Aufgabe 10-4.

a) $\tau = (\texttt{int} \xrightarrow{\{\texttt{write}(\ell)\,|\,\ell\in\Pi\}} \texttt{int})$ und $\varphi = \emptyset$.

b) $\tau = (\texttt{int} \xrightarrow{\psi} \tau) \xrightarrow{\psi\cup\{\texttt{read}(1),\texttt{write}(1)\}} \tau$ und $\varphi = \{\texttt{new}(1)\}$ für einen beliebigen Typ τ und eine beliebige Effektannotation ψ.

Herleitung für Aufgabenteil b):

$$\text{(Let)}\;\dfrac{\text{(Ref)}\;\dfrac{\text{(Con)}\;\dfrac{}{\emptyset \vdash 0\colon \texttt{int}\ \&\ \varnothing}}{\emptyset \vdash \texttt{ref}_1\ 0\colon \texttt{ref}_{\{1\}}\ \texttt{int}\ \&\ \{\texttt{new}(1)\}} \qquad (1)}{\emptyset \vdash \texttt{let x} = \texttt{ref}_1\ 0\ \texttt{in fun f} \Rightarrow (\texttt{f}\ (\texttt{x:=!x}+1))\ :\ (\texttt{int} \xrightarrow{\psi} \tau) \xrightarrow{\psi\cup\{\texttt{read}(1),\texttt{write}(1)\}} \tau\ \&\ \{\texttt{new}(1)\}}$$

Hierbei ist Γ eine Abkürzung für $[\texttt{x} \mapsto \texttt{ref}_{\{1\}}\ \texttt{int},\ \texttt{f} \mapsto (\texttt{int} \xrightarrow{\psi} \tau)]$ und die Teilherleitung (1) ist:

$$\text{(Fn)}\;\dfrac{\text{(App)}\;\dfrac{\text{(Var)}\;\dfrac{}{\Gamma \vdash \texttt{f}\colon (\texttt{int} \xrightarrow{\psi} \tau)\ \&\ \varnothing} \qquad \text{(Ass)}\;\dfrac{\text{(Sub)}\;\dfrac{\text{(Var)}\;\dfrac{}{\Gamma \vdash \texttt{x}\colon \texttt{ref}_{\{1\}}\ \texttt{int}\ \&\ \varnothing}}{\Gamma \vdash \texttt{x}\colon \texttt{ref}_{\{1\}}\ \texttt{int}\ \&\ \{\texttt{read}(1)\}} \qquad \text{(Op)}\;\dfrac{\text{(Deref)}\;\dfrac{\text{(Var)}\;\dfrac{}{\Gamma \vdash \texttt{x}\ :\ \texttt{ref}_{\{1\}}\ \texttt{int}\ \&\ \varnothing}}{\Gamma \vdash \texttt{!x}\ :\ \texttt{int}\ \&\ \{\texttt{read}(1)\}} \qquad \text{(Sub)}\;\dfrac{\text{(Con)}\;\dfrac{}{\Gamma \vdash 1\ :\ \texttt{int}\ \&\ \varnothing}}{\Gamma \vdash 1\ :\ \texttt{int}\ \&\ \{\texttt{read}(1)\}}}{\Gamma \vdash \texttt{!x}+1\ :\ \texttt{int}\ \&\ \{\texttt{read}(1)\}}}{\Gamma \vdash \texttt{x:=!x}+1\ :\ \texttt{int}\ \&\ \{\texttt{read}(1),\texttt{write}(1)\}}}{\Gamma \vdash \texttt{f}\ (\texttt{x:=!x}+1)\ :\ \tau\ \&\ \psi \cup \{\texttt{read}(1),\texttt{write}(1)\}}}{[\texttt{x} \mapsto \texttt{ref}_{\{1\}}\ \texttt{int}] \vdash \texttt{fun f} \Rightarrow (\texttt{f}\ (\texttt{x:=!x}+1))\ :\ (\texttt{int} \xrightarrow{\psi} \tau) \xrightarrow{\psi\cup\{\texttt{read}(1),\texttt{write}(1)\}} \tau\ \&\ \varnothing}$$