1. A large supermarket chain reports that 40% of all beer purchases are "light". Consider the next 20 beer purchases made.

   (a) Suppose Y is the number of those 20 beer purchases which are "light". What distribution model does Y follow? What are you assuming about the 20 beer purchases in naming this distribution?

   (b) What is the chance that between six and eight (inclusive) of these 20 purchases are "light".

a).

$$Y = Binomial\ (n = 2-, p = 0.4)$$

* Independent * Two Outcomes per trial (light or reg) * Should have the same probability

b).

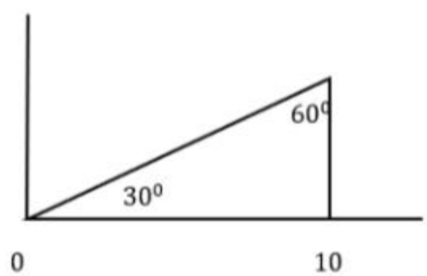
$$P(6 \le Y \ \le 8) = P(Y = 6) + P(Y = 7) + P(Y = 8)$$

$$P(6 \le Y \ \le 8) = 0.1244 + 0.1658 + 0.1779$$

$$= 0.47$$

$The\ chance\ between\ six\ and\ eight\ of\ these$ 20 $purchases\ being$ light$is\ approximately$ 47%

2. A distribution has a pdf of $f(x) = x/50$ between $x = 0$ to 10.



(a) What is the probability that $P(3 < X < 6)$?

(b) What is the probability that $P(X > 7)$?

(c) Find the three quartiles of your data set.

a)

$$P\ (3 < x < 6) = \int_{3}^{6} \left(\frac{x}{50}\right) dx$$

$$P\ (3 < x < 6) = 0.27 = 27\%$$

b).

$$P\ (x > 7) = \int_{7}^{10} \left(\frac{x}{50}\right) dx$$

$$P\ (x > 7) = 0.51 = 51\%$$

c).

$$Q_1 = \int_{0}^{Q_1} \left(\frac{x}{50}\right) dx = 0.25$$

$$Q_1 = 5.0$$

$$Q_2 = \int_{0}^{Q_3} \left(\frac{x}{50}\right) dx = 0.50$$

$$Q_2 = 7.0711$$

$$Q_3 = \int_{0}^{Q_3} \left(\frac{x}{50}\right) dx = 0.75$$

$$Q_3 = 8.6603$$

3. In a recent study, the Centers for Disease Control and Prevention report that diastolic blood pressures (in mmHg) of adult women in the United States are approximately normally distributed with mean 80.5 and standard normal deviation 9.9.

(a) What proportion of women have blood pressures lower than 70?

(b) What proportion of women have blood pressure between 75 and 90?

(c) A diastolic of women have blood pressures greater than 90 is classified as hypertension (high blood pressure). What proportion of women have hypertension?

(d) Is it unusual for a woman to have a blood pressure lower than 65? **(Hint: We say that an event with a probability less than 5% is unusual)**

a).

$$Z = \frac{70 - 80.5}{9.9} = -\frac{10.5}{9.9} = 1.0606$$

$$P(Z < 1.0606) = 1 - 0.884$$

$$P(Z < 1.0606) = P(bp < 70) = 0.1446$$

b).

$$Z_1 = \frac{75 - 80.5}{9.9} = -\frac{5.5}{9.9} = -0.5556$$

$$Z_2 = \frac{90 - 80.5}{9.9} = -\frac{9.5}{9.9} = 0.9596$$

$$P(-0.56 < Z < 0.96) = 0.8315 - 0.2877 = 0.5438$$

$$P(75 < bp < 90) = 0.5438$$

c).

$$Z = \frac{90 - 80.5}{9.9} = -\frac{9.5}{9.9} = 0.9596$$

$$P(Z > 0.96) = 1 - 0.8315$$

$$P(Z > 0.96) = P(bp > 90) = 0.1685$$

d).

$$Z = \frac{65 - 80.5}{9.9} = -\frac{15.5}{9.9} = -1.5657$$

$$P(Z < -1.57) = 0.0582 = 5.82\% - Not\ usual\ but\ rare$$

4. The weight of anodized reciprocating pistons produced by Brown Company follows a normal distribution with a mean of 10 lbs. and a standard deviation of 0.2 lb.

   (a) The heaviest 2.5% of the pistons produced are rejected as overweight. What weight, in pounds, determines the overweight classification? Give your arguments.

   (b) Suppose Brown Company can sell only those pistons weighing between 9.8 and 10.4 lbs. What proportion of pistons is lost?

a).

$$Determine\ Z - score\ for\ the\ 97.5th\ percentile: Z = 1.96$$

$$Calculate\ the\ weight: X = \mu + Z * \sigma = 10 + 1.96 * 0.2 = 10.392\ lbs$$

b).

$$Calculate\ Z - scores\ for\ 9.8\ lbs\ and\ 10.4\ lbs:$$

$$Z1 = (9.8 - 10) / 0.2 = -1$$

$$Z2 = (10.4 - 10) / 0.2 = 2$$

$$Find\ proportions\ corresponding\ to\ these\ Z - scores:$$

$$P(Z1 < Z < Z2) = P(Z < 2) - P(Z < -1) \approx 0.9772 - 0.1587 = 0.8185$$

$$Calculate\ the\ proportion\ of\ pistons\ lost: 1 - 0.8185 = 0.1815$$

$$Approximately\ 18.15\%\ of\ the\ pistons\ are\ lost.$$

5. Tofurkey is a vegan turkey substitute, usually made from tofu. At a certain restaurant, the number of calories in a serving of tofurkey with wild mushrooms stuffing and gravy is normally distributed with mean 482 and standard deviation 30.

(a) Find the 92^{nd} percentile of the number of calories.

(b) Find the 30^{th} percentile of the number of calories.

(c) Find the third quartile of the number of calories.

a).

$$Determine\ Z - score\ for\ the\ 92nd\ percentile: Z = 1.41$$

$$Calculate\ the\ calorie\ count: X = \mu + Z * \sigma = 482 + 1.41 * 30 \approx 525.3\ calories$$

b).

$$Determine\ Z - score\ for\ the\ 30th\ percentile: Z = -0.52$$

$$Calculate\ the\ calorie\ count: X = \mu + Z * \sigma = 482 + (-0.52) * 30 \approx 467.4\ calories$$

c).

$$Determine\ Z - score\ for\ the\ 75th\ percentile: Z = 0.67$$

$$Calculate\ the\ calorie\ count: X = \mu + Z * \sigma = 482 + 0.67 * 30 \approx 502.1\ calories$$

6. The time (in hours) that a technician requires to perform preventive maintenance on an air-conditioning unit is governed by the exponential distribution whose density curve is skewed to the right.

The exponential distribution arises in many engineering and industrial problems, such as the time until failure of a machine or time until success. The mean time is $\mu = 1$ hour and the standard deviation is $\sigma = 1$ hour.

Your company has a contract to maintain 70 of these units in an apartment building. You must schedule technicians' time for a visit to this building. **Let X be the time required to perform preventive maintenance.**

(a) Using the Central Limit Theorem, what is the sampling distribution of $\overline{X}$.

(b) Find the probability that it will take more than 1.1 hours for each unit on average.

(c) Find the probability that it will take more than 1.25 hours for each unit on average.

a).

$$Mean: \mu\_X = \mu = 1\ hour$$

$$Standard\ deviation: \sigma\_X = \sigma / \sqrt{n} = 1 / \sqrt{70} \approx 0.1194\ hours$$

b).

$$Z - score: Z = (1.1 - 1) / 0.1194 \approx 0.837$$

$$Probability: P(X > 1.1) \approx 1 - 0.7985 = 0.2015$$

c).

$$Z - score: Z = (1.25 - 1) / 0.1194 \approx 2.095$$

$$Probability: P(X > 1.25) \approx 1 - 0.9821 = 0.0183$$

7. In a simple random sample of 150 households, the sample mean number of personal computers was 1.32. Assume the population standard deviation is $\sigma = 0.41$.

   (a) Construct a 95% confidence interval for the mean number of personal computers.

   (b) If the sample size were 100 rather than 150, would the margin of error be larger or smaller than the result in part(a)? Explain.

   (c) If the confidence level were 98% rather than 95%, would the margin or error be larger or smaller than the result in part(a)? Explain.

   (d) Based on the confidence interval constructed in part(a), is it likely that the mean number of personal computers is greater than 1.45?

$$(a)\ 95\%\ confidence\ interval\ for\ the\ mean\ number\ of\ personal\ computers:$$

$$\bar{X} = 1.32$$

$$\sigma = 0.41$$

$$n = 150$$

$$SE = 0.41 / \sqrt{150} \approx 0.0335$$

$$Z = 1.96$$

$$ME = 1.96 * 0.0335 \approx 0.0657$$

$$CI: (1.32 - 0.0657, 1.32 + 0.0657) = (1.2543, 1.3857)$$

$$b).\ Smaller\ n \rightarrow larger\ SE \rightarrow larger\ ME$$

If the sample size were 100 instead of 159, the margin of error would be greater compared to the result in part(a). This occurs because a smaller sample size leads to a comparatively larger margin of error, as the denominator is smaller when calculating.

$$c).\ 98\%\ confidence\ level \rightarrow larger\ Z \rightarrow larger\ ME$$

If the confidence level were increased to 98% instead of 95%, the margin of error would be bigger than the outcome in part (a). This is due to the fact that the corresponding percentile in the standard normal distribution would be higher, leading to a broader confidence interval.

$$d).\, 1.45\ not\ in\ CI\ (1.2543, 1.3857)\ \rightarrow\ unlikely\ mean\ is\ greater\ than\ 1.45$$

Considering the confidence interval calculated in part (a), (1.254, 1.386), it seems improbable that the average number of personal computers exceeds 1.45 since this value is not encompassed within the given range.