We adopt the following spherical co-ordinate system (for radius = 1)

At every instance any nanobot will travel along a great circle that passes through its position and its next nanobot’s position.

First we find an equation of a great circle. Note that a great circle on unit sphere is just the intersection of spherical surface and any plane that passes through origin . So their intersection should give us the equation of the great circle.

Let and . This gives the equation of a great circle:

Due to the symmetry of the problem, of all nanobots will remain same at all times and difference between of adjacent nanobots will be , where is the total number of bots. Now we find the equation of great circle that passes through points and :

Here we choose the solution:

And to find :

So the final equation of the great circle passing through points and :

Let the position of a nanobot at any instance be and let it move on the great circle (above equation) by . Thus we have:

Expanding and :

Let and :

This is the equation followed by any nanobot. To find its length, we use the arc length formula:

Where is the total length and s is the parametrization of the curve. Einstein’s summation convention is used over and , and represents derivative with respect to .

In our equation, choosing as parameter for curve, substituting metric tensor for unit spherical surface and integrating from to :

One can find its value by numerical integration. Or, using software like Mathematica one can also find its analytical formula.

On ProjectEuler Forums, user sergej.samborskij mentions a shorter formula:

And user amcalde mentions:

Another interesting things worth mentioning:

1. To find equation of great circle in Cartesian coordinates, one can start with an equation in 2D X-Z plane: and and rotate it twice around X-axis and Z-axis by angles and . The parameters and can then be found by constraints like great circle should pass from two points. Also note that great-circle distance between two points is .
2. One can also use the geodesic’s equation to get the equation of the great circle:

Or, using , one can get:

Where should be determined from other constraints. One can also use these equations to derive previously used equation for great circle: . Check <http://vixra.org/pdf/1404.0016v1.pdf> for more details.

1. Another parametrization for great circle equation between two points in Cartesian coordinates can be:

Where is the great circle distance between two points and is the parameter defining the curve. Check <https://math.stackexchange.com/a/384719> for more details.