

Xi'an Jiaotong-Liverpool University

西交利物浦大學

Paper CODE	EXAMINER	DEPARTMENT	TEL
CSE 204		Computer Science and Software Engineering	

2nd SEMESTER 2018/19 RESIT EXAMINATION

Undergraduate – Year 3

Complexity of Algorithms

TIME ALLOWED: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This is a closed-book examination, which is to be written without books or notes.
2. Total marks available are 100. This accounts for 80% of the final mark.
3. The number in the column on the right indicates the marks for each question.
4. Answers should be written in the answer booklet(s) provided.
5. Only solutions in English are accepted.
6. All materials must be returned to the exam supervisor upon completion of the exam. Failure to do so will be deemed academic misconduct and will be dealt with accordingly.

Notes:

- To obtain full marks for each question, relevant and clear steps should be included in the answers.
- Partial marks may be awarded depending on the degree of completeness and clarity.

Question 1: Algorithm Analysis [30 marks]

- a) What is the asymptotic value of the expression $\sum_{i=1}^n \log_2 i$ as a function of n by using the big-Theta notation? [4 marks]
- b) Give a tight bound of the runtime complexity class for each of the following two code fragments in Big-Oh notation, in terms of the variable N . Justify your answers.

i. [4 marks]

```
int sum = 0;
for (int i = 1; i <= N - 5; i++) {
    for (int j = 1; j <= N - 5; j = j * 2) {
        sum++;
    }
}
```

ii. [4 marks]

```
int sum = N;
for (int i = 0; i < 1000; i++) {
    for (int j = 1; j <= i; j++) {
        sum += N;
    }
    for (int j = 1; j <= i; j++) {
        sum += N;
    }
    for (int j = 1; j <= i; j++) {
        sum += N;
    }
}
```

- c) The worst-case running time $T(N)$ of Merge-Sort on an input sequence of size N can be characterized by the following recurrence equation, wherein $a > 0$ and $b > 0$ are constants:

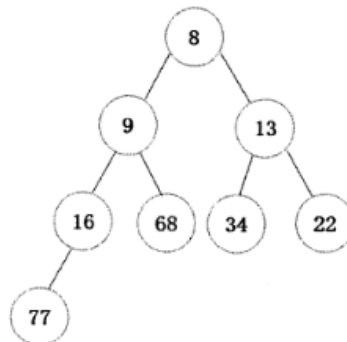
$$T(N) = \begin{cases} a & \text{if } N = 1 \\ 2T(\frac{N}{2}) + bN & \text{if } N > 1 \end{cases}$$

- i. Explain why the above recurrence equation can characterize the worst-case running time of Merge-Sort. [4 marks]
- ii. Solve the above recurrence equation and express the time complexity of Merge-Sort using Big-Oh notation. [4 marks]
- d) Given a binary search tree of height h , we wish to find out the value of its k^{th} element.
- i. Assuming we have a function that computes the size of a tree in $O(1)$, complete the following *findKth* function (using pseudo-code), which finds the k^{th} element with a time complexity in terms of the tree's height. [7 marks]
- ii. Compute the time complexity of *findKth* function by using Big-Oh notation. [3 marks]

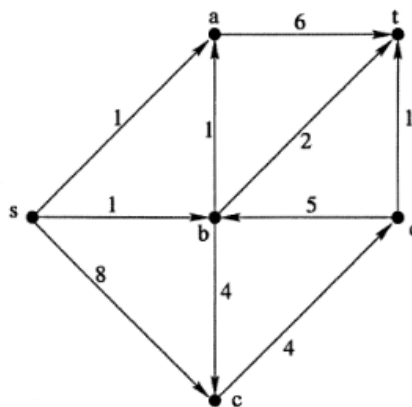
```
findKth(T; k)
Input: a binary search tree  $T$  and an integer  $k$ 
Output: the  $k^{\text{th}}$  element of  $T$ .value.
1 sLeft=size(T.left); //  $T$ .left denotes the left subtree of  $T$ .
2 _____
3 _____
...
```

Question 2: Tree and Graph [30 marks]

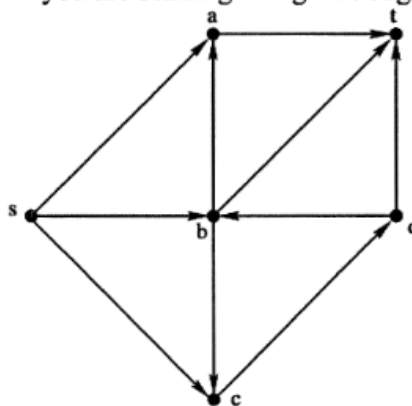
- a) Consider a binary heap. Print the keys as encountered in an inorder travel. Is the output sorted? Justify your answer. [4 marks]
- b) Delete two minimum numbers on the following min-heap. You do not need to show the array representation of the heap. You are only required to draw intermediate heaps and circle the final step. [8 marks]



- c) What is time complexity of the removal algorithm? [3 marks]
- d) Consider the following directed graph. Each edge is labelled with the capacity of that edge. For instance, the edge (s, c) has capacity 8.



- i. Find the maximum flow from s to t in this graph. Fill in the graph below with your flow: label each edge with the amount of flow you are sending along that edge. [4 marks]



- ii. What is the value of your flow? [4 marks]
- iii. Find the minimum-capacity cut between s and t in this graph. Show your answer by drawing a circle around the vertices in the above picture. [4 marks]
- iv. What is the capacity of the cut you identified in part iii? [3 marks]

Question 3: Number Theory and Cryptography [15 marks]

- a) Evaluate $27^{103} \bmod 143$. You may use the binary representation of 103. [3 marks]
- b) Consider a cryptosystem wherein $n=35$ and $e=5$.
- i. Verify that the pair (n, e) is a valid public key for an RSA cryptosystem. [3 marks]
- ii. Calculate the associated private key d . [4 marks]
- iii. Bob chooses an integer between 0 and 34, then encrypts it and sends the number 26 to Alice. Can you help Alice finding out the original integer chosen by Bob? Justify your answer. [5 marks]

Question 4: NP-Hardness [25 marks]

- a) State the definitions of 3-SAT. [5 marks]
- b) Show that the 3-SAT problem can be reduced to Vertex Cover problem.
Vertex Cover problem: Given a Graph $G(V,E)$, decide if there is k vertex such that every edge is covered by one of them? [5 marks]
- c) Deduce that Vertex Cover is NP-Complete. [5 marks]
- d) If we could solve an NP-complete problem in polynomial time, would all other problems in NP necessarily be solvable in polynomial time? Briefly justify your answer. [5 marks]
- e) If we could solve an NP-complete problem in time $O(n^{2019})$, would all other problems in NP necessarily be solvable in time $O(n^{2019})$? Briefly justify your answer. [5 marks]

END OF EXAM PAPER