# ACM TEMPLATE



UESTC\_Jungle

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8.3

#### I Datastructure

#### 1.1 Fenwick

```
/* Fenwick Tree (Binary Indexed Tree), by Abreto <m@abreto.net>. */
   #include <cstring>
 3
 4
   using namespace std;
 5
 6
    template <class T = int, int MAXN = 100001>
 7
   struct fenwick {
 8
      static inline int lowbit(int x) {
 9
        return (x&(-x));
10
      int N;
11
      T f[MAXN]; /* 1=based. */
12
13
      fenwick(void):N(MAXN) {
14
        init();
15
16
      fenwick(int n):N(n) {
17
        init();
18
      }
19
      void init(void) {
20
        memset(f,0,sizeof(f));
21
      void upd(int i, T dx) {
22
23
        while(i <= N) {</pre>
24
          f[i] += dx;
25
          i += lowbit(i);
        }
26
27
28
      T sum(int i) {
29
        T ret = 0;
30
        while(i) {
31
          ret += f[i];
32
          i -= lowbit(i);
33
34
        return ret;
35
36
   };
   /* Fenwick Tree (Binary Indexed Tree), by Abreto <m@abreto.net>. */
 2
 3
   #define MAXN 100001
 4
   #define LOWBIT(x)
                         ((x)&(-(x)))
 5
 6
   int N;
 7
   int fen[MAXN];
 8
 9
   void update(int i, int dx) {
10
      while(i <= N) {</pre>
        fen[i] += dx;
11
12
        i += LOWBIT(i);
13
14
15
16
   int sum(int i) {
      int s = 0;
17
18
      while(i > 0) {
19
        s += fen[i];
20
        i -= LOWBIT(i);
21
22
      return s;
23
```

#### 1.2 BST in pb\_ds

```
/* Red-Black tree via pb_ds. */
   #include<bits/stdc++.h>
 3
   #include<ext/pb_ds/assoc_container.hpp>
 4
   #include<ext/pb_ds/tree_policy.hpp>
 5
   using namespace __gnu_pbds;
   using namespace std;
 7
   template <typename T>
 8
   using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
       tree_order_statistics_node_update>;
 9
10
   int main() {
11
     ordered_set<int> s;
12
     s.insert(1);
13
     s.insert(3);
     cout << s.order_of_key(2) << endl; // the number of elements in the s less than 2
14
      cout << *s.find_by_order(0) << endl; // print the 0-th smallest number in s(0-based</pre>
15
16 | }
```

#### 1.3 Segment Tree

```
//* Segment tree (Interval tree, range tree), by Abreto <m@abreto.net>. */
 3
    template <int STMAX = 1000000>
 4
    struct segment_tree {
 5
      struct node_t {
 6
        static inline node_t merge(node_t n1, node_t n2) {
 7
          node_t ans;
 8
          ans.l = n1.l;
 9
          ans.r = n2.r;
          /* merge n1 and n2 to ans. */
10
11
          return ans;
        }
12
13
14
        /* Data field */
15
        int l,r;
16
      } nodes[(STMAX+1)<<2];</pre>
17
18
      struct lazy_t {
19
        int marked; /* Optional */
20
        /* lazy mark. */
21
22
        lazy_t(void) {
23
          clear();
24
25
        void clear(void) {
26
          marked=0;
27
      } marks[(STMAX+1)<<2];</pre>
28
29
30
      inline void maintain_leaf(int o, int idx) {
31
        nodes[o].l = nodes[o].r = idx;
32
        /* Operations to single elements ... */
33
34
      inline void maintain(int o) {
35
        nodes[o] = node_t::merge(nodes[o<<1], nodes[o<<1|1]);</pre>
36
37
38
      /* Usage: build(1,1,n); */
39
      void build(int o, int l, int r) \{ /* [l,r] */
```

```
40
         if( r <= l ) {
 41
           maintain_leaf(o, 1);
 42
         } else {
 43
           int mid = 1+r>>1;
 44
           build(o<<1, 1, mid);
           build(o<<1|1, mid+1, r);
 45
 46
           maintain(o);
 47
       }
 48
 49
 50
       /* Modify all elements in [l,r] */
 51
       void mark(lazy_t act, int o) {
 52
         /* do something .. */
         marks[o].marked = 1;
 53
 54
 55
 56
       /* Pass cached updates. */
 57
       void pushdown(int o) {
 58
         if( marks[o].marked ) {
 59
           mark(marks[o], o<<1);</pre>
 60
           mark(marks[o], o<<1|1);
 61
           marks[o].clear();
 62
 63
       }
 64
 65
       /* Do act on all elements in [L,R] */
 66
       void upd(int L, int R, lazy_t act, int o, int l, int r) {
 67
         if( L <= 1 && r <= R ) {
 68
           mark(act, o);
         } else if (L <= R) {</pre>
 69
           int mid = (l+r)>>1;
 70
 71
           pushdown(o);
           if( L <= mid ) upd(L, R, act, o<<1, l, mid);
 72
 73
           if( R > mid ) upd(L, R, act, o<<1|1, mid+1, r);
 74
           maintain(o);
 75
 76
       }
 77
 78
       node_t qry(int L, int R, int o, int l, int r) {
 79
         if(L \le 1 \& r \le R)
 80
           return nodes[o];
 81
         else if (L \ll R) {
 82
           int mid = (l+r)>>1;
 83
           pushdown(o);
 84
           if(R <= mid) return qry(L,R,o<<1,l,mid);</pre>
 85
           if(L > mid) return qry(L,R,o<<1|1,mid+1,r);
 86
           return node_t::merge(qry(L,R,o<<1,l,mid),qry(L,R,o<<1|1,mid+1,r));</pre>
 87
         }
 88
       }
 89
 90
       int N;
 91
 92
       segment_tree(void):N(STMAX) {}
93
       segment_tree(int n):N(n) {}
 94
       void build(int n) {
 95
         N = n;
 96
         build(1,1,N);
 97
       }
 98
       void update(int L, int R, lazy_t act) {
 99
         upd(L,R,act,1,1,N);
100
       node_t query(int L, int R) {
101
102
         return qry(L,R,1,1,N);
103
       }
```

```
104 | };
    |/* Segment tree (Interval tree, range tree), by Abreto <m@abreto.net>. */
  1
  3
    #define MAXN
                      1000001
  4
  5
     typedef struct {
       int l,r;
  6
       /* Data field */
  7
  8
    } node_t;
  9
 10
    node_t merge(node_t n1, node_t n2) {
 11
       node_t ans;
 12
       ans.l = n1.l;
 13
       ans.r = n2.r;
 14
       /* merge n1 and n2 to ans. */
 15
       return ans;
 16
    }
 17
     typedef struct {
 18
 19
       int marked; /* Optional */
 20
       /* lazy mark. */
 21
    } lazy_t;
 22
 23
    int A[MAXN];
 24
    node_t nodes[MAXN<<2];</pre>
 25
    lazy_t marks[MAXN<<2];</pre>
 26
 27
    void maintain_leaf(int o, int idx) {
 28
       nodes[o].l = nodes[o].r = idx;
 29
       /* Operations to single elements ... */
 30
 31
    void maintain(int o) {
       nodes[o] = merge(nodes[o << 1], nodes[o << 1|1]);
 32
 33
    }
 34
 35
     /* Usage: build(1,1,n); */
 36
    void build(int o, int l, int r) \{ /* [l,r] */
 37
       if( r <= l ) {
 38
         maintain_leaf(o, 1);
 39
       } else {
 40
         int mid = 1+r>>1;
 41
         build(o<<1, 1, mid);
         build(o<<1|1, mid+1, r);
 42
 43
         maintain(o);
 44
 45
       marks[o].marked = 0;
 46
 47
 48
    /* Modify all elements in [l,r] */
 49
    void mark(lazy_t act, int o) {
       /* do something .. */
 50
 51
       marks[o].marked = 1;
 52
 53
 54
    /* Pass cached updates. */
 55
    void pushdown(int o) {
 56
       if( marks[o].marked ) {
 57
         mark(marks[o], o<<1);</pre>
 58
         mark(marks[o], o<<1|1);
 59
         marks[o].marked = 0;
 60
       }
    }
 61
 62
```

```
/* **DISCARDED** */
63
   /* Set A[p]=v. Usage: modify(p, v, 1, 1, n);
65
   void modify(int p, int v, int o, int l, int r)
66
        if(r-1 < 2)
67
68
69
            maintain_leaf(o, v);
70
        } else {
71
            int mid = (l+r)/2;
72
            pushdown(o);
73
            if( p \le mid ) modify(p, v, o*2, l, mid);
74
            else modify(p, v, o*2+1, mid, r);
75
            maintain(o);
76
   }*/
77
78
79
   /* Do act on all elements in [L,R] */
80
   void update(int L, int R, lazy_t act, int o, int l, int r) {
81
     if( L <= 1 && r <= R ) {
82
        mark(act, o);
83
     } else if (L <= R) {</pre>
84
        int mid = (l+r)>>1;
85
        pushdown(o);
86
        if( L <= mid ) update(L, R, act, o<<1, l, mid);</pre>
        if( R > mid ) update(L, R, act, o<<1|1, mid+1, r);
87
88
        maintain(o);
89
     }
90 | }
```

#### 1.4 Sparse Table

```
/* RMQ with Sparse Table, by Abreto <m@abreto.net>. */
 1
 2
 3
   int min(int a, int b) {
 4
     return (a<b)?a:b;
 5
 6
 7
   #define MAXN
                    100001
 8
   #define MAXLOG
 9
10
   int N;
11
   int A[MAXN];
                    /* indexed from 0. */
12
   int st[MAXN][MAXLOG];
13
14
   void st_init() {
15
     int i = 0, j = 0, t = 0;
16
     for(i = 0; i < N; ++i) st[i][0] = A[i];
17
     for(j = 1; (t=(1<<j)) <= N; ++j)
18
        for(i = 0; (i+t-1) < N; ++i)
19
          st[i][j] = min(st[i][j-1], st[i+(t>>1)][j-1]);
20
     /* st(i,j) = min(st(i,j-1), st(i+2^(j-1),j-1)). */
21
22
   int st_query(int l, int r) {
23
24
     int k = 0;
25
     while((1 << (k+1)) <= (r-l+1)) k++;
      return min(st[l][k], st[r-(1 << k)+1][k]);
26
27 | }
```

#### 1.5 Treap

```
|#include <bits/stdc++.h>
 1
 2
 3
   using namespace std;
 5
   #define MAXN (2560000)
 6
 7
   int __treap_mem[MAXN];
 8
   void init_treap_mem(void) {
9
      for( int i = 1; i < MAXN; i++)
10
       \_treap\_mem[i-1] = i;
11
12
   int alloc_address(void) {
13
     int ret = __treap_mem[0];
14
      __treap_mem[0] = __treap_mem[ret];
15
      return ret;
16
17
   void free_address(int p) {
18
     _{\text{treap\_mem}[p]} = _{\text{treap\_mem}[0]};
19
      \_treap_mem[0] = p;
20
21
22
   typedef int key_t;
23
   typedef int val_t;
24
   struct treap {
25
     key_t x;
26
     val_t v;
27
               /* random priority */
     int r;
28
     int eq, s; /* number of equal ones, size of subtree (include root itself) */
     treap *fa; /* point to its father */
29
     treap *ch[2]; /* 0 for left child, 1 for right child. */
30
31
32
     treap(void);
33
     inline void maintain(void); /* update s */
34
     inline void set_child(int d, treap *child);
35
     inline int which(void); /* determine which child this is of its father */
36
     inline int cmp(key_t ox); /* determine which child to insert ox */
37
     treap *rotate(void); /* rotate this to its father, return this */
38
   } treap_nodes[MAXN];
39
40
   treap *new_treap(key_t x, val_t v, treap *f) {
41
     treap *ret = treap_nodes + alloc_address();
42
      ret->x = x;
43
      ret -> v = v;
44
      ret->eq = ret->s = 1;
45
      ret->fa=f;
46
      ret->ch[0] = ret->ch[1] = NULL;
47
48
   void free_treap(treap *p) {
49
      free_address( p - treap_nodes );
50
51
   void treap_clear(void) {
52
     init_treap_mem();
53
54
55
   treap::treap(void) {
56
     r = rand();
57
     eq = s = 0;
58
     fa = ch[0] = ch[1] = NULL;
59
60
   inline void treap::maintain(void) {
61
      s = eq;
     for( int i = 0 ; i < 2 ; i++ )
62
63
       if( ch[i] )
64
          s += ch[i]->s;
```

```
65
 66
    inline void treap::set_child(int d, treap *child) {
 67
       ch[d] = child;
 68
       maintain();
 69
       if( child ) child->fa = this;
 70
 71
    inline int treap::which(void) {
 72
       if( NULL == fa ) return -1; /* this is not a child */
 73
       else return ( fa->ch[1] == this );
 74
 75
    inline int treap::cmp(key_t ox) {
 76
       if( ox == x ) return -1; /* equal */
 77
       else return ( ox > x ); /* left less, right more */
 78
 79
     treap *treap::rotate(void) {
 80
       if ( NULL == fa ) return this; /* no father, already global root. */
       int d = which();
 81
 82
       fa->set_child(d, ch[d^1]);
 83
       set_child(d^1, fa);
 84
       return this;
 85
 86
 87
     // --- deprecated |
 88
    typedef int T;
 89
 90
    struct node {
91
       T v; /* value of this node */
 92
       int r; /* random priority */
 93
       int eq; /* the number of equal things */
 94
       int s; /* the size of subtree rooted at this */
       node *ch[2]; /* 0 for left child, 1 for right child. */
 95
       node(void) {
 96
 97
         r = rand();
 98
         ch[0] = ch[1] = NULL;
 99
100
      /* return where to insert x */
101
       int cmp(T x) {
102
         if(v == x) return -1;
103
         else return (x < v) ? 0 : 1;
104
105
       /* return 1 if this node is prior to other */
106
       int pri(node *o) {
107
         return (r > (o->r));
108
109
       /* maintain the s field */
110
       void maintain(void) {
111
         s = eq;
112
         if(NULL != ch[0]) s += ch[0]->s;
113
         if(NULL != ch[1]) s += ch[1]->s;
      }
114
115
116
117
     /* move o to ch[d] of o->ch[d^1] */
118
    void rotate(node *&o, int d) {
119
       node *k = o \rightarrow ch[d^1];
120
       o\rightarrow ch[d^1] = k\rightarrow ch[d];
121
       o->maintain();
122
       k\rightarrow ch[d] = o;
123
       k->maintain();
124
       o = k;
125
```

#### 1.6 Leftist Heap

```
|/* HDU 1512 Monkey King (左偏树模板题) */
   #include<iostream>
3
   #include<cstdio>
4
   using namespace std;
5
   const int maxn = 100000+5;
6
7
   int n, m;
8
9
   struct Heap {
10
     int l,r,fa,val,dis;
11
   } t[maxn];
12
13
14
   int finds(int x) {
15
     return t[x].fa == -1? x:t[x].fa = finds(t[x].fa);
16
17
   int merge(int x, int y) {
18
     if(x == 0) return y;
                            //如果为0的话,就说明是空子树,根节点当然就是另一节点了
19
20
     if(y == 0) return x;
21
     if(t[y].val>t[x].val) swap(x,y); //始终往右子树进行插入
22
     t[x].r = merge(t[x].r,y);
23
     t[t[x].r].fa = x;
24
     if(t[t[x].l].dis < t[t[x].r].dis) swap(t[x].l,t[x].r); //是否需要左右子树的对换,
        这样是为了右子树尽量短
25
     if(t[x].r == 0) t[x].dis = 0;
                                   //距离的重新分配
26
     else t[x].dis = t[t[x].r].dis + 1;
27
     return x;
   }
28
29
30
   int pop(int &root) {
31
     int l = t[root].l;
32
     int r = t[root].r;
33
     t[root].l = t[root].r = t[root].dis = 0;
34
     t[root].fa = -1;
35
     t[1].fa = t[r].fa = -1; //删除root根节点
36
                             //这样一来相当于分裂成了两棵子树,重新进行合并,最后返回值
     return merge(l,r);
        为合并后的根节点
37
   }
38
39
   int push(int x, int y) {
40
     return merge(x,y);
41
   }
42
43
   int main() {
     //freopen("in.txt","r",stdin);
44
     while(~scanf("%d",&n)) {
45
       for(int i=1; i<=n; i++) {
46
47
         t[i].l=t[i].r=t[i].dis=0;
48
         t[i].fa=-1;
49
         scanf("%d",&t[i].val);
50
       }
       scanf("%d",&m);
51
52
       while(m—) {
53
         int a,b;
         scanf("%d%d",&a,&b);
54
55
         int x=finds(a);
56
         int y=finds(b);
57
         if(x!=y) {
58
           t[x].val/=2;
59
           int xx = push(pop(x),x);
```

```
60
            t[y].val/=2;
61
            int yy = push(pop(y),y);
            printf("%d\n",t[merge(xx,yy)].val);
62
63
          } else puts("-1");
64
65
66
      return 0;
67
   1.7 Splay
   /* splay, by Abreto<m@abreto.net>. */
 2
 3
   #ifndef NULL
 4
   #define NULL 0
 5
   #endif
 6
 7
   struct node {
 8
      node *f, *ch[2];
 9
      int sz;
      node(node *fa = NULL, node *lc = NULL, node *rc = NULL) {
10
        f = fa;
11
        ch[0] = lc;
12
        ch[1] = rc;
13
14
        maintain();
15
16
      inline int szof(const int d) const {
17
        return ch[d] ? ch[d]->sz : 0;
18
19
      inline void maintain(void) {
20
        sz = szof(0) + szof(1) + 1;
21
22
      inline int which(void) {
23
        if (NULL == f) return 0;
24
        return (f\rightarrow ch[1] == this); /* f[which()] == this */
25
26
      inline node *setf(node *fa, int d = 0) {
27
        f = fa;
        if (f) {
28
          f\rightarrow ch[d] = this;
29
30
          f->maintain();
        }
31
32
        return f;
33
34
      inline node *setc(node *son, int d = 0) {
35
        ch[d] = son;
36
        if (son) son—>f = this;
37
        maintain();
38
        return this;
39
40
      /* rotate this to its fater, return this. */
41
      inline node *rotate(void) {
        if (f != NULL) {
42
          node *ff = f \rightarrow f;
43
44
          int d = which(), fd = f->which();
          setc(f->setc(ch[d \land 1], d), d \land 1);
45
46
          setf(ff, fd);
        }
47
48
        return this;
49
50
      /* splay this to child of target */
51
      inline node *splay(node * const target = NULL) {
```

```
52
        while (f != target) {
53
          if (target != f \rightarrow f) {
54
            ((which() == f->which()) ? f : this )->rotate();
55
56
          rotate();
57
58
        return this;
59
60
      /* 0-based rank */
61
      inline node *get_k_th(unsigned k) {
62
        node *p = this;
63
        int rank;
        while (k != (rank = (p->szof(0))))  {
64
          if (k < rank) {
65
66
            p = p - sh[0];
          } else {
67
68
            k = (rank + 1);
69
            p = p - > ch[1];
70
71
72
        return p->splay(f);
73
   };
74
 1
   //* HDU 3487 - Play with Chain, by Abreto<m@abreto.net>. */
   #include <bits/stdc++.h>
 3
 4
   using namespace std;
 5
   #define MAXN
                     300300
 6
 7
 8
   int n, m;
9
10
   #define LC(p)
                     ch[p][0]
11
   #define RC(p)
                     ch[p][1]
   #define TARGET(p) LC(RC(p))
12
13
14
   int nodes;
15
    int val[MAXN], ch[MAXN][2], fa[MAXN], sz[MAXN];
16
   int rev[MAXN];
17
18
   inline int new_node(int v, int f) {
19
      int p = (++nodes);
20
      val[p] = v;
      fa[p] = f;
21
22
      ch[p][0] = ch[p][1] = rev[p] = 0;
23
      sz[p] = 1;
24
      return p;
25
   inline void maintain(int p) {
26
27
      if (p) {
28
        sz[p] = sz[LC(p)] + sz[RC(p)] + 1;
29
30
31
   inline void make_child(int f, int d, int p) { /* make p the d-th ch of f */
32
      ch[f][d] = p;
33
      if(p) fa[p] = f;
34
35
   inline void myrev(int p) {
36
      if (p) {
        rev[p] ^= 1;
37
38
        swap(LC(p), RC(p));
39
40 }
```

```
inline void pushdown(int p) {
 41
 42
       if(p && rev[p]) {
 43
         if(LC(p)) myrev(LC(p));
 44
         if(RC(p)) myrev(RC(p));
 45
         rev[p] = 0;
 46
 47
 48
     int build(int f = 0, int l = 0, int r = n+1) {
 49
       if(r < l) return 0;
 50
       if(l == r) return new_node(l, f);
 51
       int mid = 1+r>>1;
 52
       int p = new_node(mid, f);
 53
       LC(p) = build(p, l, mid-1);
 54
       RC(p) = build(p, mid+1, r);
 55
       maintain(p);
 56
       return p;
 57
    inline int which(int p) { /* return 1 if p is a right child or 0 if p is a left
 58
        child. */
 59
       return (RC(fa[p]) == p);
 60
    inline int rotate(int p) { /* rotate p to its father. [!] make sure p is not global
 61
 62
       int f = fa[p], ff = fa[f];
 63
       if(0 == f) return p; /* p is global root */
 64
       pushdown(f);
 65
       pushdown(p);
 66
       int d = which(p), df = which(f);
 67
       make\_child(f, d, ch[p][d^1]);
       make\_child(p, d^1, f);
 68
 69
       maintain(f);
 70
       maintain(p);
 71
       fa[p] = ff;
       if(ff) ch[ff][df] = p;
 72
 73
       return p;
 74
 75
     inline int splay(int p, int fr) \{ /* \text{ splay p to the son of fr, return p. } */
 76
       pushdown(p);
 77
       while(fa[p] != fr) {
 78
         int f = fa[p], dp = which(p);
 79
         if(fa[f] == fr) {
 80
           return rotate(p);
 81
         } else {
 82
           int df = which(f);
 83
           if(dp == df) {
 84
             rotate(f);
 85
           } else {
 86
             rotate(p);
 87
 88
           rotate(p);
 89
         }
 90
 91
       return p;
 92
 93
     inline int get_k_th(int root, int k) {
 94
       int p = root;
 95
       int rank;
 96
       while(k != (rank = (sz[LC(p)] + 1))) {
 97
         pushdown(p);
 98
         if(k < rank) p = LC(p);
 99
         else {
100
           k = rank;
101
           p = RC(p);
         }
102
```

```
103
      return splay(p, fa[root]);
104
105
106
    inline int merge(int left, int right) {
107
      pushdown(left);
108
      if(RC(left)) left = get_k_th(left, sz[left]);
109
      RC(left) = right;
110
      maintain(left);
111
      fa[right] = left;
112
       return left;
113
    inline int split(int root, int d) { /* split ch[root][d], return the root of splited
114
        out. */
115
      pushdown(root);
116
      int child = ch[root][d];
117
       ch[root][d] = 0;
118
      maintain(root);
119
      fa[child] = 0;
120
       return child;
121
122
    inline int concat(int root, int d, int p) { /* make p be ch[root][d], return root */
123
      pushdown(root);
124
      ch[root][d] = p;
125
      fa[p] = root;
126
      maintain(root);
127
      return root;
128
129
130
    void myclear(void) {
131
      nodes = 0;
132
133
134
     int ans[MAXN];
135
    void inorder(int p, int &pos) {
136
      if(0 == p) return;
137
      pushdown(p);
138
      inorder(LC(p), pos);
139
      if( (0 < val[p]) \&\& (val[p] < n+1) ) ans[pos++] = val[p];
140
       inorder(RC(p), pos);
141
142
143
    void handle() {
144
      int i;
145
      int root;
146
      myclear();
147
       root = build(0);
148
      while(m--) {
149
         char command[8];
150
         int a, b, c;
151
         int tar;
         scanf("%s%d%d", command, &a, &b);
152
         if('C' == command[0]) {
153
           scanf("%d", &c);
154
155
           root = get_k_th(root, a);
           RC(root) = get_k_t(RC(root), b-a+2);
156
157
           tar = split(RC(root), 0);
158
           maintain(root);
159
           root = get_k_th(root, c+1);
160
           RC(root) = get_k_t(RC(root), 1);
161
           RC(root) = concat(RC(root), 0, tar);
162
           maintain(root);
163
         } else {
164
           root = get_k_th(root, a);
165
           RC(root) = get_k_t(RC(root), b-a+2);
```

```
166
           myrev(TARGET(root));
         }
167
168
169
       int pos = 0;
170
       inorder(root, pos);
       for(i = 0; i < n; i++) printf("%s%d", i ? "_{\bot}":"", ans[i]);
171
172
       puts("");
173
174
175
     int main(void) {
       while( scanf("\%d\%d\%d", &n, &m) && (n > 0) && (m > 0) )
176
177
         handle();
178
       return 0;
179 |}
```

#### 1.8 Persistent Segment Tree

- I. 首先, 给你一颗值为横坐标的线段树, 每个节点上存着该值出现了多少次, 这样的一颗线段树 你会求区间 k 大值吧. 二分即可.
- 2. 然后, 假设区间是数组 arr[n], 区间长度是 n, 那么给你 n 颗线段树, 第 i 颗线段树是第 i-1 颗线段树插入 arr[i] 得到.
- 3. 如果你有了这 n 颗线段树, 想求区间 [l,r] 中的第 k 大值, 那么你需要在第 r 颗和第 l-1 颗线段 树的差线段树上作二分, 就可以求得区间第 k 大值.
- 4. 差线段树很好理解, 比如你有一个部分和数组 sum, sum[r] sum[l-1] 就是部分和的差, 代表区间 [l,r] 的和, 差线段树同理.
- 5. 现在,可持久化线段树出现为你解决最后一个问题,空间问题. 内存很小,不能够存下 n 颗线段树. 但是,第 2 条中提到,由于第 i 颗线段是是第 i-1 颗线段是插入仅一个值得到的,两颗线段树的区别不大,仅有  $\log(n)$  个节点发生了改变,我们仅仅需要记录这  $\log(n)$  的数据就可以记录这个增量,这就是可持久化线段树.

### 2 Dynamic Programming

#### **2.1** LIS $O(n \log n)$

```
1
2
   int top = 0;
3
   for( int i=1; i<=n; i++ ) {
4
     if( ap[i] > dp[top] ) { // 如果大于 "模拟栈" 的栈顶元素直接 入栈 长度加 1
5
       dp[top] = ap[i];
6
7
       continue;
8
     int m = ap[i];
9
     // lower_bound 前闭后开 返回不小于 m 的最小值的位置
10
     pos = lower_bound(dp,dp+top,m)-dp; // 注意减去dp
11
12
     if(dp[pos] > ap[i])
13
       dp[pos] = ap[i];
14
```

#### **2.2** LCS $O(n \log n)$

总的来说,就是把 LCS 转化成 LIS,然后用 LIS 的  $O(N \log N)$  算法来求解。

实现如下: (引用)

假设有两个序列  $s_1[1...6] = abcadc, s_2[1...7] = cabedab.$ 

记录  $s_1$  中每个元素在  $s_2$  中出现的位置, 再将位置按降序排列, 则上面的例子可表示为:  $loc(a) = \{6,2\}, loc(b) = \{7,3\}, loc(c) = \{1\}, loc(d) = \{5\}.$  (倒着扫一遍  $s_2$  即可把位置扔进 vector). 将  $s_1$  中每个元素的位置按  $s_1$  中元素的顺序排列成一个序列  $s_3 = \{6,2,7,3,1,6,2,5,1\}$ . 在对  $s_3$  求 LIS 得到的值即为求 LCS 的答案。

#### 2.3 Improved by quadrilateral inequality

```
1
   /*
 2
    * 四边形不等式
 3
 4
    * 如果 dp(i,j) 满足 dp(i,j)<=dp(i,j+1)<=dp(i+1,j+1)
 5
    * 那么决策 s(i,j) 满足 s(i,j)<=s(i,j+1)<=s(i+1,j+1)
 6
    * 可以变形为:
 7
            s(i-1,j) <= s(i,j) <= s(i,j+1) // dp方向: i增j减
    *
 8
       或
    *
 9
            s(i,j-1) \ll s(i,j) \ll s(i+1,j) // dp方向: 区间长度L增
    *
10
    */
11
   #include <bits/stdc++.h>
12
13
   using namespace std;
14
15
   #define MAXN
16
   #define inf
                    (0x3fffffff)
17
18
   int n, m;
19
   int v[MAXN];
20
   int s[MAXN];
   int w[MAXN][MAXN];
21
22
   int dp[MAXN][MAXN];
23
   int c[MAXN][MAXN];
24
25
   int wa(void) {
26
     int i, j, k;
27
     for(i = 1; i \le n; ++i) {
        scanf("%d", v+i);
28
        s[i] = v[i] + s[i-1];
29
30
31
     for(i = 1; i <= n; ++i) {
32
       w[i][i] = 0;
33
       for(j = i+1; j \le n; ++j)
         w[i][j] = w[i][j-1] + v[j] * (s[j-1] - s[i-1]);
34
35
36
     /* doing dp */
37
     for(i = 1; i \le n; ++i) {
38
       dp[i][0] = w[1][i];
39
       c[i][0] = 1;
40
        c[i][i] = i-1;
41
        for(j = i-1; j > 0; j---) {
42
          dp[i][j] = inf;
43
         for(k = c[i-1][j]; k \le c[i][j+1]; ++k)
44
            if(dp[k][j-1]+w[k+1][i] \le dp[i][j]) {
45
              dp[i][j] = dp[k][j-1] + w[k+1][i];
46
              c[i][j] = k;
            }
47
48
       }
49
50
     /* dp done */
51
     return dp[n][m];
52
53
54
   int main(void) {
55
     while(EOF != scanf("%d%d", &n, &m) && n && m) {
```

#### 2.4 Improved by Slope

```
/* type 1: */
 2
   /* bzoj 1010 */
 3
   #include <bits/stdc++.h>
   using namespace std;
   typedef long double 11;
 6
 7
   #define MAXN
                     50050
 8
   #define eps
                     (1e-8)
 9
10
   int N;
   11 L;
11
12
   11 S[MAXN];
13
   11 f[MAXN];
   11 dp[MAXN];
14
15
   inline ll k(int j) {
16
      return (-2.0) * (f[j] + L);
17
18
19
   inline ll b(int j) {
20
      return dp[j] + f[j]*f[j] + 2ll*f[j]*L;
21
22
   inline ll g(int j, int i) {
23
      return k(j) * f[i] + b(j);
24
25
26
   /* check if l1 & l3 <= l2 */
   inline int check(int l1, int l2, int l3) {
27
28
      /*ll\ left = b(13)*k(11)+b(11)*k(12)+b(12)*k(13);
29
      ll right = b(l1)*k(l3)+b(l3)*k(l2)+b(l2)*k(l1);*/
30
      ll\ left = b(l3)*k(l1)-b(l1)*k(l3)
31
      ll right = k(l2)*(b(l3)-b(l1))+b(l2)*(k(l1)-k(l3));
32
      return (left <= right);
33
34
35
   int Q[MAXN], ql, qr;
36
37
    int main(void) {
38
      int i;
      scanf("%d%Lf", &N, &L);
39
40
      L += 1.0;
      for(i = 1; i \le N; ++i) {
41
        scanf("%Lf", S+i);
42
43
        S[i] += S[i-1];
44
        f[i] = S[i] + (double)i;
45
46
      Q[qr++] = 0;
      for(i = 1; i <= N; ++i) {
/* <!-- STARED */
47
48
49
        for(; ql+1 < qr && g(Q[ql],i) >= g(Q[ql+1],i); ql++);
        dp[i] = g(Q[ql], i) + f[i]*f[i] + L*L; //printf("%d: %lld,%lld\n", i, dp[i], dp[i]
50
           ]-f[i]*f[i]);
51
        for(; ql+1 < qr \&\& check(Q[qr-2], Q[qr-1], i); qr--);
52
        Q[qr++] = i;
53
        /* --> */
      }
54
```

```
55 | printf("%lld\n", (long long int)round(dp[N]));
56 | return 0;
57 |}
```

#### 2.5 Steiner Tree

令 f[i][sta] 表示 i 号节点,与其他节点的连通性为 sta 时的最小代价,这里 sta 是一个二进制数,在它二进制下的每一位中,0 表示不连通,1 表示联通 状态转移:

• 由子集转移而来

$$f[i][sta] = \min_{s \subseteq sta} \{f[i][s] + f[i][sta \setminus s] - val[i]\}$$

• 由不含该节点的状态转移而来

```
f[i][j] = \min\{f[k][j] + val[i]\}
```

```
流程:
   枚举状态集S
 2
   {
 3
         枚举S的子集s
 4
 5
             更新f[S][1~n]
 6
 7
         将 f[S][x]<inf 的x入队
 8
         spfa(S)
 9
   代码:
   |int f[1<<M][N];
   queue<int> q;
 3
   bool in[N];
 4
 5
   void spfa(int S) {
 6
     while (!q.empty()) {
 7
       int now=q.front();
 8
       q.pop();
 9
       in[now]=0;
10
        for (int i=st[now]; i; i=way[i].nxt) {
11
          int y=way[i].y;
          if (f[S][y]>f[S][now]+val[y]) {
12
13
            f[S][y]=f[S][now]+val[y];
14
            if (!in[y]) q.push(y),in[y]=1;
15
16
       }
17
     }
18
19
20
   void work() {
21
      int cnt=0;
22
     memset(f,0x7f,sizeof(f));
23
24
     for (int i=1; i<=n; i++)
25
       if (!val[i]) f[1<<cnt][i]=0,cnt++;
     for (int S=1; S<(1<<cnt); S++) {
26
27
       for (s=(S-1)\&S; s; s=(s-1)\&S)
          for (int i=1; i<=n; i++)
28
29
            f[S][i]=min(f[S][i],f[s][i]+f[S^s][i]-val[i]);
30
       for (int i=1; i<=n; i++)
31
          if (f[S][i]<INF&&!in[i])
```

```
32
           q.push(i),in[i]=1;
33
       spfa(S);
34
35
36
     int ans=INF;
37
     for (int i=1; i<=n; i++) ans=min(ans,f[(1<<cnt)-1][i]);
38
     printf("%d\n",ans);
39
40
   //
41
   // 作者: Coco_T_
42
   // 来源: CSDN
43
   |// 原文: https://blog.csdn.net/wu_tongtong/article/details/78992913
44 // 版权声明:本文为博主原创文章,转载请附上博文链接!
       Geometry
   3
   3.1 2D
   3.1.1 Point
   /* 2D Point Class, by Abreto<m@abreto.net> */
 2
   #include <cmath>
 3
 4
   /**
 5
    * Define ABG2d_USE_LL if you want to use long long int for cordnates.
 6
    */
 7
 8
   namespace ab_geometry_2d {
 9
10
   using namespace std;
11
12
   typedef double ab_float;
13
14
   const ab_float pi = acos(-1.);
15
   #ifdef ABG2d_USE_LL
16
   typedef long long int T;
17
18
   #else
19
   typedef ab_float T;
20
   const ab_float eps = 1e-8;
21
   #endif
22
23
   inline T myabs(T x) {
24
     if(x < 0) return (-x);
25
     return x;
26
   }
27
28
   inline int sgn(T x) {
     /* no difference'' in fact */
29
30
   #ifdef ABG2d_USE_LL
31
     if (0 == x) return 0;
32
   #else
33
     if (myabs(x) < eps) return 0;
34
   #endif
35
     return (x > 0) ? 1 : -1;
36
37
38
   inline T sqr(T x) {
39
     return (x * x);
40
41
42 | struct point {
```

```
43
     Tx,y;
44
     point(void):x(T()),y(T()) {}
45
     point(T xx, T yy):x(xx),y(yy) {}
46
      inline T norm2(void) {
47
        return sqr(x) + sqr(y);
48
49
     inline ab_float norm(void) {
50
        return sqrt((ab_float)(norm2()));
51
52
     inline point rotate(const ab_float &cost, const ab_float &sint) {} // TODO:
53
     inline point operator—(void) const {
54
        return point(-x,-y);
55
     inline point operator+(const point& b) const {
56
57
        return point(x+b.x,y+b.y);
58
59
     inline point operator-(const point& b) const {
        return point(x-b.x,y-b.y);
60
61
62
     inline point operator->*(const point &b) const {
63
        return (b-(*this));
64
     inline T operator*(const point& b) const {
65
66
        return ((x)*(b.x))+((y)*(b.y)); /* inner product */
67
     inline T operator^(const point& b) const {
68
69
        return ((x)*(b.y))-((b.x)*(y)); /* outter product */
70
71
     inline point& operator+=(const point& b) {
72
        point tmp=(*this)+b;
73
        (*this)=tmp;
74
        return (*this);
75
     inline point& operator—=(const point& b) {
76
77
       point tmp=(*this)-b;
78
        (*this)=tmp;
79
        return (*this);
80
81
     inline bool operator==(const point& b) const {
82
        return (0==sgn(x-b.x))&(0==sgn(y-b.y));
83
84
     inline bool operator!=(const point& b) const {
85
        return !((*this)==b);
86
87
     inline point operator<<(const ab_float& theta) const {
88
       ab_float ct = cos(theta), st = sin(theta); /* rotate counter-clockwise in radian
89
        return point(ct*x - st*y, st*x + ct*y);
90
     }
91
   };
92
93
   typedef point vec;
94
95
96 | }
       // namespace ab_geometry_2d
   3.1.2 Circle
   Base
   /* 2D Circle Base Class, by Abreto<m@abreto.net>. */
 3
   /* requirement: point.cc */
```

```
|#include "point.cc"
 5
 6
   #include <utility>
 7
 8
   namespace ab_geometry_2d {
 9
10
   using namespace std;
11
12
    struct circle {
13
      point o;
14
      Tr;
15
      circle(void) : r(T()) {}
      circle(point center, T radius) : o(center), r(radius) {}
16
17
      inline ab_float arclen(ab_float theta) {
18
19
        return theta * r;
20
21
      inline ab_float circumference(void) {
22
        return 2. * pi * r;
23
24
      inline ab_float area(void) {
25
        return pi * r * r;
26
27
28
      /* bool contain(const circle &C, const bool including_touch = false) const
29
30
          T dis2 = (o \rightarrow *(C.o)).norm2();
31
          T raw_diff = r - C.r;
32
          if (-1 == sgn(raw\_diff)) return false;
33
          T dr2 = sqr(raw_diff);
          return (dis2 < dr2) || (including_touch && (dis2 == dr2));
34
35
      inline bool in(const circle &C, const bool including_touch = false) const
36
37
38
        return C.contain(*this, including_touch);
39
      } */
40
      enum relation_t {
41
        same = 0 \times 000000,
42
        contain = 0 \times 00001,
43
        intouch = 0 \times 00010,
44
        intersect = 0x00100,
45
        outtouch = 0x01000,
46
        separate = 0x10000.
47
        unknow_relation = 0xfffff
48
      relation_t with(const circle &C) const {
49
50
        T dis2 = (o \rightarrow *(C.o)).norm2();
51
        T dr2 = sqr(r - C.r), rs2 = sqr(r + C.r);
52
        if (0 == sgn(dis2) \&\& 0 == sgn(dr2)) return same;
        if (-1 == sqn(dis2 - dr2)) return contain;
53
        if (0 == sgn(dis2 - dr2)) return intouch;
54
        if (-1 == sgn(dr2 - dis2) \&\& -1 == sgn(dis2 - rs2)) return intersect;
55
        if (0 == sgn(dis2 - rs2)) return outtouch;
56
57
        if (-1 == sgn(rs2 - dis2)) return separate;
58
        return unknow_relation;
59
      }
60
61
      enum point_relation_t {
62
        in = 0x001,
63
        on = 0 \times 010,
64
        out = 0x100,
65
        unknow_point_relation = 0xfff
66
      point_relation_t with(const point &P) const {
67
```

```
68
        T dis2 = (o\rightarrow *P).norm2();
69
        T r2 = sqr(r);
70
        int type = sgn(dis2 - r2);
71
        if (-1 == type) return in;
72
        if ( 0 == type) return on;
73
        if (+1 == type) return out;
74
        return unknow_point_relation;
75
76
77
     ab_float central_angle(const point &A, const point &B, const bool reflex = false)
         const {
78
        T dot = (A * B);
79
        if (0 == sgn(dot)) return 1. * (A != B) * pi;
80
        ab_float angle = ((ab_float)(dot)) / r / r;
81
        if (reflex) angle = 2. * pi - angle;
82
        return angle;
83
     }
84
85
     /* be sure (*this) intersect with C */
86
     pair<point, point> crosspoint(const circle &C) const {
87
        ab_float d = (o \rightarrow * (C.o)).norm();
        // TODO:
88
89
90
   };
91
92 |}
   k 次圆交
 1 | // china no.1
 2
   #pragma comment(linker, "/STACK:1024000000,1024000000")
 3
   #include <vector>
   |#include <iostream>
   #include <string>
 5
   |#include <map>
 7
   |#include <stack>
 8
   #include <cstring>
   #include <queue>
 9
   |#include <list>
10
11
   #include <stdio.h>
12 |#include <set>
13 |#include <algorithm>
14 |#include <cstdlib>
15 |#include <cmath>
16 | #include <iomanip>
17
   #include <cctype>
18
   |#include <sstream>
19
   #include <functional>
20
   #include <stdlib.h>
21
   #include <time.h>
22
   #include <bitset>
23
   using namespace std;
24
25
   #define pi acos(-1)
   #define PI acos(-1)
26
   #define endl '\n'
27
28
   #define srand() srand(time(0));
29
   #define me(x,y) memset(x,y,sizeof(x));
   #define foreach(it,a) for(__typeof((a).begin()) it=(a).begin();it!=(a).end();it++)
31
   #define close() ios::sync_with_stdio(0); cin.tie(0);
   #define FOR(x,n,i) for(int i=x;i<=n;i++)</pre>
32
   #define F0r(x,n,i) for(int i=x;i<n;i++)</pre>
33
34
   #define W while
35
   #define sgn(x) ((x) < 0 ? -1 : (x) > 0)
36 |#define bug printf("********\n");
```

```
37
    |#define db double
38
    typedef long long LL;
39
    const int INF=0x3f3f3f3f3f;
    const LL LINF=0x3f3f3f3f3f3f3f3f3f1LL;
    const int dx[] = \{-1,0,1,0,1,-1,-1,1\};
41
42
    const int dy[]= \{0,1,0,-1,-1,1,-1,1\};
43
    const int maxn=1e3+10;
44
    const int maxx=1e6+100;
    const double EPS=1e-8;
45
46
    const double eps=1e-8;
47
    const int mod=10000007;
48
    template<class T>inline T min(T a,T b,T c) {
49
       return min(min(a,b),c);
50
51
    template<class T>inline T max(T a,T b,T c) {
52
      return max(max(a,b),c);
53
54
    template<class T>inline T min(T a,T b,T c,T d) {
55
       return min(min(a,b),min(c,d));
56
57
    template<class T>inline T max(T a,T b,T c,T d) {
58
      return max(max(a,b),max(c,d));
59
60
    template <class T>
61
    inline bool scan_d(T &ret) {
62
      char c;
63
       int sgn;
64
      if (c = getchar(), c == EOF) {
65
         return 0;
66
      while (c != '-' \&\& (c < '0' || c > '9')) 
 67
68
         c = getchar();
69
70
      sgn = (c == '-') ? -1 : 1;
      ret = (c == '-') ? 0 : (c - '0');
71
72
      while (c = getchar(), c >= '0' && c <= '9') {
73
         ret = ret * 10 + (c - '0');
74
 75
      ret *= san;
 76
       return 1;
77
78
 79
    inline bool scan_lf(double &num) {
       char in;
80
      double Dec=0.1;
81
82
      bool IsN=false, IsD=false;
83
       in=getchar();
84
      if(in==EOF) return false;
85
      while(in!='-'&&in!='.'&&(in<'0'||in>'9'))in=getchar();
      if(in=='-') {
86
87
         IsN=true;
88
         num=0;
       } else if(in=='.') {
89
         IsD=true;
90
91
         num=0;
       } else num=in-'0';
92
93
      if(!IsD) {
94
         while(in=getchar(),in>='0'&&in<='9') {
95
           num*=10;
96
           num+=in-'0';
97
         }
98
99
      if(in!='.') {
100
         if(IsN) num=-num;
```

```
101
         return true;
102
       } else {
103
         while(in=getchar(),in>='0'&&in<='9') {</pre>
104
            num+=Dec*(in-'0');
105
            Dec*=0.1;
106
         }
107
       if(IsN) num=-num;
108
109
       return true;
110
111
     void Out(LL a) {
112
113
       if(a < 0) {
         putchar('-');
114
115
         a = -a;
116
117
       if(a >= 10) Out(a / 10);
118
       putchar(a % 10 + '0');
119
120
     void print(LL a) {
       Out(a),puts("");
121
122
     //freopen( "in.txt" , "r" , stdin );
//freopen( "data.txt" , "w" , stdout );
//cerr << "run time is " << clock() << endl;</pre>
123
124
125
126
     /*struct Point
127
     {
128
         double x, y;
129
         Point(const Point& rhs): x(rhs.x), y(rhs.y) { } //拷贝构造函数
130
         Point(double x = 0, double y = 0) : x(x), y(y) { }
131
         inline void input()
132
133
              scanf("%lf%lf",&x,&y);
134
135
         inline void print()
136
137
              printf("%.6lf %.6lf\n",x,y);
138
139
     };*/
140
     db sqr(db x) {
141
       return x*x;
142
143
     int dcmp(double x) {
       if(fabs(x) < EPS) return 0;
144
145
       else return x < 0 ? -1 : 1;
146
     struct Circle {
147
148
       double x, y, r, angle;
149
       int d;
150
       Circle() {}
151
       Circle(double xx, double yy, double ang = 0, int t = 0) {
152
         X = XX;
153
         y = yy;
154
         angle = ang;
155
         d = t;
156
       }
157
       void get() {
         scanf("%lf%lf%lf", &x, &y, &r);
158
159
         d = 1;
160
       }
161
162
     Circle cir[maxn],tp[maxn*2];
163
     double area[maxn];
164
     double dis(Circle a,Circle b) {
```

```
165
      return sqrt(sqr(a.x - b.x) + sqr(a.y - b.y));
166
167
    double cross(Circle p0,Circle p1,Circle p2) {
168
       return (p1.x - p0.x) * (p2.y - p0.y) - (p1.y - p0.y) * (p2.x - p0.x);
169
170
    //圆相交
    int CirCrossCir(Circle p1, double r1, Circle p2, double r2, Circle &cp1, Circle &cp2) {
171
172
      double mx = p2.x - p1.x, sx = p2.x + p1.x, mx2 = mx * mx;
173
      double my = p2.y - p1.y, sy = p2.y + p1.y, my2 = my * my;
      double sq = mx2 + my2, d = -(sq - sqr(r1 - r2)) * (sq - sqr(r1 + r2));
174
175
      if (d + eps < 0) return 0;
176
      if (d < eps) d = 0;
177
      else d = sqrt(d);
178
      double x = mx * ((r1 + r2) * (r1 - r2) + mx * sx) + sx * my2;
179
      double y = my * ((r1 + r2) * (r1 - r2) + my * sy) + sy * mx2;
180
      double dx = mx * d, dy = my * d;
181
      sq *= 2;
      cp1.x = (x - dy) / sq;
182
183
       cp1.y = (y + dx) / sq;
       cp2.x = (x + dy) / sq;
184
      cp2.y = (y - dx) / sq;
185
186
      if (d > eps) return 2;
187
      else return 1;
188
189
    bool circmp(const Circle& u, const Circle& v) {
190
      return dcmp(u.r - v.r) < 0;
191
192
    bool cmp(const Circle& u, const Circle& v) {
193
       if (dcmp(u.angle - v.angle)) return u.angle < v.angle;
194
       return u.d > v.d;
195
196
    //0.5*r*r*(K-sin(K))
197
    double calc(Circle cir,Circle cp1,Circle cp2) {
198
       double ans = (cp2.angle - cp1.angle) * sqr(cir.r)
199
                     - cross(cir, cp1, cp2) + cross(Circle(0, 0), cp1, cp2);
200
       return ans / 2;
201
202
203
    void CirUnion(Circle cir□, int n) {
204
      Circle cp1, cp2;
205
       sort(cir, cir + n, circmp);
206
       for (int i = 0; i < n; ++i)
         for (int j = i + 1; j < n; ++j)
207
208
           if (dcmp(dis(cir[i], cir[j]) + cir[i].r - cir[j].r) <= 0)</pre>
209
             cir[i].d++;
210
      for (int i = 0; i < n; ++i) {
211
        int tn = 0, cnt = 0;
212
         for (int j = 0; j < n; ++j) {
213
           if (i == j) continue;
           if (CirCrossCir(cir[i], cir[i].r, cir[j], cir[j].r,
214
215
                           cp2, cp1) < 2) continue;
216
           cp1.angle = atan2(cp1.y - cir[i].y, cp1.x - cir[i].x);
217
           cp2.angle = atan2(cp2.y - cir[i].y, cp2.x - cir[i].x);
           cp1.d = 1;
218
219
           tp[tn++] = cp1;
220
           cp2.d = -1;
221
           tp[tn++] = cp2;
222
           if (dcmp(cp1.angle - cp2.angle) > 0) cnt++;
223
224
        tp[tn++] = Circle(cir[i].x - cir[i].r, cir[i].y, pi, -cnt);
225
        tp[tn++] = Circle(cir[i].x - cir[i].r, cir[i].y, -pi, cnt);
226
         sort(tp, tp + tn, cmp);
227
        int p, s = cir[i].d + tp[0].d;
228
         for (int j = 1; j < tn; ++j) {
```

```
229
           p = s;
230
           s += tp[j].d;
231
           area[p] += calc(cir[i], tp[j - 1], tp[j]);
232
233
       }
234
235
     int n;
     void solve() {
236
237
       for(int i=0; i<n; i++)
238
         cir[i].get();
239
       me(area, 0);
240
       CirUnion(cir,n);
241
       for(int i=1; i<=n; i++) {
242
         area[i]-=area[i+1];
         printf("[%d]_{\square}=_{\square}%.3f\n", i, area[i]);
243
244
       }
245
246
    int main() {
       while(scanf("%d",&n)!=EOF)
247
248
         solve();
249 |}
    universe
  1
  2
    Point CircumCenter(Point a, Point b, Point c) { //三角形的外心
  3
       Point cp;
  4
       double a1 = b.x-a.x, b1 = b.y-a.y, c1 = (a1*a1 + b1*b1)/2;
  5
       double a2 = c.x-a.x,b2 = c.y-a.y,c2 = (a2*a2 + b2*b2)/2;
  6
       double d = a1*b2 - a2*b1;
  7
       cp.x = a.x + (c1*b2-c2*b1)/d;
  8
       cp.y = a.y + (a1*c2-a2*c1)/d;
  9
       return cp;
 10 | }
    3.1.3 Convex hull
    /* 2D Convex Hull, by Abreto <m@abreto.net>. */
  2
    #include "2d_base.hh"
  3
    #include <cmath>
    #include <algorithm>
  6
    using namespace std;
  7
  8
    point 0;
  9
 10
    bool comp_angle(point_t a, point_t b) {
       double t = (a-0).X(b-0);
 11
 12
       if(fe(t,0.0)) return fl((b-0).mag2(),(a-0).mag2());
 13
       else return fl(0.0,t);
 14
 15
     void convex_hull_graham(vp& convex, vp src) {
 16
       int i = 0, top = 0;
 17
 18
       0 = src[0];
 19
       for(auto pt : src)
 20
         if( pt.x < 0.x \mid | (pt.x == 0.x \& pt.y < 0.y))
 21
           0 = pt;
 22
       sort(src.begin(), src.end(), comp_angle);
       convex.push_back(src[0]);
 23
 24
       convex.push_back(src[1]);
       top = 1;
 25
 26
       for(i = 2; i < src.size(); ++i) {
```

```
while(top>1 && fle((convex[top]-convex[top-1]).X(src[i]-convex[top]),0.0)) {
27
28
          convex.pop_back();
29
          --top;
30
        }
31
        convex.push_back(src[i]);
32
        ++top;
33
34
    3.1.4 Intersect Area
   |#include <cstdio>
 2
   #include <cmath>
   #include <algorithm>
 3
 4
 5
   using namespace std;
 6
 7
   //#define inf 1000000000000
 8
   #define M 8
 9
   #define LL long long
10
   #define eps 1e-12
11
   #define PI acos(-1.0)
12
   using namespace std;
13
   struct node {
14
     double x,y;
15
     node() {}
16
     node(double xx,double yy) {
17
        X=XX;
18
        y=yy;
19
20
     node operator -(node s) {
21
        return node(x-s.x,y-s.y);
22
23
     node operator +(node s) {
24
        return node(x+s.x,y+s.y);
25
26
     double operator *(node s) {
27
        return x*s.x+y*s.y;
28
29
     double operator ^(node s) {
30
        return x*s.y-y*s.x;
31
32
33
   double max(double a,double b) {
34
     return a>b?a:b;
35
36
   double min(double a, double b) {
37
      return a<b?a:b;
38
39
   double len(node a) {
40
      return sqrt(a*a);
41
42
    double dis(node a, node b) { //两点之间的距离
43
      return len(b-a);
44
45
   double cross(node a, node b, node c) { //叉乘
46
     return (b-a)^(c-a);
47
48
   double dot(node a, node b, node c) { //点成
49
     return (b-a)*(c-a);
50
51
   int judge(node a,node b,node c) { //判断c是否在ab线段上(前提是c在直线ab上)
     if(c.x = min(a.x,b.x)
```

```
53
          \&c.x \le max(a.x,b.x)
 54
          \&c.y = min(a.y,b.y)
 55
          \&c.y \le max(a.y,b.y)
 56
        return 1;
 57
      return 0;
 58
 59
    double area(node b,node c,double r) {
 60
      node a(0.0,0.0);
 61
      if(dis(b,c)<eps)
 62
        return 0.0;
 63
      double h=fabs(cross(a,b,c))/dis(b,c);
      if(dis(a,b)>r-eps\&dis(a,c)>r-eps) { //两个端点都在圆的外面则分为两种情况
 64
 65
        double angle=acos(dot(a,b,c)/dis(a,b)/dis(a,c));
 66
        if(h>r-eps) {
          return 0.5*r*r*angle;
 67
 68
        } else if(dot(b,a,c)>0\&dot(c,a,b)>0) {
 69
          double angle1=2*acos(h/r);
 70
          return 0.5*r*r*fabs(angle-angle1)+0.5*r*r*sin(angle1);
 71
        } else {
 72
          return 0.5*r*r*angle;
 73
 74
      } else if(dis(a,b)<r+eps&dis(a,c)<r+eps) { //两个端点都在圆内的情况
 75
        return 0.5*fabs(cross(a,b,c));
 76
      } else { //一个端点在圆上一个端点在圆内的情况
 77
        if(dis(a,b)>dis(a,c)) { //默认b在圆内
 78
          swap(b,c);
 79
 80
        if(fabs(dis(a,b))<eps) { //ab距离为0直接返回0
 81
          return 0.0;
 82
        if(dot(b,a,c)<eps) {
 83
 84
          double angle1=acos(h/dis(a,b));
 85
          double angle2=acos(h/r)-angle1;
 86
          double angle3=acos(h/dis(a,c))-acos(h/r);
 87
          return 0.5*dis(a,b)*r*sin(angle2)+0.5*r*r*angle3;
 88
 89
        } else {
 90
          double angle1=acos(h/dis(a,b));
 91
          double angle2=acos(h/r);
 92
          double angle3=acos(h/dis(a,c))-angle2;
 93
          return 0.5*r*dis(a,b)*sin(angle1+angle2)+0.5*r*r*angle3;
 94
        }
 95
      }
 96
 97
 98
    node A, B, C;
 99
    int R;
100
101
    bool compar(node &p1, node &p2) {
102
      return (p1^p2)>eps;
103
104
105
    double f(double x, double y) {
      node 0(x,y);
106
107
      node p[8];
108
      p[0] = A-0;
109
      p[1] = B-0;
110
      p[2] = C-0;
111
      sort(p, p+3, compar);
112
      p[3] = p[0];
113
      0 = node(0,0);
114
      double sum=0;
115
      /* <!-- 求面积交部分 */
      for(int i=0; i<3; i++) { /* 按顺或逆时针顺序最后取绝对值就好 */
116
```

```
117
         int j=i+1;
118
         double s=area(p[i],p[j],(double)R);
         if(cross(0,p[i],p[j])>0)
119
120
           sum+=s;
121
         else
122
           sum-=s;
123
124
       if(sum < -eps) sum = -sum;
125
       /* --> */
126
       return sum;
127
128
129
    double trifind(double x, double y1, double y2) {
130
       double l = y1, r = y2;
131
       while(r-l>eps) {
132
         double mid = (1+r)/2.0;
         double mmid = (mid+r)/2.0;
133
134
         if( f(x,mmid) > f(x,mid) + eps )
135
           l = mid;
136
         else
137
           r = mmid;
138
139
       return f(x,l);
140
141
142
     double findmin(double x1, double x2, double y1, double y2) {
143
       double l = x1, r = x2;
144
       while(r-l>eps) {
145
         double mid = (1+r)/2.0;
         double mmid = (mid+r)/2.0;
146
147
         if( trifind(mmid,y1,y2) > trifind(mid,y1,y2)+eps )
148
           l = mid;
149
         else
150
           r = mmid;
151
152
       return trifind(l,y1,y2);
153
154
155
     double ans(int a, int b, int c, int r) {
156
       A = node(0,0);
157
       B = node((double)c, 0);
158
       R = r;
159
       double da = a, db = b, dc = c;
       double cosa = (db*db+dc*dc-da*da)/(2.0*db*dc);
160
161
       double alpha = acos(cosa);
162
       C = node(db*cosa, db*sin(alpha));
163
       return findmin(0.0, c, 0.0, db*sin(alpha));
164
165
166
     int main(void) {
       int a = 0, b = 0, c = 0, r = 0;
167
       while(EOF != scanf("%d%d%d%d",&a,&b,&c,&r) && (allbliclir))
168
         printf(%.81f\n, ans(a,b,c,r));
169
170
       return 0;
171 |}
    3.1.5 Universe
  1 |#include <bits/stdc++.h>
  2
    using namespace std;
  3
  4
    struct Point {
      double x, y;
```

```
6
     Point(double x = 0, double y = 0) : x(x), y(y) {}
 7
   };
 8
 9
   typedef Point Vector;
10
11
   Vector operator + (Vector A, Vector B) {
12
     return Vector(A.x + B.x, A.y + B.y);
13
14
   Vector operator - (Vector A, Vector B) {
15
     return Vector(A.x - B.x, A.y - B.y);
16
17
   Vector operator * (Vector A, double p) {
18
     return Vector(A.x*p, A.x*p);
19
   Vector operator / (Vector A, double p) {
20
21
     return Vector(A.x/p, A.x/p);
22
23
24
   bool operator < (const Point& a, const Point b) {
25
     return a.x < b.x | | (a.x == b.x && a.y < b.y);
26
27
28
   const double EPS = 1e-10;
29
30
   int dcmp(double x) {
31
     if(fabs(x) < EPS) return 0;
32
     else return x < 0 ? -1 : 1;
33
34
35
   bool operator == (const Point& a, const Point& b) {
36
     return dcmp(a.x-b.x) == 0 \& dcmp(a.y-b.y);
37
38
39
   //向量a的极角
   double Angle(const Vector& v) {
41
     return atan2(v.y, v.x);//\share\CodeBlocks\templates\wizard\console\cpp
42
43
44
   //向量点积
45
   double Dot(Vector A, Vector B) {
46
     return A.x*B.x + A.y*B.y;
47
48
49
   //向量长度\share\CodeBlocks\templates\wizard\console\cpp
   double Length(Vector A) {
50
51
     return sqrt(Dot(A, A));
52
53
54
   //向量夹角
   double Angle(Vector A, Vector B) {
55
     return acos(Dot(A, B) / Length(A) / Length(B));
56
57
58
59
   //向量叉积
60
   double Cross(Vector A, Vector B) {
61
     return A.x*B.y - A.y*B.x;
62
63
64
   //三角形有向面积的二倍
   double Area2(Point A, Point B, Point C) {
65
     return Cross(B-A, C-A);
66
67
68
69 | //向量逆时针旋转rad度(弧度)
```

```
| Vector Rotate(Vector A, double rad) {
     return Vector(A.x*cos(rad)-A.y*sin(rad), A.x*sin(rad)+A.y*cos(rad));
71
72
   }
73
74
   //计算向量A的单位法向量。左转90°,把长度归一。调用前确保A不是零向量。
75
   Vector Normal(Vector A) {
76
     double L = Length(A);
77
     return Vector(-A.y/L, A.x/L);
78
79
80
   81
   使用复数类实现点及向量的简单操作
82
   #include <complex>
83
84
   typedef complex<double> Point;
85
   typedef Point Vector;
86
87
   double Dot(Vector A, Vector B) { return real(conj(A)*B)}
88
   double Cross(Vector A, Vector B) { return imag(conj(A)*B);}
89
   Vector Rotate(Vector A, double rad) { return A*exp(Point(0, rad)); }
90
91
   92
93
   94
   * 用直线上的一点p0和方向向量v表示一条指向。直线上的所有点P满足P = P0+t*v;
95
   * 如果知道直线上的两个点则方向向量为B-A, 所以参数方程为A+(B-A)*t;
96
   * 当t 无限制时, 该参数方程表示直线。
97
   * 当t > 0时, 该参数方程表示射线。
98
   * 当 0 < t < 1时, 该参数方程表示线段。
99
   100
101
   //直线交点,须确保两直线有唯一交点。
102
   Point GetLineIntersection(Point P, Vector v, Point Q, Vector w) {
103
     Vector u = P - Q;
104
     double t = Cross(w, u)/Cross(v, w);
105
     return P+v*t;
106
   }
107
   //点到直线距离
108
109
   double DistanceToLine(Point P, Point A, Point B) {
110
     Vector v1 = B - A, v2 = P - A;
111
     return fabs(Cross(v1, v2) / Length(v1)); //不取绝对值, 得到的是有向距离
112
113
114
   //点到线段的距离
115
   double DistanceToSegmentS(Point P, Point A, Point B) {
116
     if(A == B) return Length(P-A);
     Vector v1 = B-A, v2 = P-A, v3 = P-B;
117
118
     if(dcmp(Dot(v1, v2)) < 0) return Length(v2);</pre>
119
     else if(dcmp(Dot(v1, v3)) > 0) return Length(v3);
120
     else return fabs(Cross(v1, v2)) / Length(v1);
121
122
123
   //点在直线上的投影
124
   Point GetLineProjection(Point P, Point A, Point B) {
125
     Vector v = B - A;
126
     return A+v*(Dot(v, P-A)/Dot(v, v));
127
128
129
   //线段相交判定,交点不在一条线段的端点
   bool SegmentProperIntersection(Point a1, Point a2, Point b1, Point b2) {
130
     double c1 = Cross(a2-a1, b1-a1), c2 = Cross(a2-a1, b2-a1);
131
132
     double c3 = Cross(b2-b1, a1-b1), c4 = Cross(b2-b1, a2-b1);
133
     return dcmp(c1)*dcmp(c2) < 0 && dcmp(c3)*dcmp(c4) < 0;
```

```
134
   |}
135
136
    //判断点是否在点段上,不包含端点
    bool OnSegment(Point P, Point a1, Point a2) {
137
138
      return dcmp(Cross(a1-P, a2-P) == 0 \& dcmp((Dot(a1-P, a2-P)) < 0));
139
140
141
    //计算凸多边形面积
    double ConvexPolygonArea(Point *p, int n) {
142
     double area = 0;
143
144
      for(int i = 1; i < n-1; i++)
145
       area += Cross(p[i] - p[0], p[i+1] - p[0]);
146
      return area/2;
147
148
149
    //计算多边形的有向面积
150
    double PolygonArea(Point *p, int n) {
151
     double area = 0;
152
     for(int i = 1; i < n-1; i++)
153
       area += Cross(p[i] - p[0], p[i+1] - p[0]);
154
      return area/2;
155
156
157
    158
    * Morley定理: 三角形每个内角的三等分线, 相交成的三角形是等边三角形。
159
    * 欧拉定理: 设平面图的定点数, 边数和面数分别为V,E,F。则V+F-E = 2;
160
    161
162
    struct Circle {
163
     Point c;
164
     double r;
165
166
     Circle(Point c, double r) : c(c), r(r) {}
     //通过圆心角确定圆上坐标
167
168
     Point point(double a) {
169
       return Point(c.x + cos(a)*r, c.y + sin(a)*r);
170
     }
171
    };
172
173
    struct Line {
174
     Point p;
175
     Vector v;
176
     double ang;
177
     Line() {}
178
     Line(Point p, Vector v) : p(p), v(v) {}
179
     bool operator < (const Line& L) const {</pre>
180
       return ang < L.ang;</pre>
181
     }
182
    };
183
    //直线和圆的交点,返回交点个数,结果存在sol中。
184
185
    //该代码没有清空sol。
    int getLineCircleIntersecion(Line L, Circle C, double& t1, double& t2, vector<Point>&
186
        sol) {
187
     double a = L.v.x, b = L.p.x - C.c.x, c = L.v.y, d = L.p.y - C.c.y;
     double e = a*a + c*c, f = 2*(a*b + c*d), g = b*b + d*d - C.r*C.r;
188
     double delta = f*f - 4*e*g;
189
190
     if(dcmp(delta) < 0) return 0; //相离
191
     if(dcmp(delta) == 0) {
                                //相切
192
       t1 = t2 = -f / (2*e);
193
       sol.push_back(C.point(t1));
194
       return 1;
195
196
     //相交
```

```
197
      t1 = (-f - sqrt(delta)) / (2*e);
198
      sol.push_back(C.point(t1));
199
      t2 = (-f + sqrt(delta)) / (2*e);
200
      sol.push_back(C.point(t2));
201
      return 2;
202
203
204
    //两圆相交
205
    int getCircleCircleIntersection(Circle C1, Circle C2, vector<Point>& sol) {
206
      double d = Length(C1.c - C2.c);
207
      if(dcmp(d) == 0) {
208
        if(dcmp(C1.r - C2.r == 0)) return -1;
                                                //两圆完全重合
209
                                                //同心圆,半径不一样
        return 0;
210
211
      if(dcmp(C1.r + C2.r - d) < 0) return 0;
212
      if(dcmp(fabs(C1.r - C2.r) == 0)) return -1;
213
214
      double a = Angle(C2.c - C1.c);
                                                  //向量C1C2的极角
215
      double da = acos((C1.r*C1.r + d*d - C2.r*C2.r) / (2*C1.r*d));
216
      //C1C2到C1P1的角
217
      Point p1 = C1.point(a-da), p2 = C1.point(a+da);
218
      sol.push_back(p1);
219
      if(p1 == p2) return 1;
220
      sol.push_back(p2);
221
      return 2;
222
223
224
    const double PI = acos(-1);
225
    //过定点做圆的切线
226
    //过点p做圆C的切线,返回切线个数。v[i]表示第i条切线
    int getTangents(Point p, Circle C, Vector* v) {
227
228
      Vector u = C.c - p;
229
      double dist = Length(u);
230
      if(dist < C.r) return 0;
231
      else if(dcmp(dist - C.r) == 0) {
232
        v[0] = Rotate(u, PI/2);
233
        return 1;
234
      } else {
235
        double ang = asin(C.r / dist);
236
        v[0] = Rotate(u, -ang);
237
        v[1] = Rotate(u, +ang);
238
        return 2;
239
      }
240
241
242
    //两圆的公切线
243
    //返回切线的个数,-1表示有无数条公切线。
244
    //a[i], b[i] 表示第i条切线在圆A, 圆B上的切点
245
    int getTangents(Circle A, Circle B, Point *a, Point *b) {
246
      int cnt = 0;
247
      if(A.r < B.r) {
        swap(A, B);
248
249
        swap(a, b);
250
      int d2 = (A.c.x - B.c.x)*(A.c.x - B.c.x) + (A.c.y - B.c.y)*(A.c.y - B.c.y);
251
252
      int rdiff = A.r - B.r;
253
      int rsum = A.r + B.r;
254
      if(d2 < rdiff*rdiff) return 0;
                                      //内含
255
      double base = atan2(B.c.y - A.c.y, B.c.x - A.c.x);
256
      if(d2 == 0 && A.r == B.r) return -1; //无限多条切线
257
                                     //内切一条切线
      if(d2 == rdiff*rdiff) {
258
        a[cnt] = A.point(base);
259
        b[cnt] = B.point(base);
260
        cnt++;
```

```
261
       return 1;
262
263
      //有外共切线
264
      double ang = acos((A.r-B.r) / sqrt(d2));
265
      a[cnt] = A.point(base+ang);
266
      b[cnt] = B.point(base+ang);
267
      cnt++:
      a[cnt] = A.point(base-ang);
268
269
      b[cnt] = B.point(base-ang);
270
      cnt++;
271
      if(d2 == rsum*rsum) { //一条公切线
272
       a[cnt] = A.point(base);
273
       b[cnt] = B.point(PI+base);
274
       cnt++;
275
      } else if(d2 > rsum*rsum) { //两条公切线
276
       double ang = acos((A.r + B.r) / sqrt(d2));
277
       a[cnt] = A.point(base+ang);
278
       b[cnt] = B.point(PI+base+ang);
279
       cnt++;
280
       a[cnt] = A.point(base-ang);
281
       b[cnt] = B.point(PI+base-ang);
282
       cnt++;
283
284
      return cnt;
285
286
287
    typedef vector<Point> Polygon;
288
289
    //点在多边形内的判定
290
    int isPointInPolygon(Point p, Polygon poly) {
291
      int wn = 0;
292
      int n = poly.size();
293
      for(int i = 0; i < n; i++) {
294
       if(OnSegment(p, poly[i], poly[(i+1)%n])) return -1; //在边界上
295
       int k = dcmp(Cross(poly[(i+1)%n]-poly[i], p-poly[i]));
       int d1 = dcmp(poly[i].y - p.y);
296
297
       int d2 = dcmp(poly[(i+1)%n].y - p.y);
298
       if(k > 0 \&\& d1 \le 0 \&\& d2 > 0) wn++;
299
       if(k < 0 \&\& d2 <= 0 \&\& d1 > 0) wn++;
300
301
      if(wn != 0) return 1;
                               //内部
302
                               //外部
      return 0;
303
304
305
    //凸包
306
    307
    * 输入点数组p, 个数为p, 输出点数组ch。 返回凸包顶点数
308
    * 不希望凸包的边上有输入点, 把两个<= 改成 <
309
    * 高精度要求时建议用dcmp比较
310
    * 输入点不能有重复点。函数执行完以后输入点的顺序被破坏
311
    312
    int ConvexHull(Point *p, int n, Point* ch) {
                       // 先比较x坐标, 再比较y坐标
313
      sort(p, p+n);
314
      int m = 0;
315
      for(int i = 0; i < n; i++) {
       while(m > 1 && Cross(ch[m-1] - ch[m-2], p[i]-ch[m-2]) <= 0) m--;
316
317
       ch[m++] = p[i];
318
      }
319
      int k = m;
320
      for(int i = n-2; i >= 0; i++) {
       while(m > k && Cross(ch[m-1] - ch[m-2], p[i]-ch[m-2]) <= 0) m--;
321
322
       ch[m++] = p[i];
323
      if(n > 1) m—;
324
```

```
325
      return m;
326
    }
327
328
    //用有向直线A->B切割多边形poly,
                                    返回"左侧"。 如果退化,可能会返回一个单点或者线段
329
    //复杂度0(n2);
330
    Polygon CutPolygon(Polygon poly, Point A, Point B) {
331
      Polygon newpoly;
332
      int n = poly.size();
      for(int i = 0; i < n; i++) {
333
334
        Point C = poly[i];
335
        Point D = poly[(i+1)\%n];
336
        if(dcmp(Cross(B-A, C-A)) >= 0) newpoly.push_back(C);
337
        if(dcmp(Cross(B-A, C-D)) != 0) {
338
          Point ip = GetLineIntersection(A, B-A, C, D-C);
339
          if(OnSegment(ip, C, D)) newpoly.push_back(ip);
340
        }
341
342
      return newpoly;
343
344
345
    //半平面交
346
347
    // 点p再有向直线L的左边。(线上不算)
348
    bool Onleft(Line L, Point p) {
349
      return Cross(L.v, p-L.p) > 0;
350
351
352
    //两直线交点,假定交点唯一存在
353
    Point GetIntersection(Line a, Line b) {
354
      Vector u = a.p - b.p;
355
      double t = Cross(b.v, u) / Cross(a.v, b.v);
356
      return a.p+a.v*t;
357
    }
358
359
    int HalfplaneIntersection(Line* L, int n, Point* poly) {
360
      sort(L, L+n);
                                  //按极角排序
361
362
      int first, last;
                                  //双端队列的第一个元素和最后一个元素
363
      Point *p = new Point[n];
                                  //p[i]为q[i]和q[i+1]的交点
364
      Line *q = new Line[n];
                                  //双端队列
365
      q[first = last = 0] = L[0]; //队列初始化为只有一个半平面L[0]
366
      for(int i = 0; i < n; i++) {
367
        while(first < last && !Onleft(L[i], p[last-1])) last--;</pre>
        while(first < last && !Onleft(L[i], p[first])) first++;</pre>
368
369
        q[++last] = L[i];
370
        if(fabs(Cross(q[last].v, q[last-1].v)) < EPS) {
371
          if(Onleft(q[last], L[i].p)) q[last] = L[i];
372
373
374
        if(first < last) p[last-1] = GetIntersection(q[last-1], q[last]);
375
      while(first < last && !Onleft(q[first], p[last-1])) last--;</pre>
376
377
      //删除无用平面
      if(last-first <= 1) return 0;</pre>
378
379
      p[last] = GetIntersection(q[last], q[first]);
380
381
      //从deque复制到输出中
382
      int m = 0;
383
      for(int i = first; i \le last; i++) poly[m++] = p[i];
384
      return m;
385 | }
```

# 4 Graph

### 4.1 Tree

### 4.1.1 Universe

```
2
   /* find root(重心) */
 3
 4
   void findroot(int u, int fa) {
 5
     int i;
 6
     size[u] = 1;
 7
     f[u] = 0;
 8
     for (i = last[u]; i; i = e[i][2]) {
 9
        if (!vis[e[i][0]] && e[i][0] != fa) {
10
          findroot(e[i][0], u);
          size[u] += size[e[i][0]];
11
12
          if (f[u] < size[e[i][0]])
13
            f[u] = size[e[i][0]];
14
        }
15
      if (f[u] < ALL - size[u])
16
17
        f[u] = ALL - size[u];
18
     if (f[u] < f[root]) root = u;
19
20
21
   /* ---- da ---- */
22
23
   int dep[MAXN+1];
24
   int ancestor[MAXN+1][MAXLGN];
25
   int minw[MAXN+1][MAXLGN];
26
   void dfs(int u, int fa) {
27
28
     ancestor[u][0] = fa;
29
     dep[u] = dep[fa] + 1;
     for(int e = u[front]; e; e = E[e].n) {
30
31
        int v = E[e].v, w = E[e].w;
32
        if(v != fa) {
33
          minw[v][0] = w;
34
          dfs(v, u);
35
        }
36
     }
37
   }
38
39
   void init_system(void) {
40
     int i = 0, w = 0;
41
     int t = 0;
42
     dep[0] = -1;
43
     dfs(1,0);
     for(w = 1; (t=(1<<w)) < N; ++w)
44
45
        for(i = 1; i <= N; ++i) if( dep[i] >= t ) {
            ancestor[i][w] = ancestor[ancestor[i][w-1]][w-1];
46
47
            minw[i][w] = min(minw[i][w-1], minw[ancestor[i][w-1]][w-1]);
48
          }
49
50
   int query(int a, int b) {
51
52
     if(dep[a] < dep[b]) return query(b,a);</pre>
     else { /* now dep[s] > dep[t] */
53
54
        int i = 0;
        int maxbit = MAXLGN-1;
55
56
        int ret = INF;
57
        //while((1<<maxbit) <= dep[a]) maxbit++;</pre>
```

```
58
        /* first up a to same dep with b. */
59
        for(i = maxbit; i >= 0; i--)
60
          if(dep[a] - (1 << i) >= dep[b]) {
61
            ret = min(ret, minw[a][i]);
62
            a = ancestor[a][i];
63
        if(a == b) return ret;
64
        for(i = maxbit; i \ge 0; i—)
65
          if(dep[a] - (1 << i) >= 0 \& ancestor[a][i] != ancestor[b][i]) {
66
67
            ret = min(ret, min(minw[a][i], minw[b][i]));
68
            a = ancestor[a][i];
69
            b = ancestor[b][i];
70
          }
        ret = min(ret, min(minw[a][0], minw[b][0]));
71
72
        return ret;
73
     }
74
   }
   4.1.2 Point Divide and Conquer
   Version 1
   //* Tree::Point divide and conquer, by Abreto<m@abreto.net>. */
 2
   #include <bits/stdc++.h>
```

```
3
 4
   using namespace std;
   typedef long long int ll;
 6
 7
   #define MAXN
                     (100001)
 8
   #define MAXV
                     (MAXN+1)
9
   #define MAXE
                     (MAXN << 1)
10
   struct edge {
11
      int v;
12
      edge *n;
13
      edge(void):v(0),n(NULL) {}
14
      edge(int vv,edge *nn):v(vv),n(nn) {}
15
16
   int nE;
17
   edge E[MAXE];
   edge *front[MAXV];
18
19
   int label[MAXV];
                        /* 0 for '(', 1 for ')' */
20
   void add_edge(int u, int v) {
21
      int ne = ++nE;
22
      E[ne] = edge(v, u[front]);
23
      u[front] = \&(E[ne]);
24
25
26
   int n;
27
   ll ans;
28
29
   char del[MAXV];
30
   namespace findroot {
31
   int ALL;
32
   int nfind;
33
   int vis[MAXV]
   int size[MAXV];
34
35
   int f[MAXV];
   int root;
36
37
   void __find(int u, int fa) {
      vis[u] = nfind;
38
39
      size[u] = 1;
40
      f[u] = 0;
41
      for(edge *e=u[front]; e; e = e->n) {
```

```
42
         int v = e \rightarrow v;
 43
         if((!del[v]) && (vis[v] != nfind) && (v != fa)) {
 44
           __find(v, u);
 45
           size[u] += size[v];
 46
           if(f[u] < size[v]) f[u] = size[v];
 47
 48
       if(f[u] < ALL - size[u]) f[u] = ALL - size[u];
 49
 50
       if(f[u] < f[root]) root = u;
 51
 52
     int find(int u, int all) {
 53
       ++nfind;
 54
       ALL = all;
       f[root = 0] = MAXV;
 55
 56
       __find(u,0);
 57
       return root;
 58
 59
    }
 60
 61
    namespace workspaces {
 62
    int maxdep;
    int dep[MAXV];
 63
     11 cntin[MAXV], cntout[MAXV];
 64
 65
     int in[2][MAXV];
                          /* 0 for '(', 1 for ')' */
 66
     int out[2][MAXV];
 67
     void getdeep(int u, int fa) {
 68
       dep[u] = dep[fa] + 1;
 69
       if(dep[u] > maxdep) maxdep = dep[u];
 70
       for(edge *e = u[front]; e; e = e->n)
 71
         if((!del[e->v]) \& (fa != e->v))
 72
           getdeep(e->v, u);
 73
 74
    void dfs(int u, int fa) {
 75
 76
         /* out from root */
 77
         out[0][u] = out[0][fa];
 78
         out[1][u] = out[1][fa];
 79
         if(0 == label[u]) { /* meet '(' */
           out[0][u]++;
 80
                     /* meet ')' */
 81
         } else {
 82
           if(out[0][u]) out[0][u]--;
 83
           else out[1][u]++;
 84
 85
         if(out[0][u] == 0)
 86
           cntout[out[1][u]]++;
 87
 88
 89
         /* in to root */
 90
         in[0][u] = in[0][fa];
 91
         in[1][u] = in[1][fa];
 92
         if(0 == label[u]) { /* meet '(' */
           if(in[1][u]) in[1][u]--;
 93
 94
           else in[0][u]++;
 95
                     /* meet ')' */
         } else {
 96
           in[1][u]++;
 97
 98
         if(0 == in[1][u])
 99
           cntin[in[0][u]]++;
100
101
       /* do something */
102
       for(edge *e = u[front]; e; e = e \rightarrow n) {
103
         int v = e \rightarrow v;
         if((!del[v]) && (v != fa)) {
104
105
           dfs(v, u);
```

```
106
         }
       }
107
108
109
     inline void init_maxdep(void) {
110
       maxdep = 0;
111
112
     inline void update_maxdep(int u) {
       dep[u] = 1;
113
       if(dep[u] > maxdep) maxdep = dep[u];
114
115
       for(edge *e = u[front]; e; e = e->n)
116
         if((!del[e->v]))
117
           getdeep(e->v, u);
118
119
     inline void clear(void) {
120
       for(int i = 0; i \le maxdep+1; ++i)
121
         cntin[i] = cntout[i] = 0;
122
123
     inline void work(int u) {
124
       in[0][u] = in[1][u] = out[0][u] = out[1][u] = 0;
125
       in[label[u]][u] = out[label[u]][u] = 1;
       if(out[0][u] == 0) cntout[out[1][u]]++;
126
       if(0 == in[1][u]) cntin[in[0][u]]++;
127
128
       /* update in and out if neccessary */
       for(edge *e = u[front]; e; e = e->n)
129
130
         if(!(del[e->v]))
131
           dfs(e->v, u);
132
133
    };
134
135
     11 count(int u, int p) {
136
       ll ret = 0;
       workspace::init_maxdep();
137
138
       workspace::update_maxdep(u);
139
       workspace::clear();
140
       if(-1 == p)  {
141
         for(edge *e = u[front]; e; e = e->n)
142
           if((!(del[e->v])))
143
             workspace::work(e->v);
         p = label[u];
144
145
         /* single end */
146
         if(0 == p) ret = workspace::cntout[1];
147
         else ret = workspace::cntin[1];
148
       } else {
149
         workspace::work(u);
150
       if(0 == p) { /* p is '(' */
151
152
         for(int i = 0; i < workspace::maxdep; ++i) /* concatenation */
153
           ret += workspace::cntin[i] * workspace::cntout[i+1];
154
                  /* p is ')' */
155
         for(int i = 0; i < workspace::maxdep; ++i) /* concatenation */
156
           ret += workspace::cntin[i+1] * workspace::cntout[i];
157
158
       return ret;
159
160
161
     void handle(int u) {
162
       del[u] = 1; /* delete current root. */
       ans += count(u, -1);
163
164
       /* do something */
165
       for(edge *e = u[front]; e; e = e \rightarrow n) {
166
         int v = e \rightarrow v;
167
         if(!del[v]) {
           ans -= count(v, label[u]);
168
169
           /* do something */
```

```
170
           int r = findroot::find(v, findroot::size[v]);
171
           handle(r);
172
         }
173
       }
174
175
176
     void proc(void) {
177
       int r = findroot::find(1,n);
178
       handle(r);
179
180
181
     char ls[MAXV+1];
     int main(void) {
182
183
       int i = 0;
       scanf("%d", &n);
scanf("%s", ls);
for(i = 0; i < n; ++i)
184
185
186
187
         label[i+1] = ls[i] - '(';
188
       for(i = 1; i < n; ++i) {
189
         int ai, bi;
         scanf("%d⊔%d", &ai, &bi);
190
191
         add_edge(ai, bi);
192
         add_edge(bi, ai);
       }
193
194
       proc();
       printf("%lld\n", ans);
195
196
       return 0;
197
     Version 2
    //* 2016 ACM/ICPC Asia Regional Dalian. Problem , by Abreto<m@abreto.net>. */
  2
    #include <bits/stdc++.h>
  3
  4
     using namespace std;
  5
     typedef long long int 11;
  6
  7
     /* offset in [1,k] */
  8
     #define GET(i,offset)
                               (((i)>>((offset)-1))&1)
                               ((i)|(1<<((offset)-1)))
  9
     #define SET(i,offset)
 10
    #define REV(i,offset)
                               ((i)^{(1<<((offset)-1))})
 11
 12
    #define MAXN
                      (50005)
 13
    #define MAXV
                      (MAXN+1)
                      (MAXN << 1)
 14
    #define MAXE
 15
    struct edge {
 16
       int v;
 17
       edge *n;
 18
       edge(void):v(0),n(NULL) {}
 19
       edge(int vv,edge *nn):v(vv),n(nn) {}
 20
    };
 21
     int nE;
     edge E[MAXE];
 22
     edge *front[MAXV];
 23
     int label[MAXV];
                          /* each kind */
 24
 25
     void add_edge(int u, int v) {
 26
       int ne = ++nE;
 27
       E[ne] = edge(v, u[front]);
 28
       u[front] = \&(E[ne]);
 29
 30
 31
    int n, k;
     ll ans;
 32
 33
    int all_kind;
 34
```

```
35
   ∣int ndel;
36
   int del[MAXV];
   namespace findroot {
37
   int ALL;
39
   ll nfind;
40
   ll vis[MAXV];
41
    int size[MAXV];
42
    int f[MAXV];
43
    int root;
44
    void __find(int u, int fa) {
45
      vis[u] = nfind;
46
      size[u] = 1;
47
      f[u] = 0;
48
      for(edge *e=u[front]; e; e = e->n) {
49
        int v = e \rightarrow v;
50
        if((del[v] != ndel) && (vis[v] != nfind) && (v != fa)) {
51
           __find(v, u);
52
          size[u] += size[v];
53
          if(f[u] < size[v]) f[u] = size[v];
        }
54
55
56
      if(f[u] < ALL - size[u]) f[u] = ALL - size[u];
57
      if(f[u] < f[root]) root = u;
58
59
    int find(int u, int all) {
60
      ++nfind;
61
      ALL = all;
62
      f[root = 0] = MAXV;
63
       _find(u,0);
64
      return root;
65
66
67
68
   namespace workspace {
69
   ll cnt[1024];
70
    int dp[MAXV];
71
    void dfs(int u, int fa) {
72
      dp[u] = dp[fa] \mid label[u];
73
      cnt[dp[u]] ++;
      /* dig into children */
74
75
      for(edge *e = u[front]; e; e = e \rightarrow n) {
76
        int v = e \rightarrow v;
77
        if((del[v] != ndel) && (v != fa)) {
78
          dfs(v, u);
79
80
      }
81
82
    inline void clear(void) {
83
      for(int i = 1; i \leftarrow all_kind; ++i)
84
        cnt[i] = 0;
85
86
    inline void work(int u) {
87
      dp[u] = label[u];
      cnt[dp[u]] ++;
88
89
      for(edge *e = u[front]; e; e = e->n)
90
        if((del[e\rightarrow v] != ndel))
91
          dfs(e\rightarrow v, u);
92
93
    inline void show(void) {
94
      for(int i = 0; i \le all_kind; ++i)
95
        printf("cnt[%d]_=_%lld\n", i, cnt[i]);
96
      for(int i = 1; i <= n; ++i)
97
        printf("dp[%d]_{\square}=_{\square}%d\n", i, dp[i]);
98 | }
```

```
99
    |};
100
101
102
     11 count(int u, int p) {
103
       ll ret = 0;
104
       workspace::clear();
105
       //printf("%d,%d :\n", u, p);
       if(-1 == p) {
106
         for(edge *e = u[front]; e; e = e->n)
107
108
           if(((del[e->v]) != ndel))
109
             workspace::work(e->v);
110
         p = label[u];
111
         /* single end */
112
         for(int i = 1; i <= all_kind; i++)
113
           if(all_kind == (i|p))
114
             ret += (workspace::cnt[i]<<1);</pre>
115
       } else {
116
         workspace::work(u);
117
118
       //workspace::show();
119
       for(int i = 1; i \leftarrow all_kind; ++i)
120
         if( workspace::cnt[i] > 0 )
           for(int j = 1; j \leftarrow all_kind; ++j)
121
122
             if(all\_kind == (i|p|j))
123
                ret += workspace::cnt[i] * workspace::cnt[j];
124
       //printf("%lld\n", ret);
125
       return ret;
126
127
128
     void handle(int u) {
       //printf("proccessing %d\n", u);
129
130
       del[u] = ndel; /* delete current root. */
131
       ans += count(u, -1);
132
       /* do something */
133
       for(edge *e = u[front]; e; e = e \rightarrow n) {
134
         int v = e \rightarrow v;
135
         if(del[v] != ndel) {
136
           ans -= count(v, label[u]);
137
           /* do something */
138
           int r = findroot::find(v, findroot::size[v]);
139
           handle(r);
140
         }
141
       }
142
143
144
     void proc(void) {
145
       int r = findroot::find(1,n);
146
       handle(r);
147
148
149
     void clear(void) {
150
       int i;
151
       ans = 0;
       nE = 0;
152
153
       for(i = 0; i \le n; ++i) {
154
         front[i] = NULL;
155
156
       //findroot::nfind = 0;
157
       ndel++;
158
159
160
     void mozhu(void) {
161
       int i = 0;
162
       int li;
```

```
163
       for(i = 1; i \le n; ++i) {
164
         scanf("%d", &li);
165
         label[i] = 1 << (li-1);
166
167
       for(i = 1; i < n; ++i) {
168
         int ai, bi;
         scanf("%d⊔%d", &ai, &bi);
169
         add_edge(ai, bi);
170
171
         add_edge(bi, ai);
172
173
       all_kind = (1 << k)-1;
       proc();
174
175
       if(1 == k) ans += n;
       printf("%lld\n", ans);
176
177
178
179
    int main(void) {
       while( EOF != scanf("%d%d", &n, &k) ) {
180
181
         clear();
182
         mozhu();
183
184
       return 0;
185
```

## 4.1.3 Hevay chain decompostion

```
1 |/* bzoj 1036 */
   /* 树链剖分 */
 2
 3
   #include <bits/stdc++.h>
 4
 5
   using namespace std;
 6
 7
                     30030
   #define MAXN
   #define MAXM
                     (MAXN << 1)
9
   struct edge {
10
      int v;
11
      edge *n;
12
      edge(void) {}
13
      edge(int vv, edge *nn):v(vv),n(nn) {}
14
   };
   typedef edge *ep;
15
16
   int nE;
   edge E[MAXM];
17
18
   ep front[MAXN];
   void add_edge(int u, int v) {
19
20
      int ne = ++nE;
21
      E[ne] = edge(v, u[front]);
22
      u[front] = \&(E[ne]);
23
   }
24
25
   int n;
    int fa[MAXN], son[MAXN], sz[MAXN], dep[MAXN];
26
27
    int top[MAXN];
   int id[MAXN];
28
29
   int tot;
30
31
   void calc(int u, int uf) {
32
      dep[u] = dep[uf] + 1;
33
      fa[u] = uf;
      sz[u] = 1;
34
35
      son[u] = -1;
      for(ep e = u[front]; e; e = e \rightarrow n) {
36
37
        if(e->v != uf) {
```

```
38
           calc(e\rightarrow v, u);
39
           sz[u] += sz[e->v];
40
           if(-1 == son[u] \mid | sz[son[u]] < sz[e \rightarrow v])
41
              son[u] = e \rightarrow v;
42
43
44
45
     void link(int u, int f) {
46
       id[u] = (++tot);
       top[u] = f;
47
48
       if(son[u] > 0) {
49
         link(son[u], f);
50
       for(ep e = u[front]; e; e = e\rightarrown) {
51
52
         if(e->v != fa[u] \&\& e->v != son[u]) {
53
           link(e->v, e->v);
54
55
       }
56
    }
57
58
    /* 其实是树链剖分 */
59
    void make_link_cut_tree(void) {
60
       calc(1, 0);
61
       link(1, 1);
62
63
64
    int w[MAXN];
65
    int sum[MAXN<<2], mx[MAXN<<2];</pre>
66
    void maintain(int o, int l, int r) {
67
       sum[o] = sum[o << 1] + sum[o << 1|1];
68
69
       mx[o] = max(mx[o<<1], mx[o<<1|1]);
70
71
    void build(int o = 1, int l = 1, int r = n) {
72
       if(r == 1) {
73
         sum[o] = w[l];
74
         mx[o] = w[l];
75
       } else {
 76
         int mid = 1+r>>1;
77
         build(o<<1, 1, mid);
78
         build(o<<1|1, mid+1, r);
79
         maintain(o, l, r);
80
       }
81
 82
     void update(int p, int x, int o = 1, int l = 1, int r = n) {
83
       if(p \le l \& r \le p) {
84
         sum[o] = x;
85
         mx[o] = x;
86
       } else {
87
         int mid = 1+r>>1;
         if(p <= mid) update(p,x,o<<1,1,mid);</pre>
88
         else update(p,x,o <<1 | 1,mid+1,r);
89
90
         maintain(o,l,r);
91
92
93
     int qs(int L, int R, int o = 1, int l = 1, int r = n) {
94
       if(R < l | l | r < L) return 0;
95
       else if (L \le l \& r \le R) {
96
         return sum[o];
97
       } else {
98
         int mid = 1+r>>1;
99
         return qs(L,R,o<<1,l,mid)+qs(L,R,o<<1|1,mid+1,r);
100
101 | }
```

```
102
     int qm(int L, int R, int o = 1, int l = 1, int r = n) {
103
       if(L \le 1 \& r \le R) {
104
         return mx[o];
105
       } else {
106
         int mid = 1+r>>1;
         if(R <= mid) return qm(L, R, o<<1, l, mid);</pre>
107
         else if (L > mid) return qm(L, R, o<<1|1, mid+1, r);
108
109
         else return max(qm(L, R, o<<1, l, mid),qm(L, R, o<<1|1, mid+1, r));
110
111
112
113
     void change(int u, int t) {
114
       update(id[u], t);
115
116
     int qmax(int u, int v) {
117
       int ret = -10000000000;
118
       while(top[u] != top[v]) {
119
         if( dep[top[u]] > dep[top[v]] ) {
120
           /* jump u */
121
           ret = max(ret, qm(id[top[u]], id[u]));
122
           u = fa[top[u]];
123
         } else {
124
           ret = max(ret, qm(id[top[v]], id[v]));
125
           v = fa[top[v]];
126
127
       }
128
       ret = max(ret, qm(min(id[u],id[v]),max(id[u],id[v])));
129
       return ret;
130
131
     int qsum(int u, int v) {
       int ret = 0;
132
133
       while(top[u] != top[v]) {
134
         if( dep[top[u]] > dep[top[v]] ) {
135
           /* jump u */
136
           ret += qs(id[top[u]], id[u]);
137
           u = fa[top[u]];
138
         } else {
139
           ret += qs(id[top[v]], id[v]);
140
           v = fa[top[v]];
141
142
       }
143
       ret += qs(min(id[u],id[v]),max(id[u],id[v]));
144
       return ret;
145
146
147
     int main(void) {
148
       int i;
       scanf("%d", &n);
149
150
       for(i = 1; i < n; ++i) {
151
         int a, b;
         scanf("%d%d", &a, &b);
152
153
         add_edge(a, b);
154
         add_edge(b, a);
155
156
       make_link_cut_tree();
157
       for(i = 1; i \le n; ++i) {
158
         scanf("%d", &(w[id[i]]));
159
160
       build();
       scanf("%d", &i);
161
162
       while(i--) {
163
         char command[8];
         int a, b;
164
         scanf("%s<sub>\\\\</sub>%d<sub>\\\</sub>%d", command, &a, &b);
165
```

```
if('C' == command[0]) change(a, b);
166
         else if ('M' == command[1]) printf("%d\n", qmax(a, b));
167
168
         else if ('S' == command[1]) printf("%d\n", qsum(a, b));
169
170
       return 0;
171
     4.2 2-SAT
    |#include <bits/stdc++.h>
  2
  3
    using namespace std;
  5
    namespace two_sat {
    const int maxn = 100000;
  7
     const int maxm = 1000000;
  8
    struct edge {
 9
       int v;
 10
       edge *n;
 11
       edge(void):v(0),n(NULL) {}
 12
       edge(int vv, edge *nn):v(vv),n(nn) {}
 13
 14
    typedef edge *ep;
    int n;
 15
 16
    int nE;
 17
    edge E[maxm];
 18
    ep front[maxn];
 19
    void add_edge(int u, int v) {
 20
       int ne = ++nE;
 21
       E[ne] = edge(v, u[front]);
 22
       u[front] = \&(E[ne]);
 23
 24
    /* (x = xval or y = yval), indexed from 0 */
 25
    void add_clause(int x, int xv, int y, int yv) {
 26
       x = x*2 + xv;
 27
       y = y*2 + yv;
       add_edge(x^1, y);
 28
 29
       add_edge(y^1, x);
    }
 30
 31
 32
     char mark[maxn<<1];</pre>
 33
    int S[maxn<<1], c;</pre>
 34
    void init(int N) {
 35
       n = N;
 36
       for(int i = 0; i < n*2; ++i) {
 37
         i[front] = NULL;
 38
         i[mark] = 0;
 39
 40
      nE = 0;
 41
 42
 43
     int dfs(int x) {
 44
       if(mark[x^1]) return 0;
 45
       if(mark[x]) return 1;
       mark[x] = 1;
 46
 47
       S[c++] = x;
 48
       for(ep e = x[front]; e; e = e->n)
 49
         if(!dfs(e->v)) return 0;
 50
       return 1;
 51
 52
 53 | int solve(void) {
```

```
54
      for(int i = 0; i < n*2; i += 2)
55
        if(!mark[i] && !mark[i+1]) {
56
          c = 0;
          if(!dfs(i)) {
57
58
            while(c > 0) mark[S[--c]] = 0;
59
            if(!dfs(i+1)) return 0;
60
61
62
      return 1;
63
64
   |}
```

# 4.3 Cut Edge and Point

```
Finding cut edges
 1
 2
   The code below works properly because the lemma above (first lemma):
 3
     h[root] = 0
 4
                par[v] = -1
 5
                          dfs (v):
 6
                          d[v] = h[v]
 7
                                 color[v] = gray
 8
                                           for u in adj[v]:
 9
                                               if color[u] == white
                                                 then par[u] = v and dfs(u) and d[v] = min(
10
                                                     d[v], d[u]
                                                     if d[u] > h[v]
11
12
                                                       then the edge v-u is a cut edge
13
                                                        else if u != par[v])
              then d[v] = min(d[v], h[u])
14
                           color[v] = black
15
16
                                       In this code, h[v] \exists height of vertex v in the DFS
                                          tree and d[\sqrt{y}] = \min(h[w]) where there is at least
                                          vertex u in subtree of v in the DFS tree where
                                          there is an edge between u and w).
17
18
                                           Finding cut vertices
19
                                           The code below works properly because the lemma
                                              above (first lemma):
20
                                           h[root] = 0
21
                                                     par[v] = -1
22
                                                          dfs (v):
23
                                                          d[v] = h[v]
24
                                                              color[v] = gray
25
                                                        for u in adj[v]:
                                                                  if color[u] == white
26
27
                                                                    then par[u] = v and dfs(
                                                                        u) and d[v] = min(d[v])
                                                                        ], d[u])
28
                                                                           if d[u] >= h[v]
                                                                              and (v != root
                                                                              number_of_children
                                                                              (v) > 1)
29
                                                                             then the edge v
                                                                                is a cut
                                                                                vertex
30
                                                                             else if u != par
                                                                                [V]
31
                    then d[v] = min(d[v], h[u])
                                 color[v] = black
32
33
                                             In this code, h[v] = 1 height of vertex v in
                                                the DFS tree and d[v] = min(h[w]) where
```

there is at least vertex u in subtree of v in the DFS tree where there is an edge between u and w).

#### 4.4 Euler Path

```
/* Euler path, by Abreto<m@abreto.net>. */
 2
   #define MAXV
                     (1024)
 3
   #define MAXE
                     (MAXV*MAXV)
 4
 5
   typedef struct {
 6
      int id;
 7
      int nxt;
 8
      int del;
 9
    } egde_t;
   int front[MAXV];
10
   egde_t edg[MAXE];
11
   int d[MAXV];
12
13
   int ind[MAXV], outd[MAXV];
14
   int nedges;
15
   void add_edge(int u, int v) {
16
      int newedge = ++nedges;
17
      edg[newedge].id = v;
18
      edg[newedge].nxt = u[front];
19
      edg[newedge].del = 0;
20
      u[front] = newedge;
21
      outd[u]++;
22
      ind[v]++;
23
      d[u]++;
24
      d[v]++;
25
26
   void del_edge(int u, int v) {
27
      int e = 0;
28
      for(e=u[front]; e; e=edg[e].nxt)
29
        if(edg[e].id==v) {
30
          edg[e].del = 1;
          outd[u]--;
31
32
          ind[v]--;
33
          d[u]--;
34
          d[v]--;
35
          return;
36
        }
37
   }
38
39
   int path[MAXV];
40
   int l;
41
42
   void add2path(int u) {
43
      path[l++] = u;
44
45
46
   /* Directed graph */
   void euler(int x) {
47
48
      if(outd[x]) {
49
        int e = 0;
        for(e=x[front]; e; e=edg[e].nxt)
50
51
          if(!edg[e].del) {
52
            int v = edg[e].id;
53
            del_edge(x,v);
54
            euler(v);
55
56
      }
```

```
57
      add2path(x);
58
   }
59
60
   /* Undirected graph */
61
   void euler(int x) {
      if(d[x]) {
62
63
        int e = 0;
        for(e=x[front]; e; e=edg[e].nxt)
64
65
          if(!edg[e].del) {
66
            int v = edg[e].id;
67
            del_edge(x,v);
            del_edge(v,x);
68
69
            euler(v);
70
71
72
      add2path(x);
73
```

## 4.5 Shortest Path

#### 4.5.1 Dijkstra

```
1 |/* Shortest Path Dijstra, by Abreto<m@abreto.net>. */
   #include <cstdio>
 3
   #include <set>
 4
   #include <utility>
 5
 6
   using namespace std;
 7
   typedef set< pair<int,int> > spii;
 8
 9
   #define MAXN
                    512
10
   #define MAXV
                    (MAXN*MAXN)
11
12
   struct egde_t {
13
     int id;
14
     int nxt;
15
16
   int front[MAXV];
17
   egde_t edg[MAXV<<3];
18
   int nedges;
19
   void add_edge(int u, int v) {
20
      int newedge = ++nedges;
21
     edg[newedge].id = v;
22
     edg[newedge].nxt = u[front];
23
     u[front] = newedge;
24
   }
25
26
   int d[MAXV];
27
   int vis[MAXN];
28
   int solid[MAXV];
29
30
   int dijstra(int s, int t) {
31
     int v = s[front];
32
     spii q;
     q.insert(make_pair(0, s));
33
34
     while(!q.empty()) {
35
        auto it = q.begin();
36
        int u = it->second;
37
        int v = u[front];
38
        q.erase(it);
39
        solid[u] = 1;
40
        if(u == t) break;
41
        while(v) {
```

```
42
          int w = edg[v].id;
43
          if(!solid[w]) {
44
            if( (0==d[w]) || (d[u] + 1 < d[w]) ) {
45
              q.erase(make_pair(d[w],w));
46
              d[w] = d[u] + 1;
47
              q.insert(make_pair(d[w],w));
48
49
            = edg[v].nxt;
50
51
52
53
      return d[t];
54
    4.5.2 Shortest Path Fast Algorithm
 1 |/* Shortest Path Fast Algorithm, by Abreto<m@abreto.net>. */
   #include <cstdio>
   #include <cstring>
   #include <queue>
 5
   #include <utility>
 6
 7
    using namespace std;
 8
 9
                     128
    #define MAXN
10
11
   struct edge {
12
      int v;
13
      int w;
14
      int n;
15
   edge edg[MAXN<<1];</pre>
16
17
   int nedg;
   int indegree[MAXN];
19
   int front[MAXN];
20
    int find_edge(int u, int v) {
21
      int e = u[front];
22
      while(e) {
23
        if(edg[e].v == v) return e;
24
        e = edg[e].n;
25
      }
26
      return 0;
27
    void add_edge(int u, int v, int w) {
28
29
      int e = find_edge(u,v);
30
      if(0==e) {
31
        int newnode = ++nedg;
32
        edq[newnode].v = v;
33
        edq[newnode].w = w;
        edg[newnode].n = u[front];
34
35
        u[front] = newnode;
36
        indegree[v]++;
37
      } else {
38
        edg[e].w = (w < edg[e].w)?w:(edg[e].w);
39
40
41
42
   int n;
43
    char inq[MAXN];
44
45
    int vis[MAXN];
```

int d[MAXN];

47 | int spfa(int s) { /\* return 1 if fuhuan exists. \*/

46

```
48
      queue<int> q;
49
      memset(inq, 0, sizeof(inq));
50
      memset(d, -1, sizeof(d));
51
      memset(vis, 0, sizeof(vis));
52
      d[s] = 0;
53
      inq[s] = 1;
54
      q.push(s);
55
      while(!q.empty()) {
56
        int u = q.front();
57
        q.pop();
58
        printf("proc<sub>\\\\</sub>d..\n", u);
59
        inq[u] = 0;
60
        if(vis[u]++ > n)
61
          return 1;
        for(int e = front[u]; e; e = edg[e].n) {
62
63
          int v = edg[e].v, w = edg[e].w;
64
          if(-1==d[v] || d[u] + w < d[v]) {
65
            d[v] = d[u] + w;
            if(!inq[v]) {
66
67
               inq[v] = 1;
68
               q.push(v);
69
70
          }
71
        }
72
73
      return 0;
74 |}
```

## 4.5.3 K-th shortest path

```
1
   /**
 2
    * poj
 3
     * Problem#2449
 4
    * Accepted
 5
    * Time: 250ms
 6
    * Memory: 9252k
 7
 8
   #include <iostream>
   #include <fstream>
9
   |#include <sstream>
10
11
   #include <algorithm>
12
   |#include <cstdio>
13 |#include <cstdlib>
14
   |#include <cstring>
15
   |#include <ctime>
   #include <cctype>
16
17
   #include <cmath>
18
   |#include <vector>
   |#include <queue>
20
   |#include <stack>
21
   #include <map>
22
   #include <set>
23
   #include <bitset>
24
   using namespace std;
25
   typedef bool boolean;
26
27
   typedef class Edge {
28
   public:
29
      int end;
30
      int next;
31
     int w;
32
33
     Edge(int end = 0, int next = -1, int w = 0):end(end), next(next), w(w) {
```

}

```
34
   |} Edge;
35
36
   const int N = 1e3, M = 1e5;
37
38
   typedef class MapManager {
39
   public:
40
      int cnt;
     int h[N + 5];
41
42
     Edge edge[M + 5];
43
44
     MapManager() {
     MapManager(int n):cnt(-1) {
45
46
   //
                  h = new int[(n + 1)];
47
   //
                  edge = new Edge[(m + 1)];
48
       memset(h, -1, sizeof(int) * (n + 1));
49
50
51
     inline void addEdge(int u, int v, int w) {
52
        edge[++cnt] = (Edge(v, h[u], w));
53
                  h[u] = (signed)edge.size() - 1;
54
        h[u] = cnt;
55
56
     inline int start(int node) {
57
58
        return h[node];
59
60
     Edge& operator [] (int pos) {
61
62
        return edge[pos];
63
64
   } MapManager;
65
   #define m_endpos -1
66
67
   int n, m;
68
   MapManager g;
69
   MapManager rg;
70
   int s, t, k;
71
   int ds[N + 5];
72
73
   inline void init() {
     scanf("%d%d", &n, &m);
74
75
     memset(g.h, -1, sizeof(int) * (n + 1));
76
     memset(rg.h, -1, sizeof(int) * (n + 1));
77
     for(int i = 1, u, v, w; i \le m; i++) {
        scanf("%d%d%d", &u, &v, &w);
78
79
        g.addEdge(u, v, w);
80
        rg.addEdge(v, u, w);
81
82
     scanf("%d%d%d", &s, &t, &k);
83
          ds = new int[(n + 1)];
84
85
86
   #define g rg
87
   #define f ds
88
   #define que que1
89
   boolean vis[N + 5];
90
   queue<int> que;
91
   boolean spfa(int s, int t) {
92
     memset(f, 0x7f, sizeof(int) * (n + 1));
93
     memset(vis, false, sizeof(boolean) * (n + 1));
94
     que.push(s);
95
     f[s] = 0;
96
     while(!que.empty()) {
97
        int e = que.front();
```

```
98
         que.pop();
 99
         vis[e] = false;
100
         for(int i = g.start(e); i != m_endpos; i = g[i].next) {
101
           int& eu = g[i].end;
                    cout << e << " " << eu << " " << i <<endl;</pre>
102
103
           if(f[e] + g[i].w < f[eu]) {
             f[eu] = f[e] + g[i].w;
104
105
             if(!vis[eu]) {
106
               que.push(eu);
107
               vis[eu] = true;
108
109
           }
         }
110
       }
111
       return (f[t] != 0x7f7f7f7f);
112
113
114
    #undef g
115
    #undef f
116
    #undef que
117
118
     typedef class Status {
119
    public:
120
       int node;
121
       int dis;
122
       int priority;
123
124
       Status(int node = 0, int dis = 0):node(node), dis(dis), priority(h()) {
                                                                                           }
125
126
       int h() {
127
         return dis + ds[node];
128
129
130
       boolean operator < (Status b) const {</pre>
131
         return priority > b.priority;
132
133
     } Status;
134
135
     int label[N + 5];
136
     priority_queue<Status> que;
137
     int bfs(int s, int t) {
138
       if(s == t)
                      k++;
139
           label = new int[(n + 1)];
140
       memset(label, 0, sizeof(int) * (n + 1));
141
       que.push(Status(s, 0));
142
       while(!que.empty()) {
143
         Status e = que.top();
144
         que.pop();
145
         label[e.node]++;
146
         if(e.node == t && label[e.node] == k)
147
           return e.dis;
148
         for(int i = g.start(e.node); i != m_endpos; i = g[i].next) {
           if(label[g[i].end] < k)
149
150
             que.push(Status(g[i].end, e.dis + g[i].w));
151
152
153
       return -1;
154
155
156
     inline void solve() {
157
       if(!spfa(t, s)) {
         puts("-1");
158
159
         return;
160
       printf("%d", bfs(s, t));
161
```

```
162
    |}
163
164
     int main() {
165
       init();
166
       solve();
167
       return 0;
168
     4.6 Maxflow
  1 | /* Max Flow Problem, by Abreto<m@abreto.net> */
     #include <bits/stdc++.h>
  4
    using namespace std;
  5
     #define MAXV
  6
                      (100000)
  7
     #define MAXE
                      (1000000)
  8
    struct edge {
  9
       static int N;
 10
       int v, w;
 11
       edge *n;
 12
       edge(void):v(0),w(0),n(NULL) {}
       edge(int vv, int ww, edge *nn):v(vv),w(ww),n(nn) {}
 13
 14
 15
    int nE;
     edge E[MAXE];
 16
     edge *front[MAXV];
 17
 18
     void add_edge(int u, int v, int w) {
 19
       int ne = ++nE;
 20
       E[ne] = edge(v, w, u[front]);
 21
       u[front] = \&(E[ne]);
 22
 23
     edge *find_edge(int u, int v) {
 24
       for(edge *e = u[front]; e != NULL; e = e->n)
 25
         if(e\rightarrow v == v)
 26
           return e;
 27
       return NULL;
 28
     void grant_e(int u, int v, int w) {
 29
       edge *e = find_edge(u, v);
 30
 31
       if(NULL == e) add_edge(u,v,w);
 32
       else e\rightarrow w += w;
 33
    }
 34
 35
     int vis[MAXV];
 36
     int path[MAXV];
 37
     int dfs(int u, int t) {
 38
       vis[u] = 1;
 39
       if(u == t) return 1;
 40
       for(edge *e = u[front]; e != NULL; e = e -> n) {
 41
         int v = e \rightarrow v;
 42
         if(!vis[v] \&\& e->w \&\& dfs(v,t)) {
 43
           path[u] = v;
 44
           return 1;
 45
         }
 46
 47
       return 0;
 48
 49
     int find_path(int s, int t) {
 50
       memset(vis, 0, sizeof(vis));
```

51

52 |}

return dfs(s,t);

```
53
    int max_flow(int s, int t) {
54
      int flow = 0;
55
      while(find_path(s,t)) {
 56
        int i = 0;
 57
        int minf = find_edge(s,path[s])->w;
58
        for(i = path[s]; i != t; i = path[i])
59
          minf = min(minf, find_edge(i,path[i])->w);
        for(i = s; i != t; i = path[i]) {
 60
61
          grant_e(i, path[i], -minf);
62
          grant_e(path[i], i, minf);
63
64
        flow += minf;
      }
65
66
      return flow;
 67
68
69
    /* Dinic */
    #define N 1000
 70
    #define INF 100000000
 71
 72
 73
    struct Edge {
74
      int from, to, cap, flow;
 75
      Edge(int u,int v,int c,int f):from(u),to(v),cap(c),flow(f) {}
 76
    };
 77
 78
    struct Dinic {
79
      int n,m,s,t;//结点数, 边数 (包括反向弧), 源点编号, 汇点编号
80
      vector<Edge>edges;//边表, dges[e]和dges[e^1]互为反向弧
      vector<int>G[N];//邻接表,G[i][j]表示结点i的第j条边在e数组中的编号
81
82
      bool vis[N]; //BFS的使用
      int d[N]; //从起点到i的距离
83
84
      int cur[N]; //当前弧下标
85
86
      void addedge(int from,int to,int cap) {
87
        edges.push_back(Edge(from,to,cap,0));
88
        edges.push_back(Edge(to,from,0,0));
89
        int m=edges.size();
90
        G[from].push_back(m-2);
91
        G[to].push_back(m-1);
92
93
94
      bool bfs() {
95
        memset(vis,0,sizeof(vis));
96
        queue<int>Q;
97
        Q.push(s);
98
        d[s]=0;
99
        vis[s]=1;
100
        while(!Q.empty()) {
101
          int x=Q.front();
102
          Q.pop();
          for(int i=0; i<G[x].size(); i++) {
103
104
            Edge&e=edges[G[x][i]];
105
            if(!vis[e.to]&&e.cap>e.flow) { //只考虑残量网络中的弧
106
              vis[e.to]=1;
107
              d[e.to]=d[x]+1;
              Q.push(e.to);
108
109
            }
110
          }
111
112
113
        return vis[t];
114
115
      int dfs(int x,int a) { //x表示当前结点, a表示目前为止的最小残量
116
```

```
117
        if(x==t||a==0)return a;//a等于0时及时退出,此时相当于断路了
118
        int flow=0,f;
119
        for(int&i=cur[x]; i<G[x].size(); i++) { //从上次考虑的弧开始, 注意要使用引用, 同
            时修改cur[x]
120
          Edge&e=edges[G[x][i]];//e是一条边
121
          if(d[x]+1==d[e.to]&&(f=dfs(e.to,min(a,e.cap-e.flow)))>0) {
122
            e.flow+=f;
123
            edges[G[x][i]^1].flow==f;
124
            flow+=f;
125
            a=f;
126
            if(!a)break;//a等于0及时退出, 当a!=0,说明当前节点还存在另一个曾广路分支。
127
128
          }
129
        }
130
        return flow;
131
      }
132
133
      int Maxflow(int s,int t) { //主过程
134
        this->s=s,this->t=t;
135
        int flow=0;
        while(bfs()) { //不停地用bfs构造分层网络, 然后用dfs沿着阻塞流增广
136
137
          memset(cur,0,sizeof(cur));
138
          flow+=dfs(s,INF);
139
140
        return flow;
141
      }
142
    };
143
144
    /* ISAP */
145
    struct Edge {
146
      int from, to, cap, flow;
147
148
    const int maxn=650;
    const int INF=0x3f3f3f3f;
149
150
    struct ISAP {
151
      int n,m,s,t;//结点数,边数(包括反向弧),源点编号,汇点编号
152
      vector<Edge>edges;
153
      vector<int>G[maxn];
154
      bool vis[maxn];
155
      int d[maxn];
156
      int cur[maxn];
157
      int p[maxn];
158
      int num[maxn];
159
      void AddEdge(int from,int to,int cap) {
160
        edges.push_back((Edge) {
161
          from, to, cap, 0
162
        });
163
        edges.push_back((Edge) {
164
          to, from, 0,0
165
        });
166
        m=edges.size();
167
        G[from].push_back(m-2);
168
        G[to].push_back(m-1);
169
      bool RevBFS() {
170
171
        memset(vis,0,sizeof(vis));
172
        queue<int>Q;
173
        Q.push(t);
174
        d[t]=0;
175
        vis[t]=1;
176
        while(!Q.empty()) {
177
          int x=Q.front();
178
          Q.pop();
179
          for(int i=0; i<G[x].size(); i++) {
```

```
180
             Edge &e =edges[G[x][i]^1;
             if(!vis[e.from]&&e.cap>e.flow) {
181
182
               vis[e.from]=1;
183
                d[e.from]=d[x]+1;
184
                Q.push(e.from);
185
             }
186
           }
187
188
         return vis[s];
189
190
       int Augment() {
191
         int x=t, a=INF;
         while(x!=s) {
192
193
           Edge &e = edges[p[x]];
194
           a= min(a,e.cap-e.flow);
           x=edges[p[x]].from;
195
196
197
         x=t;
198
         while(x!=s) {
199
           edges[p[x]].flow+=a;
200
           edges[p[x]^1].flow=a;
201
           x=edges[p[x]].from;
202
203
         return a;
204
205
       int Maxflow(int s,int t,int n) {
206
         this->s=s,this->t=t,this->n=n;
207
         int flow=0;
208
         RevBFS();
209
         memset(num,0,sizeof(num));
210
         for(int i=0; i<n; i++) {
211
           num[d[i]]++;
212
213
         int x=s;
214
         memset(cur,0,sizeof(cur));
         while(d[s]<n) {</pre>
215
216
           if(x==t) {
217
             flow+=Augment();
218
             x=s;
219
220
           int ok=0;
221
           for(int i=cur[x]; i<G[x].size(); i++) {
222
             Edge &e =edges[G[x][i]];
223
             if(e.cap>e.flow&d[x]==d[e.to]+1) {
224
                ok=1;
225
                p[e.to]=G[x][i];
226
                cur[x]=i;
227
               x=e.to;
228
               break;
             }
229
230
           if(!ok) {
231
232
             int m=n-1;
233
             for(int i=0; i<G[x].size(); i++) {</pre>
234
                Edge &e =edges[G[x][i]];
235
                if(e.cap>e.flow)
236
                 m=min(m,d[e.to]);
237
238
             if(--num[d[x]]==0)
239
               break;
240
             num[d[x]=m+1]++;
241
             cur[x]=0;
242
             if(x!=s)
243
               x=edges[p[x]].from;
```

```
244
           }
245
         }
246
         return flow;
247
248
249
     int main() {
250
       int n,m,a,b,c,res;
       while(scanf("%d%d",&m,&n)!=EOF) {
251
         ISAP tmp;
252
         for(int i=0; i<m; i++) {
253
254
           scanf("%d%d%d",&a,&b,&c);
255
           tmp.AddEdge(a,b,c);
256
         }
257
         res=tmp.Maxflow(1,n,n);
258
         printf("%d\n", res);
259
260
       return 0;
261 |}
```

## 4.7 Strongly Connected Component

```
//* Kosaraju */
 1
 2
   #define MAXN
                     10010
 3
   #define MAXM
                     100010
 4
   struct edge {
 5
      int v;
 6
      edge *n;
 7
      edge(void):v(0),n(NULL) {}
 8
      edge(int vv, edge *nn):v(vv),n(nn) {}
 9
10
   int nE;
11
   edge E[MAXM<<1];</pre>
12
   edge *ori[MAXN];
13
   edge *inv[MAXN];
14
   void add_edge(edge *front[], int u, int v) {
15
      int ne = ++nE;
16
      E[ne] = edge(v, u[front]);
      u[front] = \&(E[ne]);
17
18
19
   void connect(int u, int v) {
20
      add_edge(ori, u, v);
21
      add_edge(inv, v, u);
22
   }
23
24
   int vis[MAXN];
25
    int vst[MAXN];
26
   void first_dfs(int u, int &sig) {
27
      vis[u] = 1;
28
      for(edge *e = u[ori]; e; e = e \rightarrow n)
29
        if(!vis[e->v])
30
          first_dfs(e->v, sig);
31
      vst[++sig] = u;
32
    int mark[MAXN];
33
34
   void second_dfs(int u, int sig) {
35
      vis[u] = 1;
36
      mark[u] = sig;
37
      for(edge *e = u[inv]; e; e = e \rightarrow n)
        if(!vis[e->v])
38
39
          second_dfs(e->v, sig);
40
41
```

```
42
    |int N, M;
 43
 44
     int kosaraju(void) {
 45
       int i;
 46
       int sig = 0;
 47
       for(i = 0; i \le N; ++i) vis[i] = 0;
 48
       for(i = 1; i \le N; ++i) {
 49
         if(!vis[i])
           first_dfs(i, sig);
 50
 51
       }
 52
       sig = 1;
 53
       for(i = 0; i \le N; ++i) vis[i] = 0;
 54
       for(i = N; i > 0; --i) {
         if(!vis[vst[i]])
 55
 56
           second_dfs(vst[i], sig++);
 57
 58
       for(i = 1; i \le N; ++i)
 59
         if(mark[i] != 1)
 60
           return 0;
 61
       return 1;
 62
 63
 64
 65
     void clear(void) {
 66
       nE = 0;
 67
       for(int i = 0; i <= N; ++i) {
 68
         ori[i] = inv[i] = NULL;
 69
 70
 71
 72
     /* Tarjan */
 73
     #define MAXN
                      10010
 74
    #define MAXM
                      100010
 75
     struct edge {
 76
       int v;
 77
       edge *n;
 78
       edge(void):v(0),n(NULL) {}
 79
       edge(int vv, edge *nn):v(vv),n(nn) {}
 80
     };
 81
    typedef edge *ep;
 82
    int nE;
 83
     edge E[MAXM];
 84
     edge *front[MAXN];
 85
    void add_edge(int u, int v) {
 86
       int ne = ++nE;
 87
       E[ne] = edge(v, u[front]);
 88
       u[front] = \&(E[ne]);
 89
     }
 90
 91
     int mark[MAXN];
 92
     int dfn[MAXN], low[MAXN];
 93
     int stk[MAXN];
 94
     int stk_top;
 95
 96
     void tardfs(int u, int stamp, int &scc) {
       mark[u] = 1;
 97
98
       dfn[u] = low[u] = stamp;
 99
       stk[stk\_top++] = u;
100
       for(ep e = u[front]; e; e = e->n) {
101
         if(\emptyset == mark[e \rightarrow v]) tardfs(e \rightarrow v, ++stamp, scc);
102
         if(1 == mark[e \rightarrow v]) low[u] = min(low[u], low[e \rightarrow v]);
103
104
       if(dfn[u] == low[u]) {
105
         ++SCC;
```

```
106
         do {
107
           low[stk[stk_top-1]] = scc;
108
           mark[stk[stk\_top-1]] = 2;
109
         } while(stk[(stk_top--)-1] != u);
       }
110
111
112
113
     int tarjan(int n) {
       int scc = 0, lay = 1;
114
115
       for(int i = 1; i <= n; ++i)
116
         if(0 == mark[i])
117
           tardfs(i, lay, scc);
118
       return scc;
119
120
121
    int N, M;
122
123
    void clear(void) {
124
       nE = 0;
125
       for(int i = 0; i <= N; ++i) {
         i[front] = NULL;
126
         mark[i] = low[i] = 0;
127
128
129
       stk\_top = 0;
130
131
132
    /* Garbow */
133
    #define MAXN
                     10010
134
    #define MAXM
                     100010
135
136
    struct edge {
137
       int v;
138
       edge *n;
139
       edge(void):v(0),n(NULL) {}
140
       edge(int vv, edge *nn):v(vv),n(nn) {}
141
142
     typedef edge *ep;
143
144
     int nE;
     edge E[MAXM];
145
     edge *front[MAXN];
146
147
     void add_edge(int u, int v) {
148
       int ne = ++nE;
149
       E[ne] = edge(v, u[front]);
150
       u[front] = \&(E[ne]);
151
    }
152
    int stk1[MAXN], stk1t;
153
154
    int stk2[MAXN], stk2t;
    int low[MAXN], belg[MAXN];
155
156
     void garbowdfs(int u, int lay, int &scc) {
157
158
       stk1[++stk1t] = u;
159
       stk2[++stk2t] = u;
160
       low[u] = ++lay;
161
       for(ep e=u[front]; e; e = e->n) {
         if(!low[e->v]) garbowdfs(e->v, lay, scc);
162
163
         else if (0 == belg[e->v])
164
           while(low[stk2[stk2t]] > low[e->v])
165
             ---stk2t;
166
167
       if(stk2[stk2t] == u) {
         stk2t—;
168
169
         scc++;
```

```
170
171
           belg[stk1[stk1t]] = scc;
172
         } while(stk1[stk1t--] != u);
173
174
    }
175
176
     int grabow(int n) {
177
       int i;
178
       int scc = 0, lay = 0;
179
       for(i = 0; i <= n; ++i) {
180
         belg[i] = low[i] = 0;
181
182
       for(i = 1; i \le n; ++i)
         if(0 == low[i])
183
184
           garbowdfs(i, lay, scc);
185
       return scc;
186
    }
187
188
    int N, M;
189
190
    void clear(void) {
191
       nE = 0;
       for(int i = 0; i <= N; ++i) {
192
193
         front[i] = NULL;
194
195 |}
```

## 4.8 Perfect elimination ordering

求弦图的最大团数/最小色数的时候,只要在完美消除序列上从后往前贪心染色即可。而求最大独立集/最小团覆盖的时候,只要在完美消除序列上从前往后贪心取点即可。

```
1
   /**
 2
    * BZ0J 1006
 3
    * [HN0I2008] 神奇的国度
 4
    * 最大势法求完美消除序列
 5
    * by Abreto<m@abreto.net>.
 6
    **/
 7
   #include <cassert>
 8
   #include <cstdio>
   #include <vector>
 9
10
   #include <bitset>
11
   #include <algorithm>
12
13
   using namespace std;
   typedef vector<int> vi;
14
   typedef vi::iterator vii;
15
   #define pb push_back
16
17
   #define MAXN 10100
   #define MAXM 1000100
18
19
20
   int n;
21
   vi g[MAXN];
22
   int ans;
23
24
   struct node_t {
25
     int v;
26
     node_t *nxt;
27
   } node[MAXM << 2];</pre>
28
   |int used;
   node_t *new_node(void) {
29
30
     return node + (used ++);
31
```

```
32
33
   node_t *f[MAXN]; /* head */
34
   void lkto(int pos, int item) {
35
      node_t *t = new_node();
36
      t\rightarrow v = item;
37
      t->nxt = f[pos];
38
      f[pos] = t;
39
40
41
   int usedby[MAXN];
42
   int color[MAXN];
43
   bitset<MAXN> added;
44
   int label[MAXN], max_label;
45
   void mcs(void) {
      for (int i = 1; i \le n; i++) lkto(0, i);
46
47
      for (int i = n; i > 0; i---) {
48
        node_t *cur = f[max_label];
49
        assert(cur != NULL);
50
        while (added.test(cur->v)) { /* already added */
51
          cur = cur->nxt;
          while (NULL == cur)
52
53
            cur = f[ ---max_label ];
54
55
        f[ max_label ] = cur->nxt;
56
        while (max_label && NULL == f[max_label]) max_label—;
57
        int u = cur->v;
58
        added.set(u);
59
        /* the i-th is u */
        for (vii it = g[u].begin(); it != g[u].end(); it++) {
60
61
          int v = *it;
          if (!added.test(v)) {
62
63
            label[v] ++;
64
            max_label = max(max_label, label[v]);
65
            lkto(label[v], v);
66
          }
67
68
          usedby[color[v]] = i;
69
        for (int j = 1; j \le n; j++)
70
          if (usedby[j] != i) {
71
72
            color[u] = j;
73
            break;
74
75
        ans = max(ans, color[u]);
76
      }
77
   }
78
79
   int main(void) {
80
      int m;
      scanf("%d%d", &n, &m);
81
82
      while (m--) {
83
        int ai, bi;
        scanf("%d%d", &ai, &bi);
84
85
        g[ai].pb(bi);
        g[bi].pb(ai);
86
87
      }
88
      mcs();
      printf("%d\n", ans);
89
90
      return 0;
91
```

# 5 Math

### 5.1 Euler Function

```
| / * Euler function phi(x), by Abreto<m@abreto.net>. */
 2
 3
    #define MAXX
                       3000000
 4
 5
    int phi[MAXX];
 6
    void get_euler(void) {
 7
      int i = 0, j = 0;
 8
      phi[1] = 1;
 9
      for(i = 2; i < MAXX; ++i)
10
         if(!phi[i])
           for(j = i; j < MAXX; j += i) {
  if(!phi[j]) phi[j] = j;</pre>
11
12
13
             phi[j] = phi[j]/i * (i-1);
14
15 |}
```

### 5.2 Möbius Function

```
void sieve() {
 2
     fill(isPrime, isPrime + maxn, 1);
 3
     mu[1] = 1, num = 0;
     for (int i = 2; i < maxn; ++i) {
 4
 5
        if (isPrime[i]) primes[num++] = i, mu[i] = -1;
 6
        static int d;
 7
        for (int j = 0; j < num && (d = i * primes[j]) < maxn; ++j) {
 8
          isPrime[d] = false;
 9
          if (i % primes[j] == 0) {
            mu[d] = 0;
10
11
            break;
12
          } else mu[d] = -mu[i];
13
14
     }
15
```

### 5.3 Number Theory Inverse

```
|#include <bits/stdc++.h>
 1
 2
   using namespace std;
 3
 4
   const int n=10000000;
                                /* */
 5
   const long long mod=1e9+7; /* prime required. */
 7
   long long fact[n],fiv[n],inv[n];
 8
 9
    int main() {
10
      fact[0]=fact[1]=1;
      fiv[0]=fiv[1]=1;
11
12
      inv[1]=1;
13
      for (int i=2; i<n; i++) {
14
        fact[i]=fact[i-1]*i\%mod;
15
        inv[i]=(mod-mod/i)*inv[mod%i]%mod;
16
        fiv[i]=inv[i]*fiv[i-1]%mod;
17
      for (int i=1; i<n; i++) {
18
19
        if (fact[i]*fiv[i]%mod!=1) printf("fact\u00edwrong:\u00ed%d\n",i);
20
        if (inv[i]*i%mod!=1)
                                     printf("intv_wrong: _\%d\n",i);
21
      }
```

```
22 | cout<<"complete"<<endl;
23 | return 0;
24 |}</pre>
```

#### 5.4 Chinese Remainder Theorem

```
x \equiv a_i \pmod{m_i}
   /* Chinese Remainder Theorem, by Abreto<m@abreto.net>. */
 2
   #include "euler.c"
 3
 4
   #define MAXN
                     64
 5
 6
   typedef long long int 11;
 7
 8
    11 quickpow(ll a, ll b, ll mod) {
9
      ll ret = 1, base = a;
10
      while(b > 0) {
11
        if(b \& 1) ret = (ret * base) % mod;
12
        base = (base * base) % mod;
13
        b >>= 1;
14
15
      return ret;
16
17
18
   11 N;
   ll a[MAXN], m[MAXN]; /* a and m is indexed from 0. */
19
20
   11 \times, M;
21
22
   void naive_crt(void) {
23
      int i = 0;
24
      11 Mi[MAXN], nMi[MAXN];
25
      11 t[MAXN];
26
27
      M = 1;
28
      for(i = 0; i < N; ++i)
29
        M *= a[i];
      for(i = \overline{0}; i < N; ++i)
30
        Mi[i] = M / a[i];
31
32
      get_euler();
33
      for(i = 0; i < N; ++i)
        nMi[i] = quickpow(Mi[i], phi[a[i]]-1, a[i]);
34
35
      for(i = 0; i < N; ++i)
36
        t[i] = ((a[i] * Mi[i]) % M) * nMi[i] % M;
37
      for(i = 0; i < N; ++i)
38
        x = (x + t[i]) % M;
39 |}
```

#### 5.5 Linear congruences

```
#include <cstdio>
#include <cassert>
#include <cstdlib>

using namespace std;

class mod_equ_resolver {
 typedef long long int ll;
 ll a, m;
 inline void gurantee(void) {
```

```
11
        if ( a < 0 ) {
12
          11 k = (-a) / m;
13
          a += (k + 111) * m;
14
          a = (a + m) \% m;
15
        } else {
16
          a \% = m;
17
        // printf("x = %11d \pmod %11d\n",a, m);
18
19
20
   public:
21
      mod_equ_resolver(void) {
22
        a = 011;
23
        m = 111;
24
25
      ll exgcd(ll m, ll n, ll &x, ll &y) {
26
        if (0 == n) {
27
          x = 1;
28
          y = 0;
29
          return m;
30
31
        ll g = exgcd(n, m % n, x, y);
32
        11 t = x;
33
        x = y;
        y = t - m / n * y;
34
35
        return g;
36
37
      int onemore(ll a2, ll m2) {
38
        11 x, y;
39
        ll g = exgcd(m, m2, x, y);
        assert(x*m+y*m2==g);
40
        a2 = (a2 + m2) \% m2;
41
42
        if ( abs(a2 - a) \% g ) return -1;
43
        ll newm = m / g * m2;
44
        11 newa = a + (a2 - a) / g * x * m;
45
        a = newa;
46
        m = newm;
47
        gurantee();
48
        return 0;
49
50
      ll resolve(void) {
51
        return a;
52
53 | };
    Usage: For
                                           x \equiv a_1 \mod m_1
                                           x \equiv a_2
                                                   \mod m_2
                                                  \mod m_k
                                           x \equiv a_k
   run
 1 |mod_equ_resolver solver;
   for (int i = 1; i \le k; i++)
 2
        solver.onemore(a[i], m[i]);
   then the solution is
                                    x \equiv solver.a \mod solver.m
   5.6 FFT
 1 |#include <cmath>
```

2 using namespace std;

```
3
   namespace fft {
 4
   #define eps (1e-9)
 5
   template < typename T = double >
   struct dbl {
 6
 7
      T x;
     dbl(void):x(0.0) {}
 8
 9
     template <typename U>
10
     dbl(U a):x((T)a) {}
11
     inline char sgn(void) {
12
        return ((x>=-eps)&&(x<=eps))?(0):((x>eps)?(1):(-1));
13
14
     inline T tabs(void) {
        return ((x>=-eps)&&(x<=eps))?(0.0):((x>eps)?(x):(-x));
15
16
     inline dbl abs(void) {
17
18
        return dbl(tabs());
19
20
     template <typename U> inline dbl &operator=(const U b) {
21
22
        return (*this);
23
24
     inline T *operator&(void) {
25
        return &x;
26
27
     inline dbl operator—(void) const {
28
        return dbl(-x);
29
30
     inline dbl operator+(const dbl &b) const {
31
        return dbl(x+b.x);
32
     inline dbl operator—(const dbl &b) const {
33
34
        return dbl(x-b.x);
35
     inline dbl operator*(const dbl &b) const {
36
37
        return dbl(x*b.x);
38
39
     inline dbl operator/(const dbl &b) const {
40
        return dbl(x/b.x);
41
     template <typename U> inline dbl operator^(const U &b) const {
42
43
        T ret=1.0, base=x;
44
        while(b) {
45
          if(b&1)ret*=base;
46
          base*=base;
47
          b>>=1;
48
49
        return dbl(ret);
50
51
     inline dbl operator+=(const dbl &b) {
52
        return dbl(x+=b.x);
53
54
     inline dbl operator-=(const dbl &b) {
55
        return dbl(x-=b.x);
56
57
     inline dbl operator*=(const dbl &b) {
58
        return dbl(x*=b.x);
59
60
     inline dbl operator/=(const dbl &b) {
61
        return dbl(x/=b.x);
62
     template <typename U> inline dbl operator^=(const U &b) {
63
64
        dbl tmp=(*this)^b;
65
        *this=tmp;
66
        return tmp;
```

```
67
 68
      inline bool operator==(const dbl &b) const {
 69
         return (0 == ((*this)-b).sgn());
 70
 71
      inline bool operator!=(const dbl &b) const {
 72
         return (0 != ((*this)-b).sqn());
 73
 74
      inline bool operator<(const dbl &b) const {
 75
         return (-1 == ((*this)-b).sgn());
 76
 77
       inline bool operator<=(const dbl &b) const {
 78
         return (((*this)==b) || ((*this)<b));
 79
 80
      inline bool operator>(const dbl &b) const {
 81
         return (b < (*this));
 82
 83
      inline bool operator>=(const dbl &b) const {
 84
         return (((*this)==b) || ((*this)>b));
 85
 86
      template <typename U> inline operator U() const {
 87
         return (U)x;
 88
 89
      inline char operator[](unsigned n) {
         if(n >= 0) {
 90
 91
           long long int ret=x;
 92
           while(n--) {
 93
             ret/=10;
 94
 95
           return (ret%10);
 96
         } else {
 97
           T ret=x;
 98
           n=-n;
 99
           while(n—)ret*=10.0;
100
           return ((long long int)ret)%10;
101
         }
102
103
104
    template <typename T>
105
    struct Complex {
106
       T x,y; /* x + iy */
107
      Complex(void):x(T()),y(T()) {}
108
      Complex(T xx):x(xx) {}
      Complex(T xx,T yy):x(xx),y(yy) {}
109
110
      inline Complex operator—(void) const {
111
         return Complex(-x,-y);
112
113
      inline Complex operator+(const Complex& b) const {
114
         return Complex(x+b.x,y+b.y);
115
116
      inline Complex operator—(const Complex& b) const {
117
         return Complex(x-b.x,y-b.y);
118
119
      inline Complex operator*(const Complex& b) const {
120
         return Complex(x*b.x-y*b.y,x*b.y+y*b.x);
121
       inline Complex operator/(const Complex& b) const {
122
123
         T bo=b.x*b.x+b.y*b.y;
124
         return Complex((x*b.x+y*b.y)/bo,(y*b.x-x*b.y)/bo);
125
126
      inline Complex& operator+=(const Complex& b) {
127
         Complex tmp=(*this)+b;
128
         (*this)=tmp;
129
         return (*this);
130
      }
```

```
131
      inline Complex& operator—=(const Complex& b) {
132
         Complex tmp=(*this)_b;
133
         (*this)=tmp;
134
         return (*this);
135
136
      inline Complex& operator*=(const Complex& b) {
137
         Complex tmp=(*this)*b;
138
         (*this)=tmp;
139
         return (*this);
140
141
      inline Complex& operator/=(const Complex& b) {
142
         Complex tmp=(*this)/b;
143
         (*this)=tmp;
144
         return (*this);
145
146
      inline friend Complex operator+(const T& a, const Complex& b) {
147
         return Complex(a)+b;
148
149
      inline friend Complex operator—(const T& a, const Complex& b) {
150
         return Complex(a)_b;
151
152
      inline friend Complex operator*(const T& a, const Complex& b) {
153
         return Complex(a)*b;
154
155
      inline friend Complex operator/(const T& a, const Complex& b) {
156
         return Complex(a)/b;
157
158
    };
159
     typedef dbl<> Double;
     typedef Complex<Double> ComplexD;
160
     typedef long long int ll;
161
162
     const int maxn = 2000000; /* !! */
163
     const Double pi(acos(-1.0));
164
    void build(ComplexD _P[], ComplexD P[], int n, int m, int curr, int &cnt) {
165
      if(m == n) {
166
167
         _P[curr] = P[cnt++];
168
      } else {
         build(_P, P, n, m*2, curr, cnt);
169
170
         build(_P, _P, _n, _{m*2}, _{curr+m}, _{cnt});
171
172
173
174
    void FFT(ComplexD P[], int n, int oper) { /* n should be 2^k. */
       static ComplexD _P[maxn];
175
176
       int cnt = 0;
      build(_P, P, n, 1, 0, cnt);
177
178
      copy(P, P+n, P);
179
       for(int d = 0; (1<<d)<n; ++d) {
         int m = 1 << d;
180
181
         int m2 = m*2:
182
         Double p0 = pi / m * oper;
183
         ComplexD unit_p0(cos(p0.x), sin(p0.x));
         for(int i = 0; i < n; i += m2) {
184
185
           ComplexD unit(1,0);
           for(int j = 0; j < m; ++j) {
186
             ComplexD &P1 = P[i+j+m], &P2 = P[i+j];
187
             ComplexD t = unit * P1;
188
189
             P1 = P2 - t;
190
             P2 = P2 + t;
191
             unit *= unit_p0;
192
           }
193
         }
194
      }
```

```
195
      if(-1 == oper) {
196
         for(int i = 0; i < n; ++i)
197
           P[i] /= Double(n);
198
      }
199
200
    5.7 NTT
  1
    |#include<bits/stdc++.h>
    #define ll long long
  2
    const int N=262144;
    const ll MOD=50000000001507329LL;//998244353 1004535809
  5
    using namespace std;
  6
    int n,m;
     ll a[N],b[N],x[N],y[N];
  7
  8
    ll wn[25];
  9
    ll Mul(ll x, ll y) { //乘法超ll用快速乘, 主函数也需要用
 10
       ll ans=(x*y-(ll)((long double)x/MOD*y+1e-8)*MOD);
 11
       return ans<0?ans+MOD:ans;</pre>
 12
    ll Qpow(ll a,ll b,ll M) {
 13
 14
      ll ans=1;
 15
      a\%=M;
      while(b) {
 16
 17
         if(b&1) ans=Mul(ans,a);
 18
         a=Mul(a,a);
 19
         b>>=1;
 20
 21
      return ans;
 22
 23
     void Getwn() { //主函数预处理getwn()
 24
       for(int i=0; i<25; i++) {
 25
         wn[i]=Qpow(3,(MOD-1)/(1<< i),MOD);
 26
 27
 28
    void NTT(ll *x,int n,int rev) {
 29
       int i,j,k,ds;
 30
      ll w,u,∨;
 31
      for(i=1, j=n>>1, k=n>>1; i<n-1; i++, k=n>>1) {
 32
         if(i<j) swap(x[i],x[j]);
 33
         while(j>=k) j==k,k>>=1;
 34
         if(j< k) j+=k;
 35
       for(i=2,ds=1; i<=n; i<<=1,ds++) {
 36
 37
         for(j=0; j< n; j+=i) {
           w=1;
 38
 39
           for(k=j; k< j+i/2; k++) {
 40
             u=x[k];
 41
             v=Mul(w,x[k+i/2]);
 42
             x[k]=(u+v)%MOD;
 43
             x[k+i/2]=(u-v+MOD)%MOD;
 44
             w=Mul(w,wn[ds]);
 45
           }
         }
 46
 47
 48
      if(rev==-1) {
         for(i=1; i<n/2; i++) swap(x[i],x[n-i]);
 49
 50
         w=Qpow(n,MOD-2,MOD);
 51
         for(i=0; i<n; i++) x[i]=Mul(x[i],w);
 52
 53
    }
```

```
54
    int main() {
 55
       Getwn();
       while(~scanf("%d%d",&n,&m)) {
 56
         for(int i=0; i<n; i++)scanf("%lld",&a[i]);
 57
 58
         for(int i=0; i<m; i++)scanf("%lld",&b[i]);</pre>
 59
         int len=1,s=n+m;
 60
         while(len<s)len<<=1;</pre>
         for(int i=n; i<len; i++)a[i]=0;
for(int i=m; i<len; i++)b[i]=0;</pre>
 61
 62
 63
         NTT(a, len, 1);
 64
         NTT(b, len, 1);
 65
         for(int i=0; i<len; i++)a[i]=Mul(a[i],b[i]);
 66
         NTT(a, len, -1);
         for(int i=0; i<=s; i++)printf("%lld_{\perp}",a[i]);
 67
 68
         puts("");
 69
       }
 70
     }
 71
 72
 73
     #include<cstdio>
 74
    #include<iostream>
 75
    #include<cstring>
 76
     #include<cmath>
 77
     #include<complex>
 78
     using namespace std;
 79
     typedef long long LL;
 80
     const LL MOD=998244353, q=3, qi=332748118;
 81
    const LL N=1000005;
 82
    LL n,m;
    LL a[N],b[N];
 83
     LL pow (LL x,LL y) {
 84
 85
       if (y==1) return x;
 86
       LL lalal=pow(x,y>>1);
 87
       lalal=lalal*lalal%MOD;
 88
       if (y&1) lalal=lalal*x%MOD;
 89
       return lalal;
 90
 91
     void ntt (LL *a,LL n,LL o) {
 92
       if (n==1) return;
 93
       LL k=(n>>1);
 94
       LL w=1, wn=pow(o==1?g:gi, (MOD-1)/n), a0[k], a1[k];
 95
       for (LL u=0; u< k; u++) {
 96
         LL i=u*2;
 97
         a0[u]=a[i]
 98
         a1[u]=a[i+1];
 99
100
       ntt(a0,k,o);
101
       ntt(a1,k,o);
102
       for (LL u=0; u< k; u++) {
         a[u]=a0[u]+w*a1[u]%MOD;
103
         a[u]=(a[u]%MOD+MOD)%MOD;
104
105
         a[u+k]=a0[u]-w*a1[u];
106
         a[u+k]=(a[u+k]%MOD+MOD)%MOD;
107
         w=w*wn%MOD;
108
109
110
     void ntt(LL *a,LL n,LL op) {
111
       for (LL u=0; u<n; u++) bin[u]=(bin[u>>1]>>1)|((u&1)*(n>>1));
112
       for (LL u=0; u<n; u++) if (u<bin[u]) swap(a[u],a[bin[u]]);
113
       for (LL u=1; u<n; u<<=1) {
         LL wn=pow(op==1?g:gi,(MOD-1)/(u<<1)),w,t;
114
115
         for (LL i=0; i< n; i=i+(u<<1)) {
116
           w=1;
           for (LL k=0; k<u; k++) {
117
```

```
118
             t=w*a[u+i+k]%MOD;
119
             a[u+i+k]=(a[i+k]-t+MOD)%MOD;
120
             a[i+k]=(a[i+k]+t)%MOD;
121
             w=w*wn%MOD;
122
           }
         }
123
124
125
       if(op==-1) {
126
         LL Inv=pow(n,MOD-2);
127
         for(LL i=0; i<n; i++) a[i]=a[i]*Inv%MOD;
128
129
130
131
     int main() {
       scanf("%I64d%I64d",&n,&m);
132
133
       for (LL u=0; u<=n; u++) scanf("%I64d",&a[u]);
134
       for (LL u=0; u<=m; u++) scanf("%I64d",&b[u]);
135
       m=m+n;
136
       n=1;
137
       while (n \le m) n \le 1;
       ntt(a,n,1);
138
139
       ntt(b,n,1);
       for (LL u=0; u<=n; u++) a[u]*=b[u];
140
141
       ntt(a,n,-1);
       LL inv=pow(n,MOD-2);
142
143
       for (LL u=0; u<=m; u++)
                                   printf("%I64d<sub>\\_</sub>",a[u]*inv\%MOD);
144
       return 0;
145
```

## 5.8 Fast Walsh-Hadamard transform

异或

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\} + \mathcal{F}\{A_1\}, \mathcal{F}\{A_0\} - \mathcal{F}\{A_1\}]$$
$$\mathcal{F}^{-1}\{A\} = \left[\mathcal{F}^{-1}\{\frac{A_0 + A_1}{2}\}, \mathcal{F}^{-1}\{\frac{A_0 - A_1}{2}\}\right]$$

• 按位与

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\} + \mathcal{F}\{A_1\}, \mathcal{F}\{A_1\}]$$
$$\mathcal{F}^{-1}\{A\} = [\mathcal{F}^{-1}\{A_0\} - \mathcal{F}^{-1}\{A_1\}, \mathcal{F}^{-1}\{A_1\}]$$

• 按位或

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\}, \mathcal{F}\{A_1\} + \mathcal{F}\{A_0\}]$$
$$\mathcal{F}^{-1}\{A\} = [\mathcal{F}^{-1}\{A_0\}, \mathcal{F}^{-1}\{A_1\} - \mathcal{F}^{-1}\{A_0\}]$$

```
void FWT(int a□,int n) {
 2
      for(int d=1; d<n; d<<=1)
 3
        for(int m=d << 1, i=0; i < n; i+=m)
           for(int j=0; j<d; j++) {
  int x=a[i+j],y=a[i+j+d];</pre>
 4
 5
 6
             a[i+j]=(x+y)\mod, a[i+j+d]=(x-y+mod)\mod;
 7
             //xor:a[i+j]=x+y,a[i+j+d]=(x-y+mod)%mod;
 8
             //and:a[i+j]=x+y;
 9
             //or:a[i+j+d]=x+y;
10
11
12
    void UFWT(int a□,int n) {
13
      for(int d=1; d<n; d<<=1)
14
15
        for(int m=d<<1,i=0; i<n; i+=m)
```

```
16
          for(int j=0; j<d; j++) {
17
            int x=a[i+j], y=a[i+j+d];
18
            a[i+j]=1LL*(x+y)*rev%mod, a[i+j+d]=(1LL*(x-y)*rev%mod+mod)%mod;
19
            //xor:a[i+j]=(x+y)/2,a[i+j+d]=(x-y)/2;
20
            //and:a[i+j]=x-y;
21
            //or:a[i+j+d]=y-x;
22
23
24
   void solve(int a[],int b[],int n) {
25
     FWT(a,n);
26
     FWT(b,n);
27
     for(int i=0; i<n; i++) a[i]=1LL*a[i]*b[i]%mod;
28
     UFWT(a,n);
29 |}
   5.9 Lucas
   /* Lucas, by Abreto<m@abreto.net>. */
 1
 2
 3
   struct __lucas {
 4
     static const int maxp = 100000;
 5
     typedef long long int 11;
 6
      int p;
     int f[maxp]; // fiv[maxp], inv[maxp];
 7
 8
     inline int mul(const int a, const int b) {
 9
        ll z = 1ll * a * b;
10
        z = z / p * p;
11
        return z;
12
13
     int qow(int a, int x) {
14
        int ret = 1;
15
        while (x) {
16
          if (1 \& x) ret = mul(ret, a);
17
          a = mul(a, a);
18
          x >>= 1;
19
        }
20
       return ret;
21
22
     void init(int np) {
23
        p = np;
        // return; // uncomment this line if use binom()
24
25
        f[0] = f[1] = 1;
26
        // fiv[0] = fiv[1] = 1;
27
        // inv[1] = 1;
        for (int i = 2; i < p; i++) {
28
29
          f[i] = mul(f[i - 1], i);
30
          // inv[i] = mul(p - p / i, inv[p % i]);
31
          // fiv[i] = mul(fiv[i - 1], inv[i]);
32
33
     int C(int n, int k) {
34
35
        if (n < k) return 0;
36
        return mul(f[n], qow(mul(f[k], f[n - k]), p - 2));
37
38
     /** use following if get TLE { */
39
     int binom(int n, int k) {
40
        if (n < k) return 0;
41
        if (k > n - k) k = n - k;
42
        int a = 1, b = 1;
43
        while (k) {
          a = mul(a, n);
44
45
          b = mul(b, k);
```

```
46
         n--;
47
         k--;
48
49
       return mul(a, qow(b, p - 2));
50
     }
51
     /** } ---- */
52
     int operator()(int n, int k) {
53
       if (0 == k) return 1;
54
        if (n  return <math>C(n, k);
55
        return mul(C(n % p, k % p), (*this)(n / p, k / p));
56
  |} lucas;
57
```

## 5.10 Linear Programming

```
|/* 线性规划 */
2
   #include<bits/stdc++.h>
3
4
   using namespace std;
5
   const int Maxn=110, Maxm=59;
6
   class Simplex {
7
     /*
8
        功能:
9
        接受有n个约束, m个基本变量的方程组a[0~n][0~m]
10
        a[0][]存放需要最大化的目标函数, a[][0]存放常数
        Base □ 存放基本变量的id,初始为1~m
11
12
        Rest □ 存放松弛变量的id,初始为m+1~m+n
13
        返回此线性规划的最小值ans
        要求方案的话,Base[]中的变量值为0,Rest[]中的变量值为相应行的[0]
14
15
        如果solve
16
        返回1,说明运行正常ans是它的最大值
17
        返回0,说明无可行解
18
        返回-1,说明解没有最大值
19
        测试:
20
        m=2, n=3
21
        double a[4][3]={
22
        {0,1,3},
        \{8,-1,1\},
23
        {-3,1,1},
24
25
        \{2,1,-4\}
26
        };
27
        solve=1, ans=64/3;
28
        注意ac不了可能是eps的问题
29
      */
30
   public:
31
     static const double Inf;
32
     static const double eps;
33
     int n,m;
34
     double a[Maxn][Maxm];
35
     int Base[Maxm], Rest[Maxn];
36
     double val[Maxm];
37
     double ans;
38
     void pt() {
39
       for(int i=0; i<=n; i++) {
40
         for(int j=0; j<=m; j++)printf(^{\infty}.2f<sub>\(\dot\)</sub>,a[i][j]);
         puts("");
41
42
       }
43
44
     void pivot(int x,int y) { //将第x个非基本变量和第y个基本变量调换
45
       swap(Rest[x],Base[y]);
46
       double tmp=-1./a[x][y];
47
       a[x][y]=-1.;
```

```
48
         for(int j=0; j<=m; j++)a[x][j]*=tmp;
 49
         for(int i=0; i<=n; i++) {
 50
           if(i==x||fabs(a[i][y])<eps)continue;</pre>
 51
           tmp=a[i][y];
 52
           a[i][y]=0;
           for(int j=0; j<=m; j++)a[i][j]+=tmp*a[x][j];
 53
 54
         }
 55
 56
       bool opt() {
 57
         while(1) {
 58
           int csi=0;
 59
           for(int i=1; i<=m; i++)if(a[0][i]>eps&&(!csi||Base[i]<Base[csi]))csi=i;
 60
           if(!csi)break;
 61
           int csj=0;
           double cur;
 62
 63
           for(int j=1; j<=n; j++) {
 64
             if(a[j][csi]>-eps)continue;
 65
             double tmp=-a[j][0]/a[j][csi];
 66
             if(!csj||tmp+eps<cur||(fabs(tmp-cur)<eps&&Rest[j]<Rest[csj]))csj=j,cur=tmp;
 67
 68
           if(!csj)return 0;
 69
           pivot(csj,csi);
 70
 71
         ans=a[0][0];
 72
         return 1;
 73
       }
 74
       bool init() {
 75
         ans=0:
 76
         for(int i=1; i<=m; i++)Base[i]=i;
         for(int i=1; i<=n; i++)Rest[i]=m+i;</pre>
 77
 78
         int cs=1;
 79
         for(int i=2; i<=n; i++)if(a[i][0]<a[cs][0])cs=i;
 80
         if(a[cs][0]>=-eps)return 1;
 81
         static double tmp[Maxm];
 82
         for(int i=0; i<=m; i++)tmp[i]=a[0][i],a[0][i]=0;
         for(int i=1; i<=n; i++)a[i][m+1]=1.;
 83
 84
         a[0][m+1]=-1.;
 85
         Base[m+1]=m+n+1;
 86
         pivot(cs,++m);
 87
         opt();
 88
         m--;
 89
         if(a[0][0]<-eps)return 0;
 90
         cs=-1;
 91
         for(int i=1; i<=n; i++) {
 92
           if(Rest[i]>m+n) {
 93
             cs=i;
 94
             break;
 95
           }
 96
 97
         if(cs>=1) {
 98
           int nxt=-1;
 99
           m++;
           for(int i=1; i <= m; i++)if(a[cs][i] > eps | |a[cs][i] < -eps) {
100
101
               nxt=i;
102
               break;
103
104
           pivot(cs,nxt);
105
106
107
         for(int i=1; i<=m; i++) {
108
           if(Base[i]>m+n) {
109
             swap(Base[i],Base[m+1]);
110
             for(int j=0; j<=n; j++)a[j][i]=a[j][m+1];
111
             break;
```

```
112
           }
113
         for(int i=1; i<=m; i++)a[0][i]=0;
114
115
         a[0][0]=tmp[0];
         for(int i=1; i<=m; i++)if(Base[i]<=m)a[0][i]=tmp[Base[i]];
116
117
         for(int i=1; i<=n; i++) {
118
           if(Rest[i]<=m) {
119
             for(int j=0; j<=m; j++)a[0][j]+=tmp[Rest[i]]*a[i][j];
120
121
122
         return 1;
123
       }
124
       void getval() {
125
         for(int i=1; i<=m; i++)val[i]=0;
         for(int i=1; i<=n; i++)if(Rest[i]<=m)val[Rest[i]]=a[i][0];
126
         //for(int i=1;i<=m;i++)printf("%.2f ",val[i]);puts("");</pre>
127
128
129
       int solve() {
130
         if(!init())return 0;
131
         if(!opt())return -1;
132
         getval();
133
         return 1;
134
135
     } solver;
136
     const double Simplex:: Inf=1e80;
137
     const double Simplex:: eps=1e-8;
138
     int main() {
139
       int m,n,type;
       scanf("%d%d%d",&m,&n,&type);
140
141
       solver.a[0][0]=0;
       for(int i=1; i<=m; i++)scanf("%lf",&solver.a[0][i]);</pre>
142
143
       for(int i=1; i<=n; i++) {
144
         for(int j=1; j<=m+1; j++) {
           if(j==m+1)scanf("%lf",&solver.a[i][0]);
145
146
           else {
             scanf("%lf",&solver.a[i][j]);
147
148
             solver.a[i][j]=-solver.a[i][j];
149
         }
150
151
152
       solver.m=m, solver.n=n;
153
       int rep=solver.solve();
154
       if(rep==0)puts("Infeasible");
155
       else if(rep==-1)puts("Unbounded");
156
       else {
         printf("%.12f\n", solver.ans);
157
158
         if(type==1) {
159
           for(int i=1; i<=m; i++)printf("%.12f%c",solver.val[i],i==m?'\n':'\_');
160
161
       }
    1
162
     5.11 Big Prime Test
  1 |#include <iostream>
  2
    #include <cstdlib>
  3
    using namespace std;
    typedef long long LL;
    LL minfactor, p[11] = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\};
    LL gcd(LL a, LL b) {
  7
       return b? gcd(b, a \% b) : a;
  8
    }
```

```
9
   LL qmult(LL a, LL b, LL mod) { // 快速乘模
10
     LL sum = 0;
11
     while (b) {
12
       if (b & 1) {
13
        sum += a;
14
        if (sum >= mod) sum -= mod; // 此处无需用%, %运算比减法慢很多
15
16
       b >>= 1, a <<= 1;
17
       if (a >= mod) a == mod;
18
19
     return sum;
20
21
   LL qpow(LL a, LL b, LL mod) { // 快速幂模
22
     LL res = 1;
23
     while (b) {
24
      if (b \& 1) res = qmult(res, a, mod);
25
       b >>= 1:
26
       a = qmult(a, a, mod);
27
28
     return res;
29
30
   bool prime_test(LL n, LL a) { // 对整数n, 底数a进行测试, 返回true表示通过测试
31
     LL p = qpow(a, n - 1, n);
     if (p != 1) return false;
32
33
     else { // 二次探测
34
       LL s = n - 1;
35
       while (!(s & 1) && p == 1) {
36
        s >>= 1;
37
        p = qpow(a, s, n);
38
39
       if (p == 1 \mid | p == n - 1) return true;
40
       else return false;
41
     }
42
   bool Miller_Rabin(LL n) { // 对整数n进行Miller_Rabin素数测试,返回true表示通过测试
43
     if (n <= 29) { // if这一块其实可以不用
44
       for (int i = 0; i < 10; i++) {
45
46
        if (n == p[i]) return true;
47
48
      return false;
49
50
     for (int i = 0; i < 10; i++) { // 利用前10个素数作为底数测试的正确率已经非常高
51
      if (gcd(n, p[i]) == 1 && !prime_test(n, p[i])) return false;
52
53
     return true;
54
55
   LL randf(LL x, LL n, LL c) { // 满足要求的产生伪随机数函数
56
     return (qmult(x, x, n) + c) \% n;
57
   LL pollard_rho(LL n, LL c) { // 查找n的因数, c为上面函数要用的随机数, c也可自己指定
58
       (但要有变化)
59
     LL x = rand() % n, y = x, i = 1, k = 2, p; // 随机生成随机数的初始值,也可自己指定
60
     while (true) {
       i++;
61
       x = randf(x, n, c);
62
63
       p = gcd(y - x + n, n);
64
       if (p > 1 \&\& p < n) return p;
       if (y == x) return n;
65
                           // 判圈,返回n表示查找失败,要更新随机种子重新查找
66
       if (i == k) {
67
        y = x; // 更新范围和记录的数
68
        k <<= 1;
69
       }
70
     }
71
```

```
UESTC_Jungle
72
   void find_factor(LL n) { // 查找所有因数
73
     if (Miller_Rabin(n)) {
74
       minfactor = min(minfactor, n);
75
        return ;
76
77
     LL p = n;
     while (p == n) p = pollard_rho(n, rand() % (n - 1) + 1); // 查找失败则更新随机种子
78
         重新查找,直到找到因子
      find_factor(p);
79
                         // 递归查找更小因子
80
      find_factor(n / p);
81
82
83
   int main() {
84
     int t;
85
     cin >> t;
86
     while (t--) {
87
       LL N;
88
       cin >> N;
89
       if (Miller_Rabin(N)) cout << "Prime" << endl;</pre>
90
       else {
91
         minfactor = N;
92
         find_factor(N);
93
          cout << minfactor << endl;</pre>
94
95
96
     return 0;
97 |}
   5.11.1 Miller Rabin
   |/* Miller-Rabin Prime Test, by Abreto<m@abreto.net>. */
 2
 3
   namespace miller_rabin {
 5
   typedef long long int ll;
 6
 7
   inline II add(const II a, const II b, const II mod) {
     11 z = a + b;
 8
 9
     if (z >= mod) z == mod;
10
     return z;
11
```

12

13

14 15

16 17

18

19

20 21

22 23 24

25

26

27

28

29

30 31

32 }

33

11 z = 0;

while (b) {

b >>= 1;

ll ret = 1ll;

while (x) {

x >>= 1;

return ret;

return z;

if (a >= mod) a %= mod;

if  $(b \ge mod)$  b %= mod;

a = add(a, a, mod);

ll qow(ll a, ll x, ll mod) {

a = mul(a, a, mod);

inline ll mul(ll a, ll b, const ll mod) {

if (1 & b) z = add(z, a, mod);

if (1 & x) ret = mul(ret, a, mod);

```
34
   const int K = 5;
35
   const int p[] = {
36
      2, 3, 7, 61, 24251
37
   };
38
   const ll strong = 4685624825598111;
39
    /* 46 856 248 255 981 in (0, 1e16) */
40
41
    bool mr(ll n, int k) {
42
      ll d = n - 1;
43
      int s = 0;
44
      while (d > 1 \&\& 0 == (d \& 1))  {
45
        S++;
46
        d >>= 1;
47
      for (int i = 0; i < k; i++) {
48
49
        ll a = (i < K) ? p[i] : (1 + rand() % (n - 1));
50
        11 x = qow(a, d, n);
51
        for (int j = 0; j < s; j++) {
52
          ll xp = mul(x, x, n);
53
          if (1 == xp \&\& x != 1 \&\& x != n-1) return false;
54
55
56
        if (x != 1) return false;
57
58
      return true;
59
60
61
   /* 2,3,5,7,11,13 */
    const int pre[] = \{3, 5, 7, 11, 13\};
62
   bool test(ll n, int k = 5) {
63
64
      if (2 == n) return true;
65
      if (0 == (n \& 1)) return false;
      if (strong == n) return false;
66
      for (int i = 0; i < 5; i++) {
67
68
        if (n == pre[i]) return true;
69
        if (n == n / pre[i] * pre[i])
70
          return false;
71
72
      return mr(n, k);
73
74
75
   |}
```

### 5.11.2 Pollard's rho

```
/* Pollard's rho, by Abreto<m@abreto.net>. */
 1
 2
 3
   namespace pollards_rho {
 4
 5
   typedef long long int ll;
 6
 7
   inline ll add(const ll a, const ll b, ll mod) {
      11 z = a + b;
 8
9
     if (z \ge mod) z = mod;
10
     return z;
11
12
   inline ll mul(ll a, ll b, ll mod) {
13
     ll z = 0ll;
14
     if (a >= mod) a -= a / mod * mod;
     if (b \ge mod) b = b / mod * mod;
15
     while (b) {
16
17
        if (1 \& b) z = add(z, a, mod);
18
        a = add(a, a, mod);
```

```
19
        b >>= 1;
20
21
      return z;
22
   }
23
24
    ll gcd(ll m, ll n) {
25
      return (0 == n)? m : gcd(n, m % n);
26
27
28
   ll find(ll n, int c = -1) {
29
      ll x = rand() % n;
30
      11 y = x, k = 2;
31
      for (int i = 2; i++) {
        x = add(mul(x, x, n), (n + c) % n, n);
32
        ll d = gcd(y - x + n, n); // change to abs(y - x) if get WA
33
34
        if (1 != d && n != d) return d;
35
        if (y == x) return n;
36
        if (i == k) {
37
          y = x;
38
          k <<= 1;
39
40
41
42
43
    /** usage:
44
    * void find(ll n, int c = 107)
45
    * {
46
         if (1 == n) return;
     *
         if ( miller-rabin(n) )
47
48
     *
         {
49
     *
           n is a prime;
50
           return;
     *
51
     *
52
         ll p = n, k = c;
     *
53
         while (p \ge n) p = pollards_rho(p, k--);
54
         find(p, c);
55
         find(n/p, c);
     *
56
    * }
57
    **/
58
59 |}
```

# 5.12 Montgomery modular multiplication

```
1
 2
   /∗ — Montgomery modular algorithm { — ∗/
 3
   struct Mod64 {
 4
     typedef long long 11;
 5
     typedef unsigned long long u64;
 6
     typedef __int128_t i128;
 7
     typedef __uint128_t u128;
 8
     Mod64() :n_(0) {}
 9
     Mod64(u64 n) :n_(init(n)) {}
     static u64 init(u64 w) {
10
11
        return reduce(u128(w) * r2);
12
13
     static void set_mod(u64 m) {
14
        mod = m;
15
        assert(mod & 1);
16
        inv = m;
17
        for (int i = 0; i < 5; ++i) inv *= 2 - inv * m;
18
        r2 = -u128(m) \% m;
```

```
19
20
      static u64 reduce(u128 x) {
21
        u64 y = u64(x >> 64) - u64((u128(u64(x)*inv)*mod) >> 64);
22
        return ll(y)<0 ? y + mod : y;
23
24
      Mod64& operator += (Mod64 rhs) {
25
        n_+ = rhs.n_- mod;
        if (ll(n_{-})<0) n_{-} += mod;
26
27
        return *this;
28
29
      Mod64 operator + (Mod64 rhs) const {
30
        return Mod64(*this) += rhs;
31
32
      Mod64& operator -= (Mod64 rhs) {
33
        n_{-}= rhs.n_{-};
34
        if (ll(n_)<0) n_+=mod;
35
        return *this;
36
37
      Mod64 operator – (Mod64 rhs) const {
38
        return Mod64(*this) -= rhs;
39
40
      Mod64& operator *= (Mod64 rhs) {
41
        n_{-} = reduce(u128(n_{-})*rhs.n_{-});
42
        return *this;
43
44
      Mod64 operator * (Mod64 rhs) const {
45
        return Mod64(*this) *= rhs;
46
47
      u64 get() const {
48
        return reduce(n_);
49
50
      static u64 mod, inv, r2;
51
      u64 n_{-};
52
   };
53
54
   Mod64::u64 Mod64::mod, Mod64::inv, Mod64::r2;
55
   /* -- } Montgomery modular algorithm -- */
56
57
   /**
58
    * usage:
59
    * First, Mod64::set_mod();
60
    * Mod64 a, b, c(init_val);
61
     * a = b * c:
    * printf("%llu\n", a.get());
62
63
    5.13 Berlekamp Massey
 1 /* Berlekamp Massey by HoldZhu. */
 2
   #include <cstdio>
 3
   #include <vector>
 4
 5
   using namespace std;
 6
 7
   namespace BerlekampMassey {
 8
   const int mod = 1e9 + 7;
 9
   int L, m, b, n;
10
   vector<int> s, C, B;
11
   void init() {
12
      s.clear();
13
      C.clear();
```

14

B.clear();

```
15
      C.push_back(1);
16
      B.push_back(1);
17
      L = n = 0;
18
      m = b = 1;
19
20
   int pow_mod(int a, int k) {
21
      int s = 1;
22
      while (k) {
23
        if (k & 1)
24
          s = 111 * s * a % mod;
25
        a = 111 * a * a % mod;
26
        k >>= 1;
27
      }
28
      return s;
29
30
   void update(int d) {
31
      s.push_back(d);
32
      for (int i = 1; i <= L; ++i)
33
        d = (d + 111 * C[i] * s[n - i] % mod) % mod;
34
      if (d == 0)
35
        ++m;
36
      else if (2 * L <= n) {
37
        vector<int> T = C;
38
        C.resize(n + 1 - L + 1);
39
        for (int i = L + 1; i \le n + 1 - L; ++i)
40
          C[i] = 0;
41
        for (int i = 0; i < B.size(); ++i)
42
          C[i + m] = (C[i + m] + mod - 111 * d * pow_mod(b, mod - 2) % mod * B[i] % mod)
             % mod;
        L = n + 1 - L;
43
        B = T;
44
45
        b = d;
46
        m = 1;
47
      } else {
48
        for (int i = 0; i < B.size(); ++i)
49
          C[i + m] = (C[i + m] + mod - 1ll * d * pow_mod(b, mod - 2) % mod * B[i] % mod)
             % mod;
50
        ++m;
51
      }
52
      ++n;
53
54
   void output() {
55
      printf("F(n)=");
56
      for (int i = 1; i < C.size(); ++i) {
        int output = (mod - C[i]) % mod;
57
58
        if (output > mod / 2)
59
          output -= mod;
        printf("%s%d*F(n-%d)", (output < 0 \mid i == 1)? "" : "+", output, i);
60
61
      puts("");
62
63
64
    void output_code_for() {
      static const char *name = "dp";
65
      static const char *index = "i"
66
      static const char *upperbound = "maxn";
67
68
      puts("//_Generated_by_Berlekamp—Massey_algorithm");
69
      for (int i = 1; i < C.size(); ++i) {
70
        printf("%s[%d]=%d;\n", name, i - 1, s[i - 1]);
71
72
      printf("for(int<sub>\(\)</sub>i=\(\)d;i<\(\)s;++i)\\n", (int)C.size() - 1, upperbound);</pre>
      printf("___%s[%s]=((", name, index);
73
74
      for (int i = 1; i < C.size(); ++i) {
75
        int output = (mod - C[i]) % mod;
76
        if (output > mod / 2)
```

```
77
           output -= mod;
 78
        printf("%s%d*%s[%s-%d]%mod", (output < 0 | | i == 1) ? "" : "+", output, name,
            index, i);
79
80
      puts(")%mod+mod)%mod;");
81
    void output_code_matrix() {
82
      // TODO
83
84
85
    };
86
87
    /** usage */
88
    int usage() {
      // int arr[12] = {2, 24, 96, 416, 1536, 5504, 18944, 64000, 212992, 702464,
89
          2301952, 7512064};
      int arr[] = {3, 20, 119, 696, 4059, 23660, 137903, 803760, 4684659};
90
91
      BerlekampMassey::init();
92
      for (auto ai : arr) {
93
        BerlekampMassey::update(ai);
94
95
      printf("Formule:□");
96
      BerlekampMassey::output();
      printf("Code: \n");
97
98
      BerlekampMassey::output_code_for();
99
       return 0;
100 |}
```

# 5.14 Inclusion—exclusion principle

### 5.14.1 General form

若  $A_1, A_2, \ldots, A_n$  为有限集,则

$$\left| \bigcup_{i=1}^{n} A_i \right| = \sum_{\phi \neq J \subseteq \{1, 2, \dots, n\}} (-1)^{|J|-1} \left| \bigcap_{j \in J} A_j \right|$$

### 5.14.2 A generalization

若

$$g(A) = \sum_{S \subseteq A} f(S)$$

则

$$f(A) = \sum_{S \subseteq A} (-1)^{|A| - |S|} g(S)$$

更一般的,如果S是多重集合(multiset),那么

$$f(A) = \sum_{S \subseteq A} \mu(A - S)g(S)$$

其中

- 当 S 是含有偶数个元素的集合(没有重复元素)时, $\mu(S)=1$
- 当 S 是含有奇数个元素的集合(没有重复元素)时, $\mu(S)=-1$
- 当 S 含有重复元素时, $\mu(S)=0$ .

## 5.14.3 Applications

I. 乱序排列

如果集合 A 含有 n 个元素,则乱序排列的数目为 [n!/e], [x] 表示最接近 x 的整数.

## 5.15 Lindström-Gessel-Viennot lemma

对于一张无边权的 DAG 图,给定 n 个起点和对应的 n 个终点,这 n 条不相交路径的方案数为

```
\begin{vmatrix} e(a_1,b_1) & e(a_1,b_2) & \cdots & e(a_1,b_n) \\ e(a_2,b_1) & e(a_2,b_2) & \cdots & e(a_2,b_n) \\ \vdots & \vdots & \ddots & \vdots \\ e(a_n,b_1) & e(a_n,b_2) & \cdots & e(a_n,b_n) \end{vmatrix} (该矩阵的行列式)
```

其中 e(a,b) 为图上 a 到 b 的方案数.

# 6 String

#### 6.1 Hash

```
/* Common hash for any substrings. */
 1
 2
   typedef unsigned long long int llu;
 3
   #define MAXN 1000000
   int n;
 5
   char s[MAXN];
 6
 7
   llu H[MAXN], xP[MAXN], P = 9999111;
 8
   void init(void) {
 9
     int i = 0;
     xP[0] = 111;
10
     for(i = 1; i < MAXN; ++i) xP[i] = xP[i-1] * P;
11
     H[n] = 0;
12
     for(i = n-1; i >= 0; --i) H[i] = H[i+1]*P + s[i];
13
14
15 |\#define HASH(i,l) (H[i] - H[i+l]*xP[l])
```

#### 6.2 KMP

```
1 | /* KMP, by Abreto<m@abreto.net>. */
 2
   #include <string.h>
 3
 4
   /* !!NEED IMPROVING!! */
 5
 6
   #define MAXL (1000010)
 7
   char W[MAXL], T[MAXL];
 8
 9
   int f[MAXL];
   int lW, lT;
10
11
12
   int count(void) {
13
     int cnt = 0;
14
      int i, j;
15
     lW = strlen(W);
16
     lT = strlen(T);
     // -- self-matching
17
18
     f[0] = j = -1;
19
     for(i = 1; i < lW; i++) {
20
        while(j \ge 0 \&\& W[j+1] != W[i])
```

```
21
           j = f[j];
22
        if(W[j+1] == W[i]) j++;
23
        f[i] = j;
24
      }
      // _
25
26
      j = -1;
27
      for(i = 0; i < lT; i++) {
        while( j >= 0 && W[j+1] != T[i] )
    j = f[j];
28
29
        if(W[j+1] == T[i]) j++;
30
31
        if( j == lW-1 ) {
32
          cnt++;
33
          j = f[j];
        }
34
35
36
      return cnt;
37
```

### 6.3 exKMP

```
|#include <bits/stdc++.h>
 2
   using namespace std;
 3
 4
   namespace exkmp {
 5
 6
   const int maxn = 1000100, maxm = 1000100;
 7
 8
   int n, m;
9
   char S[maxn], T[maxm];
10
11
   /* the length of longest prefix between T[i..m—1] and T[0..m—1] */
12
   int nxt[maxm];
   /* the length of longest prefix between S[i..n-1] and T[0..m-1] */
13
14
   int ex[maxn];
15
16
   void getsize() {
17
     n = strlen(S);
18
     m = strlen(T);
19
   }
20
21
   void self(void) {
22
     int q = 1, p = 0;
23
     nxt[0] = m;
24
     while (1 + p < m \& T[1 + p] == T[p]) p++;
25
     nxt[1] = p;
      for (int i = 2; i < m; i++) {
26
27
        int l = nxt[i - q];
28
        if (i + l - 1 < p) {
29
          nxt[i] = 1;
30
        } else {
31
          int j = max(0, p - i + 1);
32
          while (i + j < m \& T[i + j] == T[j]) j++;
33
          nxt[i] = j;
          p = i + j - 1;
34
          q = i;
35
36
37
     }
38
39
40
   void run(void) {
41
     int q = 0, p = 0;
42
      self();
```

```
43
      while (p < n \&\& p < m \&\& S[p] == T[p]) p++;
44
      ex[0] = p;
45
      for (int i = 1; i < n; i++) {
46
47
        int l = nxt[i - q];
48
        if (i + l - 1 < p) {
49
          ex[i] = l;
        } else {
50
51
          int j = max(0, p - i + 1);
          while (i + j < n \&\& S[i + j] == T[j]) j++;
52
53
          ex[i] = j;
54
          p = i + j - 1;
55
          q = i;
        }
56
57
      }
58
    }
59
60
    void inspect(void) {
61
      printf("S:");
62
      for (int i = 0; i < n; i++) putchar(S[i]);
63
      puts("")
      printf("T:□");
64
65
      for (int i = 0; i < m; i++) putchar(T[i]);
      puts("");
66
67
      printf("next:");
68
      for (int i = 0; i < m; i++) printf("\lfloor \%d", nxt[i]);
69
      puts("");
70
      printf("extend:");
      for (int i = 0; i < n; i++) printf("\lfloor \%d", ex[i]);
71
72
      puts("");
73
    }
74
75 |} // exkmp
    6.4 Suffix Array
   /* Suffix Array, copied. */
 1
 2
    #define MAXN
                     (200010)
 3
 4
    namespace mzry_sa {
 5
    int wx[MAXN], wy[MAXN], *x, *y, wss[MAXN], wv[MAXN];
 7
    bool dacmp(int *r,int n,int a,int b,int l) {
 8
      return a+1< n \&\& b+1< n \&\& r[a]==r[b]\&\&r[a+1]==r[b+1];
 9
10
    void da(int str[],int sa[],int rank[],int height[],int n,int m) {
      int *s = str;
11
12
      int *x=wx,*y=wy,*t,p;
      int i,j;
13
14
      for(i=0; i<m; i++)wss[i]=0;
15
      for(i=0; i<n; i++)wss[x[i]=s[i]]++;
16
      for(i=1; i<m; i++)wss[i]+=wss[i-1];
      for(i=n-1; i>=0; i--)sa[--wss[x[i]]]=i;
for(j=1,p=1; p<n && j<n; j*=2,m=p) {
17
18
19
        for(i=n-j, p=0; i< n; i++)y[p++]=i;
        for(i=0; i<n; i++)if(sa[i]-j>=0)y[p++]=sa[i]-j;
20
21
        for(i=0; i<n; i++)wv[i]=x[y[i]];
```

22

23

24

25

26

for(i=0; i<m; i++)wss[i]=0;

for(i=0; i<n; i++)wss[wv[i]]++;

for(i=1; i<m; i++)wss[i]+=wss[i-1];

for(i=n-1; i>=0; i--)sa[--wss[wv[i]]]=y[i];

for(t=x,x=y,y=t,p=1,i=1,x[sa[0]]=0; i<n; i++)

```
27
          x[sa[i]]=dacmp(y,n,sa[i-1],sa[i],j)?p-1:p++;
28
29
      for(int i=0; i<n; i++) rank[sa[i]]=i;
30
      for(int i=0, j=0, k=0; i< n; height[rank[i++]]=k)
31
        if(rank[i]>0)
32
          for(k?k-:0, j=sa[rank[i]-1];
33
              i+k < n \& j+k < n \& str[i+k] == str[j+k];
34
35
36
37
38
39
   Suffix array 0(n lg^2 n)
40
   LCP table O(n)
41
42
   #include <cstdio>
43
   #include <algorithm>
44
   #include <cstring>
45
46
   using namespace std;
47
48
   #define REP(i, n) for (int i = 0; i < (int)(n); ++i)
49
50
   namespace SuffixArray {
51
    const int MAXN = 1 \ll 21;
52
   char * S;
53
   int N, gap;
54
   int sa[MAXN], pos[MAXN], tmp[MAXN], lcp[MAXN];
55
56
   bool sufCmp(int i, int j) {
57
      if (pos[i] != pos[j])
58
        return pos[i] < pos[j];
59
      i += gap;
60
      j += gap;
      return (i < N && j < N) ? pos[i] < pos[j] : i > j;
61
62
63
64
   void buildSA() {
65
      N = strlen(S);
66
      REP(i, N) sa[i] = i, pos[i] = S[i];
67
      for (gap = 1;; gap <<= 1) {
68
        sort(sa, sa + N, suf(mp));
69
        REP(i, N-1) tmp[i+1] = tmp[i] + sufCmp(sa[i], sa[i+1]);
70
        REP(i, N) pos[sa[i]] = tmp[i];
71
        if (tmp[N-1] == N-1) break;
72
      }
73
   }
74
75
    void buildLCP() {
      for (int i = 0, k = 0; i < N; ++i) if (pos[i] != N - 1) {
76
77
          for (int j = sa[pos[i] + 1]; S[i + k] == S[j + k];)
78
            ++k;
79
          lcp[pos[i]] = k;
80
          if (k)—k;
81
82
83
   } // end namespace SuffixArray
84
85
   namespace HashSuffixArray {
86
   const int
87
   MAXN = 1 << 21;
88
89
   typedef unsigned long long hash;
90
```

```
91
    | const | hash | BASE | = 137;
 92
 93
    int N;
 94
    char * S;
 95
    int sa[MAXN];
 96
    hash h[MAXN], hPow[MAXN];
 97
98
    #define getHash(lo, size) (h[lo] - h[(lo) + (size)] * hPow[size])
 99
100
    inline bool sufCmp(int i, int j) {
101
      int lo = 1, hi = min(N - i, N - j);
102
      while (lo <= hi) {
         int mid = (lo + hi) >> 1;
103
104
         if (getHash(i, mid) == getHash(j, mid))
105
           lo = mid + 1;
106
        else
107
           hi = mid - 1;
108
109
       return S[i + hi] < S[j + hi];</pre>
110
111
112
    void buildSA() {
113
      N = strlen(S);
114
      hPow[0] = 1;
115
      for (int i = 1; i <= N; ++i)
116
        hPow[i] = hPow[i - 1] * BASE;
117
      h[N] = 0;
118
      for (int i = N - 1; i >= 0; —i)
119
        h[i] = h[i + 1] * BASE + S[i], sa[i] = i;
120
121
      stable_sort(sa, sa + N, sufCmp);
122
123
124
    } // end namespace HashSuffixArray
125
126
    namespace lrj_sa {
127
    const int MAXN = 1000;
    char s[MAXN]; /* 原始字符数组 (最后一个字符应必须是0, 而前面的字符必须非0) */
128
129
    int sa[MAXN], t[MAXN], t2[MAXN], c[MAXN], n; /* n seems to be the length of s. */
130
    /* every charactor is in [0,m-1] */
131
    void build_sa(int m) {
132
      int i, *x = t, *y = t2;
133
      for(i = 0; i < m; ++i) c[i] = 0;
134
      for(i = 0; i < n; i++) c[x[i]=s[i]]++;
135
      for(i = 1; i < m; ++i) c[i] += c[i-1];
136
       for(i = n-1; i >= 0; --i) sa[--c[x[i]]] = i;
137
       for(int k = 1; k <= n; k <<= 1) {
138
        int p = 0;
139
         for(i = n-k; i < n; ++i) y[p++] = i;
140
         for(i = 0; i < n; ++i) if(sa[i] >= k) y[p++] = sa[i]-k;
         for(i = 0; i < m; i++) c[i] = 0;
141
142
         for(i = 0; i < n; i++) c[x[y[i]]]++;
143
         for(i = 0; i < m; ++i) c[i]+=c[i-1];
144
         for(i = n-1; i >= 0; --i) sa[--c[x[y[i]]]] = y[i];
145
        swap(x,y);
146
        p = 1;
147
        x[sa[0]] = 0;
148
         for(i = 1; i < n; ++i)
           x[sa[i]] = y[sa[i-1]] == y[sa[i]] && y[sa[i-1]+k] == y[sa[i]+k] ? p-1:p++;
149
150
         if(p >= n) break;
151
        m = p;
152
      }
153
154 | int rank[MAXN], height[MAXN];
```

```
155
    void get_height(void) {
156
      int i,j,k = 0;
157
       for(i = 0; i < n; ++i) rank[sa[i]] = i;
158
       for(i = 0; i < n; ++i) {
159
         if(k) k-;
160
         j = sa[rank[i]-1];
161
         while(s[i+k]==s[j+k]) k++;
162
         height[rank[i]] = k;
163
164
165 |} // end namespace lrj_sa
```

## 6.5 Aho-Corasick Automata

```
//* Aho-Corasick automaton algorithm, by Abreto<m@abreto.net>. */
 3
    #define MAXN 500500
 4
   #define NALPHA 26
 5
   #define FIRSTA 'a'
 6
 7
    /* pointer version => { */
 8
   struct vtx {
 9
      vtx *nxt[NALPHA];
10
      vtx *fail;
11
      int end;
12
    } vtxs[MAXN];
13
    int nvtxs;
    void myclr(void) {
14
15
      nvtxs = 0;
16
17
    vtx *new_vtx(void) {
18
      vtx *ret = vtxs+(nvtxs++);
19
      for(int i = 0; i < NALPHA; i++)
20
        ret->nxt[i] = NULL;
21
      ret->fail = NULL;
22
      ret->end = 0;
23
      return ret;
24
25
    void myins(vtx *root, char const *s) {
26
      for(; *s; s++) {
27
        int of = (*s) – FIRSTA;
28
        if ( NULL == root->nxt[of] ) {
29
          root->nxt[of] = new_vtx();
30
31
        root = root->nxt[of];
32
33
      root->end++;
34
35
    void build_ac(vtx *root) {
36
      queue<vtx *> q;
37
      q.push(root);
38
      while(!q.empty()) {
39
        vtx *p = q.front();
40
        q.pop();
41
        for(int i = 0; i < NALPHA; i++) {
42
          if( NULL == p->nxt[i] ) continue;
43
          if( root == p ) p->nxt[i]->fail = root;
44
          else {
            vtx *t = p->fail;
45
            while ( t && NULL == t\rightarrow nxt[i] ) {
46
47
              t = t \rightarrow fail;
48
```

```
49
             if (t) p->nxt[i]->fail = t->nxt[i];
50
             else p->nxt[i]->fail = root;
51
52
           /* version[1] { */
           p->nxt[i]->end += p->nxt[i]->fail->end; /* update this sum, add its existing
53
              prefix to this. */
           /* } */
54
55
           q.push(p->nxt[i]);
56
57
58
59
    int qry(vtx *root, char const *s) {
60
      vtx *p = root;
      int cnt = 0;
61
      for(; *s; s++) {
62
63
         int of = (*s) - FIRSTA;
64
         while( p != root && NULL == p->nxt[of] ) {
65
           p = p \rightarrow fail;
66
         if (p->nxt[of]) p = p->nxt[of];
67
         cnt += p->end; // correct when version[1] exists.
68
         // if version[1] not exists, you need to add all ends from this vertex up.
69
70
         //for( vtx *t = p ; t ; t = t->fail )
 71
         // cnt += t->end;
72
73
      return cnt;
74
75
    /* } */
76
    /* — usage (of pointer version) — */
77
78
    #include <bits/stdc++.h>
79
    using namespace std;
80
81
    char S[1000100];
82
    char pat[64];
83
84
    int main(void) {
85
      int T, N;
86
      vtx *root = NULL;
87
      scanf("%d", &T);
88
      while(T---) {
89
         myclr();
         root = new_vtx();
90
        scanf("%s", S);
scanf("%d", &N);
91
92
93
         while(N---) {
           scanf("%s", pat);
94
95
           myins(root, pat);
96
97
         build_ac(root);
98
         printf("%d\n", qry(root, S));
99
100
       return 0;
101
```

## 6.6 Manacher

```
1 | char t[MAXL<<1];
2 | int p[MAXL<<1];
3 | int manacher(char *s) {
4 | int i;
5 | int sl = strlen(s);</pre>
```

```
6
      int pos = 0, mxr = 0;
 7
      int ret = 0;
 8
      t[0] = '^;
      for(i = 0; i < sl; ++i) {
 9
10
        t[i*2+1] = '#';
11
        t[i*2+2] = s[i];
12
      t[sl*2+1] = '#';
13
      t[sl*2+2] = '$';
14
      sl = sl*2+2;
15
16
      for(i = 1; i < sl; ++i) {
17
        if(i <= mxr) {
18
          p[i] = min(p[2*pos - i], mxr-i+1);
19
        } else {
20
          p[i] = 1;
21
22
        while( t[i-p[i]] == t[i+p[i]] ) p[i]++;
23
        if(i + p[i] - 1 > mxr) {
24
          mxr = i+p[i]-1;
25
          pos = i;
26
27
        ret = max(ret, p[i]-1);
28
29
      return ret;
30 | }
```

# 7 Utility

# 7.1 IO plug-in

```
/* I/O Plug-in, by Abreto <m@abreto.net>. */
   #include <stdio.h>
 3
 4
   #if ( _WIN32 || __WIN32__ || _WIN64 || __WIN64__ )
 5
   #define INT64 "%I64d"
 6
   #else
   #define INT64 "%lld"
 7
 8
   #endif
 9
10
   #if ( _WIN32 || __WIN32__ || _WIN64 || __WIN64__ )
   #define UNS64 "%I64u"
11
12
   #else
   #define UNS64 "%llu"
13
14
   #endif
15
16
   #define ISDIGIT(x) ((x>='0')\&\&(x<='9'))
17
   int readn(int *n) {
18
     int c=0;
19
     *n=0;
20
     for(; !ISDIGIT(c); c=getchar());
21
     for(; ISDIGIT(c); c=getchar())*n=(*n)*10+c-'0';
22
      return (*n);
23
24
   void putn(int n) {
25
     int ns[16] = \{0, n\%10\}, nd=1;
26
     while(n/=10)ns[++nd]=n%10;
27
     while(nd)putchar(ns[nd--]+'0');
28
 1
   |#include <cstdio>
 3 | class abio {
```

```
static const unsigned BUF_SZ = 65536;
 4
 5
      FILE *istream, *ostream;
      char ibuf[BUF_SZ], obuf[BUF_SZ];
 6
     bool reached_eof;
 7
 8
     size_t ip, isz;
 9
      size_t op, osz;
10
     inline void clear_ibuf(void) {
11
        ip = isz = 0u;
12
13
     inline void clear_obuf(void) {
14
        op = osz = 0u;
15
     inline void clear_buffer(void) {
16
        reached_eof = false;
17
        clear_ibuf();
18
19
        clear_obuf();
20
21
     inline size_t read_buffer(void) {
22
        isz = std::fread(ibuf, sizeof(char), BUF_SZ, istream);
23
        ip = 0;
24
        return isz;
25
26
     inline size_t write_buffer(void) {
27
        if(osz) {
28
          size_t ret = std::fwrite(obuf+op, sizeof(char), osz-op, ostream);
29
30
          if(op == osz) clear_obuf();
31
          return ret;
32
        }
33
        return 0;
34
35
     inline abio &reach_eof(void) {
36
        reached_eof = true;
37
        return (*this);
38
39
   public:
40
     static const char endl = '\n';
     abio(FILE *input = stdin, FILE *output = stdout) {
41
42
        this->istream = input;
43
        this->ostream = output;
44
        clear_buffer();
45
46
     abio(const char *input, const char *output) {
47
        this->istream = std::fopen(input, "r");
        this->istream = std::fopen(output, "w+");
48
49
        clear_buffer();
50
51
     ~abio(void) {
52
        write_buffer();
53
        std::fclose(istream);
54
        std::fclose(ostream);
55
     }
56
     operator bool() const {
57
        return (!reached_eof);
58
59
      inline int getchar(void) {
60
        if(isz == ip) read_buffer();
61
        if(isz == ip) return EOF;
62
        return ibuf[ip++];
63
64
     inline int putchar(int ch) {
        if(osz == BUF_SZ) write_buffer();
65
        if(osz == BUF_SZ) return EOF;
66
        return (obuf[osz++] = ch);
67
```

```
68
 69
      abio &read_int(int &x) {
 70
         int flag = 0, ch = getchar();
 71
         for (; (EOF!=ch)&&((ch<'0')||(ch>'9')); ch=getchar()) if ('-' == ch) flag = 1;
 72
         if (EOF == ch) return (this->reach_eof());
 73
         x = 0;
 74
         for (; (ch>='0')&(ch<='9'); ch=qetchar()) x = x * 10 + (ch - '0');
 75
         if (flag) x *= (-1);
 76
         return (*this);
 77
 78
      abio &read_ll(long long int &x) {
 79
         int flag = 0, ch = getchar();
         for (; (EOF!=ch)\&((ch<'0')||(ch>'9')); ch=getchar()) if ('-' == ch) flag = 1;
 80
         if (EOF == ch) return (this->reach_eof());
 81
 82
         x = 011;
 83
         for (; (ch>='0')&(ch<='9'); ch=getchar()) x = x * 10ll + (ch - '0');
 84
         if (flag) x *= (-111);
 85
         return (*this);
 86
 87
      abio &read_unsigned(unsigned &x) {
 88
         int ch = getchar();
 89
         for(; (EOF != ch) && ((ch < '0') || (ch > '9')); ch = getchar());
 90
         if (EOF == ch) return (this->reach_eof());
 91
         x = 0u;
 92
         for(; (ch >= '0') && (ch <= '9'); ch = getchar()) x = x * 10u + (ch - '0');
 93
         return (*this);
 94
 95
      abio &read_ull(unsigned long long int &x) {
 96
         int ch = getchar();
         for(; (EOF != ch) && ((ch < '0') || (ch > '9')); ch = getchar());
 97
 98
         if (EOF == ch) return (this->reach_eof());
 99
         x = 0ull:
100
         for(; (ch >= '0') && (ch <= '9'); ch = getchar()) x = x * 10ull + (ch - '0');
101
         return (*this);
102
103
      /* set interrupt as '\n' to read a whole line. */
104
      abio &read_s(char *s, const char interrupt = '□') {
105
         int ch = getchar();
         while((EOF!=ch)&&(ch<'!'||ch>'~'))ch=aetchar();
106
107
         if(EOF==ch) return (this->reach_eof());
108
         for(; (EOF!=ch)&&(interrupt!=ch)&&(ch>='\_''&&ch<='\~'); ch=getchar())(*s++)=ch;
109
         (*s)=0;
110
         return (*this);
111
      abio &write_int(int x, char append = 0) {
112
113
         int d[20],nd=0;
114
         if(0==x) putchar('0');
115
         if(x<0) {
           putchar('-');
116
117
           X=-X;
118
119
         while(x) {
120
           d[nd++]=x%10;
121
           x/=10;
122
123
         while(nd—)putchar('0'+d[nd]);
124
         if(append)putchar(append);
125
         return (*this);
126
127
      abio write_ll(long long int x, char append = 0) {
128
         int d[20], nd=0;
129
         if(0==x) putchar('0');
130
         if(x<0) {
           putchar('-');
131
```

```
132
           X=-X;
133
134
         while(x) {
135
           d[nd++]=x%10;
136
           x/=10;
137
         while(nd—)putchar('0'+d[nd]);
138
139
         if(append)putchar(append);
140
         return (*this);
141
142
       abio \&write\_unsigned(unsigned x, char append = 0) {
143
         int d[20], nd=0;
144
         if(0==x) putchar('0');
145
         while(x) {
146
           d[nd++]=x%10;
147
           x/=10;
148
149
         while(nd—)putchar('0'+d[nd]);
150
         if(append)putchar(append);
151
         return (*this);
152
       abio write_ull(unsigned long long int x, char append = 0) {
153
154
         int d[20],nd=0;
         if(0==x) putchar('0');
155
156
         while(x) {
157
           d[nd++]=x%10;
158
           x/=10;
159
160
         while(nd—)putchar('0'+d[nd]);
         if(append)putchar(append);
161
162
         return (*this);
163
164
       abio &write_s(const char *s, char append = 0) {
165
         while(*s) putchar(*s++);
166
         if(append) putchar(append);
167
         return (*this);
168
       }
169
       abio &operator>>(char &ch) {
         ch = getchar();
170
171
         if(E0F==ch) return (this->reach_eof());
172
         return (*this);
173
       }
174
       abio &operator>>(int &x) {
175
         return read_int(x);
176
       abio &operator>>(long long int &x) {
177
178
         return read_ll(x);
179
180
       abio &operator>>(unsigned &x) {
181
         return read_unsigned(x);
182
183
       abio &operator>>(unsigned long long int &x) {
184
         return read_ull(x);
185
186
       abio &operator>>(char *s) {
187
         return read_s(s);
188
189
       abio &operator<<(const char ch) {</pre>
190
         putchar(ch);
191
         return (*this);
192
193
       abio &operator<<(const int x) {
194
         return write_int(x);
195
       }
```

```
196
       abio &operator<<(const long long int x) {</pre>
197
         return write_ll(x);
198
199
       abio &operator<<(const unsigned x) {
200
         return write_unsigned(x);
201
202
       abio &operator<<(const unsigned long long int x) {</pre>
203
         return write_ull(x);
204
       abio &operator<<(const char *s) {
205
206
         return write_s(s);
207
208
    } io;
```

### 7.2 Random Numbers

```
1 | #include <algorithm>
   #include <chrono>
   |#include <iostream>
 3
 4
   #include <random>
   #include <vector>
 5
   using namespace std;
 6
 7
 8
   const int N = 3000000;
 9
   double average_distance(const vector<int> &permutation) {
10
11
     double distance_sum = 0;
12
13
     for (int i = 0; i < N; i++)
14
        distance_sum += abs(permutation[i] - i);
15
16
      return distance_sum / N;
17
18
19
   int main() {
20
     /* use mt19937_64 if you want 64-bit random numbers */
21
     mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
22
     vector<int> permutation(N);
23
24
     for (int i = 0; i < N; i++)
25
        permutation[i] = i;
26
      shuffle(permutation.begin(), permutation.end(), rng);
27
28
     cout << average_distance(permutation) << '\n';</pre>
29
30
      for (int i = 0; i < N; i++)
31
        permutation[i] = i;
32
33
     for (int i = 1; i < N; i++)
        swap(permutation[i], permutation[uniform_int_distribution<int>(0, i)(rng)]);
34
35
36
      cout << average_distance(permutation) << '\n';</pre>
37
38
39
   // -
40
   // rand() * rand() % M
41
   // or
42
   // rand() << 16 | rand()
43 // maybe also work in some scene;
```

# 8 Appendix

```
8.1 C++ Reference
  8.1.1 Debug
  |#ifdef JDEBUGMODE
  #define jdebug(format, ...) fprintf(stderr, "L%d:" format, __LINE__, __VA_ARGS__)
  #define jshow(...) fprintf(stderr, "%s\n", #__VA_ARGS__)
5
  #define jdebug(...) (void (0))
  #define jshow(...) (void (0))
  #endif
  8.1.2 STL
  bitset
1 |template <size_t N> class bitset;
  access
  count Count bits set
  test Return bit value
  any Test if any bit is set
  none Test if no bit is set
  all Test if all bits are set
  operations
  set Set bits 1
  reset Reset bits o
  flip Flip bits
  to_string
  to_ullong to unsigned long long
  could be used for unordered_set or unordered_map
1 | template <class T> struct hash;
                                                // unspecialized
 template <size_t N> struct hash<br/>bitset<N>>; // bitset specializatio
  unordered_map 相对于map,插入慢,查询快.
  multiset 对于 erase 操作,如果传值进去,会把所有该值的元素删去;如果传迭代器,则只删一
  个元素。
  other
1
  template <class InputIterator1, class InputIterator2>
2
    bool includes ( InputIterator1 first1, InputIterator1 last1,
3
                     InputIterator2 first2, InputIterator2 last2 );
4
  template <class InputIterator1, class InputIterator2, class Compare>
5
6
    bool includes ( InputIterator1 first1, InputIterator1 last1,
7
                    InputIterator2 first2, InputIterator2 last2, Compare comp );
  auto il = { 10, 20, 30 }; // the type of il is an initializer_list
```

### 8.2 Java Reference

#### 8.2.1 Basic

```
Structure
   |import java.io.*;
 1
   import java.util.*;
   import java.math.*;
 5
   public class Main {
     public static final int maxn = 50050;
 6
     public static int[] int_array = new int[maxn]; /** Array */
 7
 8
     public static int[] another_arr = {1, 2, 3, 5};
 9
     public static void main(String[] args) throws Exception {
10
        Scanner cin = new Scanner(System.in);
11
        int a = cin.nextInt(), b = cin.nextInt();
        System.out.println(a + b);
12
13
        for (int i = 0; i < 4; i++)
14
          System.out.println(another_arr[i]);
15
16 | }
   Constant
       public static final int A = 0;
   Array
        int [] a = new int[5];
        int [] b = \{10, 35, 45, 89, 90\};
```

## Bit op

The Java programming language also provides operators that perform bitwise and bit shift operations on integral types. The operators discussed in this section are less commonly used. Therefore, their coverage is brief; the intent is to simply make you aware that these operators exist.

The unary bitwise complement operator "~" inverts a bit pattern; it can be applied to any of the integral types, making every "0" a "1" and every "1" a "0". For example, a byte contains 8 bits; applying this operator to a value whose bit pattern is "00000000" would change its pattern to "11111111".

The signed left shift operator "<<" shifts a bit pattern to the left, and the signed right shift operator ">>" shifts a bit pattern to the right. The bit pattern is given by the left-hand operand, and the number of positions to shift by the right-hand operand. The unsigned right shift operator ">>>" shifts a zero into the leftmost position, while the leftmost position after ">>" depends on sign extension. The bitwise & operator performs a bitwise AND operation.

The bitwise operator performs a bitwise exclusive OR operation.

The bitwise | operator performs a bitwise inclusive OR operation.

### 8.2.2 BigInteger

Immutable arbitrary-precision integers. All operations behave as if BigIntegers were represented in two's-complement notation (like Java's primitive integer types). BigInteger provides analogues to all of Java's primitive integer operators, and all relevant methods from java.lang.Math. Additionally, BigInteger provides operations for modular arithmetic, GCD calculation, primality testing, prime generation, bit manipulation, and a few other miscellaneous operations.

### API

- ZERO, ONE, TEN
- Constructors
- 1 | public BigInteger(String val, int radix)

```
    primes

 1 \mid /* \text{ prob of composite} <= 2^{-100} */
   public BigInteger nextProbablePrime()
   /* return true if the probability that it is prime exceeds (1 - 1/(2^{crtainty}))
 4 public boolean isProbablePrime(int certainty)

    valueOf

  • 算术运算
 1 | public BigInteger add(BigInteger val)
   public BigInteger subtract(BigInteger val)
   public BigInteger multiply(BigInteger val)
   public BigInteger divide(BigInteger val)
   public BigInteger[] divideAndRemainder(BigInteger val) // [quotient, remainder]
   public BigInteger remainder(BigInteger val)
   public BigInteger pow(int exponent)
   public BigInteger gcd(BigInteger val)
   public BigInteger abs()
  public BigInteger negate()
  |public int signum() // -1, 0 or 1
11
   public BigInteger mod(BigInteger m) // always returns a non-negative BigInteger.
   public BigInteger modPow(BigInteger exponent, BigInteger m)
14 | public BigInteger modInverse(BigInteger m)
  位运算
  public BigInteger shiftLeft(int n)
   public BigInteger shiftRight(int n)
 3
   public BigInteger and(BigInteger val)
   public BigInteger or(BigInteger val)
   public BigInteger xor(BigInteger val)
   public BigInteger not()
 7
   public boolean testBit(int n)
   public BigInteger setBit(int n)
   public BigInteger clearBit(int n)
   public BigInteger flipBit(int n)
  public int getLowestSetBit() // lowbit
12 public int bitLength()
13 public int bitCount()
 比较
 1 | public int compare To (Biq Integer val) // -1, 0 or 1 if this () val
  public BigInteger min(BigInteger val)
 3 public BigInteger max(BigInteger val)

    transform

 1 |public int hashCode()
   public String toString(int radix)
   |public int intValue()
 4 | public long longValue()
```

#### 8.2.3 BigDecimal

Immutable, arbitrary-precision signed decimal numbers. A BigDecimal consists of an arbitrary precision integer unscaled value and a 32-bit integer scale. If zero or positive, the scale is the number of digits to the right of the decimal point. If negative, the unscaled value of the number is multiplied by ten to the power of the negation of the scale. The value of the number represented by the BigDecimal is therefore ( $unscaledValue \times 10^{-scale}$ ).

#### Constructors

```
public BigDecimal(String val)
public BigDecimal(double val)
public BigDecimal(BigInteger val)
public BigDecimal(int val)
public static BigDecimal valueOf(long unscaledVal, int scale) // u * 10^(-scale)
public static BigDecimal valueOf(double val)
```

## arithmetic operation

```
public BigDecimal add(BigDecimal augend)
   public BigDecimal subtract(BigDecimal subtrahend)
 3
   public BigDecimal multiply(BigDecimal multiplicand)
   public BigDecimal divide(BigDecimal divisor)
   public BigDecimal divideToIntegralValue(BigDecimal divisor)
   public BigDecimal remainder(BigDecimal divisor)
   public BigDecimal[] divideAndRemainder(BigDecimal divisor)
 8
   public BigDecimal pow(int n)
9
   public BigDecimal abs()
   public BigDecimal negate()
10
  public int signum()
11
   public int scale()
13 public int precision()
```

### transform

```
1  |// RoundingMode.
2  |// UP, DOWN, CEILING, FLOOR, HALF_UP, HALF_DOWN, HALF_EVEN
3  | public MathContext(int setPrecision[, RoundingMode setRoundingMode])
4  | public BigDecimal round(MathContext mc)
5  | public int hashCode()
6  | public String toString()
7  | public String toPlainString()
8  | public double doubleValue()
```

# comparison

```
1 | public int compareTo(BigDecimal val)
2 | public BigDecimal min(BigDecimal val)
3 | public BigDecimal max(BigDecimal val)
```

#### 8.2.4 Sorting

java.util.Arrays

### 8.3 Environment test items

- 一秒运算次数,带模一秒跑多少
- 行末空格
- assert (0) 是 WA 还是 RE
- RAND MAX大小