**MA 2621-E01 Probability for Applications**
**E1-Term Spring 2023 - Homework 01**

**Due: F 05/26 by 11.59 PM**

**Show all work as described in class. Partial credits will be given. Submit your solutions in a pdf format to canvas. Please write your name.**

1. Suppose $A$ and $B$ are disjoint with $P(A) = 0.3$ and $P(B) = 0.5$. What is $P(A^c \cap B^c)$?

$$A^c \cap B^c = (A \cup B)^c$$

$$P(A \cup B) = P(A) + P(B) = 0.3 + 0.5 = 0.8$$

$$P((A \cup B)^c) = 1 - P(A \cup B) = 1 - 0.8 = 0.2$$

$$P(A^c \cap B^c) = 0.2$$

2. The weather forecaster says that the probability of rain on Saturday is 20% and the probability of rain on Sunday is 30%.
   (a) Can you determine the probability of rain over the weekend (Saturday or Sunday)? Explain.
   (b) If it rains on Saturday, the probability of rain on Sunday is 60%. Can you now determine the probability of rain over the weekend?

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a).

$$P(Saturday\ or\ Sunday) = P(Saturday) + P(Sunday) - \underbrace{P(Saturday)P(Sunday)}_{\text{independent ?}}$$

$$= 0.2 + 0.3 - 0.2 * 0.3$$

$$= 0.44\ or\ 44\%$$

b).

P(A)

$$P(Saturday\ or\ Sunday) = P(Saturday) + \underbrace{P(Sunday|Saturday)P(Saturday)}_{\neq}$$

−2

$$- P(Saturday)P(Sunday|Saturday)$$

$$= 0.2 + 0.6 * 0.2 - 0.2 * 0.6$$

$$= 0.2\ or\ 20\%$$

P(Sunday) = 0.3.

3. Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products.

   (a) What is the probability that a randomly selected adult regularly consumes both coffee and soda?

   (b) What is the probability that a randomly selected adult does not regularly consume at least one of these two products?

a).

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.70 = 0.55 + 0.45 - P(A \cap B)$$

$$P(A \cap B) = 0.55 + 0.45 - 0.70 = 0.30 \text{ or } 30\%$$

b).

$$1 - P(A \cup B) = 1 - 0.70 = 0.30 \text{ or } 30\%$$

4. Given two events $A$ and $B$ with $P(A) = 0.2$ and $P(B) = 0.5$, what are the maximum and minimum possible values for $P(A \cup B)$?

$$P(A \cup B) = P(B)$$

$$= 0.5 \ (maximum)$$

typo

$$P(A \cup B) = P(A) + P(B)$$

$$= 0.2 + 0.5 = 0.7 \ (minimum)$$

5. Let $A$ and $B$ be two independent events with $P(A) = 0.4$ and $P(A \cup B) = 0.64$. What is $P(B)$?

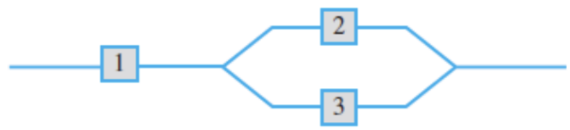
$$P(A \cup B) = P(A) + P(B) - P(A)P(B)$$

$$0.64 = 0.4 + P(B) - 0.4P(B) \qquad 0.24 = P(B) - 0.4P(B)$$

$$0.24 = 0.6 * P(B) \qquad P(B) = 0.24 / 0.6 \qquad = 0.4 \ (or\ 40\%)$$

6. Three components are connected to form a system as shown in the accompanying diagram. Because the components in the 2–3 subsystem

are connected in parallel, that subsystem will function if at least one of the two individual components functions. For the entire system to function, component 1 must function and so must the 2–3 subsystem.




The experiment consists of determining the condition of each component [$S$ (success) for a functioning component and $F$ (failure) for a non-functioning component]. Consider the following events.

$A$ : exactly two out of the three components function

$B$ : at least two of the three components function

$C$ : the system functions

(a) Determine the sample space, $\Omega$.

(b) List the outcomes in events: $A$, $B$, $C$, $C^c$, $A \cup C$, and $B \cap C$

(c) Determine the probabilities of the events in part ($b$).

a).

$$2^3 = 8 \text{ Possible Outcomes}$$

$$\Omega = \{SSS,\ SSF,\ SFS,\ SFF,\ FSS,\ FFS,\ FSF,\ FFF\}$$

b).

$$Event\ A\text{: } exactly\ two\ out\ of\ the\ three\ components\ function.$$

$$A = \{SSF,\ SFS,\ FSS\}$$

$$Event\ B\text{: } at\ least\ two\ of\ the\ three\ components\ function.$$

$$B = \{SSS,\ SSF,\ SFS,\ FSS\}$$

*Event $C$: the system functions, which requires component 1 to function and at least one of components 2 or 3 to function.*

$$C = \{SSS,\ SSF,\ SFS\}$$

*Event $Cc$: the system does not function. This is the complement of $C$.*

$$Cc = \{SFF,\ FSS,\ FFS,\ FSF,\ FFF\}$$

*Event $A \cup C$: the union of events $A$ and $C$ (outcomes that belong to either $A$ or $C$, or both)*

$$A \cup C = \{SSS,\ SSF,\ SFS,\ FSS\}$$

*Event $B \cap C$: the intersection of events $B$ and $C$ (outcomes that belong to both $B$ and $C$)*

$$B \cap C = \{SSS,\ SSF,\ SFS\}$$

c).

*For Event $A$, there are 3 outcomes in event $A$ and 8 possible outcomes, so:*

$$P(A) = |A|\ /\ |\Omega| = 3\ /\ 8 = 0.375$$

$$P(B) = |B|\ /\ |\Omega| = 4\ /\ 8 = 0.5$$

$$P(C) = |C|\ /\ |\Omega| = 3\ /\ 8 = 0.375$$

$$P(C^c) = |C^c|\ /\ |\Omega| = 5\ /\ 8 = 0.625$$

$$P(A \cup C) = |A \cup C|\ /\ |\Omega| = 4\ /\ 8 = 0.5$$

$$P(B \cap C) = |B \cap C|\ /\ |\Omega| = 3\ /\ 8 = 0.375$$

7. Suppose a 6-sided die is repeatedly rolled until "2" is obtained.
   (a) Determine the sample space. *(Use $f$ for any failure)*
   (b) Calculate the probability of the first "2" occurs at the $2^{\text{nd}}$ roll.
   (c) Find a general formula for the probability of the first 2 occurs at the $n^{\text{th}}$ roll or later.

a).

*The sample space, $\Omega$, is the set of all possible outcomes were we roll the die until we get a "2".*

*Each outcome consists of several "failures"*

$$\Omega = \{2,\ f2,\ ff2,\ fff2,\ ffff2,\ fffff2,\ \ldots\}$$

b).

*The probability of not getting a "2" on any roll is $5/6$ and the probability of getting a "2" is $1/6$*

$$P(first\ 2\ on\ 2nd\ roll) = P(f)\ *\ P(2)$$

$$= (5/6) \ * \ (1/6) \qquad = 5/3$$ 6 ← typo

c).

*The probability of failure (not rolling a* 2*) on any roll is* 5/6 *and the probability of success is* 1/6.
*For* $n - 1$ *failures followed by a success, the probability is*:

$$P(first\ 2\ on\ nth\ roll) \ = \ (P(f))\text{^}(n - 1) \ * \ P(2)$$

$$= (5/6)\text{^}(n - 1) \ * \ (1/6).$$

−2

*This applies to "at the nth roll or later" because the first* 2 *can be rolled on the nth roll, the* $(n + 1)$*th roll, and so on.*

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