Linear Regression

Contents

1. Discussion Questions

2. Linear Regression fundamentals

3. Performance measures

4. Pros & cons

Questions to discuss

1. What is the relationship between Machine Learning (ML) and Supervised Learning (SL)?

2. What is Linear Regression and how does it work?

3. What are the evaluation metrics for regression?

4. What are the pros and cons of linear regression?

What is the relationship between ML and SL?

Machine Learning (ML)

- Machine Learning is the ability of a computer to do some task without being explicitly programmed.
- The ability to do the tasks comes from the underlying model which is the result of the learning process.
- The model is generated by learning from huge volumes (both in breadth and depth) of historical data reflecting the real world in which the processes are performed.

Examples of what machine learning algorithms can do

- Search through the data to look for patterns in the form of trends, cycles, associations, etc.
- Express these patterns as mathematical structures (model)
- Using those patterns to test the unseen data

Supervised Learning (SL)

- It builds a mathematical model using data that contains both the inputs and the desired output (labels or ground truth)
- There are basically two types of supervised learning:
  - Regression - where the desired output is in the form of continuous values
    - **e.g.** predicting the house prices based on some features like area, the number of rooms, etc.
  - Classification - where the desired output is in the form of categories
    - **e.g.** predicting if the person is likely to default on a loan based on the features like age, past transactions, etc.
- The model learns from the training data using these 'target variables' as reference variables.
- The model thus generated is used to make predictions about data not seen by the model before.
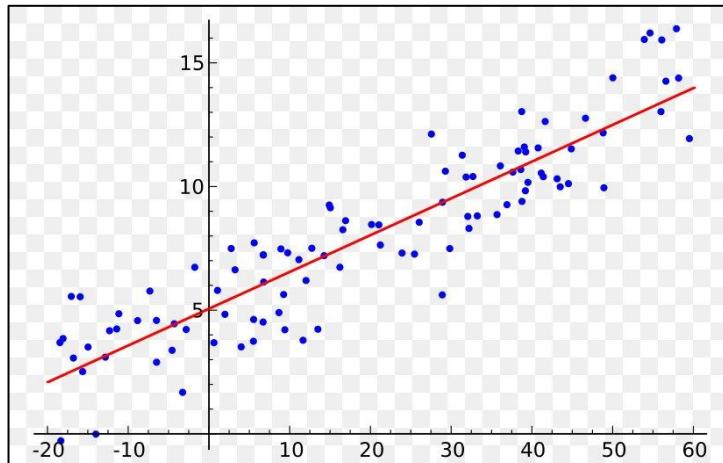
What is Linear Regression and how does it work?
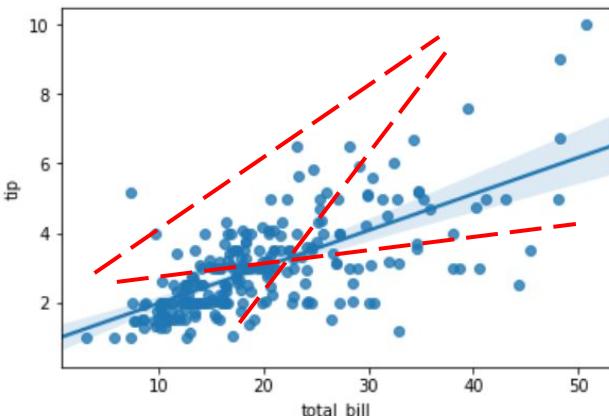
- Linear regression is a way to identify a relationship between the independent variable(s) and dependent variable

- We can use these relationships to predict values for one variable for the given value(s) of other variable(s)

- It assumes the relationship between variables can be modeled through a linear equation or an equation of a line.

- The variable, which is used in prediction is termed as independent/explanatory/regressor where the predicted variable is termed as dependent/target/response variable.

- In the case of linear regression with a single explanatory variable, the linear combination can be expressed as :

response = intercept + constant * explanatory variable

What is the best fit line in linear regression?

- Learning from the data, the model generates a line that fits the data.

- Our aim is to find a regression line that best fits the data

- By best fit, it means that the line will be such that the cumulative distance of all the points from the line is minimized

- Mathematically, the line that minimizes the sum of squared error of residuals is called Regression Line or the Best Fit Line.

- In the example here, you can see a scatter plot between the *tip* amount and the *total_bill* amount

- We can see that there is a positive correlation between these two - as the bill amount increases, the tip increases

- The line in blue that you see is the 'best fit' line - those in red are some examples of all other lines that are not the 'best fit'

What is Multiple Linear Regression?

- This is just the extension of the concept of simple linear regression with one variable

- In the real world, any phenomenon or outcomes could be driven by many different independent variables

- Therefore the need to have a mathematical model that can capture this relationship
  - Ex: predicting the price of a house, we need to consider various attributes related to the house, such as area, number of rooms, number of kitchens, etc.

- Such a regression problem is an example of multiple regression.

- It can be represented by :

  target = intercept + constant1*feature1 + constant2*feature2 + constant3*feature3 + …..

- The model aims to find the constants and intercept such that this line is the best fit.

- Evaluating a model is very important as it helps us understand the model performance.

- Evaluation metrics allow us to quantify our model's performance using a single number.

- Comparing the metric values for train and test sets helps us get an idea about the fit of the model.

  - If the model performance is low on the train and test sets, then the model is said to underfit the data
  - If the model performance is high on the train set but low on the test set, then the model is said to overfit the data.

- The aim is to find the model which best fits our data.

What are the evaluation metrics for regression?

R-squared	Adjusted R-squared	Mean Absolute Error	Root Mean Square Error
• Measure of the % of variance in the target variable explained by the model • Generally the first metric to look at for linear regression model performance • Higher the better	• Conceptually, very similar to R-squared but penalizes for the addition of too many variables • Generally used when you have too many variables as adding more variables always increases R^2 but not Adjusted R^2 • Higher the better	• Simplest metric to check prediction accuracy • Same unit as the dependent variable • Not sensitive to outliers i.e. errors doesn't increase too much if there are outliers • Difficult to optimize from a mathematical point of view (pure maths logic) • Lower the better	• Another metric to measure the accuracy of prediction • Same unit as the dependent variable • Sensitive to outliers - errors will be magnified due to the square function • But has other mathematical advantages that will be covered later • Lower the better

What are the pros and cons of linear regression?

Pros:

- Simple to implement and easier to interpret the outputs coefficient.

- Helpful if the relationship between the independent and dependent variable is linear

Cons:

- Has a lot of statistical assumptions which are not always true for real-world data

- Outliers can have huge effects on regression

- Assumes that the input variables are independent. It gets highly affected by multicollinearity.

Happy Learning !

Appendix: Regression Model Evaluation Metrics

Metric	Formula
R-squared	$$R^2 = 1 - \frac{\frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2}{\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2}$$
Adjusted R-squared	$$Adj.\,R^2 = 1 - \left[\frac{(1 - R^2)(n - 1)}{n - k - 1}\right]$$
Mean Absolute Error	$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \widehat{y}_i|$$
Root Mean Square Error	$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2}$$