# 4.3.5 Probit regression

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp - \frac{(x-\mu)^2}{2\sigma^3}$$

$$\Phi(a) = \int_{-\infty}^{a} \mathcal{N}(\theta \mid 0, 1) \mathrm{d}\theta$$

$$\operatorname{erf}(a) = \frac{2}{\sqrt{\pi}} \int_{0}^{a} \exp\left(-\theta^2/2\right) \mathrm{d}\theta$$

$$\mathbf{\Phi}(a) = \frac{1}{2}\left\{1 + \frac{1}{\sqrt{2}} \operatorname{erf}(a)\right\}$$

$$\begin{aligned} p(t \mid \mathbf{x}) &= (1-\epsilon)\sigma(\mathbf{x}) + \epsilon(1-\sigma(\mathbf{x})) \\ &= \epsilon + (1-2\epsilon)\sigma(\mathbf{x}) \end{aligned}$$

# 4.3.6 Canonical link functions

$$p(t \mid \eta, s) = \frac{1}{s} h\left(\frac{t}{s}\right) g(\eta) \exp\left\{\frac{\eta t}{s}\right\}$$

$$y \equiv \mathbb{E}[t \mid \eta] = -s \frac{d}{d\eta} \ln g(\eta)$$

$$y = f\left(\mathbf{w}^{\mathrm{T}} \boldsymbol{\phi}\right)$$

$$\ln p(\mathbf{t} \mid \eta, s) = \sum_{n=1}^{N} \ln p\left(t_n \mid \eta, s\right) = \sum_{n=1}^{N} \left\{\ln g\left(\eta_n\right) + \frac{\eta_n t_n}{s}\right\} + \text{ const}$$

$$\nabla_{\mathbf{w}} \ln p(\mathbf{t} \mid \eta, s) = \sum_{n=1}^{N} \left\{ \frac{d}{d\eta_n} \ln g\left(\eta_n\right) + \frac{t_n}{s} \right\} \frac{d\eta_n}{dy_n} \frac{dy_n}{da_n} \nabla a_n$$

$$= \sum_{n=1}^{N} \frac{1}{s} \left\{t_n - y_n\right\} \psi'\left(y_n\right) f'\left(a_n\right) \boldsymbol{\phi}_n$$

$$f^{-1}(y) = \psi(y)$$

$$\nabla \ln E(\mathbf{w}) = \frac{1}{s} \sum_{n=1}^{N} \left\{y_n - t_n\right\} \phi_n$$

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