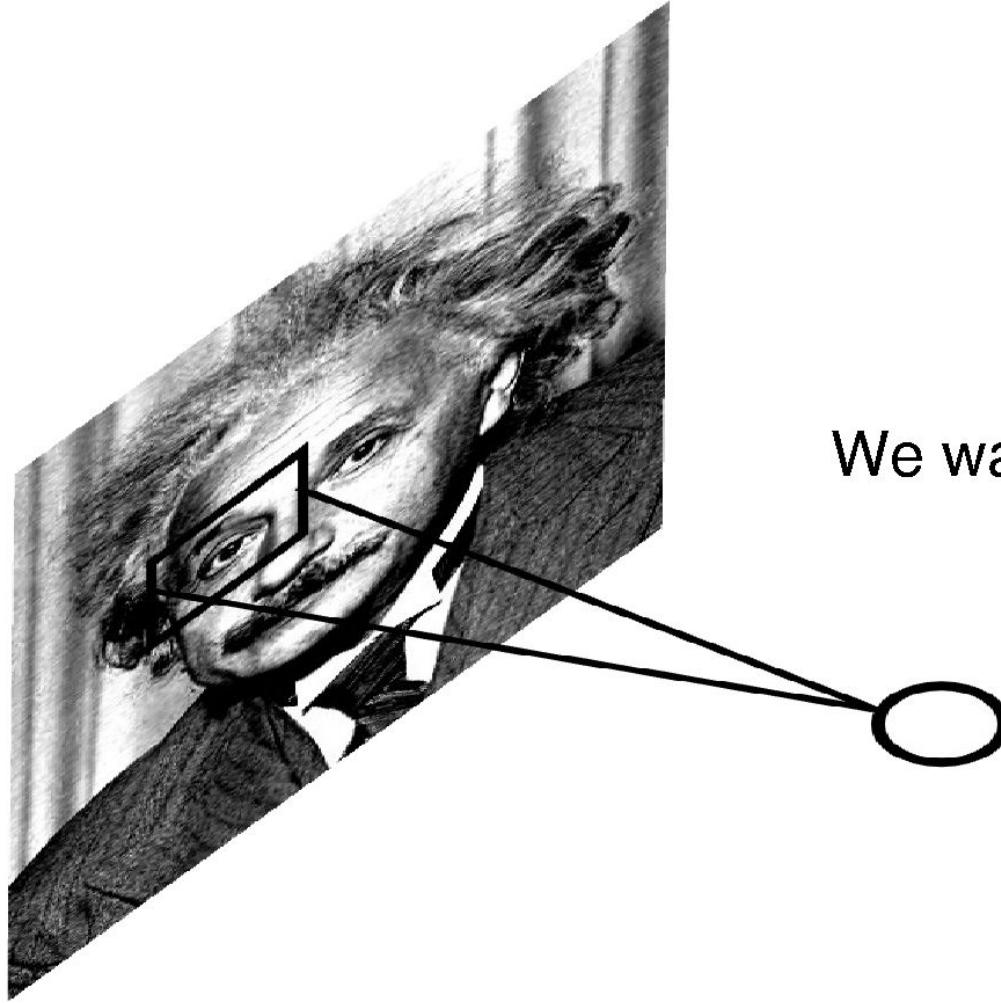
Local Contrast Normalization

$$h^{i+1}(x, y) = \frac{h^i(x, y) - m^i(N(x, y))}{\sigma^i(N(x, y))}$$



Local Contrast Normalization

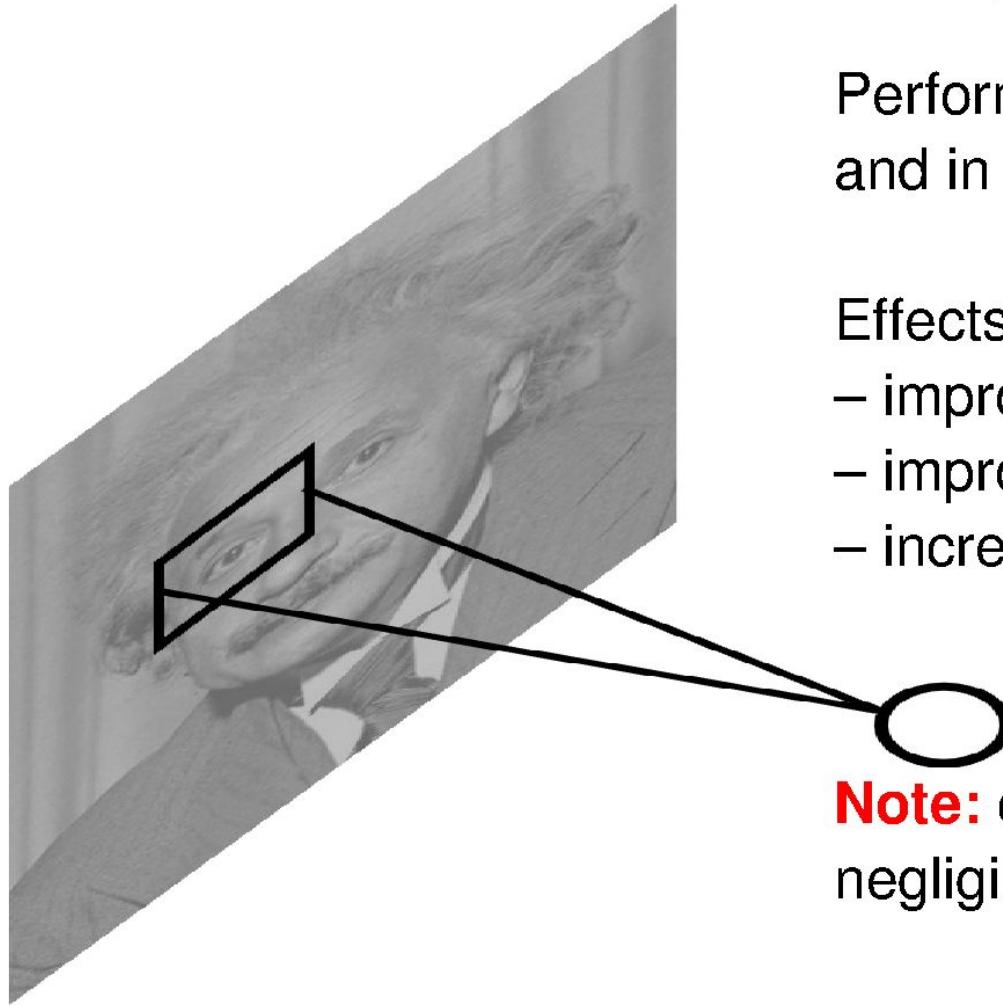
$$h^{i+1}(x, y) = \frac{h^i(x, y) - m^i(N(x, y))}{\sigma^i(N(x, y))}$$



We want the same response.

Local Contrast Normalization

$$h^{i+1}(x, y) = \frac{h^i(x, y) - m^i(N(x, y))}{\sigma^i(N(x, y))}$$



Performed also across features
and in the higher layers..

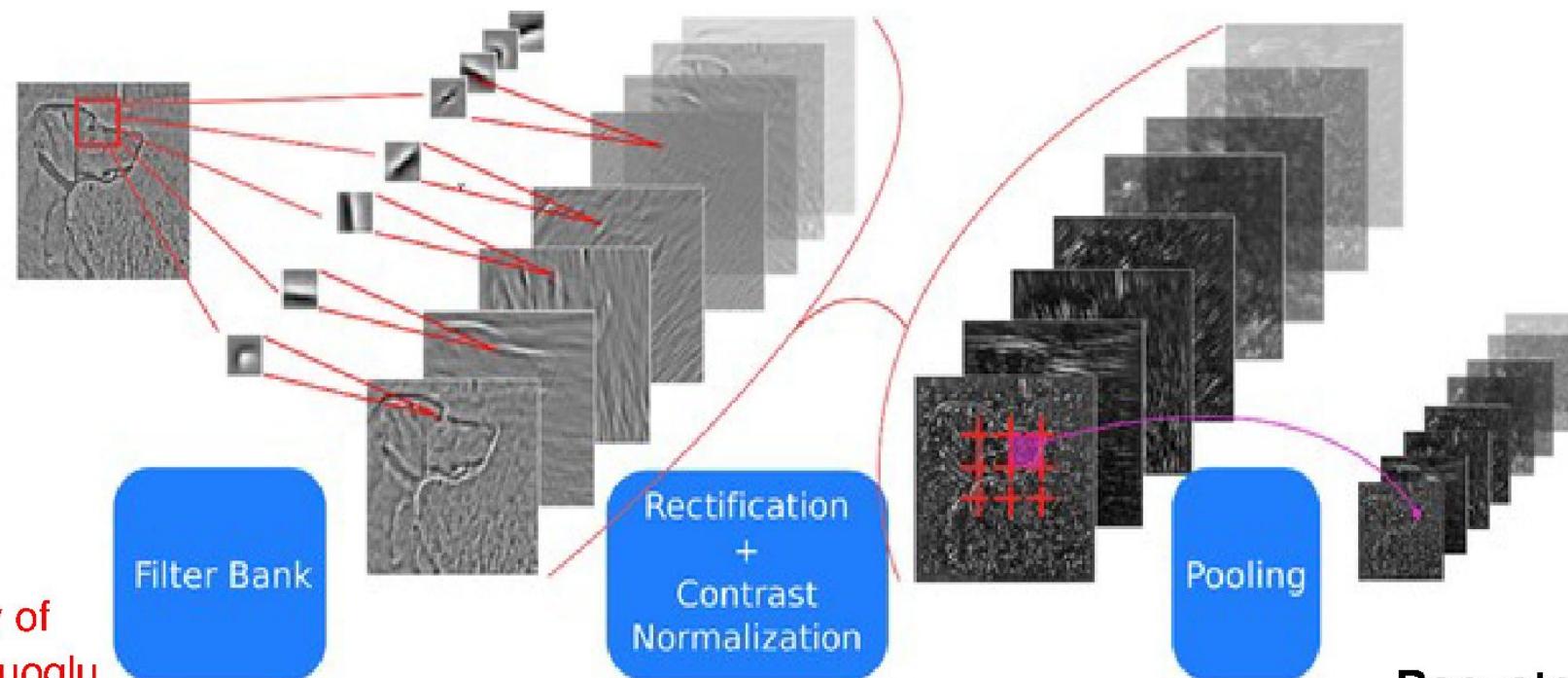
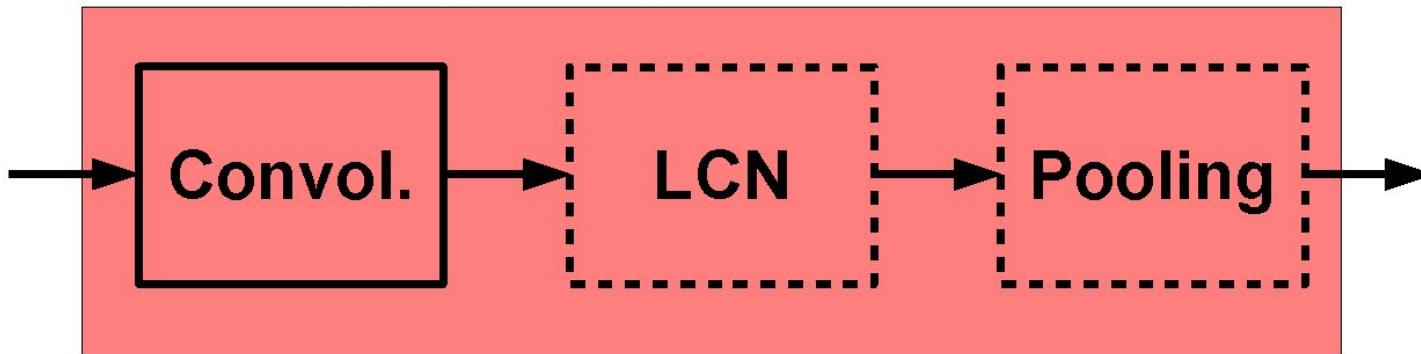
Effects:

- improves invariance
- improves optimization
- increases sparsity

Note: computational cost is negligible w.r.t. conv. layer.

ConvNets: Typical Stage

One stage (zoom)

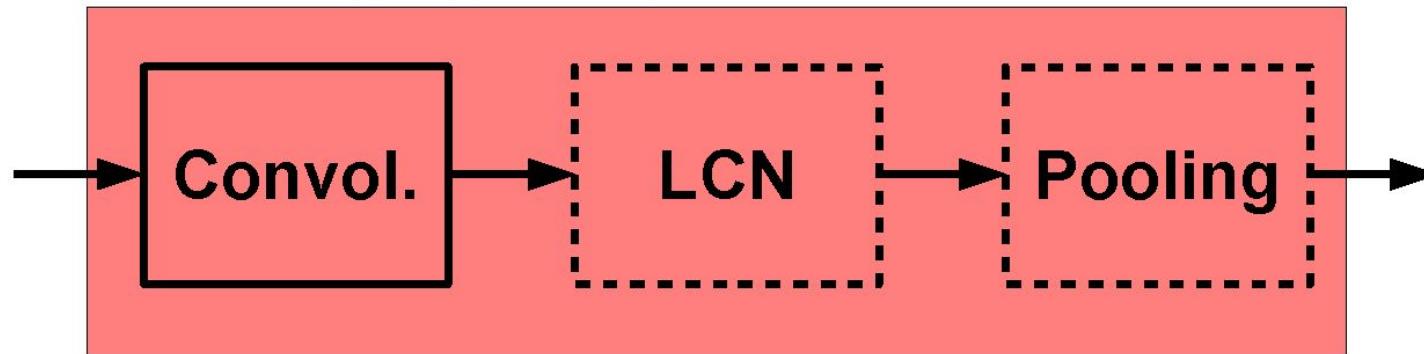


courtesy of
K. Kavukcuoglu

Ranzato

ConvNets: Typical Stage

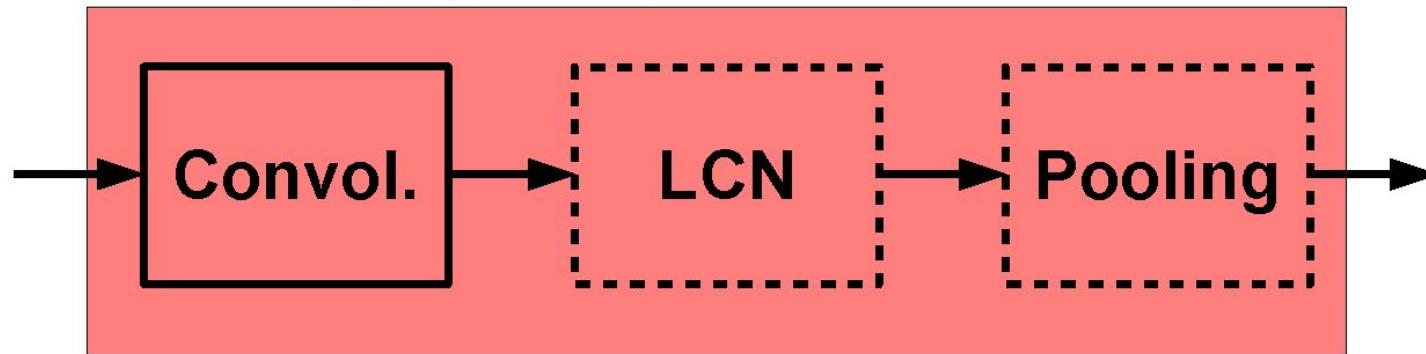
One stage (zoom)



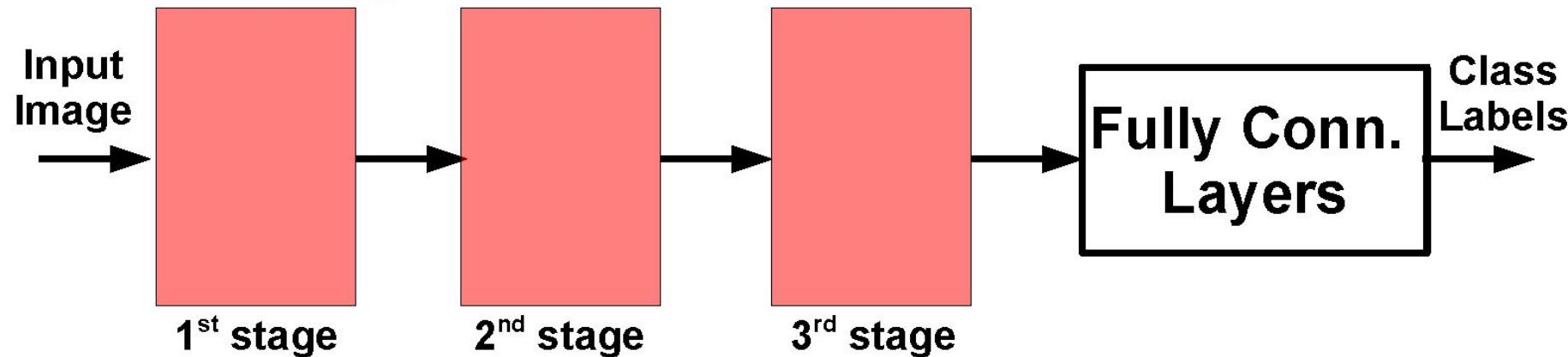
Conceptually similar to: SIFT, HoG, etc.

ConvNets: Typical Architecture

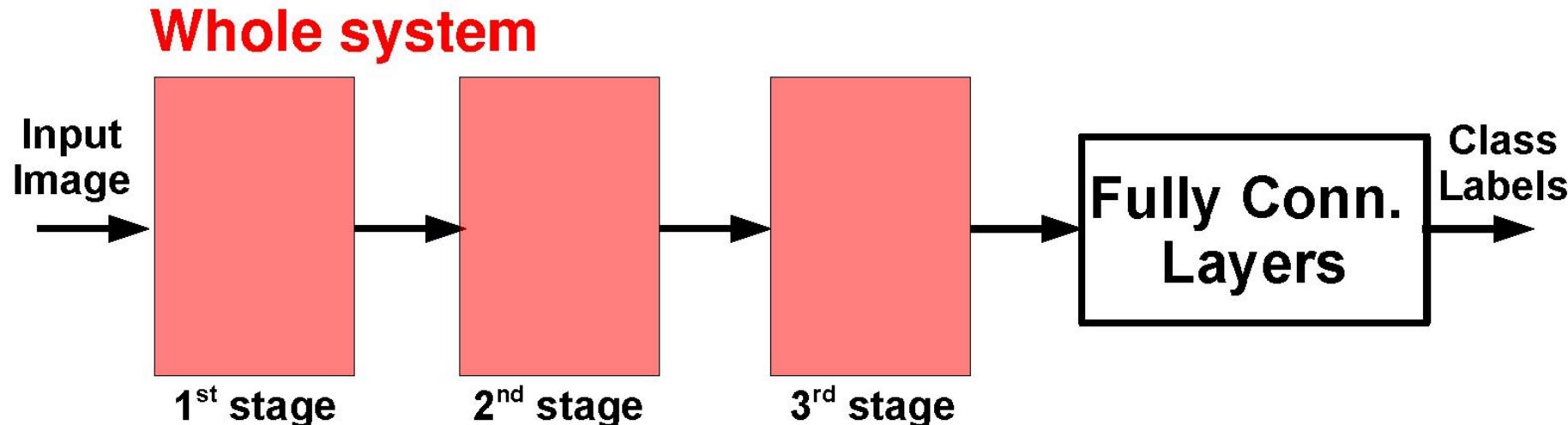
One stage (zoom)



Whole system



ConvNets: Typical Architecture



Conceptually similar to:

SIFT → K-Means → Pyramid Pooling → SVM

Lazebnik et al. "...Spatial Pyramid Matching..." CVPR 2006

SIFT → Fisher Vect. → Pooling → SVM

Sanchez et al. "Image classification with F.V.: Theory and practice" IJCV 2012

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration
Neural Network	1
Nearest Neighbor	10
Linear SVM	10
Non-linear SVM	5
Decision Tree or Random Forest	4

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation
Neural Network	1	1
Nearest Neighbor	10	10
Linear SVM	10	9
Non-linear SVM	5	4
Decision Tree or Random Forest	4	4

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation	Speed / memory when training
Neural Network	1	1	1
Nearest Neighbor	10	10	8
Linear SVM	10	9	10
Non-linear SVM	5	4	2
Decision Tree or Random Forest	4	4	4

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation	Speed / memory when training	Speed / memory at test time
Neural Network	1	1	1	6
Nearest Neighbor	10	10	8	4
Linear SVM	10	9	10	10
Non-linear SVM	5	4	2	2
Decision Tree or Random Forest	4	4	4	8

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation	Speed / memory when training	Speed / memory at test time	Accuracy w/ lots of data
Neural Network	1	1	1	6	10
Nearest Neighbor	10	10	8	4	7
Linear SVM	10	9	10	10	5
Non-linear SVM	5	4	2	2	8
Decision Tree or Random Forest	4	4	4	8	7

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Neural Network	1	1	1	6	10
Nearest Neighbor	10	10	8	4	7
Linear SVM	10				
Non-linear SVM	5				
Decision Tree or Random Forest	4				

Representation design matters
more for all of these