8- Hypothesis testing with qualitative variables

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Readme

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Introduction

- Categorical variables represent facts that can be better described with *labels* than with numbers.
  - Example: `Sex`, better choose from {`Male` , `Female`} than from: {1,2}.
- Sometimes ordering of labels makes sense, although it is not reasonable to assign numbers to categories:
  - Example: `Tumor stage`: {1,2,3,4}, burt $1 + 2 \neq 3$!!!
- `Sex` is an example of a categorical variable in `nominal` scale
- `Stage` is an example of a categorical variable in `ordinal` scale

```
sex <- factor(c("Female", "Male"))
stage <- factor(1:4, ordered=TRUE)
```

Example

- Select one of the datasets that you have worked with during the course
    - diabetes.xls
    - osteopòrosis.csv
    - demora.xls
- Read the dataset and check that the categorical variables you are interested in are converted into factors.
- Confirm the conversion by summarizing the variables

The analysis of categorical variables

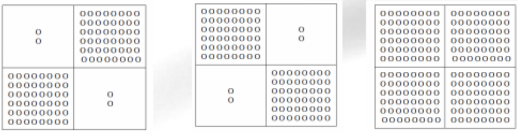
- The analysis of categorical data proceeds as usual:
  - Start exploring the data with the tables and graphics
  - Proceed to estimation and testing if *appropriate*
- Estimation
  - Proportions: Point estimates, confidence intervals, Sample Size
- Testing
  - One variable (tests with proportions)
    - Does the proportion (% affected) match a given value?
    - Is the proportion (% affected) the same in two populations?
  - With two variables (chi-square and related)
    - Is there an association between two categorical variables?
    - Is there a relationship between the values of a categorical variable before and after treatment?

Example

Consider the following study relating smoking and cancer

Load data: “dadescancer.csv”

	Smoking X=1	Non smoking X=0	TOTAL
CANCER Y=1	190	87	277
NO CANCER Y=0	60	163	223
TOTAL	250	250	500



Crosstabulating a dataset

```
dadescancer <- read.csv("datasets/dadescancer.csv", strings
```

```
attach(dadescancer)
mytable <-table(cancer, fumar)
mytable
```

```
##             fumar
## cancer       Fuma No fuma
##   Cancer      190      87
##   No cancer    60     163
```

Crosstabulation (2): Marginal tables

```r
margin.table(mytable, 1) # A frequencies (summed over B)
```

```
## cancer
##    Cancer No cancer
##       277       223
```

```r
margin.table(mytable, 2) # B frequencies (summed over A)
```

```
## fumar
##    Fuma No fuma
##     250     250
```

Crosstabulation (2): In percentages

```r
prop.table(mytable) # cell percentages
```

```
##             fumar
## cancer        Fuma No fuma
##   Cancer     0.380   0.174
##   No cancer  0.120   0.326
```

```r
prop.table(mytable, 1) # row percentages
```

```
##             fumar
## cancer            Fuma   No fuma
##   Cancer     0.6859206 0.3140794
##   No cancer  0.2690583 0.7309417
```

```r
# prop.table(mytable, 2) # column percentages
```

Exercice

- With the osteoporosis dataset repeat the crosstabulation done above using
  - Two categorical variables
  - Variable "MENOP" and a newly created variable "catBUA" created by properly categorizing variable BUA.

One variable: Proportion tests

- According to medical literature, in the period 1950-1980, the proportion of obese individuals (defined by medical criteria: BMI ≥ 30) was 15% in the population of men over 55 years old.
- A random sample obtained from the same population between 2000 and 2003 showed that, over a total of 723 men older than 55, 142 were obese.
- Considering that the significance level that we use is 5%, can we say that the population of men older than 55 in 2000-2003 had the same proportion of obese cases than that population had in 50'-80'?

Proportion tests with R

```r
prop.test(x=142, n=723, p=0.15)
```

```
##
##  1-sample proportions test with continuity correction
##
## data:  142 out of 723, null probability 0.15
## X-squared = 11.849, df = 1, p-value = 0.0005768
## alternative hypothesis: true p is not equal to 0.15
## 95 percent confidence interval:
##  0.1684325 0.2276606
## sample estimates:
##         p
## 0.1964039
```

Estimation comes with proportion test

- `prop.test` does **three** distinct calculations
  - A test for the hypothesis $H_0 : p = p_0$ is performed
  - A confidence interval for p is built based on the sample
  - A point estimate for p is also provided.

```
> prop.test(x=142, n=723, p=0.15)

        1-sample proportions test with continuity correction

data:  142 out of 723, null probability 0.15
X-squared = 11.849, df = 1, p-value = 0.0005768
alternative hypothesis: true p is not equal to 0.15
95 percent confidence interval:
 0.1684325 0.2276606
sample estimates:
        p
0.1964039
```

Hypothesis Test

Confidence interval

Point estimate

Exercise

- In the osteoporosis dataset
  - Test the hypothesis that the proportion of women with `osteoporosis` is higher than 7%
    - In the global population of the study
    - Only in women with osteoporosis
  - Select a sample of size 100 and repeat the test. How do the results change?
  - What sample size should we have taken so that th precision of the confidence intervals would have been at most 3% with a probability of 95%?

Contingency tables

- A contingency table (a.k.a cross tabulation or cross tab) is a matrix-like table that displays the (multivariate) frequency distribution of the variables.
- It is bidimensional, and classifies all observations according with two qualitative variables (A and B, rows and columns).

Clasif	B_1	B_2	...	B_s	Total
A_1	n_{11}	n_{12}	...	n_{1s}	$n_{1\bullet}$
A_2	n_{21}	n_{22}	...	n_{2s}	$n_{2\bullet}$
...	...	...	...	...	
A_r	n_{r1}	n_{r2}	...	n_{rs}	$n_{r\bullet}$
Total	$n_{\bullet 1}$	$n_{\bullet 2}$		$n_{\bullet s}$	N

Chi-squared test

Chi squared independence test

- When the sample comes from a single population with 2 qualitative variables, the aim is to determine if there is relationship between vars:

Chi squared homogeneity test

- When each row is a sample from distinct populations (groups, subgroups...), the aim is to determine if both groups have significative differences in that variable

Chi-squared tests

- When we have:
  - quantitative data,
  - one or more categories,
  - independent observations,
  - adequate sample size (>10)
- and our questions are like. . .
  - *Do the number of individuals or objects that fall in each category differ significantly from the number you would expect?*
  - *Is this difference between the expected and observed due to sampling variation, or is it a real difference?*

Chi squared.test: Observed vs expected

Observades	Braf -	Braf +
Grau 1	97	5
Grau 2	81	7
Grau 3	32	18

Esperades	Braf -	Braf +
Grau 1	89.25	12.75
Grau 2	77.00	11.00
Grau 3	43.75	6.25

Chi squared tests with R

```
mytable <-table(cancer, fumar)
chisq.test (mytable)
```

```
##
##  Pearson's Chi-squared test with Yates' continuity corre
##
## data:  mytable
## X-squared = 84.214, df = 1, p-value < 2.2e-16
```

Alternatively use Fisher test

```
fisher.test(mytable)
```

```
##
##  Fisher's Exact Test for Count Data
##
## data:  mytable
## p-value < 2.2e-16
## alternative hypothesis: true odds ratio is not equal to
```

Exercise

- Use the osteoporosis dataset to study if it can be detected an association between the variables `menop` and `classific` in the osteoporosis dataset.
- Do not start with a test but with an appropriate summarization and visualization!