Xi'an Jiaotong-LiverpoolUniversity　　西交利物浦大學

PAPER CODE	EXAMINER	DEPARTMENT	TEL
CSE204		Computer Science and Software Engineering	

2ⁿᵈ SEMESTER 2019/20 RESIT EXAMINATIONS

Undergraduate – Year 3

COMPLEXITY OF ALGORITHMS

TIME ALLOWED:　2 Hours

INSTRUCTIONS TO CANDIDATES

1、　Total marks available are 100. This accounts for 100% of the final mark.

2、　Answer all questions. Relevant and clear steps should be included in the answers.

3、　The full marks for each question are indicated in round brackets at the end of the question.

4、　The answer should be written in English in the answer sheets. Answer sheets (with labeled pages and question numbers) should be scaned/snapshotted then convert to single Word/PDF file for submission.

5、　Students should use the format "Module Code-Student ID.filetype" to name their files before submitting to ICE. For example, "CSE204-18181881.pdf" or "CSE204-18191615.doc"

6、　Students should download the question paper, write down answers on their scripts, scan/snapshot their scripts for submission within the 2-hour exam.

Notes:

- To obtain full marks for each question, relevant and clear steps should be included in the answers.
- Partial marks may be awarded depending on the degree of completeness and clarity.

Question 1: Algorithm Analysis [24 marks]

a) Prove that $\log n + \log\log n$ is $\Theta(\log n)$.

[6 marks]

b) Give a tight asymptotic upper bound (Θ notation) on the solution to each of the following function. You need to justify your answers. (Hint: $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$)

$$T(n) = \begin{cases} 1, & if\ n = 1 \\ T(n-1) + n(n-2), & if\ n \geq 2 \end{cases}$$

[6 marks]

c) Find the complexity of the function *selectkth* used to find the kth smallest integer in an unordered array of n integers, writing down the running time function and express it using "Big-Oh" notation. (Hint: using mathematical induction.)

[6 marks]

```
int Select_k'th (int a[], int k, int n) {
    int i, j, mini, tmp;
    for (i = 0; i < k; i++) {
        mini = i;
        for (j = i+1; j < n; j++)
            if (a[j]<a[mini])
                mini = j;
        tmp = a[i];
        a[i] = a[mini];
        a[mini] = tmp;
    }
    return a[k-1];
}
```

d) Given a weighted digraph with $|V|$ vertices and $|E|$ edges, analyse the running time to find single-source shortest paths with Dijkstra's algorithm by using different data structures.

[6 marks]

Question 2: Trees and Heaps [18 marks]

a) Assuming we have a function that computes the size of a tree in O(1), describe an algorithm (using pseudo-code), which finds the kth element with a time complexity in terms of the tree's height. Justify your answer.

[6 marks]

FINDKTH(T, k)

    Input: a binary search tree T and an integer k
    Output: the kth element of T.
1.
2.
3.
.
…

b) Consider a binary heap. Print the keys as encountered in an in-order travel. Is the output sorted? Justify your answer. **[6 marks]**

c) Suppose we are given a sequence S of n elements, on which a total order relation is defined. Describe an efficient method for determining whether there are two equal elements in S. What is the running time of your algorithm? Justify your answer.
[6 marks]

Question 3: Graphs [18 marks]
a) Use the following graph for this problem. Where needed and not determined by the algorithm, assume that any algorithm begins at node A.
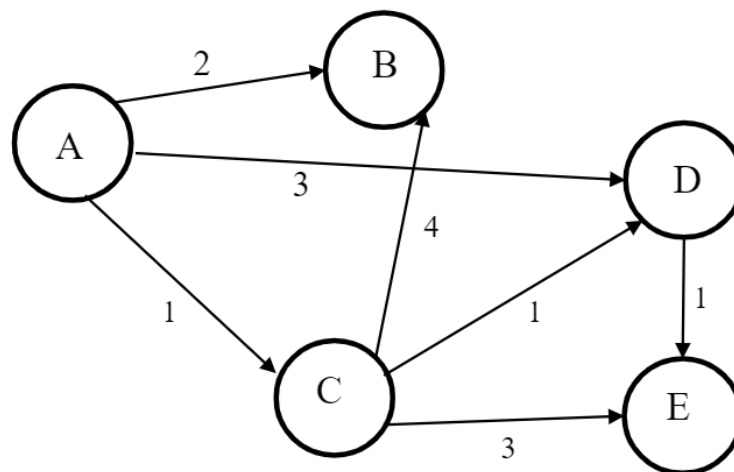
    i.Step through Dijkstra's Algorithm to calculate the single source shortest path from A to every other vertex. You only need to show your final table, but you should show your steps in the table below for partial credit. Show your steps by crossing through values that are replaced by a new value. Note that the next question asks you to recall what order vertices were declared known.
[7 marks]

    ii. What is the shortest (weighted) path from A to E? What is the length (weighted cost) of the shortest path?
[6 marks]

iii. Suppose the given graph is an undirected graph G, find the minimum spanning tree (MST) of G by using Prim's Algorithm. You only need to draw the MST. What is the total weight of edges belong to the MST?

[5marks]

Question 4: Number Theory and Cryptography [20marks]

a) In the RSA method, suppose that p = 5; q = 17, and e = 13.
   i. Find the private key d corresponding to these parameters.

   [6 marks]

   ii. Decrypt the ciphertext messages, C, below to find the original (plaintext) messages.
      1. C = 12 **[4 marks]**
      2. C = 9   **[4 marks]**

b) Compute $\phi(165)$ where ϕ is the Euler's totient function. How many numbers between 1 and 164 do not have a multiplicative inverse modulo 165? **[6 marks]**

Question 5: NP-Completeness [20marks]

a) Show that the 3-SAT problem can be reduced to vertex cover problem.
   Vertex Cover Problem:Given a Graph G(V,E), decide if there is k vertex such that everty edge is covered by one of them?

   [10 marks]

b) Show that 3-SAT belongs to the class NP and reduce the SAT problem to 3-SAT.

   [10 marks]

END OF EXAM PAPER