

Let  $t$  be total number of discs and  $b$  be total blue discs. We want:

$$\begin{aligned}\frac{b}{t} \cdot \frac{b-1}{t-1} &= \frac{1}{2} \\ \Rightarrow (2t-1)^2 - 2(2b-1)^2 &= -1 \\ \Rightarrow y^2 - 2x^2 &= -1 \quad \{\text{let}\}\end{aligned}$$

Which is a form of Pell's Equation. We can see that  $(1,1)$  satisfies the equation. To get further solutions, let  $(p, q) = (1,1)$  denote the first solution. Then

$$\begin{aligned}2x^2 - y^2 &= (2p^2 - q^2)^n = 1 \\ \Rightarrow (\sqrt{2}x - y)(\sqrt{2}x + y) &= (\sqrt{2}p - q)^n (\sqrt{2}p + q)^n = 1 \\ \Rightarrow [x, y] &= \left[ \frac{(\sqrt{2}p + q)^n - (\sqrt{2}p - q)^n}{2\sqrt{2}}, \frac{(\sqrt{2}p + q)^n + (\sqrt{2}p - q)^n}{2} \right]\end{aligned}$$

Least  $n$  for which  $y > 2 \times 10^{12} - 1 \Rightarrow t > 10^{12}$  is  $n = 33$ .

For  $n = 33$ , we get  $b = 756872327473$ .