MA 2621-E1 Probability for Applications
C-Term Spring 2023 - Homework 02

Due: F 05/02 by 11.59 p.m.

Show all work as described in class. Partial credits will be given. Submit your solutions in a pdf format to canvas. Please write your name.

1. Using the letters in the word "SQUARE", How many 6 letter arrangements, with no repetitions, are possible if,
   (a) there is not any restriction,
   (b) the first letter is a vowel,
   (c) vowels and consonants are alternate, beginning with a consonant,

a).

$$6! \;=\; 6 \times 5 \times 4 \times 3 \times 2 \times 1 \;=\; 720$$

b).

$$3 \;*\; 5! \;=\; 3 \;*\; (5 \times 4 \times 3 \times 2 \times 1) \;=\; 360$$

c).

$$3 \;*\; 3 \;*\; 2 \;*\; 2 \;*\; 1 \;*\; 1 \;=\; 36\; arrangements\; possible$$

2. A committee of 5 people is to be chosen from a group of 6 men and 4 women. How many committees are possible if:
   (a) there are to be 3 men and 2 women?
   (b) there is to be a majority of women?

a).

$$"6\; choose\; 3"\; is\; calculated\; as\; 6!\; / \,[3!\,(6 - 3)!] \;=\; 20$$

$$"4\; choose\; 2"\; is\; calculated\; as\; 4!\; / \,[2!\,(4 - 2)!] \;=\; 6$$

$$The\; total\; number\; of\; committees\; possible\; is\; the\; product\; of\; these\; two\; combinations:\; 20 \;*\; 6 \;=\; 120$$

b).

$$3 \; women \; and \; 2 \; men:$$

$$"4 \; choose \; 3" \; for \; the \; women \; and \; "6 \; choose \; 2" \; for \; the \; men \; gives \; 4 \;\; * \;\; 15 \;\; = \;\; 60 \; possibilities$$

$$4 \; women \; and \; 1 \; men:$$

$$"4 \; choose \; 4" \; for \; the \; women \; and \; "6 \; choose \; 1" \; for \; the \; men \; gives \; 1 \;\; * \;\; 6 \;\; = \;\; 6 \; possibilities$$

$$5 \; women \; and \; 0 \; men:$$

$$This \; is \; not \; possible \; because \; there \; are \; only \; 4 \; women \; available$$

$$So, \; the \; total \; number \; of \; committees \; possible \; with \; a \; majority \; of \; women \; is \; 60 \;\; + \;\; 6 \;\; = \;\; 66.$$

3. A mail-order computer business has six telephone lines. Let X denote the number of lines in use at a specified time. The PMF of X is:

$X = x$	0	1	2	3	4	5	6
$P(X = x)$	0.10	0.15	0.20	0.25	0.20	?	0.04

(a) Find the missing value so that $P(X = x)$ represent a probability mass function of X.

(b) Calculate $E(X), Var(X)$, and the standard deviation of X.

a).

$$0.10 \; + \; 0.15 \; + \; 0.20 \; + \; 0.25 \; + \; 0.20 \; + \; 0.04 \; = \; 0.94$$

$$1 \; - \; 0.94 \; = \; 0.06$$

$$Therefore, \; the \; missing \; probability \; is: \; 0.06$$

$$E(X) \; = \; \sum [x \; * \; P(X \; = \; x)]$$

$$= \; 00.10 \; + \; 10.15 \; + \; 20.20 \; + \; 30.25 \; + \; 40.20 \; + \; 50.06 \; + \; 6 * 0.04$$

$$= \; 0 \; + \; 0.15 \; + \; 0.40 \; + \; 0.75 \; + \; 0.80 \; + \; 0.30 \; + \; 0.24$$

−1

$$E(X) \; = \; 2.84$$

x

$$Var(X) \; = \; \sum [(x \; - \; E(X))^2 \; * \; P(X \; = \; x)]$$

$$= \; [(0 - 2.84)^2 \; * \; 0.10] \; + \; [(1 - 2.84)^2 \; * \; 0.15] \; + \; [(2 - 2.84)^2 \; * \; 0.20]$$

$$+ \; [(3 - 2.84)^2 \; * \; 0.25] \; + \; [(4 - 2.84)^2 \; * \; 0.20] \; + \; [(5 - 2.84)^2 \; * \; 0.06] \; + [(6 - 2.84)^2 \; * \; 0.04]$$

$$= [8.0656 * 0.10] + [3.3856 * 0.15] + [0.7056 * 0.20] + [0.0256 * 0.25] + [1.3456 * 0.20] + [4.6656 * 0.06] + [9.9856 * 0.04]$$

$$= 0.80656 + 0.50784 + 0.14112 + 0.0064 + 0.26912 + 0.279936 + 0.399424$$

$$Var(X) = 2.410416$$

$$SD(X) = \sqrt{(Var(X))} = \sqrt{2.410416} = 1.552231$$

4. Suppose X is a random variable such that $E[X] = 50$ and $Var(X) = 12$. Calculate the following quantities.
   (a) $E[X^2]$
   (b) $E[3X + 2]$
   (c) $E[(X + 2)^2]$
   (d) $Var(2X + 1)$

a).

$$E[X^2] = Var(X) + (E[X])^2$$

$$= 12 + (50)^2 \quad = 12 + 2500 \quad = 2512$$

$$E[X^2] = 2512$$

b).

$$E[3X + 2] = 3E[X] + 2$$

$$= 3 * 50 + 2 \quad = 152$$

$$E[3X + 2] = 152$$

c).

$$E[(X + 2)^2] = E[X^2 + 4X + 4]$$

$$= E[X^2] + 4E[X] + 4$$

$$= 2512 + 4 * 50 + 4 \quad = 2512 + 200 + 4 \quad = 2716$$

$$E[(X + 2)^2] = 2716$$

d).

$$Var(2X + 1) = 2^2 * Var(X)$$

$$= 4 * 12 \quad = 48$$

5. In 1997, 10.8% of female smokers smoked cigars. In a sample of size 10 female smokers, what is the probability that

   (a) What is the probability that exactly 3 of the women smoke cigars?

   (b) What is the probability that at least 2 women smokes cigars?

a).

$$P(X = 3) = C(10,\ 3) * (0.108)^{3} * (1 - 0.108)^{(10-3)}$$

$$= (10!\ /\ 3!\,(10 - 3)!) * (0.108)^{3} * (0.892)^{7}$$

−1

$$= 120 * (0.108)^{3} * (0.892)^{7}$$

$$\approx 0.141$$

The probability that exactly 3 *of the women smoke cigars is about* 0.141, *or* 14.1%

b).

$$P(X = 0) = C(10,\ 0) * (0.108)^{0} * (1 - 0.108)^{(10-0)}$$

$$= 1 * (0.892)\text{^}10$$

$$\approx 0.316$$

$$P(X = 1) = C(10,\ 1) * (0.108)^{1} * (1 - 0.108)^{(10-1)}$$

−1

$$= 10 * 0.108 * (0.892)^{9}$$

$$\approx 0.376$$

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$$

$$= 1 - 0.316 - 0.376$$

$$\approx 0.308$$

The probability that at least 2 *of the women smoke cigars is about* 0.308, *or* 30.8%.

6. In a certain town, 40% of the eligible voters prefer candidate A, 10% prefer candidate B, and the remaining 50% have no preference. You randomly sample 10 eligible voters. What is the probability that 4 will prefer candidate A, 1 will prefer candidate B, and the remaining 5 will have no preference?

$$P(X1 = x1,\ X2 = x2,\ ...,\ Xk = xk) = \frac{n!}{(x1!x2!...xk!) * p1\text{^}x1 * p2\text{^}x2 * ... * pk\text{^}xk}$$

$$P(X1 = 4,\ X2 = 1,\ X3 = 5) = 10!\ /\ (4!\,1!\,5!) * 0.4^{4} * 0.1^{1} * 0.5^{5}$$

$$= 252 * 0.4^4 * 0.1 * 0.5^5$$

$$\approx 0.252$$

−1

×

The probability that 4 *voters will prefer candidate* A, 1 *will prefer candidate* B, *and the remaining* 5 *will have no preference is approximately* 0.252, *or* 25.2%

7. Consider writing onto a computer disk and then sending it through a certifier that counts the number of missing pulses. Suppose this number X has a Poisson distribution with parameter $\lambda = 0.2$.
   (a) What is the probability that a disk has exactly one missing pulse?
   (b) What is the probability that a disk has at least two missing pulses?
   (c) If two disks are independently selected, what is the probability that neither contains a missing pulse?

$$P(X = k) = (\lambda\text{^}k * e\text{^} - \lambda) / k!$$

a).

$$For\ k = 1:$$

$$P(X = 1) = (0.2^1 * e^{-0.2}) / 1!$$

$$= 0.2 * e^{-0.2}$$

$$\approx 0.163$$

The probability that a disk has exactly one missing pulse is about 0.163, *or* 16.3%

b).

$$P(X = 0) = (0.2^0 * e^{-0.2}) / 0!$$

$$= e^{-0.2}$$

$$\approx 0.818$$

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$$

$$= 1 - 0.818 - 0.163$$

$$\approx 0.019$$

The probability that a disk has at least two missing pulses is about 0.019, *or* 1.9%

$$P(both\ disks\ have\ X = 0) = P(X = 0)^2$$

$$= (0.818)^2$$

$$\approx 0.669$$

The probability that neither disk contains a missing pulse is about 0.669, *or* 66.9%.