

USMANU DANFODIYO UNIVERSITY SOKOTO
DEPARTMENT OF MATHEMATICS

SECOND SEMESTER EXAMINATION 2014/2015 SESSION

COURSE: MAT 102

COURSE TITLE: ELEMENTARY MATHEMATICS

COURSE UNIT: 3 units

TIME ALLOWED: 2 hours

INSTRUCTION: Answer any four questions and do not write on the question paper

1. (a) Find the magnitude, direction ratio and direction cosine of the vector.
 $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

- (b) If $\vec{A} = 3\hat{i} - \hat{j} - 4\hat{k}$, $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{C} = \hat{i} + 2\hat{j}$. Find

(i) $2\vec{A} - \vec{B} + 3\vec{C}$

(ii) $|\vec{A} + \vec{B} + \vec{C}|$

(iii) $|\vec{A} - 2\vec{B} + 4\vec{C}|$

2. (a) Determine the value of x so that $\vec{a} = 2\hat{i} + 3\hat{j} + x\hat{k}$ and $\vec{b} = x\hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular.

- (b) Find the projection of the vector $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ on the vector $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$.

3. (a) If $\vec{a} = \hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$. Find

$\vec{a} \times \vec{b}$

(ii) $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

(iii) $\vec{b} \times \vec{a}$

- (b) Find the vector perpendicular to the plane ABC where A, B and C are the points $(4, -1, 3)$, $(4, -3, 1)$ and $(1, -1, -3)$.

4. (a) If A and B are differentiable function of a scalar U, prove

(i) $\frac{d}{du}(A \cdot B) = A \frac{dB}{du} + B \frac{dA}{du}$ (ii) $\frac{d}{du}(A \times B) = A \times \frac{dB}{du} + B \times \frac{dA}{du}$

- (b) If $\vec{A} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{B} = \sin t\hat{i} - \cos t\hat{j}$. Find

(i) $\frac{d}{dt}(\vec{A} \cdot \vec{B})$

(ii) $\frac{d}{dt}(\vec{A} \times \vec{B})$

5. A particle move so that its position vector is given by $\vec{r} = \cos wt\hat{i} + \sin wt\hat{j}$ where w is a constant. Show that

- (a) The velocity \vec{v} of the particle is perpendicular to \vec{r} .
(b) The acceleration \vec{a} is directed toward the origin and has magnitude proportional to the distance from the origin.
(c) $\vec{r} \times \vec{v} = \text{a constant vector}$

6. (a) If $\vec{R}(u) = (u - u^2)\hat{i} + 2u^3\hat{j} - 3u\hat{k}$. Find (i) $\int \vec{R}(u) du$ (ii) $\int |\vec{R}(u)| du$

- (b) The acceleration of a particle at any time $t \geq 0$ is given by $\vec{a} = \frac{dv}{dt} = 12 \cos 2t\hat{i} - 8 \sin 2t\hat{j} + 16t\hat{k}$. If the velocity \vec{v} and displacement \vec{r} are zero at $t = 0$. Find \vec{v} and \vec{r} at any time.

1. [a] [b] [c] [d]
2. [a] [b] [c] [d]
3. [a] [b] [c] [d]
4. [a] [b] [c] [d]

5. [a] [b] [c] [d]
6. [a] [b] [c] [d]
7. [a] [b] [c] [d]
8. [a] [b] [c] [d]

9. [a] [b] [c] [d]
10. [a] [b] [c] [d]
11. [a] [b] [c] [d]
12. [a] [b] [c] [d]

$$\sqrt{9+4+36} = 7$$

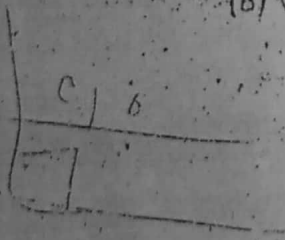
13. A unit vector in the direction of $\vec{b} = 3\hat{i} - 2\hat{j} + 6\hat{k}$ is
(A) $\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$ (B) $\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$ (C) $\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$ (D) $\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$
14. Determine the value of x so that $\vec{a} = 2\hat{i} + 3\hat{j} + x\hat{k}$ and $\vec{b} = 4\hat{i} - 2\hat{j} - 3\hat{k}$ is perpendicular
(A) 4 (B) 1 (C) 0 (D) 2
15. Find the area of the parallelogram determined by the vector $\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$
(A) 126 (B) $4\sqrt{3}$ (C) $\frac{1}{2}\sqrt{192}$ (D) $4\sqrt{3}$
16. Find the volume of a parallelepiped with sides $\vec{a} = 3\hat{i} - \hat{j}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 5\hat{j} + 4\hat{k}$
(A) 6 (B) 4 (C) 26 (D) 15
17. Find the area of the triangle whose vertices are $A(3, -1, 2)$, $B(1, -1, -2)$ and $C(3, -3, 1)$ is
(A) $\frac{1}{2}\sqrt{165}$ (B) $\frac{1}{2}[-5\hat{i} - 5\hat{j} + 4\hat{k}]$ (C) $\frac{1}{2}\sqrt{-34}$ (D) $-5\hat{i} - 5\hat{j} + 4\hat{k}$
18. Given $\vec{r} = \sin t\hat{i} + \cos t\hat{j} + 3t\hat{k}$ Find $\left|\frac{d\vec{r}}{dt}\right|$
(A) $-\sin t\hat{i} + \cos t\hat{j} - 3$ (B) 2 (C) 4 (D) $\cos t\hat{i} + \sin t\hat{j} + 3$
19. A particle moves so that its position vector is given by $\vec{r} = \cos 3t\hat{i} + \sin 6t\hat{j} - 2e^{-3t}\hat{k}$. Find its acceleration
(A) $6(\cos 6t\hat{i} + 6\sin 6t\hat{j}) + e^{-3t}\hat{k}$ (B) $20\hat{i} - 20\hat{j} + 16\hat{k}$
(C) $-6(\cos 3t\hat{i} + 6\sin 6t\hat{j}) + 2e^{-3t}\hat{k}$ (D) $6(\cos 6t\hat{i} + 6\sin 6t\hat{j}) - 2e^{-3t}\hat{k}$
20. The acceleration of a particle at any time $t \geq 0$ is given by $\vec{a} = 4s^4\hat{i} + 4\cos 4t\hat{j} + e^{-t}\hat{k}$. Find its velocity at any time t .
(A) $-2\cos t + \sin 4t + e^{-t} + c$ (B) $-2\cos t + \sin 4t - e^{-t} + c$ (C) $20\hat{i} - 20\hat{j} + 16\hat{k}$
(D) $-24\hat{i} - 15\hat{j} - 16\hat{k}$
21. $\vec{A} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$, and $\vec{B} = \sin t\hat{i} - \cos t\hat{j}$. Find the derivative of $\vec{A} \cdot \vec{B}$.
(A) $(5t^2 + 1)\cos t + 11t\sin t$ (B) $(5t^2 - 1)\cos t + 11t\sin t$
(C) $(5t^2 - 1)\cos t + 11t\sin t$ (D) $(5t^2 - 1)\cos t - 11t\sin t$
22. Vectors which have the same terminal point are called
(A) Zero Vectors (B) Like Vectors (C) Coterminal Vectors
(D) Unlike Vectors

+32k.

21 + 15

$$\cos t - \sin t$$

- Find the area of the parallelogram determined by the vectors $\vec{i} - 2\vec{j} + 3\vec{k}$ and $4\vec{i} - \vec{j} + 2\vec{k}$
- (A) 150 (B) $6\sqrt{5}$ (C) $\frac{1}{2}\sqrt{150}$ (D) $5\sqrt{6}$
2. The projection of the vector $5\vec{i} - 3\vec{j} + 2\vec{k}$ on the vector $-\vec{i} + 2\vec{j} - \vec{k}$ is
- (A) $\frac{-13}{\sqrt{6}}$ (B) $\frac{13}{\sqrt{38}}$ (C) $\frac{13}{\sqrt{6}}$ (D) $\frac{-13}{\sqrt{38}}$
3. Find $|\vec{a} - \vec{b}|$, if $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 9$.
- (A) $\sqrt{13}$ (B) $\sqrt{-9}$ (C) 4 (D) $\sqrt{31}$
4. The direction cosine of the vector $\vec{a} = 3\vec{i} - 4\vec{k}$ is
- (A) $2\sqrt{5}$ (B) $\frac{3}{5}, 0, \frac{-4}{5}$ (C) $\frac{7}{8}$ (D) $\frac{0}{6}, \frac{-4}{6}, \frac{1}{6}$
5. Determine the value of x so that $\vec{a} = 2\vec{i} + x\vec{j} + 1\vec{k}$ and $\vec{b} = 4\vec{i} - 7\vec{j} - 2\vec{k}$ are perpendicular.
- (A) -1 (B) 3 (C) 0 (D) 1
6. If $\vec{a} \cdot \vec{b} = 0$, then this implies that \vec{a} is
- (A) Parallel to \vec{b} (B) Co-initial vector (C) Perpendicular to \vec{b} (D) Unit vector
7. Find the vector perpendicular to the plane where A, B and C are the points $(4, 1, 3)$, $(4, -3, -1)$ and $C(1, 1, 3)$ respectively
- (A) $\frac{2\vec{i} + 5\vec{j} - \vec{k}}{6}\sqrt{6}$ (B) $\frac{2\vec{i} - \vec{j} - \vec{k}}{3}\sqrt{3}$ (C) $\frac{1}{2}\sqrt{-4}$ (D) $\frac{2\vec{i} + \vec{j} - \vec{k}}{6}\sqrt{6}$
8. The area of parallelogram formed by the vectors with diagonals $\vec{i} - \vec{j} + 2\vec{k}$ & $-2\vec{i} + 3\vec{j} + 4\vec{k}$ is
- (A) $5\sqrt{3}$ (B) $2\sqrt{38}$ (C) $3\sqrt{2}$ (D) $\sqrt{5}$
9. If \vec{a} is a vector such that $|\frac{3}{4}\vec{a}| = 1$. Find the magnitude of \vec{a}
- (A) $-\frac{3}{4}$ (B) $-\frac{4}{3}$ (C) $\frac{4}{3}$ (D) $\frac{3}{4}$
10. Find the unit tangent vector at $t = 0$ on the curve $x = t^3 - 1 = 0$, $y = 4t^2 - 3$, and $z = 2t^2 + 6t$.
- (A) $\frac{11\vec{k}}{\sqrt{37}}$ (B) $\frac{-\vec{i} + 6\vec{k}}{\sqrt{37}}$ (C) $\frac{-\vec{i} - 6\vec{k}}{\sqrt{37}}$ (D) $\frac{4\vec{i} + 8\vec{j} + 10\vec{k}}{\sqrt{12}}$
11. Find the magnitude of the vector $\vec{a} = 3\vec{i} - 4\vec{j} + 5\vec{k}$.
- (A) $5\sqrt{2}$ (B) 7 (C) $2\sqrt{5}$ (D) $\sqrt{7}$
12. Find $\vec{a} \cdot \vec{b}$, if $\vec{a} = 2\vec{i} - 3\vec{j} - \vec{k}$ and $\vec{b} = \vec{i} + 4\vec{j} - 2\vec{k}$
- (A) $10\vec{i} + 3\vec{j} + 11\vec{k}$ (B) $-10\vec{i} - 3\vec{j} - 11\vec{k}$ (C) $10\vec{i} + 3\vec{j} - 11\vec{k}$ (D) $10\vec{i} - 3\vec{j} - 11\vec{k}$
13. If \vec{a} and \vec{b} are non-zero vectors the projection of \vec{b} on \vec{a} is
- (A) $\vec{a} \cdot \vec{b}$ (B) zero (C) $\vec{a}\vec{b}$ (D) $\vec{b} \cdot \vec{a}$
14. Find the area of the triangle whose vertices are $A(3, -1, 2)$, $B(1, -1, -3)$ and $C(4, -2, 1)$
- (A) $\frac{1}{2}\sqrt{66}$ (B) $\frac{1}{2}[-5\vec{i} - 5\vec{j} + 4\vec{k}]$ (C) $\frac{1}{2}\sqrt{-34}$ (D) $-5\vec{i} - 5\vec{j} + 4\vec{k}$
15. If $R = \cos t \vec{i} + \cos t \vec{j} - 2\sin 2t \vec{k}$. Find $\left|\frac{dR}{dt}\right|$ at $t = 0$.
- (A) 3 (B) $\sqrt{3}$ (C) $\sqrt{4}$ (D) -3



$$A \times B = |A||B|\sin\theta$$

$$A \times B = |A||B|\sqrt{1 - \cos^2\theta} = |A||B|\sin\theta$$

ADMISSION NUMBER.....

DEPT/UNIT.....

USMANU DANFODIYO UNIVERSITY, SOKOTO
DEPARTMENT OF MATHEMATICS

MAT 102 C/A TEST

COURSE: MAT 102

COURSE UNIT: 3 Units

COURSE TITLE: Elementary Mathematics II

TIME ALLOWED: 40 Min.

INSTRUCTIONS: Answer All Questions and shade the Correct Answer.
Do not write on this Question Paper.

1. [a] [b] [c] [d]
2. [a] [b] [c] [d]
3. [a] [b] [c] [d]
4. [a] [b] [c] [d]
5. [a] [b] [c] [d]
6. [a] [b] [c] [d]
7. [a] [b] [c] [d]
8. [a] [b] [c] [d]
9. [a] [b] [c] [d]
10. [a] [b] [c] [d]
1. The unit vector in the direction of $\vec{b} = 3\vec{i} - 2\vec{j} + 6\vec{k}$ is
(A) 7 (B) $\frac{3}{7}\vec{i} - \frac{2}{7}\vec{j} + \frac{6}{7}\vec{k}$ (C) $\frac{3\vec{i} - 2\vec{j} + 6\vec{k}}{7}$ (D) $\frac{1}{7}(3\vec{i} - 2\vec{j} + 6\vec{k})$
2. Determine the value of x so that $\vec{a} = 2\vec{i} + 3\vec{j} + x\vec{k}$ and $\vec{a} = 4\vec{i} - 2\vec{j} - 7\vec{k}$ is perpendicular
(A) -1 (B) 2 (C) 0 (D) 7
3. Find the area of the parallelogram determined by the vector $\vec{i} + 2\vec{j} + 3\vec{k}$ and $3\vec{i} - 2\vec{j} + \vec{k}$
(A) 196 (B) $9\sqrt{3}$ (C) $\frac{1}{2}\sqrt{192}$ (D) $4\sqrt{3}$
4. Find the volume of a parallelepiped with sides $\vec{a} = 3\vec{i} - \vec{j}$, $\vec{b} = \vec{j} + 2\vec{k}$ and $\vec{c} = \vec{i} - 5\vec{j} + 4\vec{k}$
(A) 6 (B) 4 (C) 24 (D) 5
5. The area of the triangle whose vertices are $A(3, -1, 2)$, $B(1, -1, -3)$ and $C(4, -3, 1)$ is
(A) $\frac{1}{2}\sqrt{165}$ (B) $\frac{1}{2}[-5\vec{i} - 5\vec{j} + 4\vec{k}]$ (C) $\frac{1}{2}\sqrt{-34}$ (D) $-5\vec{i} - 5\vec{j} + 4\vec{k}$
6. Given $\vec{R} = \sin t\vec{i} - \cos t\vec{j} + 3\vec{k}$ find $\left|\frac{d\vec{R}}{dt}\right|$
(A) $-\sin t\vec{i} + \cos t\vec{j} - 3$ (B) 2 (C) 4 (D) $\cos t\vec{i} + \sin t\vec{j} + 3$
7. A particle moves so that its position vector is given by $\vec{r} = \cos 3t\vec{i} + \sin 6t\vec{j} + 2e^{-3t}\vec{k}$
Find its acceleration
(A) $6(\cos 6t\vec{i} + 6\sin 6t\vec{j} + e^{-3t}\vec{k})$ (B) $20\vec{i} - 20\vec{j} + 16\vec{k}$
(C) $-6(\cos 3t\vec{i} + 6\sin 6t\vec{j} + 2e^{-3t}\vec{k})$ (D) $6(\cos 6t\vec{i} + 6\sin 6t\vec{j} - 2e^{-3t}\vec{k})$
8. The acceleration of a particle at any time $t \geq 0$ is given by $\vec{a} = 4\sin 2t\vec{i} + 4\cos 4t\vec{j} + e^{-t}\vec{k}$
Find its velocity at any time t .
(A) $-2\cos t + \sin 4t + e^{-t} + c$ (B) $-2\cos t + \sin 4t - e^{-t} + c$ (C) $20\vec{i} - 20\vec{j} + 16\vec{k}$
(D) $-24\vec{i} - 15\vec{j} - 16\vec{k}$
9. $\vec{A} = 5t^2\vec{i} + t\vec{j} - t^3\vec{k}$, and $\vec{B} = \sin t\vec{i} - \cos t\vec{j}$. Find the derivative of $\vec{A} \cdot \vec{B}$
(A) $(5t^2 + 1)\cos t + 11t\sin t$ (B) $(5t^2 - 1)\cos t + 11t\sin t$
(C) $(5t^2 - 1)\cos t + 11\sin t$ (D) $(5t^2 - 1)\cos t - 11t\sin t$
10. The Vectors which have the same terminal point are called
(A) Zero Vectors (B) Like Vectors (C) Collinear Vectors
(D) Unlike Vectors

$$\cos t - \sin t + 3$$

1. The vector perpendicular to the plane where A, B and C are the points (3, -1, 2), (1, -1, -2) and (2, 3, 1) respectively.

- (A) $\frac{-i+j+k}{\sqrt{21}}$ (B) $\frac{-2i-j+4k}{21}\sqrt{21}$ (C) $\frac{1}{2}\sqrt{4}$ (D) $\frac{-i-j+4k}{7}\sqrt{7}$

2. The direction cosine of the vector $\vec{a} = 7\hat{i} + 6\hat{j} - 6\hat{k}$ is

- (A) $\frac{7}{5}, 0, -1$ (B) $\frac{3}{5}, 0, -1$ (C) $2\sqrt{7}$ (D) $\frac{7}{11}, \frac{6}{11}, \frac{6}{11}$

3. The area of the parallelogram determined by the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$.

- (A) $8\sqrt{3}$ (B) $\frac{1}{2}\sqrt{192}$ (C) $2\sqrt{6}$ (D) $2\sqrt{192}$

4. The projection of the vector $-\hat{i} + 2\hat{j} - \hat{k}$ on the vector $5\hat{i} - 3\hat{j} + 2\hat{k}$ is.

- (A) $\frac{13}{\sqrt{6}}$ (B) $\frac{13}{\sqrt{6}}$ (C) $\frac{-13}{\sqrt{38}}$ (D) $\frac{-13}{\sqrt{6}}$

5. Find the value of x so that $\vec{a} = 4\hat{i} + x\hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular.

- (A) 0 (B) 0 (C) 3 (D) 1

6. If $\vec{a} = \vec{b}$, then this implies that \vec{a} is

- (A) Parallel to \vec{b} (B) Co-initial vector (C) Perpendicular to \vec{b} (D) A vector

7. A vector such that $|\vec{a}| = 1$. Find the magnitude of \vec{a} .

- (A) 1 (B) 1 (C) 2 (D) 1

8. The magnitude of the vector $\vec{a} = \hat{i} - 2\hat{k}$.

- (A) $\sqrt{5}$ (B) 2 (C) $2\sqrt{5}$ (D) $\sqrt{5}$

9. Find $\vec{a} \times \vec{b}$, if $\vec{a} = \hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} - \hat{k}$.

- (A) $10\hat{i} + 3\hat{j} + 11\hat{k}$ (B) $-10\hat{i} - 3\hat{j} - 11\hat{k}$ (C) $10\hat{i} + 3\hat{j} - 11\hat{k}$ (D) $10\hat{i} - 3\hat{j} - 11\hat{k}$

10. For non-zero vectors \vec{a} and \vec{b} , the projection of \vec{a} on \vec{b} is

- (A) zero (B) $\vec{b} \cdot \vec{a}$ (C) $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$ (D) \vec{a}

11. The area of the triangle whose vertices are A(3, -1, 2), B(1, -1, -3) and C(2, 3, 1).

- (A) $\frac{1}{2}\sqrt{35}$ (B) $\frac{1}{2}[-5\hat{i} - 5\hat{j} + 4\hat{k}]$ (C) $\frac{1}{2}\sqrt{-3}$ (D) $[-5\hat{i} - 5\hat{j} + 4\hat{k}]$

12. The area of parallelogram formed by the vectors with diagonals $-3\hat{i} - \hat{j} + 2\hat{k}$ & $-\hat{i} + 3\hat{j} + 4\hat{k}$.

- (A) $5\sqrt{5}$ (B) $\sqrt{3}$ (C) $3\sqrt{5}$ (D) $\sqrt{5}$

13. A particle moves along a curve whose parametric equations are: $x = -e^{-t}$, $y = 2e^{-t}$, $z = 0$. Find the magnitude of the velocity at $t = 0$.

- (A) 25 (B) 81.006 (C) $\sqrt{37}$ (D) $\sqrt{10}$

14. The unit tangent vector at $t = 0$ on the curve $x - t^2 - 1 = 0$, $-y = -4t^2 + 3$, $z = 2t(t - 1)$.

- (A) $\frac{-i+j+k}{\sqrt{3}}$ (B) $\frac{-i+6k}{\sqrt{32}}$ (C) $\frac{-4j-6k}{\sqrt{52}}$ (D) $\frac{4j-6k}{\sqrt{32}}$

15. Find $|\vec{a} + \vec{b}|$, if $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 9$.

- (A) 5 (B) $\sqrt{-3}$ (C) 5 (D) $\sqrt{17}$

Handwritten notes and calculations at the bottom of the page, including $\vec{a} =$, $\vec{b} =$, and various vector operations.