# Edge Detection

## Audiovisual Processing CMP-6026A

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# Content

- Edges from image derivatives
- Laplacian matrices
- Line detection operators
- Canny edge detector

# Edge Detection

Convert an image into a set of **curves**.

- Extracts salient *features* of the image.
- Far more *compact* than pixels.

# Edges

An *edge* in an image is a significant local change or discontinuity in the image intensity.

# Edges

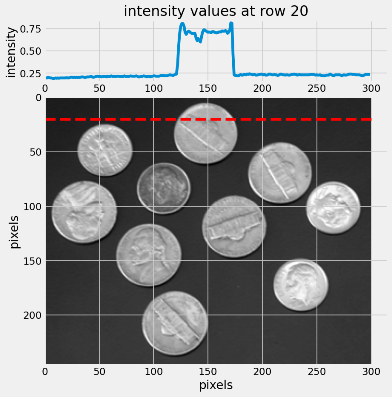
Edges come from discontinuity in:

- surface normal
- depth
- surface color
- illumination

# Edges

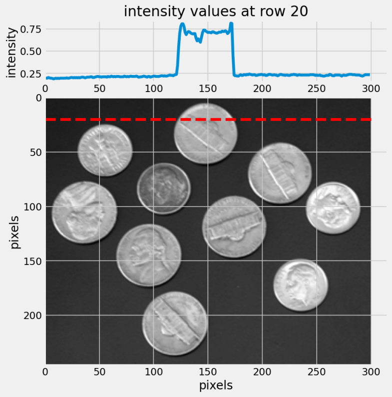

An image is a 2D matrix of intensities.

# Edges




We can look at those intensities in a single row.

# Edges




We can see how edges are defined by these changes in intensity.

# Derivatives

The derivative is the rate of change of a function.

– 1D *first* order derivative: **difference** in consecutive pixels:

$$\frac{\delta f}{\delta x} \approx f(x+1) - f(x)$$

# Derivatives

The derivative is the rate of change of a function.

- 1D *second* order derivative: **acceleration** of pixel intensity change:

$$\frac{\delta^2 f}{\delta x^2} \approx f(x+1) + f(x-1) - 2f(x)$$

# Derivatives

Required properties of first derivatives:

- Zero in regions of constant intensity
- Non-zero at onset of a ramp or step
- Non-zero along intensity ramps

# Derivatives

Required properties of second derivatives:

- Zero in regions of constant intensity
- Non-zero at the onset **and** end of an intensity step or ramp.
- *Zero* along intensity ramps.

# Derivatives

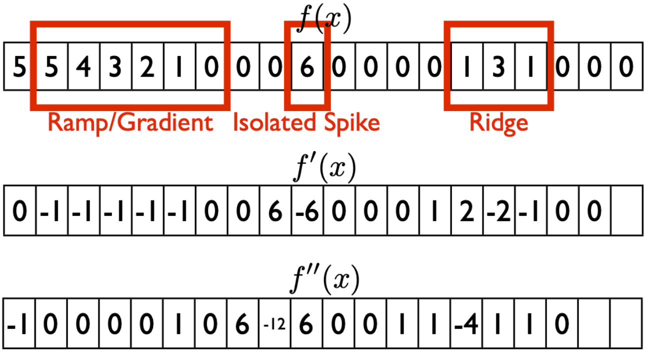



Figure 1: Example from Gonzalez and Woods.

# Derivatives

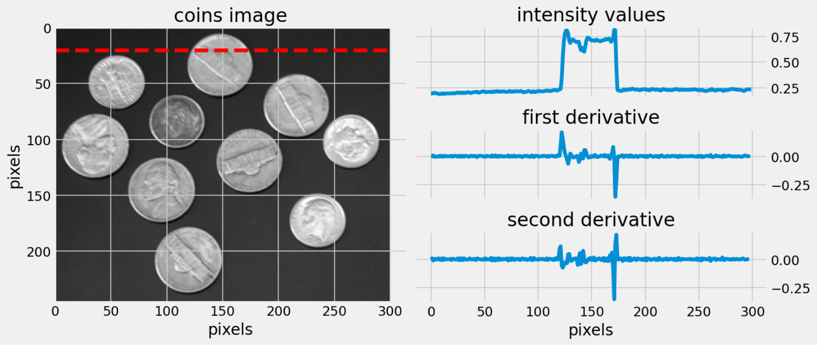



Figure 2: Intensity, first and second derivatives

# Image Derivatives

For images, we must consider the derivative in both directions:

$$\frac{\delta f}{\delta x} \approx f(x+1,y) - f(x,y)$$

$$\frac{\delta f}{\delta y} \approx f(x,y+1) - f(x,y)$$

# Image Derivatives

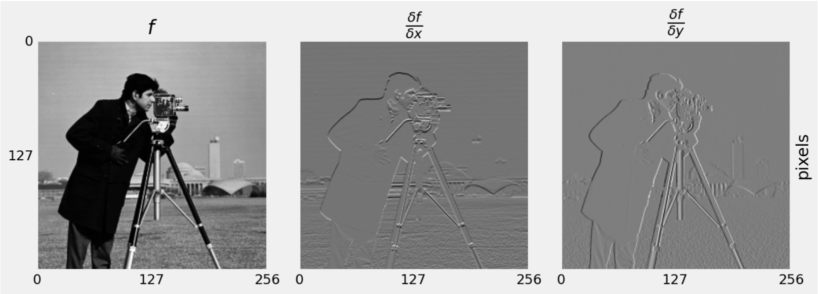



Figure 3: x and y first derivatives

# Image Derivatives

An image *gradient* is formed of two components:

$$\nabla f = \left[\frac{\delta f}{\delta x}, \frac{\delta f}{\delta y}\right]$$

# Image Derivatives

Image gradient is a vector:

$$\nabla f = \left[ \frac{\delta f}{\delta x}, \frac{\delta f}{\delta y} \right]$$

# Image Derivatives

A vector has magnitude...

$$|\nabla f| = \sqrt{\left(\frac{\delta f}{\delta x}\right)^2 + \left(\frac{\delta f}{\delta y}\right)^2}$$

Magnitude is the *strength* of the edge.

# Image Derivatives

A vector has direction. . .

$$\theta = \tan^{-1}\left(\frac{\delta f}{\delta y} / \frac{\delta f}{\delta x}\right)$$

Direction of an edge is **perpendicular** to the gradient direction.

# Image Derivatives

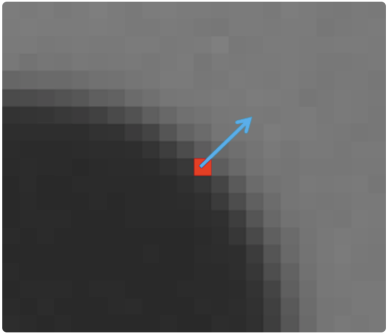


Figure 4: gradient direction

- The gradient points in the direction of most rapid change in intensity.
- **Perpendicular** to the edge direction.

# Image Derivatives

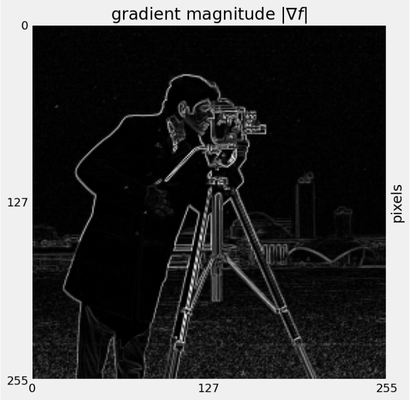



Figure 5: gradient magnitude as greyscale

# Image Derivatives

First order derivatives:

- produce thicker edges in images
- have a stronger response to stepped intensity changes

# Second Order Derivatives

Second order derivatives:

- have a stronger response to fine detail
- are more aggressive at enhancing detail
- Generally, second-order derivatives are *preferred.*

# Second Order Derivatives

$$\nabla^2 f = \frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2}$$

Derivative in this form is known as the **Laplacian**.

# Laplacian

We know:

$$\frac{\delta^2 f}{\delta x^2} \approx f(x+1) + f(x-1) - 2f(x)$$

$$\frac{\delta^2 f}{\delta y^2} \approx f(y+1) + f(y-1) - 2f(y)$$

# Laplacian

So, the Laplacian is calculated as:

$$\nabla^2 f = f(x+1) + f(x-1) + f(y+1) + f(y-1) - 4f(x,y)$$

# Laplacian

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

The Laplacian can also be calculated by **convolving** the image with this filter.

# Laplacian

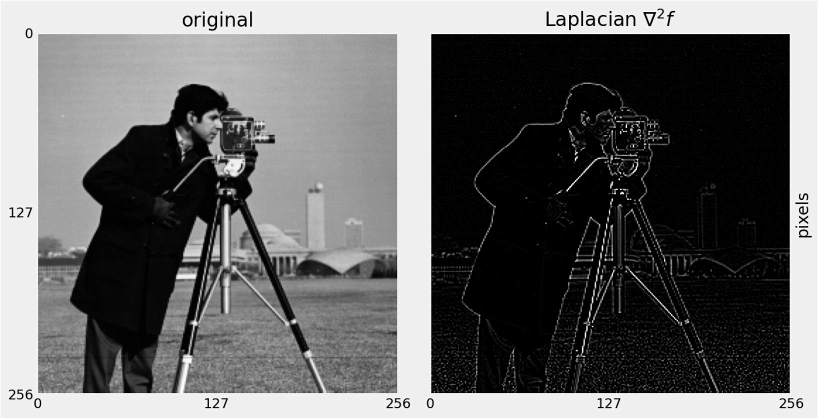



Figure 6: Laplacian

# Laplacian

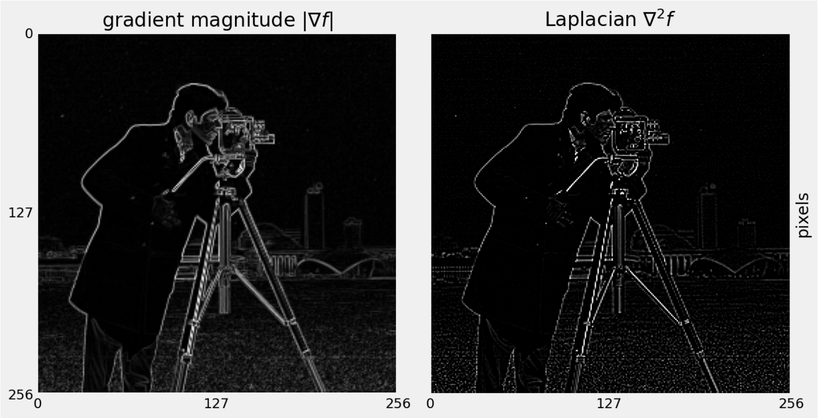



Figure 7: Gradient magnitude and Laplacian

# Line Detection

The Laplacian responds strongly to *any* detail in the image.

# Line Detection

What if we only wanted to detect lines that point in a certain direction?

$$\begin{bmatrix} -1 & 2 & -1 \\ -1 & 2 & -1 \\ -1 & 2 & -1 \end{bmatrix}$$

# Line Detection

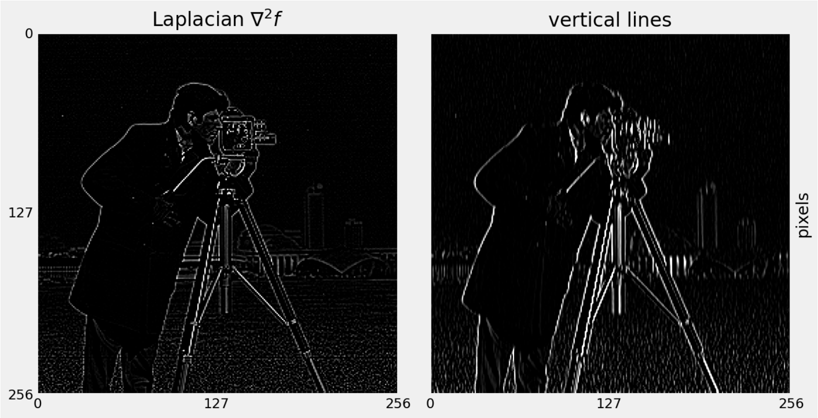



Figure 8: Line Detection

# Line Detection

What about detecting edges in other directions?

vertical

| | | |
|---|---|---|
| -1 | 2 | -1 |
| -1 | 2 | -1 |
| -1 | 2 | -1 |

forward diagonal

| | | |
|---|---|---|
| -1 | -1 | 2 |
| -1 | 2 | -1 |
| 2 | -1 | -1 |

horizontal

| | | |
|---|---|---|
| -1 | -1 | -1 |
| 2 | 2 | 2 |
| -1 | -1 | -1 |

backward diagonal

| | | |
|---|---|---|
| 2 | -1 | -1 |
| -1 | 2 | -1 |
| -1 | -1 | 2 |

Figure 9: Line directions

# Line Detection

What about detecting edges in other directions?

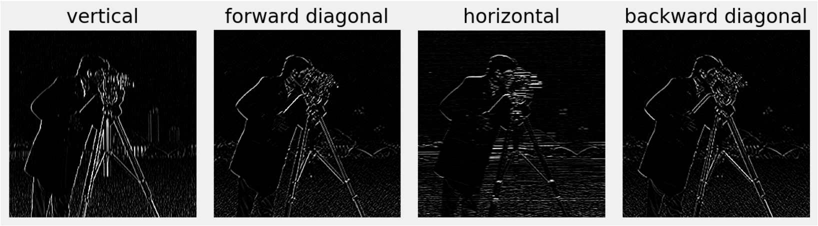



Figure 10: Line directions

# Line Detection

Previous filter gives strong response along a line.

- **But...** also responds at isolated pixels.
- Edge detector should respond *only* to edges

# Line Detection

Look either side of candidate pixel. . .

- but ignore the pixel itself.

# Line Detection

Two popular *first-order* operators are **Prewitt** and **Sobel**.

Both provide approximations of derivatives.

# Line Detection

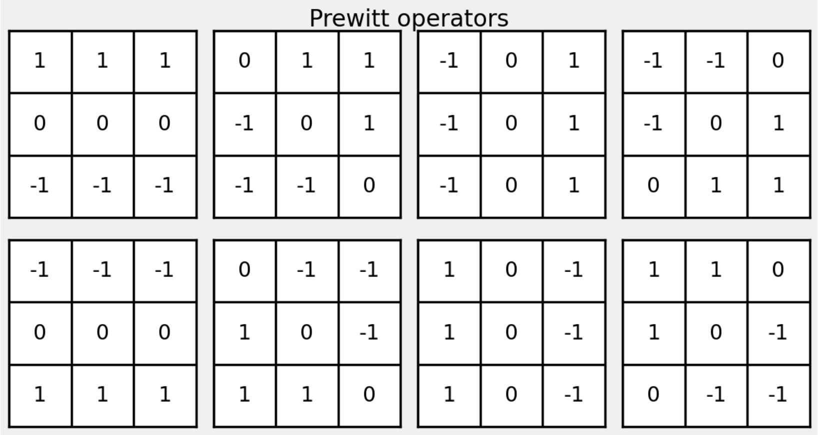



Figure 11: Prewitt, J.M.S. (1970). "Object Enhancement and Extraction"

# Line Detection

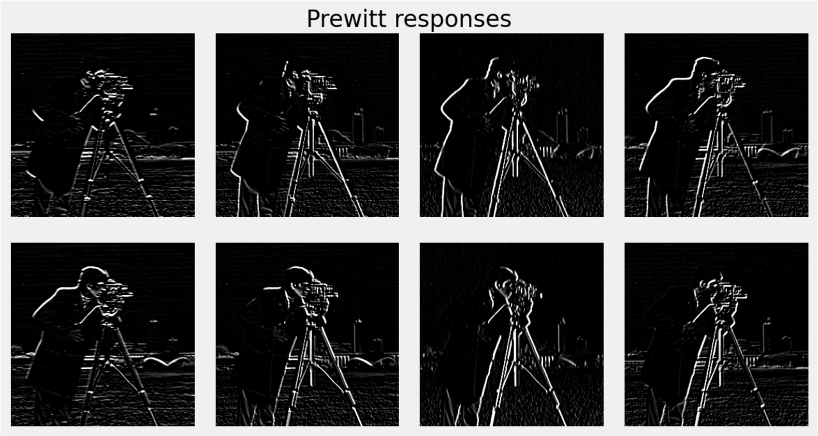



Figure 12: Prewitt responses

# Line Detection

Sobel operators

$$
\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}
\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}
\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}
\begin{bmatrix} -2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}
$$

$$
\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}
\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}
\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}
\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -2 \end{bmatrix}
$$

Figure 13: Sobel, I. (1968) “An Isotropic 3x3 Image Gradient Operator”

# Line Detection

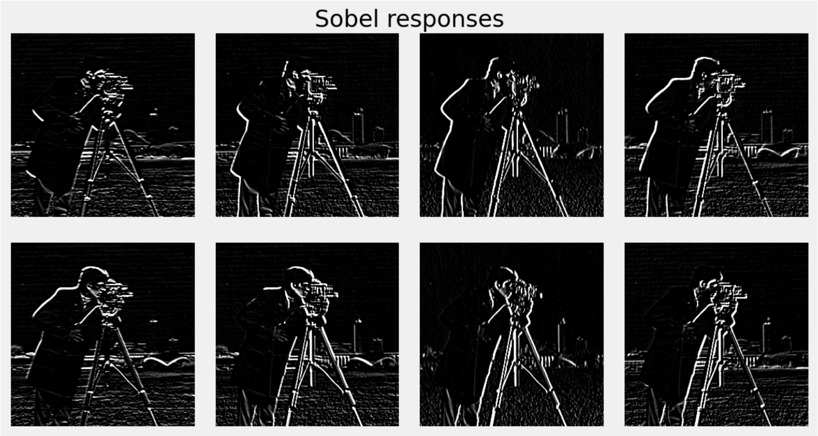



Figure 14: Sobel responses

For each pixel, find the maximum value from all of the filter responses, and then threshold.

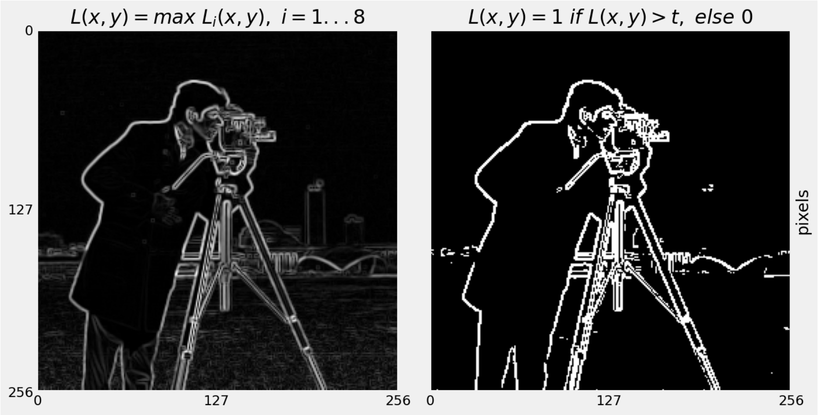



Figure 15: Sobel maximum

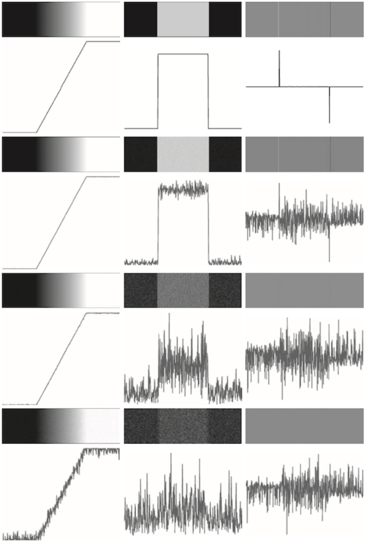


Figure 16: Gonzalez and Woods

We rarely observe *ideal* edges in real images.

- Lens imperfections
- sensor noise, etc.
- Edges appear more like noisy ramps.

# Edge Detection

Four limitations with basic gradient-based edge detection:

- Hard to set the optimal value for the threshold.
- Edges are broken (known as streaking)
- Edges can be poorly localised
- An edge might produce more than one response

# Canny Edge Detector

The **Canny Edge Detector** is *optimal* with respect to gradient-based limitations.

# Canny Edge Detector

Requirements for a *good* edge detector:

- Good detection - respond to edges, not noise.
- Good localisation - detected edge near real edge.
- Single response - only one response per edge.

# Canny Edge Detector

Canny provides an elegant solution to edge detection.

- Canny provides a *hacky* solution to edge detection!

# Canny Edge Detector

Canny Edge Detection is a four step process:

1. Convolve image with Gaussians of particular scales.
2. Compute gradient magnitude and direction.
3. Perform **non-maximal** suppression to thin the edges.
4. Threshold edges with **hysteresis**.

# Canny Edge Detector

Step 1: Convolve image with Gaussians of particular scales.

- Smoothing helps ensure robustness to noise.
- The size of the Gaussian kernel affects the performance of the detector.

# Canny Edge Detector

Step 2: Compute gradient *magnitude* and *direction*:

– Using Sobel operators.

**Quantise** the angle of the gradient:

– Discrete nature of image limits the possible angle.
– Angle can only be $\{0, 45, 90, 135\}$ degrees.

# Canny Edge Detector

Step 3: Perform **non-maximal** suppression.

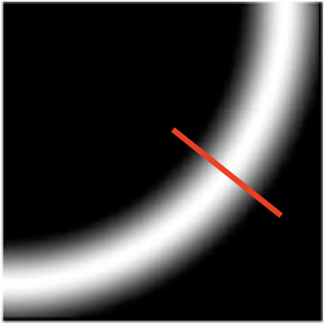


Figure 17: direction of gradient

- An edge-thinning technique.
- Searches for maximum value along direction of gradient and sets all others to zero.
- Result is a one pixel wide curve.

# Canny Edge Detector

Step 4: Threshold edges with **hysteresis**.

- Hysteresis is the dependence of the state of a system on its history.

# Canny Edge Detector

Step 4: Threshold edges with **hysteresis**.

Use **two** thresholds: $T_{min}$ and $T_{max}$.

$$E(x,y) = \begin{cases} 1 & E(x,y) \geq T_{max} \\ 0 & E(x,y) < T_{min} \end{cases}$$

# Canny Edge Detector

Step 4: Threshold edges with **hysteresis**.

$$E(x, y) = \begin{cases} 1 & T_{min} \leq E(x, y) < T_{max} \iff \text{linked to an edge} \\ 0 & T_{min} \leq E(x, y) < T_{max} \;\; \text{otherwise} \end{cases}$$

# Canny Edge Detector

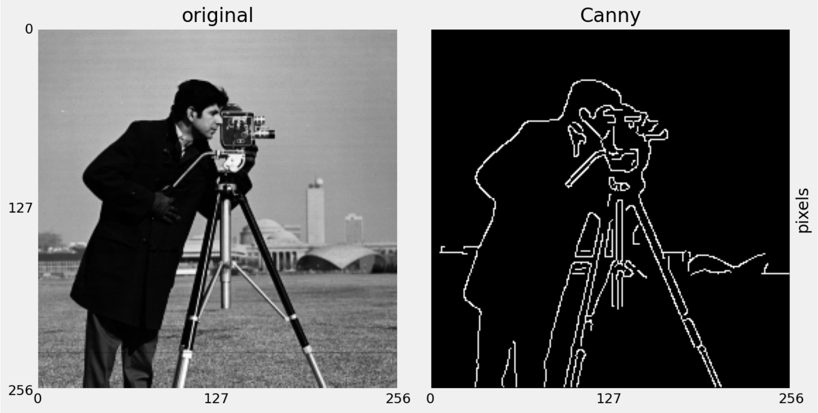



Figure 18: Canny edge detection

# Canny Edge Detector

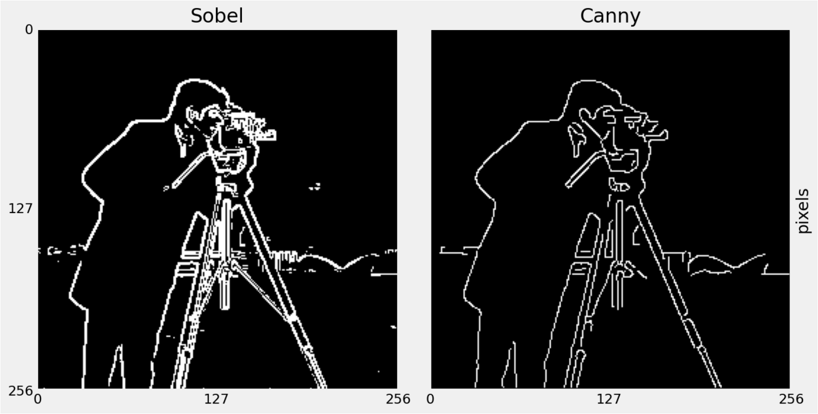



Figure 19: Max Sobel compared to Canny

# Summary

- Image derivatives
- Laplacian operator
- Line detection kernels
- Canny Edge Detector