1. The average sleep time of college students is studied. A random sample of 225 students nationwide had a mean of 6.8 hours with a standard deviation of 2.5 hours. Create a 90% confidence interval for mean hours of sleep for all college students nationally.

$$\text{Find the } z-\text{score corresponding to the } 90\% \text{ confidence level}: z \approx 1.645$$

$$\text{Compute the standard error}: SE = \sigma / \sqrt{n} = 2.5 / \sqrt{225} \approx 0.1667$$

$$\text{Calculate the margin of error}: \text{Margin of Error} = z * SE \approx 1.645 * 0.1667 \approx 0.274$$

$$\text{Calculate the lower limit of the confidence interval}: \bar{x} - \text{Margin of Error} = 6.8 - 0.274 \approx 6.526$$

$$\text{Calculate the upper limit of the confidence interval}: \bar{x} + \text{Margin of Error} = 6.8 + 0.274 \approx 7.074$$

$$\text{The } 90\% \text{ confidence interval is approximately } (6.526 \text{ hours}, 7.074 \text{ hours}).$$

2. A Pew Research report indicates that 73% of teenagers aged 13-17 own smart-phones. A random sample of 150 teenagers is drawn.
   (a) Find the mean of $\hat{p}$.
   (b) Find the standard deviation of $\hat{p}$.
   (c) Find the probability that more that 70% of the sampled teenagers own a smartphone.
   (d) Find the probability that the proportion of the sampled teenagers who own a smart phone is between 0.76 and 0.80.
   (e) Find the probability that less than 75% of the sampled teenagers own smartphones.
   (f) Would it be unusual if less than 68% of the sample teenagers owned smartphones?

$$Check\ if\ conditions\ for\ normal\ distribution\ are\ met:$$

$$np = 150 * 0.73 = 109.5$$

$$n(1-p) = 150 * 0.27 = 40.5$$

$$(\boldsymbol{a})\ \boldsymbol{Mean\ of\ \hat{p}}: \mu(\hat{p}) = p = 0.73$$

$$(\boldsymbol{b})\boldsymbol{Standard\ deviation\ of\ \hat{p}}:$$

$$\sigma(\hat{p}) = \sqrt{}(p(1-p)\ /\ n) = \sqrt{}(0.73 * 0.27\ /\ 150) \approx 0.0383$$

$$(\boldsymbol{c})\ \boldsymbol{P(\hat{p} > 0.70)}:$$

$$z = (0.70 - 0.73)\ /\ 0.0383 \approx -0.78$$

$$P(\hat{p} > 0.70) = 1 - P(z = -0.78) \approx 0.7823$$

$$(\boldsymbol{d})\ \boldsymbol{P(0.76 < \hat{p} < 0.80)}:$$

$$z1 = (0.76 - 0.73)\ /\ 0.0383 \approx 0.78$$

$$z2 = (0.80 - 0.73)\ /\ 0.0383 \approx 1.83$$

$$P(0.76 < \hat{p} < 0.80) = P(z = 1.83) - P(z = 0.78) \approx 0.1841$$

$$(\boldsymbol{e})\ \boldsymbol{P(\hat{p} < 0.75)}:$$

$$z = (0.75 - 0.73)\ /\ 0.0383 \approx 0.52$$

$$P(\hat{p} < 0.75) = P(z = 0.52) \approx 0.6985$$

$$(\boldsymbol{f})\ \boldsymbol{Unusual\ if\ less\ than\ 68\%\ own\ smartphones}?$$

$$z = (0.68 - 0.73)\ /\ 0.0383 \approx -1.30$$

$$P(\hat{p} < 0.68) = P(z = -1.30) \approx 0.0968$$

3. A quality control specialist expects a process to produce gizmos of length 12.25 mm. A random sampling of 36 gizmos gave us a mean of 12.163mm and a standard deviation of 0.240 mm. How would a 95% confidence interval help analyze the process? Calculate and $\boxed{\text{conclude}}$ your result.

$$\textit{Calculate the margin of error:}$$

$$\textit{Margin of error} = \textit{Critical value} * (\textit{Standard deviation} / \sqrt{\textit{Sample size}})$$
$$= 1.96 * (0.240 / \sqrt{36})$$

$$\textit{Calculate the lower limit of the confidence interval:}$$

$$\textit{Lower limit} = \textit{Sample mean} - \textit{Margin of error} = 12.163 - 0.0784 \approx 12.0846\ mm$$

$$\textit{Calculate the upper limit of the confidence interval:}$$

$$\textit{Upper limit} = \textit{Sample mean} + \textit{Margin of error} = 12.163 + 0.0784 \approx 12.2414\ mm$$

$$\textit{The } 95\% \textit{ confidence interval is approximately } (12.0846\ mm, 12.2414\ mm).$$

4. According to the U.S. Census Bureau, 43% of men who worked at home were college graduates. In a sample of 500 women who worked at home, 162 were college graduates.

   (a) Find a point estimator for the proportion of college graduate among women who work at home.

   (b) Construct a 98% confidence interval for the proportion of women who work at home who are college graduates.

   (c) Based on the confidence interval, is it reasonable to believe that the proportion of college graduates among women who work at home is the same as the proportion of college graduates among men who work at home? Explain.

$$(\boldsymbol{a})\ \boldsymbol{Calculate\ the\ point\ estimator}:$$

$$Point\ estimator\ =\ 162\ /\ 500\ =\ 0.324$$

$$(\boldsymbol{b})\ \boldsymbol{Calculate\ the\ margin\ of\ error}:$$

$$Margin\ of\ error\ =\ 2.33\ *\ \sqrt{}(0.324\ *\ (1\ -\ 0.324)\ /\ 500)\ \approx\ 0.0604$$

$$(\boldsymbol{c})\ \boldsymbol{Calculate\ the\ 98\%\ confidence\ interval}:$$

$$Confidence\ interval$$
$$=\ Sample\ proportion\ \pm\ (Critical\ value\ *\ \sqrt{}(Sample\ proportion\ *\ (1\ -\ Sample\ proportion)\ /\ Sample\ size))$$

$$For\ a\ 98\%\ confidence\ interval, the\ critical\ value\ (z)\ is\ approximately\ 2.33.$$

$$Sample\ proportion\ (p)\ =\ 0.324$$

$$Sample\ size\ (n)\ =\ 500$$

$$Margin\ of\ error = 2.33 * \sqrt{}(0.324 * (1 - 0.324) / 500) \approx 0.0604$$

$$Lower\ limit = 0.324 - 0.0604 \approx 0.2636$$

$$Upper\ limit = 0.324 + 0.0604 \approx 0.3844$$

$The\ 98\%\ confidence\ interval\ is\ approximately\ (0.2636, 0.3844).$

5. Making sure that the scales used by businesses in the United States are accurate is the responsibility of the National Institute for Standards and Technology (NIST) in Washington, D. C. Suppose that NIST technicians are testing a scale by using a weight known to weigh exactly 1000 grams.

   They weigh this weight on the scales 50 times and read the result each time. The 50 scale readings have a sample mean of $\overline{X} = 1000.6$ grams. The scale is out of calibration if the mean scale reading differs from 1000 grams. The technicians want to perform a hypothesis test to determine whether the scale is out of calibration.

   (a) State the appropriate null and alternative hypotheses.

   (b) The standard deviation of scale reading is known to be $\sigma = 2$. Compute the value of the test statistic.

   (c) State a conclusion. Use $\alpha = 0.05$ level of significance.

$$a).Calculate\ the\ test\ statistic\ using\ the\ z - test\ formula:$$

$$z = (\bar{X} - \mu) / (\sigma / \sqrt{}n)$$

$$H0: \mu = 1000$$

$$H1: \mu \neq 1000$$

$$b).Computing\ the\ value\ of\ test\ statistics$$

$$z = (\bar{X} - \mu) / (\sigma / \sqrt{n})$$

$$Where:$$

$$\bar{X} = sample\ mean = 1000.6\ grams$$

$$\mu = population\ mean\ (known\ weight) = 1000\ grams$$

$$\sigma = standard\ deviation = 2\ grams$$

$$n = sample\ size = 50$$

$$z = (1000.6 - 1000) / (2 / \sqrt{50}) = 0.6 / (2 / \sqrt{50}) \approx 2.12132$$

$$c).\ Conclusion\ of\ a = 0.05\ level\ of\ significance$$

Using a two-tailed hypothesis test with a 0.05 level of significance, we calculated a test statistic of z ≈ 4.24, which is greater than the critical z-value of +1.96. Since the test statistic falls in the rejection region, we reject the null hypothesis. Thus, there is sufficient evidence to conclude that the scale is out of calibration at a 0.05 level of significance.

6. The market research firm Salary.com reported that the mean annual earnings of all family practitioners in the United States was \$178,258. A random sample of 55 family practitioners in Los Angeles that month had a mean earning of $\overline{X} = \$192,140$ with a standard deviation of \$42,387. Do the data provide sufficient evidence to conclude that the mean salary for family practitioners in Los Angeles is greater than the national average?

(a) State the null and alternative hypothesis.

(b) Compute the value of the t-statistic. How many degrees of freedom are there?

(c) State your conclusion. Use $\alpha = 0.05$ level of significance.

$$a). Calculate\ the\ t - statistic\ using\ the\ t - test\ formula:$$

$$t = (\bar{X} - \mu) / (s / \sqrt{n})$$

$$H0: \mu\_LA = \mu\_US$$

$$H1: \mu\_LA > \mu\_US$$

$$b).\ Plug\ in\ the\ given\ values:$$

$$t = (\bar{X} - \mu) / (s / \sqrt{n})$$

$$\bar{X} = \$192{,}140$$

$$\mu = \$178{,}258$$

$$s = \$42{,}387$$

$$n = 55$$

$$t = (\$192{,}140 - \$178{,}258) / (\$42{,}387 / \sqrt{55}) \approx 2.22$$

$$Degrees\ of\ freedom\ (df)\ =\ n\ -\ 1\ =\ 55\ -\ 1\ =\ 54$$

$$c).\ Conclusion\ of\ a\ =\ 0.05\ level\ of\ confidence$$

Using a one-tailed hypothesis test with a 0.05 level of significance and 54 degrees of freedom, we calculated a t-statistic of approximately 2.22, which is greater than the critical t-value of 1.675. Since the t-statistic falls in the rejection region, we reject the null hypothesis. Thus, there is sufficient evidence to conclude that the mean salary for family practitioners in Los Angeles is greater than the national average at a 0.05 level of significance.

7. **(BONUS)** According to Secure List, 71.8% of all emails sent is spam. A system manager at a large corporation believes that the percentage at his company may be 80%. He examines a random sample of 500 emails received at an email server and finds that 382 of the messages are spam.

(a) State the appropriate null and alternative hypothesis.

(b) Compute the test statistic.

(c) Use $\alpha = 0.05$ level of significance, can you conclude that the percentage of emails that are spam differs from 80%?

(d) Use $\alpha = 0.1$ level of significance, can you conclude that the percentage of emails that are spam differs from 80%?

$$(a)\ State\ the\ appropriate\ null\ and\ alternative\ hypothesis.$$

$$H0: p\ =\ 0.80$$

$$H1: p\ \neq\ 0.80$$

$$(b)\ Compute\ the\ test\ statistic.$$

$$\hat{p}\ =\ number\ of\ spam\ emails\ /\ total\ number\ of\ emails\ =\ 382\ /\ 500\ =\ 0.764$$

$$z = (\hat{p} - p) / \sqrt{}(p(1 - p) / n)$$

$$Plug\ in\ the\ given\ values:$$

$$\hat{p} = 0.764$$

$$p = 0.80$$

$$n = 500$$

$$z \approx -2.01$$

$$(c). Use\ a - 0.05\ level\ of\ significance$$

$$P - value = 2P\ (Z > |Z0|)$$

$$P - value = 0.0444 < a = 0.05$$

$$(d). Use\ a = 0.1\ level\ of\ significance$$

$$P - value = 2P\ (Z > |Z0|)$$

$$P - value = 0.0444 < a = 0.10$$

$(d)\ Use\ \alpha$

$= 0.1\ level\ of\ significance, can\ you\ conclude\ that\ the\ percentage\ of\ emails\ that\ are\ spam\ differs\ from\ 80\%$

Our calculated test statistic (z ≈ -1.74) now falls in the rejection region, as it is less than the critical z-value of -1.645. Since the test statistic falls in the rejection region, we reject the null hypothesis.

Conclusion: At a 0.1 level of significance, we can conclude that the percentage of emails that are spam differs from 80%.