5- Introduction to Statistical Inference

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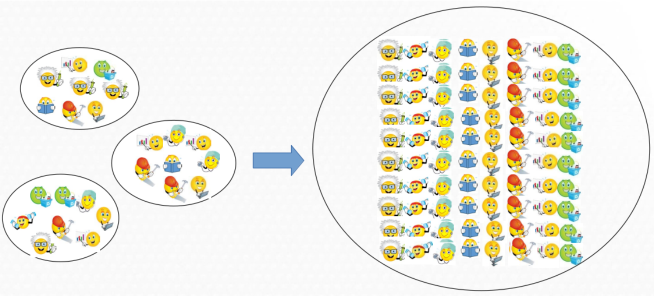
Outline

- The objectives of statistical inference
- Examples
- Point estimation. On incidence and prevalence
- Confidence intervals
- Sample size calculations

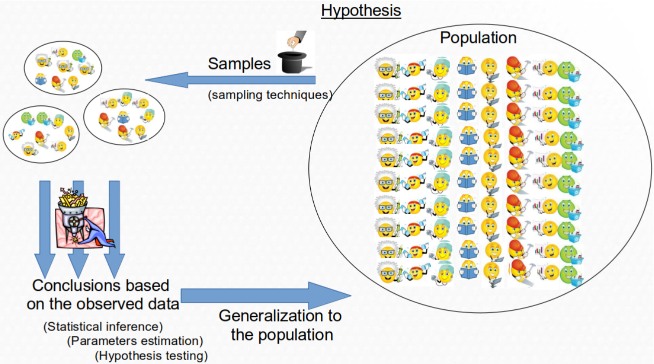
The objectives of Statistical Inference (I)

Taking the observed (measured) values of a group of samples...



we aim at determining the properties of the entire population.

The objectives of Statistical Inference (II)



Example

- Consider the data in the "osteoporosis.csv" dataset.
- It can be useful to provide information such as:
    - The percentage of menopausic women with osteoporosis
    - The mean bone density in menopausic or non-menopausic women
    - The existence of significance differences:
        - Observed % of osteoporosis vs "theoretical" population values
        - BUA in menopasuic vs non menopausic
- Answering these questions (and questions like these) is the main goal of Statistical Inference

Two types of statistical inference problems

- ESTIMATION
  - When we wish to *learn some characteristics of our population*, such as
    - The percentage of non osteopenic or menopausic women
    - The mean bone density in each of these groups
- HYPOTHESIS TESING
  - When we wish to *check about some statement on some characteristic of the population* or we *wish to make some comparisons*
    - Is it true that the mean bone density is smaller than 75 in menopausic
    - Can we state that non menopausic women have a higher bone density than menopausic?

Estimators: Aproximating the value of population parameters

- Numerical values calculated on a sample that we believe to be a good approximation of a certain real value (parameter) in the population.
- Intuitively, we work with many estimators, such as the mean or a computed percentage of a given sample, that we assume that are somehow characterizing a population.
- It is **not always obvious to decide which is the best estimator for each parameter**
- In order to decide which estimator we use we can rely on the *properties* of the estimators such as **the bias** or the **precision (the variance)** of the estimator.

Exercise

- Read the diabetes dataset. Convert characters into factors before continuing.
- Provide an estimate of
  - The distribution of a numerical variable.
  - a proportion of at least one categorical variable and
  - the mean value of at least one numerical variable.
- Could you have used different estimators?
- How would you decide?

How precise is an estimator?

- We all are familar with “forks” associated with voting results.
  - They usually start “wide” and tend to disappear as more votes are counted.
- Imagine you are given an estimate of 18% for the incidence of a certain disease.
- Is it a good estimate?
- Hard to know without more information
  - 18 ± 2 is probably useful
  - 18 ± 12 is probably too wide to be considered useful
- So given an estimator and a n estimation (a value) **how can we provide a measure of how precise this estimation is**?

The *Standard Error* of an estimator

- An obvious question when we choose an estimator is *how precise it is to approximate the value of the population parameter*.
- This can be answered using the **standard error of the estimator**
- The standard error is a great quantity :
  - It informs about the *precision* of our estimates
  - Helps build another type of estimators: *confidence intervals*
  - Helps find formulae to compute *sample size* for estimation

Some standard errors

- Standard error of the sample mean

$$SEM = \frac{\hat{s}}{\sqrt{n}}$$

- Standard error of the sample proportion

$$SEP = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Confidence intervals

- Confidence intervals are based on standard errors
- Confidence interval for the mean

$$\overline{X} - t_{\epsilon/2}\frac{\hat{s}}{\sqrt{n}} \leq \mu \leq \overline{X} + t_{\epsilon/2}\frac{\hat{s}}{\sqrt{n}} = \mathbf{\overline{X}} \pm \mathbf{t_{\epsilon/2}} \cdot \mathrm{SEM}$$

- Confidence interval for the proportion

$$\hat{p} - z_{\epsilon/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq \mu \leq \hat{p} + z_{\epsilon/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \widehat{\mathbf{p}} \pm \mathbf{z_{\epsilon/2}} \cdot \mathrm{SEM}$$