Aufgabe 10-1.

Abkürzungen:

Γ steht für $[i \mapsto \forall\alpha.\,(\alpha \to \alpha) \to (\alpha \to \alpha)]$.

$I(\alpha)$ steht für $(\alpha \to \alpha) \to (\alpha \to \alpha)$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{Annahme}}{[x \mapsto \texttt{int}] \vdash e\colon (\alpha \to \alpha) \to (\alpha \to \alpha)}\,(\textsc{Gen})}{[x \mapsto \texttt{int}] \vdash e\colon \forall\alpha.\,(\alpha \to \alpha) \to (\alpha \to \alpha)} \quad (\textsc{App})\ \dfrac{(1) \qquad (\textsc{Const})\ \dfrac{}{\Gamma \vdash 0\colon \texttt{int}}}{\Gamma \vdash i\ i\ (\texttt{fun}\ x \Rightarrow x+1)\ 0\colon \texttt{int}}}{\emptyset \vdash \texttt{let}\ i = e\ \texttt{in}\ i\ i\ (\texttt{fun}\ x \Rightarrow x+1)\ 0\colon \texttt{int}}\,(\textsc{Let})$$

Die Herleitung (1) ist:

$$\dfrac{\dfrac{\dfrac{(\textsc{Var})\ \dfrac{}{\Gamma \vdash i\colon \forall\alpha.\,I(\alpha)}}{\Gamma \vdash i\colon I(\texttt{int}) \to I(\texttt{int})}\,(\textsc{Inst}) \qquad \dfrac{(\textsc{Var})\ \dfrac{}{\Gamma \vdash i\colon \forall\alpha.\,I(\alpha)}}{\Gamma \vdash i\colon I(\texttt{int})}\,(\textsc{Inst})}{\Gamma \vdash i\ i\colon (\texttt{int} \to \texttt{int}) \to \texttt{int} \to \texttt{int}}\,(\textsc{App}) \qquad \dfrac{\dfrac{\dfrac{}{\Gamma \vdash x\colon \texttt{int}} \quad \dfrac{}{\Gamma \vdash 1\colon \texttt{int}}}{\Gamma \vdash x+1\colon \texttt{int}}}{\Gamma \vdash \texttt{fun}\ x \Rightarrow x+1\colon \texttt{int} \to \texttt{int}}}{\Gamma \vdash i\ i\ (\texttt{fun}\ x \Rightarrow x+1)\colon \texttt{int} \to \texttt{int}}$$

Beachte:

$$\begin{aligned} I(\texttt{int}) \to I(\texttt{int}) &= ((\texttt{int} \to \texttt{int}) \to (\texttt{int} \to \texttt{int})) \to ((\texttt{int} \to \texttt{int}) \to (\texttt{int} \to \texttt{int})) \\ &= I(\texttt{int} \to \texttt{int}) \end{aligned}$$