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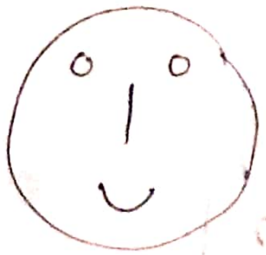

Ans to the ques no:- 2

a

The given function, $f(x) = e^x - x$

On the interval $[1,3]$, the actual integral value of the function $= \int_1^3 f(x)\,dx$

$$= \int_1^3 (e^x - x)\,dx$$

$$= \int_1^3 e^x\,dx - \int_1^3 x\,dx$$

$$= \left[e^x\right]_1^3 - \left[\frac{x^2}{2}\right]_1^3$$

$$= e^3 - e - \left(\frac{3^2}{2} - \frac{1^2}{2}\right)$$

$$= e^3 - e - \frac{9}{2} + \frac{1}{2}$$

$$= \frac{2e^3 - 2e - 9 + 1}{2}$$

$$= \frac{2e^3 - 2e - 8}{2}$$

$$= e^3 - e - 4$$

(Ans)

$\boxed{b}$

No of segments, $m = 4$

$\therefore$ Height $= \dfrac{3-1}{4} = 0.5$

$\therefore$ Points $\Rightarrow$ $1+(0.5\times0)$, $1+(0.5\times1)$, $1+(0.5\times2)$, $1+(0.5\times3)$

$= 1$, $= 1.5$, $= 2$, $= 2.5$

And, $1+(0.5\times4)$

$= 3$

$$\therefore C_{1,m}(f) = \frac{1}{2} \cdot h \cdot \left[f(1) + 2\times f(1.5) + 2\times f(2) + 2\times f(2.5) + f(3)\right]$$

$$= \frac{1}{2}\times 0.5 \left[e-1 + 2e^{1.5}-3 + 2e^{2}-4 + 2e^{2.5}-5 + e^{3}-3\right]$$

$$= \frac{1}{4}\left[e + 2e^{1.5} + 2e^{2} + 2e^{2.5} + e^{3} - 16\right]$$

(Ans)

$\boxed{c}$

Result obtained from symbolic integration

$= e^3 - e - 4$

And, from Newton-Cote's composite formulation using 4 segments $= \frac{1}{4}\left[e + 2e^{1.5} + 2e^{2} + 2e^{2.5} + e^{3} - 16\right]$

$\therefore$ Error in percentage =

$$\left| \frac{(e^3 - e - 4) - \frac{1}{4}\left[e + 2e^{1.5} + 2e^{2} + 2e^{2.5} + e^{3} - 16\right]}{(e^3 - e - 4)} \right| \times 100\%$$

$= 2.6955\ \%$

To decrease the error more, we need to increase the number of segments in our given interval. If the number of segments are large enough, then the error will be minimized more.

$\boxed{d}$

Using the Simpson rule ~~of~~ on finding

$$\int_1^3 f(x)\,dx = \frac{3-1}{6} \cdot \left[f(1) + 4 \cdot f\left(\frac{1+3}{2}\right) + f(3)\right]$$

$$= \frac{1}{3}\left[f(1) + 4 \cdot f(2) + f(3)\right]$$

$$= \frac{1}{3}\left[e - 1 + 4e^2 - 8 + e^3 - 3\right]$$

$$= \frac{1}{3}\left[e^3 + 4e^2 + e - 12\right]$$

(Ans)

— o — x — o —

Ans. to the ques. no: 3

$\boxed{a}$

Given function, $f(x) = 6x^2 - 4x - 9$

Given interval $[-2, 2]$

$$\therefore \int_{-2}^{2} f(x)\,dx = \frac{1}{2} \cdot b-a \cdot \left(f(a) + f(b)\right)$$

$$= \frac{1}{2} \cdot (2-(-2)) \cdot [f(-2) + f(2)]$$

$$= \frac{1}{2} \times 4 \cdot [6\times(-2)^2 - 4\times(-2) - 9 + 6\times 2^2 - 4\times 2 - 9]$$

$$= 2 \cdot [6\times 4 + 8 - 9 + 6\times 4 - 8 - 9]$$

$$= 60$$

(Ans)

(b)

Exact integrated value,

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{2} (6x^2 - 4x - 9)\,dx$$

$$= \left[6 \cdot \frac{x^3}{3} - 4 \cdot \frac{x^2}{2} - 9x\right]_{-2}^{2}$$

$$= \left[2x^3 - 2x^2 - 9x\right]_{-2}^{2}$$

$$= (2\times 2^3 - 2\times 2^2 - 9\times 2) - (2\times(-2)^3 - 2\times(-2)^2 - 9\times(-2))$$

$$= -4$$

(Ans)

$\boxed{c}$

Relative error is $\left| \frac{-4 - 60}{-4} \right| \times 100\%$

$= 1600\%$

(Ans)

— o — x — o —

Ans to the ques no :- 1

$\boxed{a}$

The linear equation will be :-

x_1 = number of PSG jersey

x_2 = number of Barcelona jersey

$\therefore x_1 + x_2 = 30$ ——— (i)

$\therefore 400x_1 + 400x_2 = 12000$ ——— (ii)

$\therefore 500x_1 + 300x_2 = 13000$ ——— (iii)

$\boxed{b}$

$$A = \begin{bmatrix} 1 & 1 \\ 400 & 400 \\ 500 & 300 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$b = \begin{bmatrix} 30 \\ 12000 \\ 13000 \end{bmatrix}$$

$\boxed{c}$

Here, $A = [a_1 \quad a_2]$

$$\therefore \quad a_1 = \begin{bmatrix} 1 \\ 400 \\ 500 \end{bmatrix} = u_1$$

$$\therefore r_{11} = \sqrt{1^2 + 400^2 + 500^2} = 640.3132046$$

$$\therefore \quad v_1 = \frac{1}{640.3132046} \begin{bmatrix} 1 \\ 400 \\ 500 \end{bmatrix} = \begin{bmatrix} 1.561 \times 10^{-3} \\ 0.624694 \\ 0.780868 \end{bmatrix}$$

$$\therefore q_2 = \begin{bmatrix} 1 \\ 400 \\ 300 \end{bmatrix}$$

$$\therefore u_2 = q_2 - (q_2 \cdot v_1) \cdot v_1$$

$$= \begin{bmatrix} 1 \\ 400 \\ 300 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 400 \\ 300 \end{bmatrix} \times \begin{bmatrix} 1.561 \times 10^{-3} \\ 0.624694 \\ 0.780868 \end{bmatrix} \right) \begin{bmatrix} 1.561 \times 10^{-3} \\ 0.624694 \\ 0.780868 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 400 \\ 300 \end{bmatrix} - 484.139561 \begin{bmatrix} 1.561 \times 10^{-3} \\ 0.624694 \\ 0.780868 \end{bmatrix}$$

$$= \begin{bmatrix} 0.244258 \\ 97.560921 \\ -78.049090 \end{bmatrix}$$

$$\therefore r_{12} = 484.139561$$

$$\therefore r_{22} = \sqrt{0.244258^2 + 97.560921^2 + (-78.049090)^2}$$

$$= 124.9393994$$

$$\therefore v_2 = \frac{1}{124.9393994} \begin{bmatrix} 0.244258 \\ 97.560921 \\ -78.04909 \end{bmatrix}$$

$$= \begin{bmatrix} 1.955 \times 10^{-3} \\ 0.780866 \\ -0.6246956 \end{bmatrix}$$

$\therefore Q = [v_1 \quad v_2]$

$$= \begin{bmatrix} 1.561 \times 10^{-3} & 1.955 \times 10^{-3} \\ 0.624694 & 0.780866 \\ 0.780868 & -0.624696 \end{bmatrix}$$

$\therefore R = \begin{bmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{bmatrix}$

$$= \begin{bmatrix} 640.3132046 & 484.139561 \\ 0 & 124.939399 \end{bmatrix}$$

(Ans)

$\boxed{d}$

We know, $R \cdot x = Q^T \cdot b$

$$\begin{bmatrix} 640.3132046 & 484.139561 \\ 0 & 124.939399 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.561 \times 10^{-3} & 0.624694 & 0.780868 \\ 1.955 \times 10^{-3} & 0.780866 & -0.624696 \end{bmatrix} \cdot \begin{bmatrix} 30 \\ 12000 \\ 13000 \end{bmatrix}$$

$$= \begin{bmatrix} 17647.65883 \\ 1249.40265 \end{bmatrix}$$

$\therefore 124.939399 \, x_2 = 1249.40265$

$\Rightarrow x_2 = \dfrac{1249.40265}{124.939399} \approx 10$

$\therefore 640 \cdot 3132046\, x_1 + 484 \cdot 139561\, x_2 = 17647 \cdot 65883$

$$\Rightarrow x_1 = \frac{17647 \cdot 65883 - 484 \cdot 139561 \times 10}{640 \cdot 3132046}$$

$$\approx 20$$

$\therefore x_1 = 20$

$x_2 = 10$

(Ans)

— o — x — o —