MA 2611-DL01 Applied Statistics I
D-Term Spring 2023 - Homework 03

Due: R 04/06 by 11.59 p.m.

Show all work as described in class. Partial credits will be given. When applicable, provide your final answer with four decimal places. Submit your solutions in a pdf format to canvas. Please write your name.

1. Each of a sample of three home mortgages is classified as fixed rate (F) or variable rate (V).

$A =$ exactly one of the three is a variable-rate mortgage
$B =$ at most two of the selected mortgages are fixed rate
$C =$ all three mortgages are of the same type

(a) List the sample space.
(b) List the outcomes of the events: $A, B, C, A \cup B, B \cap C, C'$.
(c) Find the probability of events in part (b).

a). The sample space consists of all possible combinations of three mortgages being either fixed or variable. So the sample spaces are as follows.

$$\{FFF, FFV, FVF, FVV, VFF, VFV, VVF, VVV\}$$

b). The outcome of the events are as follows:

$A = \{FFV, FVF, VFF\}$

$B = \{VVV, FFV, FVF, VFF, VFV, FVV, VVF\}$

$C = \{FFF, VVV\}$

$A \cup B = \{VVV, FFV, FVF, VFF, VFV, FVV, VFV, VVF\}$

$B \cap C = \{FFF\}$

$C' = \{FFV, FVF, VFF, FVV, VFV, VVF\}$

c. The probability of events are as follows:

$P(A) = 3/8$

$P(B) = 7/8$

$P(C) = \frac{1}{4}$

$P(A \cup B) = \frac{7}{8}$

$P(B \cap C) = \frac{1}{8}$

$C' = \frac{6}{8} = \frac{3}{4}$

2. To better understand the website builder market, Clutch surveyed individuals who created a website using a do-it-yourself (DIY) website builder. Responders, categorized by the type of website they build–business or personal, were asked to indicate the primary purpose for building their website. The following table summarizes the findings:

	Type of Website		
Primary Purpose	Business	Personal	Total
Online Business Presence	52	4	56
Online Sales	32	13	45
Creative Display	28	54	82
Informational	9	24	33
Resources Blog	8	52	60
Total	129	147	276

If a website builder is selected at random, what is the probability that he or she,

(a) indicated creative display as the primary purpose for building his/her website?

(b) indicated creative display or informational resources as the primary purpose for building his/her website?

(c) is a business website builder or indicated online sales as the primary purpose for building his/her website?

a). Probability of selecting Creative Display at random:

$$P_{(Creative\ Display)} = \frac{Number\ of\ Individuals\ who\ selected\ Creative\ Display}{Total\ Number\ of\ Individuals\ Surveyed}$$

$$P_{(Creative\ Display)} = \frac{82}{276}$$

$$P_{(Creative\ Display)} = 0.2971\ or\ 29.71\%$$

b). Probability of selecting Creative Display or Informational at Random:

$$P_{(Creative\ Display+Informational)} = \frac{Number\ of\ Individuals\ who\ selected\ Creative\ Display + Informational}{Total\ Number\ of\ Individuals\ Surveyed}$$

$$P_{(Creative\ Display\ +\ Informational)} = \frac{82+33}{276}$$

$$P_{(Creative\ Display+Informational)} = 04167\ or\ 41.67\%$$

c). Probability of selecting Creative Display or Informational at Random:

$$P_{(Online\ Sales+Business)} = \frac{Number\ of\ Individuals\ who\ does\ Online\ Sales + Business}{Total\ Number\ of\ Business\ Website\ Builder}$$

$$P_{(Online\ Sales+Business)} = \frac{129+13}{276}$$

$$P_{(Online\ Sales+Business)} = 0.5145\ or\ 51.45\%$$

3. Do Millennials or Gen-Xers feel more tense or stressed out at work? A survey of employed adults conducted online by Harris Interactive on behalf of the American Psychological Association revealed the following:

Age Group	Felt Tense or Stressed Out at Work	
	Yes	No
Millennials	175	206
Gen-Xers	183	390

(a) Given that the employed adult felt tense or stressed out at work, what is the probability that the employed adult was a millennial?

(b) Given that the employed adult is a millennial, what is the probability that the person felt tense or stressed out at work?

(c) Explain the difference between in the results in a) and b).

(d) Is feeling tense or stressed out at work and age group independent?

a). Probability that employed adult was a millennial and stressed:

$$P_{(Millennial)} = \frac{175}{(175\ 185)} = 0.4888 = 48.8\%$$

b). Probability that a Millennial felt tense or stressed out at work

$$P_{(Felt\ Tense\ |\ Millennial)} = \frac{175}{(175\ +\ 206)} = 0.4593 = 45.93\%$$

c). The difference in result between part a and part b

The reason why there is a variation in outcomes between (a) and (b) is because of the order in which the events are conditioned. In (a), the scenario assumes that the employed adult feels tense or stressed out at work and seeks to determine the likelihood that they are a millennial, whereas in (b), the scenario assumes that the employed adult is a millennial and seeks to establish the likelihood that they feel tense or stressed out at work. The order of conditioning has an impact on how the probabilities are calculated.

d). Is feeling tense or stressed out at work and age group independent?

$$= \frac{175 + 183}{178 + 183 + 206 + 340}$$

$$= \frac{358}{954} \approx 0.3753\ \approx 37.53\%$$

We can say that feeling tense or stressed out at work and age group are not independent, since the equation does not hold true. This means that the probability of feeling tense or stressed out at work depends on the age group of the employed adult, and the two variables are related. Specifically, the data suggests that Gen-Xers are more likely to feel tense or stressed out at work compared to Millennials.

4. A particular model of a jet engine is used in a two-engine plane and a four-engine plane. The probability that one of these engines fails in flight is p. The two-engine plane will crash only if both engines fail, and the four-engine plane will crash only if three or more engines fail. Assume engine failure is the only cause of plane crashes, and that the engines operate independently.
   (a) What is the probability that the two-engine plane crashes?
   (b) What is the probability that the four-engine plane crashes?

a). Probability that the two-engine plane crashes:

Considering the engines operate independently, the probability that both engines fail is the product of the probability that each engine fails

$$P_{Both\ Engine\ Failing} = P \times P = P^2$$

b). Probability that the four-engine plane crashes:

The four-engine plane will crash only if three or more engines fail. The probability of exactly three engine failures is given by

$$P_{Exactly\ Three\ Engine\ Fails} = \frac{4!}{3!\,(4-3)!} \times P^3 \times (1-P)$$

$$P_{All\ four\ Engines\ Failing} = P \times P \ \times P \times P = P^4$$

$$P_{All\ four\ Engines\ Failing} = 4P^{3(1-P)} + P^4$$

5. A fair coin is independently tossed three times or until a head appears, whichever comes first. Let the random variable X denote the number of heads and let the random variable Y denote the number of tails obtained.
   (a) Find the probability mass function of X.
   (b) Find the probability mass function of Y.
   (c) What is the expected number of heads?
   (d) What is the expected number of tails?
   (e) What is the probability that at least two tails are observed?
   (f) What is the probability that exactly one head is observed?

a). The probable values of X are 0, 1, 2, and 3. Let P be the probability of getting a head on any given toss. Then, the probability of the Mass function of X is as follows:

$$P_{x=0} = \frac{1}{8}$$

$$P_{x=1} = \frac{7}{8}$$

b). The possible values of Y are 0, 1, 2, 3. To find the probability of mass function of Y, we can use the information that $Y = X - 1$. Therefore, the probability mass function of Y could be expressed as follows.

$$P_{x=0} = \frac{1}{2}$$

$$P_{x=1} = \frac{1}{4}$$

$$P_{x=2} = \frac{1}{8}$$

$$P_{x=3} = \frac{1}{8}$$

c). The expected number of head are as follows:

$$E(X) = 0P_{x=0} + 1P_{x=1}$$

$$E(X) = \frac{7}{8} + 0(\frac{1}{8})$$

$$E(X) = \frac{7}{8}$$

d). The expected number of tail are as follows:

$$E(Y) = 0P_{(Y=0)} + 1P_{(Y=1)}2P_{(Y=2)} + 3P_{(Y=3)}$$

$$E(Y) = 0 + 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{8}\right) + 3(\frac{1}{8})$$

$$E(Y) = \frac{7}{8}$$

e). The probability that at least 2 tails are observed are:

$$E(Y) = 2P_{(Y=2)} + 3P_{(Y=3)}$$

$$E(Y) = \frac{1}{8} + \frac{1}{8}$$

$$E(Y) = \frac{1}{4}$$

f). The probability that exactly one head is observed:

$$P_{(X=1)} = \frac{7}{8}$$

6. The number of arrivals per minute at a bank located in the central business district of a large city was recorded over a period of 200 minutes, with the following results.

Arrivals	0	1	2	3	4	5	6	7	8
Frequency	14	31	47	41	29	21	10	5	2

(a) What is the probability that there will be fewer than 2 arrivals in a given minutes?

(b) Compute the expected number of arrivals per minute.

(c) Compute the variance of number of arrivals per minute.

a). The total number of arrivals over the 200-minute period:

$$14(0) + 31(1) + 47(2) + 41(3) + 29(4) + 21(5) + 10(6) + 5(7) + 2(8) = 664$$

$$P_{(X<2)} = P_{(x=0)} + P_{(x=1)}$$

$$P_{(X<2)} = \frac{14}{200} + \frac{31}{200}$$

$$P_{(X<2)} = 0.225$$

$$\textit{Therefore, the probability that there will be fewer than 2 arrivals in a}$$

$$\textit{given minute is approximately } 0.225 \textit{ or } 22.5\%.$$

b). The expected number of arrivals per minute:

$$E(X) = \sum x\, P_{(X=x)}$$

$$= 0(\frac{14}{200}) + 1(\frac{31}{200}) + 2(\frac{47}{200}) + 3(\frac{41}{200}) + 4(\frac{29}{200}) + 5(\frac{21}{200}) + 6(\frac{10}{200}) + 7(\frac{5}{200}) + 8(\frac{2}{200})$$

$$E(X) = 2.9$$

$$Therefore, we\ can\ expect\ approximately\ 2.9\ arrivals\ per\ minute\ on\ average.$$

c). The variance of the number of the number of arrivals per minute:

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = 0^2(\frac{14}{200}) + 1^2(\frac{31}{200}) + 2^2(\frac{47}{200}) + 3^2(\frac{41}{200}) + 4^2(\frac{29}{200}) + 5^2(\frac{21}{200}) + 6^2(\frac{10}{200}) + 7^2(\frac{5}{200}) + 8^2(\frac{2}{200})$$

$$\frac{628}{200} = \frac{157}{50} = 3.14$$

$$Therefore, the\ variance\ of\ the\ number\ of\ arrivals\ per\ minute\ is\ 3.14.$$

7. The following table contains the probability distribution for the number of traffic accidents daily in a small town:

Number of Accidents Daily ($X = x$)	0	1	2	3	4	5
$P(X = x)$	0.10	0.20	0.45	0.15	0.05	c

(a) Find the constant c such that the $P(X)$ is a probability mass function (pmf).

(b) What is the probability that there will be at least 2 accidents on a given day?

(c) Compute the standard deviation of X.

a). Find the constant "c"

$$0.10 + 0.20 + 0.45 + 0.15 + 0.05 = 1$$

$$c = 0.05$$

b). Probability that there will be at least 2 accidents on a given day

$$P_{(X\geq 2)} = P_{(X=2)} + P_{(X=3)} + P_{(X=4)} + P_{(X=5)}$$

$$P_{(X\geq 2)} = 0.45 + 0.15 + 0.05 + 0.05$$

$$P_{(X\geq 2)} = 0.70$$

$$Therefore, the\ probability\ that\ there\ will\ be\ at\ least\ 2\ accidents\ on\ a\ given\ day\ is\ 0.70\ or\ 70\%.$$

c). The Variance of X is given by:

$$E(X) = 0(0.10) + 1(0.20) + 2(0.45) + 3(0.15) + 4(0.05) + 5(0.05)$$

$$E(X) = 2.0$$

$$E(X^2) = 0^2(0.10) + 1^2(0.20) + 2^2(0.45) + 3^2(0.15) + 4^2(0.05) + 5^2(0.05)$$

$$E(X^2) = 5.4$$

$$\sqrt{x^2\big(E(X)\big) - x((E(x)))^2}$$

$$\sqrt{5.4 - 2.0^2}$$

$$\sqrt{1.4}$$

$$1.18321$$

$$Therefore, the\ standard\ deviation\ of\ X\ is\ approximately\ 1.1832.$$
