ACM TEMPLATE

UESTC_Jungle

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Contents

1 Datastructure

1.1 Fenwick

```
1  /* Fenwick Tree (Binary Indexed Tree), by Abreto <m@abreto.net>. */
2  #include <cstring>
3
4  using namespace std;
5
6  template <class T = int, int MAXN = 100001>
7  struct fenwick {
8    static inline int lowbit(int x) {
9      return (x&(-x));
10   }
11   int N;
12   T f[MAXN]; /* 1=based. */
13   fenwick(void):N(MAXN) {
14     init();
15   }
16   fenwick(int n):N(n) {
17     init();
18   }
19   void init(void) {
20     memset(f,0,sizeof(f));
21   }
22   void upd(int i, T dx) {
23     while(i <= N) {
24       f[i] += dx;
25       i += lowbit(i);
26     }
27   }
28   T sum(int i) {
29     T ret = 0;
30     while(i) {
31       ret += f[i];
32       i -= lowbit(i);
33     }
34     return ret;
35   }
36 };
```

```
1  /* Fenwick Tree (Binary Indexed Tree), by Abreto <m@abreto.net>. */
2
3  #define MAXN 100001
4  #define LOWBIT(x)   ((x)&(-(x)))
5
6  int N;
7  int fen[MAXN];
8
9  void update(int i, int dx) {
10   while(i <= N) {
11     fen[i] += dx;
12     i += LOWBIT(i);
13   }
14 }
15
16 int sum(int i) {
17   int s = 0;
18   while(i > 0) {
19     s += fen[i];
20     i -= LOWBIT(i);
21   }
22   return s;
23 }
```

1.2 BST in pb_ds

```
1  /* Red-Black tree via pb_ds. */
2  #include<bits/stdc++.h>
3  #include<ext/pb_ds/assoc_container.hpp>
4  #include<ext/pb_ds/tree_policy.hpp>
5  using namespace __gnu_pbds;
6  using namespace std;
7  template <typename T>
8  using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
       tree_order_statistics_node_update>;
9
10 int main() {
11   ordered_set<int>  s;
12   s.insert(1);
13   s.insert(3);
14   cout << s.order_of_key(2) << endl; // the number of elements in the s less than 2
15   cout << *s.find_by_order(0) << endl; // print the 0-th smallest number in s(0-based
        )
16 }
```

1.3 Segment Tree

```
1  /* Segment tree (Interval tree, range tree), by Abreto <m@abreto.net>. */
2
3  template <int STMAX = 1000000>
4  struct segment_tree {
5    struct node_t {
6      static inline node_t merge(node_t n1, node_t n2) {
7        node_t ans;
8        ans.l = n1.l;
9        ans.r = n2.r;
10       /* merge n1 and n2 to ans. */
11       return ans;
12     }
13
14     /* Data field */
15     int l,r;
16   } nodes[(STMAX+1)<<2];
17
18   struct lazy_t {
19     int marked; /* Optional */
20     /* lazy mark. */
21
22     lazy_t(void) {
23       clear();
24     }
25     void clear(void) {
26       marked=0;
27     }
28   } marks[(STMAX+1)<<2];
29
30   inline void maintain_leaf(int o, int idx) {
31     nodes[o].l = nodes[o].r = idx;
32     /* Operations to single elements ... */
33   }
34   inline void maintain(int o) {
35     nodes[o] = node_t::merge(nodes[o<<1], nodes[o<<1|1]);
36   }
37
38   /* Usage: build(1,1,n); */
39   void build(int o, int l, int r) { /* [l,r] */
```

```
40      if( r <= l ) {
41        maintain_leaf(o, l);
42      } else {
43        int mid = l+r>>1;
44        build(o<<1, l, mid);
45        build(o<<1|1, mid+1, r);
46        maintain(o);
47      }
48    }
49
50    /* Modify all elements in [l,r] */
51    void mark(lazy_t act, int o) {
52      /* do something .. */
53      marks[o].marked = 1;
54    }
55
56    /* Pass cached updates. */
57    void pushdown(int o) {
58      if( marks[o].marked ) {
59        mark(marks[o], o<<1);
60        mark(marks[o], o<<1|1);
61        marks[o].clear();
62      }
63    }
64
65    /* Do act on all elements in [L,R] */
66    void upd(int L, int R, lazy_t act, int o, int l, int r) {
67      if( L <= l && r <= R ) {
68        mark(act, o);
69      } else if (L <= R) {
70        int mid = (l+r)>>1;
71        pushdown(o);
72        if( L <= mid ) upd(L, R, act, o<<1, l, mid);
73        if( R > mid ) upd(L, R, act, o<<1|1, mid+1, r);
74        maintain(o);
75      }
76    }
77
78    node_t qry(int L, int R, int o, int l, int r) {
79      if(L <= l && r <= R)
80        return nodes[o];
81      else if (L <= R) {
82        int mid = (l+r)>>1;
83        pushdown(o);
84        if(R <= mid) return qry(L,R,o<<1,l,mid);
85        if(L > mid) return qry(L,R,o<<1|1,mid+1,r);
86        return node_t::merge(qry(L,R,o<<1,l,mid),qry(L,R,o<<1|1,mid+1,r));
87      }
88    }
89
90    int N;
91
92    segment_tree(void):N(STMAX) {}
93    segment_tree(int n):N(n) {}
94    void build(int n) {
95      N = n;
96      build(1,1,N);
97    }
98    void update(int L, int R, lazy_t act) {
99      upd(L,R,act,1,1,N);
100   }
101   node_t query(int L, int R) {
102     return qry(L,R,1,1,N);
103   }
```

```
104 };

  1 /* Segment tree (Interval tree, range tree), by Abreto <m@abreto.net>. */
  2
  3 #define MAXN    1000001
  4
  5 typedef struct {
  6   int l,r;
  7   /* Data field */
  8 } node_t;
  9
 10 node_t merge(node_t n1, node_t n2) {
 11   node_t ans;
 12   ans.l = n1.l;
 13   ans.r = n2.r;
 14   /* merge n1 and n2 to ans. */
 15   return ans;
 16 }
 17
 18 typedef struct {
 19   int marked; /* Optional */
 20   /* lazy mark. */
 21 } lazy_t;
 22
 23 int A[MAXN];
 24 node_t nodes[MAXN<<2];
 25 lazy_t marks[MAXN<<2];
 26
 27 void maintain_leaf(int o, int idx) {
 28   nodes[o].l = nodes[o].r = idx;
 29   /* Operations to single elements ... */
 30 }
 31 void maintain(int o) {
 32   nodes[o] = merge(nodes[o<<1], nodes[o<<1|1]);
 33 }
 34
 35 /* Usage: build(1,1,n); */
 36 void build(int o, int l, int r) { /* [l,r] */
 37   if( r <= l ) {
 38     maintain_leaf(o, l);
 39   } else {
 40     int mid = l+r>>1;
 41     build(o<<1, l, mid);
 42     build(o<<1|1, mid+1, r);
 43     maintain(o);
 44   }
 45   marks[o].marked = 0;
 46 }
 47
 48 /* Modify all elements in [l,r] */
 49 void mark(lazy_t act, int o) {
 50   /* do something .. */
 51   marks[o].marked = 1;
 52 }
 53
 54 /* Pass cached updates. */
 55 void pushdown(int o) {
 56   if( marks[o].marked ) {
 57     mark(marks[o], o<<1);
 58     mark(marks[o], o<<1|1);
 59     marks[o].marked = 0;
 60   }
 61 }
 62
```

```
63  /* **DISCARDED** */
64  /* Set A[p]=v. Usage: modify(p, v, 1, 1, n); */
65  void modify(int p, int v, int o, int l, int r)
66  {
67      if( r - l < 2 )
68      {
69          maintain_leaf(o, v);
70      } else {
71          int mid = (l+r)/2;
72          pushdown(o);
73          if( p <= mid ) modify(p, v, o*2, l, mid);
74          else modify(p, v, o*2+1, mid, r);
75          maintain(o);
76      }
77  }*/
78
79  /* Do act on all elements in [L,R] */
80  void update(int L, int R, lazy_t act, int o, int l, int r) {
81    if( L <= l && r <= R ) {
82      mark(act, o);
83    } else if (L <= R) {
84      int mid = (l+r)>>1;
85      pushdown(o);
86      if( L <= mid ) update(L, R, act, o<<1, l, mid);
87      if( R > mid ) update(L, R, act, o<<1|1, mid+1, r);
88      maintain(o);
89    }
90  }
```

1.4   Sparse Table

```
1  /* RMQ with Sparse Table, by Abreto <m@abreto.net>. */
2
3  int min(int a, int b) {
4      return (a<b)?a:b;
5  }
6
7  #define MAXN    100001
8  #define MAXLOG  32
9
10 int N;
11 int A[MAXN];      /* indexed from 0. */
12 int st[MAXN][MAXLOG];
13
14 void st_init() {
15   int i = 0, j = 0, t = 0;
16   for(i = 0; i < N; ++i) st[i][0] = A[i];
17   for(j = 1; (t=(1<<j)) <= N; ++j)
18     for(i = 0; (i+t-1) < N; ++i)
19       st[i][j] = min(st[i][j-1], st[i+(t>>1)][j-1]);
20   /* st(i,j) = min(st(i,j-1), st(i+2^(j-1),j-1)). */
21 }
22
23 int st_query(int l, int r) {
24   int k = 0;
25   while((1<<(k+1)) <= (r-l+1)) k++;
26   return min(st[l][k], st[r-(1<<k)+1][k]);
27 }
```

1.5   Treap

```
1  #include <bits/stdc++.h>
2
3  using namespace std;
4
5  #define MAXN  (2560000)
6
7  int __treap_mem[MAXN];
8  void init_treap_mem(void) {
9    for( int i = 1 ; i < MAXN ; i++ )
10     __treap_mem[i-1] = i;
11 }
12 int alloc_address(void) {
13   int ret = __treap_mem[0];
14   __treap_mem[0] = __treap_mem[ret];
15   return ret;
16 }
17 void free_address(int p) {
18   __treap_mem[p] = __treap_mem[0];
19   __treap_mem[0] = p;
20 }
21
22 typedef int key_t;
23 typedef int val_t;
24 struct treap {
25   key_t x;
26   val_t v;
27   int r;     /* random priority */
28   int eq, s;  /* number of equal ones, size of subtree (include root itself) */
29   treap *fa;  /* point to its father */
30   treap *ch[2]; /* 0 for left child, 1 for right child. */
31
32   treap(void);
33   inline void maintain(void); /* update s */
34   inline void set_child(int d, treap *child);
35   inline int which(void); /* determine which child this is of its father */
36   inline int cmp(key_t ox); /* determine which child to insert ox */
37   treap *rotate(void);  /* rotate this to its father, return this */
38 } treap_nodes[MAXN];
39
40 treap *new_treap(key_t x, val_t v, treap *f) {
41   treap *ret = treap_nodes + alloc_address();
42   ret->x = x;
43   ret->v = v;
44   ret->eq = ret->s = 1;
45   ret->fa = f;
46   ret->ch[0] = ret->ch[1] = NULL;
47 }
48 void free_treap(treap *p) {
49   free_address( p - treap_nodes );
50 }
51 void treap_clear(void) {
52   init_treap_mem();
53 }
54
55 treap::treap(void) {
56   r = rand();
57   eq = s = 0;
58   fa = ch[0] = ch[1] = NULL;
59 }
60 inline void treap::maintain(void) {
61   s = eq;
62   for( int i = 0 ; i < 2 ; i++ )
63     if( ch[i] )
64       s += ch[i]->s;
```

```
65  }
66  inline void treap::set_child(int d, treap *child) {
67     ch[d] = child;
68     maintain();
69     if( child ) child->fa = this;
70  }
71  inline int treap::which(void) {
72     if( NULL == fa ) return -1; /* this is not a child */
73     else return ( fa->ch[1] == this );
74  }
75  inline int treap::cmp(key_t ox) {
76     if( ox == x ) return -1;  /* equal */
77     else return ( ox > x );   /* left less, right more */
78  }
79  treap *treap::rotate(void) {
80     if ( NULL == fa ) return this;  /* no father, already global root. */
81     int d = which();
82     fa->set_child(d, ch[d^1]);
83     set_child(d^1, fa);
84     return this;
85  }
86
87  // —— deprecated |
88  typedef int T;
89
90  struct node {
91     T v;  /* value of this node */
92     int r;  /* random priority */
93     int eq; /* the number of equal things */
94     int s;  /* the size of subtree rooted at this */
95     node *ch[2];  /* 0 for left child, 1 for right child. */
96     node(void) {
97        r = rand();
98        ch[0] = ch[1] = NULL;
99     }
100    /* return where to insert x */
101    int cmp(T x) {
102       if(v == x) return -1;
103       else return (x < v) ? 0 : 1;
104    }
105    /* return 1 if this node is prior to other */
106    int pri(node *o) {
107       return (r > (o->r));
108    }
109    /* maintain the s field */
110    void maintain(void) {
111       s = eq;
112       if(NULL != ch[0]) s += ch[0]->s;
113       if(NULL != ch[1]) s += ch[1]->s;
114    }
115 };
116
117 /* move o to ch[d] of o->ch[d^1] */
118 void rotate(node *&o, int d) {
119    node *k = o->ch[d^1];
120    o->ch[d^1] = k->ch[d];
121    o->maintain();
122    k->ch[d] = o;
123    k->maintain();
124    o = k;
125 }
```

1.6 Leftist Heap

```
1  /* HDU 1512 Monkey King（左偏树模板题） */
2  #include<iostream>
3  #include<cstdio>
4  using namespace std;
5  const int maxn = 100000+5;
6
7  int n, m;
8
9  struct Heap {
10    int l,r,fa,val,dis;
11 } t[maxn];
12
13
14 int finds(int x) {
15    return t[x].fa == −1? x:t[x].fa = finds(t[x].fa);
16 }
17
18 int merge(int x, int y) {
19   if(x == 0)  return y;   //如果为0的话，就说明是空子树，根节点当然就是另一节点了
20   if(y == 0)  return x;
21   if(t[y].val>t[x].val)  swap(x,y);  //始终往右子树进行插入
22   t[x].r = merge(t[x].r,y);
23   t[t[x].r].fa = x;
24   if(t[t[x].l].dis < t[t[x].r].dis) swap(t[x].l,t[x].r);   //是否需要左右子树的对换，
           这样是为了右子树尽量短
25   if(t[x].r == 0)  t[x].dis = 0;   //距离的重新分配
26   else t[x].dis = t[t[x].r].dis + 1;
27   return x;
28 }
29
30 int pop(int &root) {
31    int l = t[root].l;
32    int r = t[root].r;
33    t[root].l = t[root].r = t[root].dis = 0;
34    t[root].fa = −1;
35    t[l].fa = t[r].fa = −1;  //删除root根节点
36    return merge(l,r);          //这样一来相当于分裂成了两棵子树，重新进行合并，最后返回值
           为合并后的根节点
37 }
38
39 int push(int x, int y) {
40    return merge(x,y);
41 }
42
43 int main() {
44   //freopen("in.txt","r",stdin);
45   while(~scanf("%d",&n)) {
46     for(int i=1; i<=n; i++) {
47       t[i].l=t[i].r=t[i].dis=0;
48       t[i].fa=−1;
49       scanf("%d",&t[i].val);
50     }
51     scanf("%d",&m);
52     while(m−−) {
53       int a,b;
54       scanf("%d%d",&a,&b);
55       int x=finds(a);
56       int y=finds(b);
57       if(x!=y) {
58         t[x].val/=2;
59         int xx = push( pop(x),x);
60         t[y].val/=2;
```

```
61          int yy = push( pop(y),y);
62          printf("%d\n",t[merge(xx,yy)].val);
63        } else puts("−1");
64      }
65    }
66    return 0;
67 }
```

1.7 Splay

```
1  /* splay, by Abreto<m@abreto.net>. */
2
3  #ifndef NULL
4  #define NULL 0
5  #endif
6
7  struct node {
8    node *f, *ch[2];
9    int sz;
10   node(node *fa = NULL, node *lc = NULL, node *rc = NULL) {
11     f = fa;
12     ch[0] = lc;
13     ch[1] = rc;
14     maintain();
15   }
16   inline int szof(const int d) const {
17     return ch[d] ? ch[d]→sz : 0;
18   }
19   inline void maintain(void) {
20     sz = szof(0) + szof(1) + 1;
21   }
22   inline int which(void) {
23     if (NULL == f) return 0;
24     return (f→ch[1] == this);   /* f[which()] == this */
25   }
26   inline node *setf(node *fa, int d = 0) {
27     f = fa;
28     if (f) {
29       f→ch[d] = this;
30       f→maintain();
31     }
32     return f;
33   }
34   inline node *setc(node *son, int d = 0) {
35     ch[d] = son;
36     if (son) son→f = this;
37     maintain();
38     return this;
39   }
40   /* rotate this to its fater, return this. */
41   inline node *rotate(void) {
42     if (f != NULL) {
43       node *ff = f→f;
44       int d = which(), fd = f→which();
45       setc(f→setc(ch[d ^ 1], d), d ^ 1);
46       setf(ff, fd);
47     }
48     return this;
49   }
50   /* splay this to child of target */
51   inline node *splay(node * const target = NULL) {
52     while (f != target) {
53       if (target != f→f) {
```

```
54          ( (which() == f→which()) ? f : this )→rotate();
55        }
56        rotate();
57      }
58      return this;
59    }
60    /* 0-based rank */
61    inline node *get_k_th(unsigned k) {
62      node *p = this;
63      int rank;
64      while (k != (rank = (p→szof(0)))) {
65        if (k < rank) {
66          p = p→ch[0];
67        } else {
68          k -= (rank + 1);
69          p = p→ch[1];
70        }
71      }
72      return p→splay(f);
73    }
74 };
```

```
 1 /* HDU 3487 - Play with Chain, by Abreto<m@abreto.net>. */
 2 #include <bits/stdc++.h>
 3
 4 using namespace std;
 5
 6 #define MAXN    300300
 7
 8 int n, m;
 9
10 #define LC(p)    ch[p][0]
11 #define RC(p)    ch[p][1]
12 #define TARGET(p) LC(RC(p))
13
14 int nodes;
15 int val[MAXN], ch[MAXN][2], fa[MAXN], sz[MAXN];
16 int rev[MAXN];
17
18 inline int new_node(int v, int f) {
19   int p = (++nodes);
20   val[p] = v;
21   fa[p] = f;
22   ch[p][0] = ch[p][1] = rev[p] = 0;
23   sz[p] = 1;
24   return p;
25 }
26 inline void maintain(int p) {
27   if (p) {
28     sz[p] = sz[LC(p)] + sz[RC(p)] + 1;
29   }
30 }
31 inline void make_child(int f, int d, int p) { /* make p the d-th ch of f */
32   ch[f][d] = p;
33   if(p) fa[p] = f;
34 }
35 inline void myrev(int p) {
36   if (p) {
37     rev[p] ^= 1;
38     swap(LC(p), RC(p));
39   }
40 }
41 inline void pushdown(int p) {
42   if(p && rev[p]) {
```

```
43        if(LC(p)) myrev(LC(p));
44        if(RC(p)) myrev(RC(p));
45        rev[p] = 0;
46    }
47 }
48 int build(int f = 0, int l = 0, int r = n+1) {
49    if(r < l) return 0;
50    if(l == r) return new_node(l, f);
51    int mid = l+r>>1;
52    int p = new_node(mid, f);
53    LC(p) = build(p, l, mid−1);
54    RC(p) = build(p, mid+1, r);
55    maintain(p);
56    return p;
57 }
58 inline int which(int p) {  /* return 1 if p is a right child or 0 if p is a left
       child. */
59    return (RC(fa[p]) == p);
60 }
61 inline int rotate(int p) { /* rotate p to its father. [!] make sure p is not global
       root. */
62    int f = fa[p], ff = fa[f];
63    if(0 == f) return p;  /* p is global root */
64    pushdown(f);
65    pushdown(p);
66    int d = which(p), df = which(f);
67    make_child(f, d, ch[p][d^1]);
68    make_child(p, d^1, f);
69    maintain(f);
70    maintain(p);
71    fa[p] = ff;
72    if(ff) ch[ff][df] = p;
73    return p;
74 }
75 inline int splay(int p, int fr) { /* splay p to the son of fr, return p. */
76    pushdown(p);
77    while(fa[p] != fr) {
78        int f = fa[p], dp = which(p);
79        if(fa[f] == fr) {
80            return rotate(p);
81        } else {
82            int df = which(f);
83            if(dp == df) {
84                rotate(f);
85            } else {
86                rotate(p);
87            }
88            rotate(p);
89        }
90    }
91    return p;
92 }
93 inline int get_k_th(int root, int k) {
94    int p = root;
95    int rank;
96    while(k != (rank = (sz[LC(p)] + 1))) {
97        pushdown(p);
98        if(k < rank) p = LC(p);
99        else {
100           k −= rank;
101           p = RC(p);
102       }
103   }
104   return splay(p, fa[root]);
```

```
105  }
106  inline int merge(int left, int right) {
107    pushdown(left);
108    if(RC(left)) left = get_k_th(left, sz[left]);
109    RC(left) = right;
110    maintain(left);
111    fa[right] = left;
112    return left;
113  }
114  inline int split(int root, int d) { /* split ch[root][d], return the root of splited
         out. */
115    pushdown(root);
116    int child = ch[root][d];
117    ch[root][d] = 0;
118    maintain(root);
119    fa[child] = 0;
120    return child;
121  }
122  inline int concat(int root, int d, int p) { /* make p be ch[root][d], return root */
123    pushdown(root);
124    ch[root][d] = p;
125    fa[p] = root;
126    maintain(root);
127    return root;
128  }
129
130  void myclear(void) {
131    nodes = 0;
132  }
133
134  int ans[MAXN];
135  void inorder(int p, int &pos) {
136    if(0 == p) return;
137    pushdown(p);
138    inorder(LC(p), pos);
139    if( (0 < val[p]) && (val[p] < n+1) ) ans[pos++] = val[p];
140    inorder(RC(p), pos);
141  }
142
143  void handle() {
144    int i;
145    int root;
146    myclear();
147    root = build(0);
148    while(m--) {
149      char command[8];
150      int a, b, c;
151      int tar;
152      scanf("%s%d%d", command, &a, &b);
153      if('C' == command[0]) {
154        scanf("%d", &c);
155        root = get_k_th(root, a);
156        RC(root) = get_k_th(RC(root), b-a+2);
157        tar = split(RC(root), 0);
158        maintain(root);
159        root = get_k_th(root, c+1);
160        RC(root) = get_k_th(RC(root), 1);
161        RC(root) = concat(RC(root), 0, tar);
162        maintain(root);
163      } else {
164        root = get_k_th(root, a);
165        RC(root) = get_k_th(RC(root), b-a+2);
166        myrev(TARGET(root));
167      }
```

```
168    }
169    int pos = 0;
170    inorder(root, pos);
171    for(i = 0; i < n; i++) printf("%s%d", i ? "␣":"", ans[i]);
172    puts("");
173  }
174
175  int main(void) {
176    while( scanf("%d%d", &n, &m) && (n > 0) && (m > 0) )
177      handle();
178    return 0;
179  }
```

2   Dynamic Programming

2.1   LIS $O(n \log n)$

```
1
2  int top = 0;
3  for( int i=1; i<=n; i++ ) {
4    if( ap[i] > dp[top] ) { // 如果大于 "模拟栈" 的栈顶元素直接 入栈 长度加 1
5      top++;
6      dp[top] = ap[i];
7      continue;
8    }
9    int m = ap[i];
10   // lower_bound 前闭后开 返回不小于 m 的最小值的位置
11   pos = lower_bound(dp,dp+top,m)-dp; // 注意减去dp
12   if( dp[pos] > ap[i])
13     dp[pos] = ap[i];
14 }
```

2.2   LCS $O(n \log n)$

总的来说，就是把 LCS 转化成 LIS，然后用 LIS 的 $\mathcal{O}(N \log N)$ 算法来求解。
实现如下：（引用）
假设有两个序列 $s_1[1 \dots 6] = abcadc$, $s_2[1 \dots 7] = cabedab$.
记录 s_1 中每个元素在 s_2 中出现的位置, 再将位置按降序排列, 则上面的例子可表示为:
$loc(a) = \{6,2\}$, $loc(b) = \{7,3\}$, $loc(c) = \{1\}$, $loc(d) = \{5\}$. (倒着扫一遍 s_2 即可把位置扔进 vector).
将 s_1 中每个元素的位置按 s_1 中元素的顺序排列成一个序列 $s_3 = \{6,2,7,3,1,6,2,5,1\}$.
在对 s_3 求 LIS 得到的值即为求 LCS 的答案。

2.3   Improved by quadrilateral inequality

```
1  /*
2   * 四边形不等式
3   *
4   * 如果 dp(i,j) 满足 dp(i,j)<=dp(i,j+1)<=dp(i+1,j+1)
5   * 那么决策 s(i,j) 满足 s(i,j)<=s(i,j+1)<=s(i+1,j+1)
6   * 可以变形为:
7   *     s(i-1,j) <= s(i,j) <= s(i,j+1)  // dp方向: i增j减
8   *   或
9   *     s(i,j-1) <= s(i,j) <= s(i+1,j)  // dp方向: 区间长度L增
10  */
11 #include <bits/stdc++.h>
12
13 using namespace std;
14
15 #define MAXN    1024
```

```
16  #define inf        (0x3fffffff)
17
18  int n, m;
19  int v[MAXN];
20  int s[MAXN];
21  int w[MAXN][MAXN];
22  int dp[MAXN][MAXN];
23  int c[MAXN][MAXN];
24
25  int wa(void) {
26    int i, j, k;
27    for(i = 1; i <= n; ++i) {
28      scanf("%d", v+i);
29      s[i] = v[i] + s[i-1];
30    }
31    for(i = 1; i <= n; ++i) {
32      w[i][i] = 0;
33      for(j = i+1; j <= n; ++j)
34        w[i][j] = w[i][j-1] + v[j] * (s[j-1] - s[i-1]);
35    }
36    /* doing dp */
37    for(i = 1; i <= n; ++i) {
38      dp[i][0] = w[1][i];
39      c[i][0] = 1;
40      c[i][i] = i-1;
41      for(j = i-1; j > 0; j--) {
42        dp[i][j] = inf;
43        for(k = c[i-1][j]; k <= c[i][j+1]; ++k)
44          if(dp[k][j-1]+w[k+1][i] <= dp[i][j]) {
45            dp[i][j] = dp[k][j-1] + w[k+1][i];
46            c[i][j] = k;
47          }
48      }
49    }
50    /* dp done */
51    return dp[n][m];
52  }
53
54  int main(void) {
55    while(EOF != scanf("%d%d", &n, &m) && n && m) {
56      printf("%d\n", wa());
57    }
58    return 0;
59  }
```

2.4   Improved by Slope

```
1   /* type 1: */
2   /* bzoj 1010 */
3   #include <bits/stdc++.h>
4
5   using namespace std;
6   typedef long double ll;
7   #define MAXN    50050
8   #define eps      (1e-8)
9
10  int N;
11  ll L;
12  ll S[MAXN];
13  ll f[MAXN];
14  ll dp[MAXN];
15
16  inline ll k(int j) {
```

```
17      return (-2.0) * (f[j] + L);
18  }
19  inline ll b(int j) {
20      return dp[j] + f[j]*f[j] + 2ll*f[j]*L;
21  }
22  inline ll g(int j, int i) {
23      return k(j) * f[i] + b(j);
24  }
25
26  /* check if l1 & l3 <= l2 */
27  inline int check(int l1, int l2, int l3) {
28      /*ll left = b(l3)*k(l1)+b(l1)*k(l2)+b(l2)*k(l3);
29      ll right = b(l1)*k(l3)+b(l3)*k(l2)+b(l2)*k(l1);*/
30      ll left = b(l3)*k(l1)-b(l1)*k(l3);
31      ll right = k(l2)*(b(l3)-b(l1))+b(l2)*(k(l1)-k(l3));
32      return (left <= right);
33  }
34
35  int Q[MAXN], ql, qr;
36
37  int main(void) {
38      int i;
39      scanf("%d%Lf", &N, &L);
40      L += 1.0;
41      for(i = 1; i <= N; ++i) {
42          scanf("%Lf", S+i);
43          S[i] += S[i-1];
44          f[i] = S[i] + (double)i;
45      }
46      Q[qr++] = 0;
47      for(i = 1; i <= N; ++i) {
48          /* <!-- STARED */
49          for(; ql+1 < qr && g(Q[ql],i) >= g(Q[ql+1],i); ql++);
50          dp[i] = g(Q[ql], i) + f[i]*f[i] + L*L; //printf("%d: %lld,%lld\n", i, dp[i], dp[i
                ]-f[i]*f[i]);
51          for(; ql+1 < qr && check(Q[qr-2], Q[qr-1], i); qr--);
52          Q[qr++] = i;
53          /* --> */
54      }
55      printf("%lld\n", (long long int)round(dp[N]));
56      return 0;
57  }
```

3 Geometry

3.1 2D

3.1.1 Point

```
1  /* 2D Point Class, by Abreto<m@abreto.net> */
2  #include <cmath>
3
4  /**
5   * Define ABG2d_USE_LL if you want to use long long int for cordnates.
6   */
7
8  namespace ab_geometry_2d {
9
10 using namespace std;
11
12 typedef double ab_float;
13
```

```
14  const ab_float pi = acos(−1.);
15
16  #ifdef ABG2d_USE_LL
17  typedef long long int T;
18  #else
19  typedef ab_float T;
20  const ab_float eps = 1e−8;
21  #endif
22
23  inline T myabs(T x) {
24    if(x < 0) return (−x);
25    return x;
26  }
27
28  inline int sgn(T x) {
29    /* no difference'' in fact */
30  #ifdef ABG2d_USE_LL
31    if (0 == x) return 0;
32  #else
33    if (myabs(x) < eps) return 0;
34  #endif
35    return (x > 0) ? 1 : −1;
36  }
37
38  inline T sqr(T x) {
39    return (x * x);
40  }
41
42  struct point {
43    T x,y;
44    point(void):x(T()),y(T()) {}
45    point(T xx, T yy):x(xx),y(yy) {}
46    inline T norm2(void) {
47      return sqr(x) + sqr(y);
48    }
49    inline ab_float norm(void) {
50      return sqrt((ab_float)(norm2()));
51    }
52    inline point rotate(const ab_float &cost, const ab_float &sint) {} // TODO:
53    inline point operator−(void) const {
54      return point(−x,−y);
55    }
56    inline point operator+(const point& b) const {
57      return point(x+b.x,y+b.y);
58    }
59    inline point operator−(const point& b) const {
60      return point(x−b.x,y−b.y);
61    }
62    inline point operator−>*(const point &b) const {
63      return (b−(*this));
64    }
65    inline T operator*(const point& b) const {
66      return ((x)*(b.x))+((y)*(b.y)); /* inner product */
67    }
68    inline T operator^(const point& b) const {
69      return ((x)*(b.y))−((b.x)*(y)); /* outter product */
70    }
71    inline point& operator+=(const point& b) {
72      point tmp=(*this)+b;
73      (*this)=tmp;
74      return (*this);
75    }
76    inline point& operator−=(const point& b) {
77      point tmp=(*this)−b;
```

```
78      (*this)=tmp;
79      return (*this);
80    }
81    inline bool operator==(const point& b) const {
82      return (0==sgn(x-b.x))&&(0==sgn(y-b.y));
83    }
84    inline bool operator!=(const point& b) const {
85      return !((*this)==b);
86    }
87    inline point operator<<(const ab_float& theta) const {
88      ab_float ct = cos(theta), st = sin(theta);  /* rotate counter-clockwise in radian
            */
89      return point(ct*x - st*y, st*x + ct*y);
90    }
91  };
92
93  typedef point vec;
94
95
96  }   // namespace ab_geometry_2d
```

3.1.2 Circle

Base

```
1   /* 2D Circle Base Class, by Abreto<m@abreto.net>. */
2
3   /* requirement: point.cc */
4   #include "point.cc"
5
6   #include <utility>
7
8   namespace ab_geometry_2d {
9
10  using namespace std;
11
12  struct circle {
13    point o;
14    T r;
15    circle(void) : r(T()) {}
16    circle(point center, T radius) : o(center), r(radius) {}
17
18    inline ab_float arclen(ab_float theta) {
19      return theta * r;
20    }
21    inline ab_float circumference(void) {
22      return 2. * pi * r;
23    }
24    inline ab_float area(void) {
25      return pi * r * r;
26    }
27
28    /* bool contain(const circle &C, const bool including_touch = false) const
29    {
30        T dis2 = (o->*(C.o)).norm2();
31        T raw_diff = r - C.r;
32        if ( -1 == sgn(raw_diff) ) return false;
33        T dr2 = sqr(raw_diff);
34        return (dis2 < dr2) || (including_touch && (dis2 == dr2));
35    }
36    inline bool in(const circle &C, const bool including_touch = false) const
37    {
38      return C.contain(*this, including_touch);
```

```
39      } */
40      enum relation_t {
41        same = 0x00000,
42        contain = 0x00001,
43        intouch = 0x00010,
44        intersect = 0x00100,
45        outtouch = 0x01000,
46        separate = 0x10000,
47        unknow_relation = 0xfffff
48      };
49      relation_t with(const circle &C) const {
50        T dis2 = (o->*(C.o)).norm2();
51        T dr2 = sqr(r - C.r), rs2 = sqr(r + C.r);
52        if ( 0 == sgn(dis2) && 0 == sgn(dr2) ) return same;
53        if ( -1 == sgn(dis2 - dr2) ) return contain;
54        if ( 0 == sgn(dis2 - dr2) ) return intouch;
55        if ( -1 == sgn(dr2 - dis2) && -1 == sgn(dis2 - rs2) ) return intersect;
56        if ( 0 == sgn(dis2 - rs2) ) return outtouch;
57        if ( -1 == sgn(rs2 - dis2) ) return separate;
58        return unknow_relation;
59      }
60
61      enum point_relation_t {
62        in = 0x001,
63        on = 0x010,
64        out = 0x100,
65        unknow_point_relation = 0xfff
66      };
67      point_relation_t with(const point &P) const {
68        T dis2 = (o->*P).norm2();
69        T r2 = sqr(r);
70        int type = sgn(dis2 - r2);
71        if (-1 == type) return in;
72        if ( 0 == type) return on;
73        if (+1 == type) return out;
74        return unknow_point_relation;
75      }
76
77      ab_float central_angle(const point &A, const point &B, const bool reflex = false)
            const {
78        T dot = (A * B);
79        if (0 == sgn(dot)) return 1. * (A != B) * pi;
80        ab_float angle = ((ab_float)(dot)) / r / r;
81        if ( reflex ) angle = 2. * pi - angle;
82        return angle;
83      }
84
85      /* be sure (*this) intersect with C */
86      pair<point,point> crosspoint(const circle &C) const {
87        ab_float d = (o ->* (C.o)).norm();
88        // TODO:
89      }
90   };
91
92   }
```

k 次圆交

```
1   //china no.1
2   #pragma comment(linker, "/STACK:1024000000,1024000000")
3   #include <vector>
4   #include <iostream>
5   #include <string>
6   #include <map>
7   #include <stack>
```

```
 8  #include <cstring>
 9  #include <queue>
10  #include <list>
11  #include <stdio.h>
12  #include <set>
13  #include <algorithm>
14  #include <cstdlib>
15  #include <cmath>
16  #include <iomanip>
17  #include <cctype>
18  #include <sstream>
19  #include <functional>
20  #include <stdlib.h>
21  #include <time.h>
22  #include <bitset>
23  using namespace std;
24
25  #define pi acos(-1)
26  #define PI acos(-1)
27  #define endl '\n'
28  #define srand() srand(time(0));
29  #define me(x,y) memset(x,y,sizeof(x));
30  #define foreach(it,a) for(__typeof((a).begin()) it=(a).begin();it!=(a).end();it++)
31  #define close() ios::sync_with_stdio(0); cin.tie(0);
32  #define FOR(x,n,i) for(int i=x;i<=n;i++)
33  #define FOr(x,n,i) for(int i=x;i<n;i++)
34  #define W while
35  #define sgn(x) ((x) < 0 ? -1 : (x) > 0)
36  #define bug printf("**********\n");
37  #define db double
38  typedef long long LL;
39  const int INF=0x3f3f3f3f;
40  const LL LINF=0x3f3f3f3f3f3f3f3fLL;
41  const int dx[]= {-1,0,1,0,1,-1,-1,1};
42  const int dy[]= {0,1,0,-1,-1,1,-1,1};
43  const int maxn=1e3+10;
44  const int maxx=1e6+100;
45  const double EPS=1e-8;
46  const double eps=1e-8;
47  const int mod=10000007;
48  template<class T>inline T min(T a,T b,T c) {
49      return min(min(a,b),c);
50  }
51  template<class T>inline T max(T a,T b,T c) {
52      return max(max(a,b),c);
53  }
54  template<class T>inline T min(T a,T b,T c,T d) {
55      return min(min(a,b),min(c,d));
56  }
57  template<class T>inline T max(T a,T b,T c,T d) {
58      return max(max(a,b),max(c,d));
59  }
60  template <class T>
61  inline bool scan_d(T &ret) {
62      char c;
63      int sgn;
64      if (c = getchar(), c == EOF) {
65          return 0;
66      }
67      while (c != '-' && (c < '0' || c > '9')) {
68          c = getchar();
69      }
70      sgn = (c == '-') ? -1 : 1;
71      ret = (c == '-') ? 0 : (c - '0');
```

```
72    while (c = getchar(), c >= '0' && c <= '9') {
73      ret = ret * 10 + (c - '0');
74    }
75    ret *= sgn;
76    return 1;
77  }
78
79  inline bool scan_lf(double &num) {
80    char in;
81    double Dec=0.1;
82    bool IsN=false,IsD=false;
83    in=getchar();
84    if(in==EOF) return false;
85    while(in!='-'&&in!='.'&&(in<'0'||in>'9'))in=getchar();
86    if(in=='-') {
87      IsN=true;
88      num=0;
89    } else if(in=='.') {
90      IsD=true;
91      num=0;
92    } else num=in-'0';
93    if(!IsD) {
94      while(in=getchar(),in>='0'&&in<='9') {
95        num*=10;
96        num+=in-'0';
97      }
98    }
99    if(in!='.') {
100     if(IsN) num=-num;
101     return true;
102   } else {
103     while(in=getchar(),in>='0'&&in<='9') {
104       num+=Dec*(in-'0');
105       Dec*=0.1;
106     }
107   }
108   if(IsN) num=-num;
109   return true;
110 }
111
112 void Out(LL a) {
113   if(a < 0) {
114     putchar('-');
115     a = -a;
116   }
117   if(a >= 10) Out(a / 10);
118   putchar(a % 10 + '0');
119 }
120 void print(LL a) {
121   Out(a),puts("");
122 }
123 //freopen( "in.txt" , "r" , stdin );
124 //freopen( "data.txt" , "w" , stdout );
125 //cerr << "run time is " << clock() << endl;
126 /*struct Point
127 {
128     double x, y;
129     Point(const Point& rhs): x(rhs.x), y(rhs.y) { } //拷贝构造函数
130     Point(double x = 0, double y = 0) : x(x), y(y) { }
131     inline void input()
132     {
133         scanf("%lf%lf",&x,&y);
134     }
135     inline void print()
```

```
136         {
137             printf("%.6lf %.6lf\n",x,y);
138         }
139  };*/
140  db sqr(db x) {
141      return x*x;
142  }
143  int dcmp(double x) {
144      if(fabs(x) < EPS) return 0;
145      else return x < 0 ? -1 : 1;
146  }
147  struct Circle {
148      double x, y, r, angle;
149      int d;
150      Circle() {}
151      Circle(double xx, double yy, double ang = 0, int t = 0) {
152          x = xx;
153          y = yy;
154          angle = ang;
155          d = t;
156      }
157      void get() {
158          scanf("%lf%lf%lf", &x, &y, &r);
159          d = 1;
160      }
161  };
162  Circle cir[maxn],tp[maxn*2];
163  double area[maxn];
164  double dis(Circle a,Circle b) {
165      return sqrt(sqr(a.x - b.x) + sqr(a.y - b.y));
166  }
167  double cross(Circle p0,Circle p1,Circle p2) {
168      return (p1.x - p0.x) * (p2.y - p0.y) - (p1.y - p0.y) * (p2.x - p0.x);
169  }
170  //圆相交
171  int CirCrossCir(Circle p1, double r1,Circle p2, double r2,Circle &cp1,Circle &cp2) {
172      double mx = p2.x - p1.x, sx = p2.x + p1.x, mx2 = mx * mx;
173      double my = p2.y - p1.y, sy = p2.y + p1.y, my2 = my * my;
174      double sq = mx2 + my2, d = -(sq - sqr(r1 - r2)) * (sq - sqr(r1 + r2));
175      if (d + eps < 0) return 0;
176      if (d < eps) d = 0;
177      else d = sqrt(d);
178      double x = mx * ((r1 + r2) * (r1 - r2) + mx * sx) + sx * my2;
179      double y = my * ((r1 + r2) * (r1 - r2) + my * sy) + sy * mx2;
180      double dx = mx * d, dy = my * d;
181      sq *= 2;
182      cp1.x = (x - dy) / sq;
183      cp1.y = (y + dx) / sq;
184      cp2.x = (x + dy) / sq;
185      cp2.y = (y - dx) / sq;
186      if (d > eps) return 2;
187      else return 1;
188  }
189  bool circmp(const Circle& u, const Circle& v) {
190      return dcmp(u.r - v.r) < 0;
191  }
192  bool cmp(const Circle& u, const Circle& v) {
193      if (dcmp(u.angle - v.angle)) return u.angle < v.angle;
194      return u.d > v.d;
195  }
196  //0.5*r*r*(K-sin(K))
197  double calc(Circle cir,Circle cp1,Circle cp2) {
198      double ans = (cp2.angle - cp1.angle) * sqr(cir.r)
199                  - cross(cir, cp1, cp2) + cross(Circle(0, 0), cp1, cp2);
```

```
200    return ans / 2;
201  }
202
203  void CirUnion(Circle cir[], int n) {
204    Circle cp1, cp2;
205    sort(cir, cir + n, circmp);
206    for (int i = 0; i < n; ++i)
207      for (int j = i + 1; j < n; ++j)
208        if (dcmp(dis(cir[i], cir[j]) + cir[i].r – cir[j].r) <= 0)
209          cir[i].d++;
210    for (int i = 0; i < n; ++i) {
211      int tn = 0, cnt = 0;
212      for (int j = 0; j < n; ++j) {
213        if (i == j) continue;
214        if (CirCrossCir(cir[i], cir[i].r, cir[j], cir[j].r,
215                        cp2, cp1) < 2) continue;
216        cp1.angle = atan2(cp1.y – cir[i].y, cp1.x – cir[i].x);
217        cp2.angle = atan2(cp2.y – cir[i].y, cp2.x – cir[i].x);
218        cp1.d = 1;
219        tp[tn++] = cp1;
220        cp2.d = –1;
221        tp[tn++] = cp2;
222        if (dcmp(cp1.angle – cp2.angle) > 0) cnt++;
223      }
224      tp[tn++] = Circle(cir[i].x – cir[i].r, cir[i].y, pi, –cnt);
225      tp[tn++] = Circle(cir[i].x – cir[i].r, cir[i].y, –pi, cnt);
226      sort(tp, tp + tn, cmp);
227      int p, s = cir[i].d + tp[0].d;
228      for (int j = 1; j < tn; ++j) {
229        p = s;
230        s += tp[j].d;
231        area[p] += calc(cir[i], tp[j – 1], tp[j]);
232      }
233    }
234  }
235  int n;
236  void solve() {
237    for(int i=0; i<n; i++)
238      cir[i].get();
239    me(area,0);
240    CirUnion(cir,n);
241    for(int i=1; i<=n; i++) {
242      area[i]–=area[i+1];
243      printf("[%d]␣=␣%.3f\n", i, area[i]);
244    }
245  }
246  int main() {
247    while(scanf("%d",&n)!=EOF)
248      solve();
249  }
```

universe

```
1
2  Point CircumCenter(Point a,Point b,Point c) { //三角形的外心
3    Point cp;
4    double a1 = b.x–a.x,b1 = b.y–a.y,c1 = (a1*a1 + b1*b1)/2;
5    double a2 = c.x–a.x,b2 = c.y–a.y,c2 = (a2*a2 + b2*b2)/2;
6    double d = a1*b2 – a2*b1;
7    cp.x = a.x + (c1*b2–c2*b1)/d;
8    cp.y = a.y + (a1*c2–a2*c1)/d;
9    return cp;
10 }
```

3.1.3 Convex hull

```
1  /* 2D Convex Hull, by Abreto <m@abreto.net>. */
2  #include "2d_base.hh"
3  #include <cmath>
4  #include <algorithm>
5
6  using namespace std;
7
8  point O;
9
10 bool comp_angle(point_t a, point_t b) {
11    double t = (a-O).X(b-O);
12    if(fe(t,0.0)) return fl((b-O).mag2(),(a-O).mag2());
13    else return fl(0.0,t);
14 }
15
16 void convex_hull_graham(vp& convex, vp src) {
17    int i = 0, top = 0;
18    O = src[0];
19    for(auto pt : src)
20      if( pt.x < O.x || (pt.x == O.x && pt.y < O.y))
21        O = pt;
22    sort(src.begin(), src.end(), comp_angle);
23    convex.push_back(src[0]);
24    convex.push_back(src[1]);
25    top = 1;
26    for(i = 2; i < src.size(); ++i) {
27      while(top>1 && fle((convex[top]-convex[top-1]).X(src[i]-convex[top]),0.0)) {
28        convex.pop_back();
29        --top;
30      }
31      convex.push_back(src[i]);
32      ++top;
33    }
34 }
```

3.1.4 Intersect Area

```
1  #include <cstdio>
2  #include <cmath>
3  #include <algorithm>
4
5  using namespace std;
6
7  //#define inf 1000000000000
8  #define M 8
9  #define LL long long
10 #define eps 1e-12
11 #define PI acos(-1.0)
12 using namespace std;
13 struct node {
14    double x,y;
15    node() {}
16    node(double xx,double yy) {
17      x=xx;
18      y=yy;
19    }
20    node operator -(node s) {
21      return node(x-s.x,y-s.y);
22    }
23    node operator +(node s) {
24      return node(x+s.x,y+s.y);
```

```
25      }
26      double operator *(node s) {
27        return x*s.x+y*s.y;
28      }
29      double operator ^(node s) {
30        return x*s.y-y*s.x;
31      }
32    };
33    double max(double a,double b) {
34      return a>b?a:b;
35    }
36    double min(double a,double b) {
37      return a<b?a:b;
38    }
39    double len(node a) {
40      return sqrt(a*a);
41    }
42    double dis(node a,node b) { //两点之间的距离
43      return len(b-a);
44    }
45    double cross(node a,node b,node c) { //叉乘
46      return (b-a)^(c-a);
47    }
48    double dot(node a,node b,node c) { //点成
49      return (b-a)*(c-a);
50    }
51    int judge(node a,node b,node c) { //判断c是否在ab线段上（前提是c在直线ab上）
52      if(c.x>=min(a.x,b.x)
53          &&c.x<=max(a.x,b.x)
54          &&c.y>=min(a.y,b.y)
55          &&c.y<=max(a.y,b.y))
56        return 1;
57      return 0;
58    }
59    double area(node b,node c,double r) {
60      node a(0.0,0.0);
61      if(dis(b,c)<eps)
62        return 0.0;
63      double h=fabs(cross(a,b,c))/dis(b,c);
64      if(dis(a,b)>r-eps&&dis(a,c)>r-eps) { //两个端点都在圆的外面则分为两种情况
65        double angle=acos(dot(a,b,c)/dis(a,b)/dis(a,c));
66        if(h>r-eps) {
67          return 0.5*r*r*angle;
68        } else if(dot(b,a,c)>0&&dot(c,a,b)>0) {
69          double angle1=2*acos(h/r);
70          return 0.5*r*r*fabs(angle-angle1)+0.5*r*r*sin(angle1);
71        } else {
72          return 0.5*r*r*angle;
73        }
74      } else if(dis(a,b)<r+eps&&dis(a,c)<r+eps) { //两个端点都在圆内的情况
75        return 0.5*fabs(cross(a,b,c));
76      } else { //一个端点在圆上一个端点在圆内的情况
77        if(dis(a,b)>dis(a,c)) { //默认b在圆内
78          swap(b,c);
79        }
80        if(fabs(dis(a,b))<eps) { //ab距离为0直接返回0
81          return 0.0;
82        }
83        if(dot(b,a,c)<eps) {
84          double angle1=acos(h/dis(a,b));
85          double angle2=acos(h/r)-angle1;
86          double angle3=acos(h/dis(a,c))-acos(h/r);
87          return 0.5*dis(a,b)*r*sin(angle2)+0.5*r*r*angle3;
88
```

```
89        } else {
90          double angle1=acos(h/dis(a,b));
91          double angle2=acos(h/r);
92          double angle3=acos(h/dis(a,c))-angle2;
93          return 0.5*r*dis(a,b)*sin(angle1+angle2)+0.5*r*r*angle3;
94        }
95      }
96    }
97
98    node A, B, C;
99    int R;
100
101   bool compar(node &p1, node &p2) {
102      return (p1^p2)>eps;
103   }
104
105   double f(double x, double y) {
106      node O(x,y);
107      node p[8];
108      p[0] = A-O;
109      p[1] = B-O;
110      p[2] = C-O;
111      sort(p, p+3, compar);
112      p[3] = p[0];
113      O=node(0,0);
114      double sum=0;
115      /* <!-- 求面积交部分 */
116      for(int i=0; i<3; i++) { /* 按顺或逆时针顺序最后取绝对值就好 */
117        int j=i+1;
118        double s=area(p[i],p[j],(double)R);
119        if(cross(O,p[i],p[j])>0)
120          sum+=s;
121        else
122          sum-=s;
123      }
124      if(sum < -eps) sum = -sum;
125      /* --> */
126      return sum;
127   }
128
129   double trifind(double x, double y1, double y2) {
130      double l = y1, r = y2;
131      while(r-l>eps) {
132        double mid = (l+r)/2.0;
133        double mmid = (mid+r)/2.0;
134        if( f(x,mmid) > f(x,mid)+eps )
135          l = mid;
136        else
137          r = mmid;
138      }
139      return f(x,l);
140   }
141
142   double findmin(double x1, double x2, double y1, double y2) {
143      double l = x1, r = x2;
144      while(r-l>eps) {
145        double mid = (l+r)/2.0;
146        double mmid = (mid+r)/2.0;
147        if( trifind(mmid,y1,y2) > trifind(mid,y1,y2)+eps )
148          l = mid;
149        else
150          r = mmid;
151      }
152      return trifind(l,y1,y2);
```

```
153 | }
154 |
155 | double ans(int a, int b, int c, int r) {
156 |   A = node(0,0);
157 |   B = node((double)c,0);
158 |   R = r;
159 |   double da = a, db = b, dc = c;
160 |   double cosa = (db*db+dc*dc−da*da)/(2.0*db*dc);
161 |   double alpha = acos(cosa);
162 |   C = node(db*cosa, db*sin(alpha));
163 |   return findmin(0.0, c, 0.0, db*sin(alpha));
164 | }
165 |
166 | int main(void) {
167 |   int a = 0, b = 0, c = 0, r = 0;
168 |   while(EOF != scanf("%d%d%d%d",&a,&b,&c,&r) && (a||b||c||r))
169 |     printf("%.8lf\n", ans(a,b,c,r));
170 |   return 0;
171 | }
```

3.1.5 Universe

```
 1 | #include <bits/stdc++.h>
 2 | using namespace std;
 3 |
 4 | struct Point {
 5 |   double x, y;
 6 |   Point(double x = 0, double y = 0) : x(x), y(y) {}
 7 | };
 8 |
 9 | typedef Point Vector;
10 |
11 | Vector operator + (Vector A, Vector B) {
12 |   return Vector(A.x + B.x, A.y + B.y);
13 | }
14 | Vector operator − (Vector A, Vector B) {
15 |   return Vector(A.x − B.x, A.y − B.y);
16 | }
17 | Vector operator * (Vector A, double p) {
18 |   return Vector(A.x*p, A.x*p);
19 | }
20 | Vector operator / (Vector A, double p) {
21 |   return Vector(A.x/p, A.x/p);
22 | }
23 |
24 | bool operator < (const Point& a, const Point b) {
25 |   return a.x < b.x || (a.x == b.x && a.y < b.y);
26 | }
27 |
28 | const double EPS = 1e−10;
29 |
30 | int dcmp(double x) {
31 |   if(fabs(x) < EPS) return 0;
32 |   else return x < 0 ? −1 : 1;
33 | }
34 |
35 | bool operator == (const Point& a, const Point& b) {
36 |   return dcmp(a.x−b.x) == 0 && dcmp(a.y−b.y);
37 | }
38 |
39 | //向量a的极角
40 | double Angle(const Vector& v) {
41 |   return atan2(v.y, v.x);//\share\CodeBlocks\templates\wizard\console\cpp
```

```
42  }
43
44  //向量点积
45  double Dot(Vector A, Vector B) {
46      return A.x*B.x + A.y*B.y;
47  }
48
49  //向量长度\share\CodeBlocks\templates\wizard\console\cpp
50  double Length(Vector A) {
51      return sqrt(Dot(A, A));
52  }
53
54  //向量夹角
55  double Angle(Vector A, Vector B) {
56      return acos(Dot(A, B) / Length(A) / Length(B));
57  }
58
59  //向量叉积
60  double Cross(Vector A, Vector B) {
61      return A.x*B.y - A.y*B.x;
62  }
63
64  //三角形有向面积的二倍
65  double Area2(Point A, Point B, Point C) {
66      return Cross(B-A, C-A);
67  }
68
69  //向量逆时针旋转rad度(弧度)
70  Vector Rotate(Vector A, double rad) {
71      return Vector(A.x*cos(rad)-A.y*sin(rad), A.x*sin(rad)+A.y*cos(rad));
72  }
73
74  //计算向量A的单位法向量。左转90°，把长度归一。调用前确保A不是零向量。
75  Vector Normal(Vector A) {
76      double L = Length(A);
77      return Vector(-A.y/L, A.x/L);
78  }
79
80  /********************************************************************
81  使用复数类实现点及向量的简单操作
82
83  #include <complex>
84  typedef complex<double> Point;
85  typedef Point Vector;
86
87  double Dot(Vector A, Vector B) { return real(conj(A)*B)}
88  double Cross(Vector A, Vector B) { return imag(conj(A)*B);}
89  Vector Rotate(Vector A, double rad) { return A*exp(Point(0, rad)); }
90
91  *********************************************************************/
92
93  /********************************************************************
94  * 用直线上的一点p0和方向向量v表示一条指向。直线上的所有点P满足P = P0+t*v;
95  * 如果知道直线上的两个点则方向向量为B-A，所以参数方程为A+(B-A)*t;
96  * 当t 无限制时， 该参数方程表示直线。
97  * 当t > 0时， 该参数方程表示射线。
98  * 当 0 < t < 1时， 该参数方程表示线段。
99  *********************************************************************/
100
101 //直线交点,须确保两直线有唯一交点。
102 Point GetLineIntersection(Point P, Vector v, Point Q, Vector w) {
103     Vector u = P - Q;
104     double t = Cross(w, u)/Cross(v, w);
105     return P+v*t;
```

```
106 }
107
108 //点到直线距离
109 double DistanceToLine(Point P, Point A, Point B) {
110     Vector v1 = B − A, v2 = P − A;
111     return fabs(Cross(v1, v2) / Length(v1)); //不取绝对值，得到的是有向距离
112 }
113
114 //点到线段的距离
115 double DistanceToSegmentS(Point P, Point A, Point B) {
116     if(A == B) return Length(P−A);
117     Vector v1 = B−A, v2 = P−A, v3 = P−B;
118     if(dcmp(Dot(v1, v2)) < 0) return Length(v2);
119     else if(dcmp(Dot(v1, v3)) > 0) return Length(v3);
120     else return fabs(Cross(v1, v2)) / Length(v1);
121 }
122
123 //点在直线上的投影
124 Point GetLineProjection(Point P, Point A, Point B) {
125     Vector v = B − A;
126     return A+v*(Dot(v, P−A)/Dot(v, v));
127 }
128
129 //线段相交判定，交点不在一条线段的端点
130 bool SegmentProperIntersection(Point a1, Point a2, Point b1, Point b2) {
131     double c1 = Cross(a2−a1, b1−a1), c2 = Cross(a2−a1, b2−a1);
132     double c3 = Cross(b2−b1, a1−b1), c4 = Cross(b2−b1, a2−b1);
133     return dcmp(c1)*dcmp(c2) < 0 && dcmp(c3)*dcmp(c4) < 0;
134 }
135
136 //判断点是否在点段上，不包含端点
137 bool OnSegment(Point P, Point a1, Point a2) {
138     return dcmp(Cross(a1−P, a2−P) == 0 && dcmp((Dot(a1−P, a2−P)) < 0));
139 }
140
141 //计算凸多边形面积
142 double ConvexPolygonArea(Point *p, int n) {
143     double area = 0;
144     for(int i = 1; i < n−1; i++)
145         area += Cross(p[i] − p[0], p[i+1] − p[0]);
146     return area/2;
147 }
148
149 //计算多边形的有向面积
150 double PolygonArea(Point *p, int n) {
151     double area = 0;
152     for(int i = 1; i < n−1; i++)
153         area += Cross(p[i] − p[0], p[i+1] − p[0]);
154     return area/2;
155 }
156
157 /*******************************************************************
158 * Morley定理：三角形每个内角的三等分线，相交成的三角形是等边三角形。
159 * 欧拉定理：设平面图的定点数，边数和面数分别为V,E,F。则V+F−E = 2;
160 *******************************************************************/
161
162 struct Circle {
163     Point c;
164     double r;
165
166     Circle(Point c, double r) : c(c), r(r) {}
167     //通过圆心角确定圆上坐标
168     Point point(double a) {
169         return Point(c.x + cos(a)*r, c.y + sin(a)*r);
```

```
170       }
171     };
172
173     struct Line {
174       Point p;
175       Vector v;
176       double ang;
177       Line() {}
178       Line(Point p, Vector v) : p(p), v(v) {}
179       bool operator < (const Line& L) const {
180         return ang < L.ang;
181       }
182     };
183
184     //直线和圆的交点，返回交点个数，结果存在sol中。
185     //该代码没有清空sol。
186     int getLineCircleIntersecion(Line L, Circle C, double& t1, double& t2, vector<Point>&
            sol) {
187       double a = L.v.x, b = L.p.x − C.c.x, c = L.v.y, d = L.p.y − C.c.y;
188       double e = a*a + c*c, f = 2*(a*b + c*d), g = b*b + d*d − C.r*C.r;
189       double delta = f*f − 4*e*g;
190       if(dcmp(delta) < 0) return 0; //相离
191       if(dcmp(delta) == 0) {          //相切
192         t1 = t2 = −f / (2*e);
193         sol.push_back(C.point(t1));
194         return 1;
195       }
196       //相交
197       t1 = (−f − sqrt(delta)) / (2*e);
198       sol.push_back(C.point(t1));
199       t2 = (−f + sqrt(delta)) / (2*e);
200       sol.push_back(C.point(t2));
201       return 2;
202     }
203
204     //两圆相交
205     int getCircleCircleIntersection(Circle C1, Circle C2, vector<Point>& sol) {
206       double d = Length(C1.c − C2.c);
207       if(dcmp(d) == 0) {
208         if(dcmp(C1.r − C2.r == 0)) return −1;      //两圆完全重合
209         return 0;                                  //同心圆，半径不一样
210       }
211       if(dcmp(C1.r + C2.r − d) < 0) return 0;
212       if(dcmp(fabs(C1.r − C2.r) == 0)) return −1;
213
214       double a = Angle(C2.c − C1.c);              //向量C1C2的极角
215       double da = acos((C1.r*C1.r + d*d − C2.r*C2.r) / (2*C1.r*d));
216       //C1C2到C1P1的角
217       Point p1 = C1.point(a−da), p2 = C1.point(a+da);
218       sol.push_back(p1);
219       if(p1 == p2) return 1;
220       sol.push_back(p2);
221       return 2;
222     }
223
224     const double PI = acos(−1);
225     //过定点做圆的切线
226     //过点p做圆C的切线，返回切线个数。v[i]表示第i条切线
227     int getTangents(Point p, Circle C, Vector* v) {
228       Vector u = C.c − p;
229       double dist = Length(u);
230       if(dist < C.r) return 0;
231       else if(dcmp(dist − C.r) == 0) {
232         v[0] = Rotate(u, PI/2);
```

```
233        return 1;
234      } else {
235        double ang = asin(C.r / dist);
236        v[0] = Rotate(u, -ang);
237        v[1] = Rotate(u, +ang);
238        return 2;
239      }
240    }
241
242    //两圆的公切线
243    //返回切线的个数, -1表示有无数条公切线。
244    //a[i], b[i] 表示第i条切线在圆A,圆B上的切点
245    int getTangents(Circle A, Circle B, Point *a, Point *b) {
246      int cnt = 0;
247      if(A.r < B.r) {
248        swap(A, B);
249        swap(a, b);
250      }
251      int d2 = (A.c.x - B.c.x)*(A.c.x - B.c.x) + (A.c.y - B.c.y)*(A.c.y - B.c.y);
252      int rdiff = A.r - B.r;
253      int rsum = A.r + B.r;
254      if(d2 < rdiff*rdiff) return 0;   //内含
255      double base = atan2(B.c.y - A.c.y, B.c.x - A.c.x);
256      if(d2 == 0 && A.r == B.r) return -1;   //无限多条切线
257      if(d2 == rdiff*rdiff) {         //内切一条切线
258        a[cnt] = A.point(base);
259        b[cnt] = B.point(base);
260        cnt++;
261        return 1;
262      }
263      //有外共切线
264      double ang = acos((A.r-B.r) / sqrt(d2));
265      a[cnt] = A.point(base+ang);
266      b[cnt] = B.point(base+ang);
267      cnt++;
268      a[cnt] = A.point(base-ang);
269      b[cnt] = B.point(base-ang);
270      cnt++;
271      if(d2 == rsum*rsum) {  //一条公切线
272        a[cnt] = A.point(base);
273        b[cnt] = B.point(PI+base);
274        cnt++;
275      } else if(d2 > rsum*rsum) {   //两条公切线
276        double ang = acos((A.r + B.r) / sqrt(d2));
277        a[cnt] = A.point(base+ang);
278        b[cnt] = B.point(PI+base+ang);
279        cnt++;
280        a[cnt] = A.point(base-ang);
281        b[cnt] = B.point(PI+base-ang);
282        cnt++;
283      }
284      return cnt;
285    }
286
287    typedef vector<Point> Polygon;
288
289    //点在多边形内的判定
290    int isPointInPolygon(Point p, Polygon poly) {
291      int wn = 0;
292      int n = poly.size();
293      for(int i = 0; i < n; i++) {
294        if(OnSegment(p, poly[i], poly[(i+1)%n])) return -1; //在边界上
295        int k = dcmp(Cross(poly[(i+1)%n]-poly[i], p-poly[i]));
296        int d1 = dcmp(poly[i].y - p.y);
```

```
297      int d2 = dcmp(poly[(i+1)%n].y - p.y);
298      if(k > 0 && d1 <= 0 && d2 > 0) wn++;
299      if(k < 0 && d2 <= 0 && d1 > 0) wn++;
300    }
301    if(wn != 0) return 1;        //内部
302    return 0;                    //外部
303  }
304
305  //凸包
306  /***********************************************************
307   * 输入点数组p，  个数为p，  输出点数组ch。  返回凸包顶点数
308   * 不希望凸包的边上有输入点，把两个<= 改成 <
309   * 高精度要求时建议用dcmp比较
310   * 输入点不能有重复点。函数执行完以后输入点的顺序被破坏
311   ***********************************************************/
312  int ConvexHull(Point *p, int n, Point* ch) {
313    sort(p, p+n);        //先比较x坐标，再比较y坐标
314    int m = 0;
315    for(int i = 0; i < n; i++) {
316      while(m > 1 && Cross(ch[m-1] - ch[m-2], p[i]-ch[m-2]) <= 0) m--;
317      ch[m++] = p[i];
318    }
319    int k = m;
320    for(int i = n-2; i >= 0; i++) {
321      while(m > k && Cross(ch[m-1] - ch[m-2], p[i]-ch[m-2]) <= 0) m--;
322      ch[m++] = p[i];
323    }
324    if(n > 1) m--;
325    return m;
326  }
327
328  //用有向直线A->B切割多边形poly，  返回"左侧"。  如果退化，可能会返回一个单点或者线段
329  //复杂度O(n2);
330  Polygon CutPolygon(Polygon poly, Point A, Point B) {
331    Polygon newpoly;
332    int n = poly.size();
333    for(int i = 0; i < n; i++) {
334      Point C = poly[i];
335      Point D = poly[(i+1)%n];
336      if(dcmp(Cross(B-A, C-A)) >= 0) newpoly.push_back(C);
337      if(dcmp(Cross(B-A, C-D)) != 0) {
338        Point ip = GetLineIntersection(A, B-A, C, D-C);
339        if(OnSegment(ip, C, D)) newpoly.push_back(ip);
340      }
341    }
342    return newpoly;
343  }
344
345  //半平面交
346
347  //点p再有向直线L的左边。（线上不算）
348  bool Onleft(Line L, Point p) {
349    return Cross(L.v, p-L.p) > 0;
350  }
351
352  //两直线交点，假定交点唯一存在
353  Point GetIntersection(Line a, Line b) {
354    Vector u = a.p - b.p;
355    double t = Cross(b.v, u) / Cross(a.v, b.v);
356    return a.p+a.v*t;
357  }
358
359  int HalfplaneIntersection(Line* L, int n, Point* poly) {
360    sort(L, L+n);              //按极角排序
```

```
361
362    int first, last;          //双端队列的第一个元素和最后一个元素
363    Point *p = new Point[n];     //p[i]为q[i]和q[i+1]的交点
364    Line *q = new Line[n];       //双端队列
365    q[first = last = 0] = L[0]; //队列初始化为只有一个半平面L[0]
366    for(int i = 0; i < n; i++) {
367      while(first < last && !Onleft(L[i], p[last-1])) last--;
368      while(first < last && !Onleft(L[i], p[first])) first++;
369      q[++last] = L[i];
370      if(fabs(Cross(q[last].v, q[last-1].v)) < EPS) {
371        last--;
372        if(Onleft(q[last], L[i].p)) q[last] = L[i];
373      }
374      if(first < last) p[last-1] = GetIntersection(q[last-1], q[last]);
375    }
376    while(first < last && !Onleft(q[first], p[last-1])) last--;
377    //删除无用平面
378    if(last-first <= 1) return 0;   //空集
379    p[last] = GetIntersection(q[last], q[first]);
380
381    //从deque复制到输出中
382    int m = 0;
383    for(int i = first; i <= last; i++) poly[m++] = p[i];
384    return m;
385  }
```

4 Graph

4.1 Tree

4.1.1 Universe

```
1
2  /* find root(重心) */
3
4  void findroot(int u, int fa) {
5    int i;
6    size[u] = 1;
7    f[u] = 0;
8    for (i = last[u]; i; i = e[i][2]) {
9      if (!vis[e[i][0]] && e[i][0] != fa) {
10       findroot(e[i][0], u);
11       size[u] += size[e[i][0]];
12       if (f[u] < size[e[i][0]])
13         f[u] = size[e[i][0]];
14     }
15   }
16   if (f[u] < ALL - size[u])
17     f[u] = ALL - size[u];
18   if (f[u] < f[root]) root = u;
19 }
20
21 /* —— da —— */
22
23 int dep[MAXN+1];
24 int ancestor[MAXN+1][MAXLGN];
25 int minw[MAXN+1][MAXLGN];
26
27 void dfs(int u, int fa) {
28   ancestor[u][0] = fa;
29   dep[u] = dep[fa] + 1;
30   for(int e = u[front]; e; e = E[e].n) {
```

```
31        int v = E[e].v, w = E[e].w;
32        if(v != fa) {
33          minw[v][0] = w;
34          dfs(v, u);
35        }
36      }
37    }
38
39    void init_system(void) {
40      int i = 0, w = 0;
41      int t = 0;
42      dep[0] = -1;
43      dfs(1,0);
44      for(w = 1; (t=(1<<w)) < N; ++w)
45        for(i = 1; i <= N; ++i) if( dep[i] >= t ) {
46            ancestor[i][w] = ancestor[ancestor[i][w-1]][w-1];
47            minw[i][w] = min(minw[i][w-1], minw[ancestor[i][w-1]][w-1]);
48        }
49    }
50
51    int query(int a, int b) {
52      if(dep[a] < dep[b]) return query(b,a);
53      else {  /* now dep[s] > dep[t] */
54        int i = 0;
55        int maxbit = MAXLGN-1;
56        int ret = INF;
57        //while((1<<maxbit) <= dep[a]) maxbit++;
58        /* first up a to same dep with b. */
59        for(i = maxbit; i >= 0; i--)
60          if(dep[a] - (1<<i) >= dep[b]) {
61            ret = min(ret, minw[a][i]);
62            a = ancestor[a][i];
63          }
64        if(a == b) return ret;
65        for(i = maxbit; i >= 0; i--)
66          if(dep[a] - (1<<i) >= 0 && ancestor[a][i] != ancestor[b][i]) {
67            ret = min(ret, min(minw[a][i], minw[b][i]));
68            a = ancestor[a][i];
69            b = ancestor[b][i];
70          }
71        ret = min(ret, min(minw[a][0], minw[b][0]));
72        return ret;
73      }
74    }
```

4.1.2 Point Divide and Conquer

Version 1

```
1  /* Tree::Point divide and conquer, by Abreto<m@abreto.net>. */
2  #include <bits/stdc++.h>
3
4  using namespace std;
5  typedef long long int ll;
6
7  #define MAXN    (100001)
8  #define MAXV    (MAXN+1)
9  #define MAXE    (MAXN<<1)
10 struct edge {
11   int v;
12   edge *n;
13   edge(void):v(0),n(NULL) {}
14   edge(int vv,edge *nn):v(vv),n(nn) {}
```

```
15 };
16 int nE;
17 edge E[MAXE];
18 edge *front[MAXV];
19 int label[MAXV];    /* 0 for '(', 1 for ')' */
20 void add_edge(int u, int v) {
21   int ne = ++nE;
22   E[ne] = edge(v, u[front]);
23   u[front] = &(E[ne]);
24 }
25
26 int n;
27 ll ans;
28
29 char del[MAXV];
30 namespace findroot {
31 int ALL;
32 int nfind;
33 int vis[MAXV];
34 int size[MAXV];
35 int f[MAXV];
36 int root;
37 void __find(int u, int fa) {
38   vis[u] = nfind;
39   size[u] = 1;
40   f[u] = 0;
41   for(edge *e=u[front]; e; e = e→n) {
42     int v = e→v;
43     if((!del[v]) && (vis[v] != nfind) && (v != fa)) {
44       __find(v, u);
45       size[u] += size[v];
46       if(f[u] < size[v])  f[u] = size[v];
47     }
48   }
49   if(f[u] < ALL − size[u])  f[u] = ALL − size[u];
50   if(f[u] < f[root])  root = u;
51 }
52 int find(int u, int all) {
53   ++nfind;
54   ALL = all;
55   f[root = 0] = MAXV;
56   __find(u,0);
57   return root;
58 }
59 }
60
61 namespace workspaces {
62 int maxdep;
63 int dep[MAXV];
64 ll cntin[MAXV], cntout[MAXV];
65 int in[2][MAXV];    /* 0 for '(', 1 for ')' */
66 int out[2][MAXV];
67 void getdeep(int u, int fa) {
68   dep[u] = dep[fa] + 1;
69   if(dep[u] > maxdep) maxdep = dep[u];
70   for(edge *e = u[front]; e; e = e→n)
71     if((!del[e→v]) && (fa != e→v))
72       getdeep(e→v, u);
73 }
74 void dfs(int u, int fa) {
75   {
76     /* out from root */
77     out[0][u] = out[0][fa];
78     out[1][u] = out[1][fa];
```

```
 79       if(0 == label[u]) { /* meet '(' */
 80         out[0][u]++;
 81       } else {     /* meet ')' */
 82         if(out[0][u]) out[0][u]--;
 83         else out[1][u]++;
 84       }
 85       if(out[0][u] == 0)
 86         cntout[out[1][u]]++;
 87     }
 88     {
 89       /* in to root */
 90       in[0][u] = in[0][fa];
 91       in[1][u] = in[1][fa];
 92       if(0 == label[u]) { /* meet '(' */
 93         if(in[1][u]) in[1][u]--;
 94         else in[0][u]++;
 95       } else {     /* meet ')' */
 96         in[1][u]++;
 97       }
 98       if(0 == in[1][u])
 99         cntin[in[0][u]]++;
100     }
101     /* do something */
102     for(edge *e = u[front]; e; e = e->n) {
103       int v = e->v;
104       if((!del[v]) && (v != fa)) {
105         dfs(v, u);
106       }
107     }
108   }
109   inline void init_maxdep(void) {
110     maxdep = 0;
111   }
112   inline void update_maxdep(int u) {
113     dep[u] = 1;
114     if(dep[u] > maxdep) maxdep = dep[u];
115     for(edge *e = u[front]; e; e = e->n)
116       if((!del[e->v]))
117         getdeep(e->v, u);
118   }
119   inline void clear(void) {
120     for(int i = 0; i <= maxdep+1; ++i)
121       cntin[i] = cntout[i] = 0;
122   }
123   inline void work(int u) {
124     in[0][u] = in[1][u] = out[0][u] = out[1][u] = 0;
125     in[label[u]][u] = out[label[u]][u] = 1;
126     if(out[0][u] == 0) cntout[out[1][u]]++;
127     if(0 == in[1][u]) cntin[in[0][u]]++;
128     /* update in and out if neccessary */
129     for(edge *e = u[front]; e; e = e->n)
130       if(!(del[e->v]))
131         dfs(e->v, u);
132   }
133 };
134
135 ll count(int u, int p) {
136   ll ret = 0;
137   workspace::init_maxdep();
138   workspace::update_maxdep(u);
139   workspace::clear();
140   if(-1 == p) {
141     for(edge *e = u[front]; e; e = e->n)
142       if((!(del[e->v])))
```

```
143        workspace::work(e→v);
144      p = label[u];
145      /* single end */
146      if(0 == p) ret = workspace::cntout[1];
147      else ret = workspace::cntin[1];
148    } else {
149      workspace::work(u);
150    }
151    if(0 == p) { /* p is '(' */
152      for(int i = 0; i < workspace::maxdep; ++i)  /* concatenation */
153        ret += workspace::cntin[i] * workspace::cntout[i+1];
154    } else {    /* p is ')' */
155      for(int i = 0; i < workspace::maxdep; ++i)  /* concatenation */
156        ret += workspace::cntin[i+1] * workspace::cntout[i];
157    }
158    return ret;
159 }
160
161 void handle(int u) {
162    del[u] = 1; /* delete current root. */
163    ans += count(u, −1);
164    /* do something */
165    for(edge *e = u[front]; e; e = e→n) {
166      int v = e→v;
167      if(!del[v]) {
168        ans −= count(v, label[u]);
169        /* do something */
170        int r = findroot::find(v, findroot::size[v]);
171        handle(r);
172      }
173    }
174 }
175
176 void proc(void) {
177    int r = findroot::find(1,n);
178    handle(r);
179 }
180
181 char ls[MAXV+1];
182 int main(void) {
183    int i = 0;
184    scanf("%d", &n);
185    scanf("%s", ls);
186    for(i = 0; i < n; ++i)
187      label[i+1] = ls[i] − '(';
188    for(i = 1; i < n; ++i) {
189      int ai, bi;
190      scanf("%d %d", &ai, &bi);
191      add_edge(ai, bi);
192      add_edge(bi, ai);
193    }
194    proc();
195    printf("%lld\n", ans);
196    return 0;
197 }
```

Version 2

```
1 /* 2016 ACM/ICPC Asia Regional Dalian. Problem , by Abreto<m@abreto.net>. */
2 #include <bits/stdc++.h>
3
4 using namespace std;
5 typedef long long int ll;
6
7 /* offset in [1,k] */
```

```
 8  #define GET(i,offset)    (((i)>>((offset)−1))&1)
 9  #define SET(i,offset)    ((i)|(1<<((offset)−1)))
10  #define REV(i,offset)    ((i)^(1<<((offset)−1)))
11
12  #define MAXN     (50005)
13  #define MAXV     (MAXN+1)
14  #define MAXE     (MAXN<<1)
15  struct edge {
16    int v;
17    edge *n;
18    edge(void):v(0),n(NULL) {}
19    edge(int vv,edge *nn):v(vv),n(nn) {}
20  };
21  int nE;
22  edge E[MAXE];
23  edge *front[MAXV];
24  int label[MAXV];    /* each kind */
25  void add_edge(int u, int v) {
26    int ne = ++nE;
27    E[ne] = edge(v, u[front]);
28    u[front] = &(E[ne]);
29  }
30
31  int n, k;
32  ll ans;
33  int all_kind;
34
35  int ndel;
36  int del[MAXV];
37  namespace findroot {
38  int ALL;
39  ll nfind;
40  ll vis[MAXV];
41  int size[MAXV];
42  int f[MAXV];
43  int root;
44  void __find(int u, int fa) {
45    vis[u] = nfind;
46    size[u] = 1;
47    f[u] = 0;
48    for(edge *e=u[front]; e; e = e→n) {
49      int v = e→v;
50      if((del[v] != ndel) && (vis[v] != nfind) && (v != fa)) {
51        __find(v, u);
52        size[u] += size[v];
53        if(f[u] < size[v])  f[u] = size[v];
54      }
55    }
56    if(f[u] < ALL − size[u])  f[u] = ALL − size[u];
57    if(f[u] < f[root])  root = u;
58  }
59  int find(int u, int all) {
60    ++nfind;
61    ALL = all;
62    f[root = 0] = MAXV;
63    __find(u,0);
64    return root;
65  }
66  }
67
68  namespace workspace {
69  ll cnt[1024];
70  int dp[MAXV];
71  void dfs(int u, int fa) {
```

```
72    dp[u] = dp[fa] | label[u];
73    cnt[dp[u]] ++;
74    /* dig into children */
75    for(edge *e = u[front]; e; e = e->n) {
76      int v = e->v;
77      if((del[v] != ndel) && (v != fa)) {
78        dfs(v, u);
79      }
80    }
81 }
82 inline void clear(void) {
83    for(int i = 1; i <= all_kind; ++i)
84      cnt[i] = 0;
85 }
86 inline void work(int u) {
87    dp[u] = label[u];
88    cnt[dp[u]] ++;
89    for(edge *e = u[front]; e; e = e->n)
90      if((del[e->v] != ndel))
91        dfs(e->v, u);
92 }
93 inline void show(void) {
94    for(int i = 0; i <= all_kind; ++i)
95      printf("cnt[%d]␣=␣%lld\n", i, cnt[i]);
96    for(int i = 1; i <= n; ++i)
97      printf("dp[%d]␣=␣%d\n", i, dp[i]);
98 }
99 };
100
101
102 ll count(int u, int p) {
103    ll ret = 0;
104    workspace::clear();
105    //printf("%d,%d   :\n", u, p);
106    if(-1 == p) {
107      for(edge *e = u[front]; e; e = e->n)
108        if(((del[e->v]) != ndel))
109          workspace::work(e->v);
110      p = label[u];
111      /* single end */
112      for(int i = 1; i <= all_kind; i++)
113        if(all_kind == (i|p))
114          ret += (workspace::cnt[i]<<1);
115    } else {
116      workspace::work(u);
117    }
118    //workspace::show();
119    for(int i = 1; i <= all_kind; ++i)
120      if( workspace::cnt[i] > 0 )
121        for(int j = 1; j <= all_kind; ++j)
122          if(all_kind == (i|p|j))
123            ret += workspace::cnt[i] * workspace::cnt[j];
124    //printf("%lld\n", ret);
125    return ret;
126 }
127
128 void handle(int u) {
129    //printf("proccessing %d\n", u);
130    del[u] = ndel; /* delete current root. */
131    ans += count(u, -1);
132    /* do something */
133    for(edge *e = u[front]; e; e = e->n) {
134      int v = e->v;
135      if(del[v] != ndel) {
```

```
136          ans -= count(v, label[u]);
137          /* do something */
138          int r = findroot::find(v, findroot::size[v]);
139          handle(r);
140       }
141    }
142  }
143
144  void proc(void) {
145    int r = findroot::find(1,n);
146    handle(r);
147  }
148
149  void clear(void) {
150    int i;
151    ans = 0;
152    nE = 0;
153    for(i = 0; i <= n; ++i) {
154      front[i] = NULL;
155    }
156    //findroot::nfind = 0;
157    ndel++;
158  }
159
160  void mozhu(void) {
161    int i = 0;
162    int li;
163    for(i = 1; i <= n; ++i) {
164      scanf("%d", &li);
165      label[i] = 1<<(li-1);
166    }
167    for(i = 1; i < n; ++i) {
168      int ai, bi;
169      scanf("%d %d", &ai, &bi);
170      add_edge(ai, bi);
171      add_edge(bi, ai);
172    }
173    all_kind = (1<<k)-1;
174    proc();
175    if(1 == k) ans += n;
176    printf("%lld\n", ans);
177  }
178
179  int main(void) {
180    while( EOF != scanf("%d%d", &n, &k) ) {
181      clear();
182      mozhu();
183    }
184    return 0;
185  }
```

4.1.3 Hevay chain decompostion

```
1  /* bzoj 1036 */
2  /* 树链剖分 */
3  #include <bits/stdc++.h>
4
5  using namespace std;
6
7  #define MAXN    30030
8  #define MAXM    (MAXN<<1)
9  struct edge {
10    int v;
```

```
11    edge *n;
12    edge(void) {}
13    edge(int vv, edge *nn):v(vv),n(nn) {}
14  };
15  typedef edge *ep;
16  int nE;
17  edge E[MAXM];
18  ep front[MAXN];
19  void add_edge(int u, int v) {
20    int ne = ++nE;
21    E[ne] = edge(v, u[front]);
22    u[front] = &(E[ne]);
23  }
24
25  int n;
26  int fa[MAXN], son[MAXN], sz[MAXN], dep[MAXN];
27  int top[MAXN];
28  int id[MAXN];
29  int tot;
30
31  void calc(int u, int uf) {
32    dep[u] = dep[uf] + 1;
33    fa[u] = uf;
34    sz[u] = 1;
35    son[u] = -1;
36    for(ep e = u[front]; e; e = e->n) {
37      if(e->v != uf) {
38        calc(e->v, u);
39        sz[u] += sz[e->v];
40        if( -1 == son[u] || sz[son[u]] < sz[e->v] )
41          son[u] = e->v;
42      }
43    }
44  }
45  void link(int u, int f) {
46    id[u] = (++tot);
47    top[u] = f;
48    if(son[u] > 0) {
49      link(son[u], f);
50    }
51    for(ep e = u[front]; e; e = e->n) {
52      if(e->v != fa[u] && e->v != son[u]) {
53        link(e->v, e->v);
54      }
55    }
56  }
57
58  /* 其实是树链剖分 */
59  void make_link_cut_tree(void) {
60    calc(1, 0);
61    link(1, 1);
62  }
63
64  int w[MAXN];
65  int sum[MAXN<<2], mx[MAXN<<2];
66
67  void maintain(int o, int l, int r) {
68    sum[o] = sum[o<<1] + sum[o<<1|1];
69    mx[o] = max(mx[o<<1], mx[o<<1|1]);
70  }
71  void build(int o = 1, int l = 1, int r = n) {
72    if(r == l) {
73      sum[o] = w[l];
74      mx[o] = w[l];
```

```
75  │     } else {
76  │       int mid = l+r>>1;
77  │       build(o<<1, l, mid);
78  │       build(o<<1|1, mid+1, r);
79  │       maintain(o, l, r);
80  │     }
81  │ }
82  │ void update(int p, int x, int o = 1, int l = 1, int r = n) {
83  │     if(p <= l && r <= p) {
84  │       sum[o] = x;
85  │       mx[o] = x;
86  │     } else {
87  │       int mid = l+r>>1;
88  │       if(p <= mid) update(p,x,o<<1,l,mid);
89  │       else update(p,x,o<<1|1,mid+1,r);
90  │       maintain(o,l,r);
91  │     }
92  │ }
93  │ int qs(int L, int R, int o = 1, int l = 1, int r = n) {
94  │     if(R < l || r < L) return 0;
95  │     else if (L <= l && r <= R) {
96  │       return sum[o];
97  │     } else {
98  │       int mid = l+r>>1;
99  │       return qs(L,R,o<<1,l,mid)+qs(L,R,o<<1|1,mid+1,r);
100 │     }
101 │ }
102 │ int qm(int L, int R, int o = 1, int l = 1, int r = n) {
103 │     if(L <= l && r <= R) {
104 │       return mx[o];
105 │     } else {
106 │       int mid = l+r>>1;
107 │       if(R <= mid) return qm(L, R, o<<1, l, mid);
108 │       else if ( L > mid ) return qm(L, R, o<<1|1, mid+1, r);
109 │       else return max(qm(L, R, o<<1, l, mid),qm(L, R, o<<1|1, mid+1, r));
110 │     }
111 │ }
112 │
113 │ void change(int u, int t) {
114 │     update(id[u], t);
115 │ }
116 │ int qmax(int u, int v) {
117 │     int ret = -1000000000;
118 │     while(top[u] != top[v]) {
119 │       if( dep[top[u]] > dep[top[v]] ) {
120 │         /* jump u */
121 │         ret = max(ret, qm(id[top[u]], id[u]));
122 │         u = fa[top[u]];
123 │       } else {
124 │         ret = max(ret, qm(id[top[v]], id[v]));
125 │         v = fa[top[v]];
126 │       }
127 │     }
128 │     ret = max(ret, qm(min(id[u],id[v]),max(id[u],id[v])));
129 │     return ret;
130 │ }
131 │ int qsum(int u, int v) {
132 │     int ret = 0;
133 │     while(top[u] != top[v]) {
134 │       if( dep[top[u]] > dep[top[v]] ) {
135 │         /* jump u */
136 │         ret += qs(id[top[u]], id[u]);
137 │         u = fa[top[u]];
138 │       } else {
```

```
139        ret += qs(id[top[v]], id[v]);
140        v = fa[top[v]];
141      }
142    }
143    ret += qs(min(id[u],id[v]),max(id[u],id[v]));
144    return ret;
145  }
146
147  int main(void) {
148    int i;
149    scanf("%d", &n);
150    for(i = 1; i < n; ++i) {
151      int a, b;
152      scanf("%d%d", &a, &b);
153      add_edge(a, b);
154      add_edge(b, a);
155    }
156    make_link_cut_tree();
157    for(i = 1; i <= n; ++i) {
158      scanf("%d", &(w[id[i]]));
159    }
160    build();
161    scanf("%d", &i);
162    while(i--) {
163      char command[8];
164      int a, b;
165      scanf("%s %d %d", command, &a, &b);
166      if('C' == command[0]) change(a, b);
167      else if ('M' == command[1]) printf("%d\n", qmax(a, b));
168      else if ('S' == command[1]) printf("%d\n", qsum(a, b));
169    }
170    return 0;
171  }
```

4.2   2-SAT

```
1  #include <bits/stdc++.h>
2
3  using namespace std;
4
5  namespace two_sat {
6  const int maxn = 100000;
7  const int maxm = 1000000;
8  struct edge {
9    int v;
10    edge *n;
11    edge(void):v(0),n(NULL) {}
12    edge(int vv, edge *nn):v(vv),n(nn) {}
13  };
14  typedef edge *ep;
15  int n;
16  int nE;
17  edge E[maxm];
18  ep front[maxn];
19  void add_edge(int u, int v) {
20    int ne = ++nE;
21    E[ne] = edge(v, u[front]);
22    u[front] = &(E[ne]);
23  }
24  /* (x = xval or y = yval), indexed from 0 */
25  void add_clause(int x, int xv, int y, int yv) {
26    x = x*2 + xv;
27    y = y*2 + yv;
```

```
28 |    add_edge(x^1, y);
29 |    add_edge(y^1, x);
30 | }
31 |
32 | char mark[maxn<<1];
33 | int S[maxn<<1], c;
34 | void init(int N) {
35 |    n = N;
36 |    for(int i = 0; i < n*2; ++i) {
37 |      i[front] = NULL;
38 |      i[mark] = 0;
39 |    }
40 |    nE = 0;
41 | }
42 |
43 | int dfs(int x) {
44 |    if(mark[x^1]) return 0;
45 |    if(mark[x]) return 1;
46 |    mark[x] = 1;
47 |    S[c++] = x;
48 |    for(ep e = x[front]; e; e = e→n)
49 |      if(!dfs(e→v)) return 0;
50 |    return 1;
51 | }
52 |
53 | int solve(void) {
54 |    for(int i = 0; i < n*2; i += 2)
55 |      if(!mark[i] && !mark[i+1]) {
56 |        c = 0;
57 |        if(!dfs(i)) {
58 |          while(c > 0) mark[S[--c]] = 0;
59 |          if(!dfs(i+1)) return 0;
60 |        }
61 |      }
62 |    return 1;
63 | }
64 | }
```

4.3 Cut Edge and Point

```
 1 | Finding cut edges
 2 | The code below works properly because the lemma above (first lemma):
 3 |   h[root] = 0
 4 |            par[v] = −1
 5 |                    dfs (v):
 6 |                    d[v] = h[v]
 7 |                            color[v] = gray
 8 |                                    for u in adj[v]:
 9 |                                        if color[u] == white
10 |                                          then par[u] = v and dfs(u) and d[v] = min(
                                                    d[v], d[u])
11 |                                            if d[u] > h[v]
12 |                                              then the edge v—u is a cut edge
13 |                                              else if u != par[v])
14 |          then d[v] = min(d[v], h[u])
15 |                    color[v] = black
16 |                                In this code, h[v] = height of vertex v in the DFS
                                      tree and d[v] = min(h[w] where there is at least
                                      vertex u in subtree of v in the DFS tree where
                                      there is an edge between u and w).
17 |
18 |                                      Finding cut vertices
```

```
19                                      The code below works properly because the lemma
                                           above (first lemma):
20                                      h[root] = 0
21                                              par[v] = −1
22                                                  dfs (v):
23                                                  d[v] = h[v]
24                                                      color[v] = gray
25                                              for u in adj[v]:
26                                                          if color[u] == white
27                                                          then par[u] = v and dfs(
                                                              u) and d[v] = min(d[v
                                                              ], d[u])
28                                                              if d[u] >= h[v]
                                                                  and (v != root
                                                                  or
                                                                  number_of_children
                                                                  (v) > 1)
29                                                              then the edge v
                                                                  is a cut
                                                                  vertex
30                                                              else if u != par
                                                                  [v])
31                  then d[v] = min(d[v], h[u])
32                          color[v] = black
33                                      In this code, h[v] =  height of vertex v in
                                           the DFS tree and d[v] =  min(h[w] where
                                           there is at least vertex u in subtree of v
                                           in the DFS tree where there is an edge
                                           between u and w).
```

4.4 Euler Path

```
1  /* Euler path, by Abreto<m@abreto.net>. */
2  #define MAXV    (1024)
3  #define MAXE    (MAXV*MAXV)
4
5  typedef struct {
6    int id;
7    int nxt;
8    int del;
9  } egde_t;
10 int front[MAXV];
11 egde_t edg[MAXE];
12 int d[MAXV];
13 int ind[MAXV], outd[MAXV];
14 int nedges;
15 void add_edge(int u, int v) {
16   int newedge = ++nedges;
17   edg[newedge].id = v;
18   edg[newedge].nxt = u[front];
19   edg[newedge].del = 0;
20   u[front] = newedge;
21   outd[u]++;
22   ind[v]++;
23   d[u]++;
24   d[v]++;
25 }
26 void del_edge(int u, int v) {
27   int e = 0;
28   for(e=u[front]; e; e=edg[e].nxt)
29     if(edg[e].id==v) {
30       edg[e].del = 1;
31       outd[u]--;
```

```
32        ind[v]--;
33        d[u]--;
34        d[v]--;
35        return;
36     }
37 }
38
39 int path[MAXV];
40 int l;
41
42 void add2path(int u) {
43   path[l++] = u;
44 }
45
46 /* Directed graph */
47 void euler(int x) {
48   if(outd[x]) {
49     int e = 0;
50     for(e=x[front]; e; e=edg[e].nxt)
51       if(!edg[e].del) {
52         int v = edg[e].id;
53         del_edge(x,v);
54         euler(v);
55       }
56   }
57   add2path(x);
58 }
59
60 /* Undirected graph */
61 void euler(int x) {
62   if(d[x]) {
63     int e = 0;
64     for(e=x[front]; e; e=edg[e].nxt)
65       if(!edg[e].del) {
66         int v = edg[e].id;
67         del_edge(x,v);
68         del_edge(v,x);
69         euler(v);
70       }
71   }
72   add2path(x);
73 }
```

4.5 Shortest Path

4.5.1 Dijkstra

```
1 /* Shortest Path Dijstra, by Abreto<m@abreto.net>. */
2 #include <cstdio>
3 #include <set>
4 #include <utility>
5
6 using namespace std;
7 typedef set< pair<int,int> > spii;
8
9 #define MAXN    512
10 #define MAXV    (MAXN*MAXN)
11
12 struct egde_t {
13   int id;
14   int nxt;
15 };
16 int front[MAXV];
```

```
17  egde_t edg[MAXV<<3];
18  int nedges;
19  void add_edge(int u, int v) {
20    int newedge = ++nedges;
21    edg[newedge].id = v;
22    edg[newedge].nxt = u[front];
23    u[front] = newedge;
24  }
25
26  int d[MAXV];
27  int vis[MAXN];
28  int solid[MAXV];
29
30  int dijstra(int s, int t) {
31    int v = s[front];
32    spii q;
33    q.insert(make_pair(0, s));
34    while(!q.empty()) {
35      auto it = q.begin();
36      int u = it->second;
37      int v = u[front];
38      q.erase(it);
39      solid[u] = 1;
40      if(u == t) break;
41      while(v) {
42        int w = edg[v].id;
43        if(!solid[w]) {
44          if( (0==d[w]) || (d[u] + 1 < d[w]) ) {
45            q.erase(make_pair(d[w],w));
46            d[w] = d[u] + 1;
47            q.insert(make_pair(d[w],w));
48          }
49        }
50        v = edg[v].nxt;
51      }
52    }
53    return d[t];
54  }
```

4.5.2 Shortest Path Fast Algorithm

```
1   /* Shortest Path Fast Algorithm, by Abreto<m@abreto.net>. */
2   #include <cstdio>
3   #include <cstring>
4   #include <queue>
5   #include <utility>
6
7   using namespace std;
8
9   #define MAXN    128
10
11  struct edge {
12    int v;
13    int w;
14    int n;
15  };
16  edge edg[MAXN<<1];
17  int nedg;
18  int indegree[MAXN];
19  int front[MAXN];
20  int find_edge(int u, int v) {
21    int e = u[front];
22    while(e) {
```

```
23       if(edg[e].v == v) return e;
24       e = edg[e].n;
25     }
26     return 0;
27   }
28   void add_edge(int u, int v, int w) {
29     int e = find_edge(u,v);
30     if(0==e) {
31       int newnode = ++nedg;
32       edg[newnode].v = v;
33       edg[newnode].w = w;
34       edg[newnode].n = u[front];
35       u[front] = newnode;
36       indegree[v]++;
37     } else {
38       edg[e].w = (w < edg[e].w)?w:(edg[e].w);
39     }
40   }
41
42   int n;
43
44   char inq[MAXN];
45   int vis[MAXN];
46   int d[MAXN];
47   int spfa(int s) { /* return 1 if fuhuan exists. */
48     queue<int> q;
49     memset(inq, 0, sizeof(inq));
50     memset(d, −1, sizeof(d));
51     memset(vis, 0, sizeof(vis));
52     d[s] = 0;
53     inq[s] = 1;
54     q.push(s);
55     while(!q.empty()) {
56       int u = q.front();
57       q.pop();
58       printf("proc %d..\n", u);
59       inq[u] = 0;
60       if(vis[u]++ > n)
61         return 1;
62       for(int e = front[u]; e; e = edg[e].n) {
63         int v = edg[e].v, w = edg[e].w;
64         if( −1==d[v] || d[u] + w < d[v] ) {
65           d[v] = d[u] + w;
66           if(!inq[v]) {
67             inq[v] = 1;
68             q.push(v);
69           }
70         }
71       }
72     }
73     return 0;
74   }
```

4.6   Maxflow

```
1  /* Max Flow Problem, by Abreto<m@abreto.net> */
2
3  #include <bits/stdc++.h>
4  using namespace std;
5
6  #define MAXV    (100000)
7  #define MAXE    (1000000)
8  struct edge {
```

```
 9 │   static int N;
10 │   int v, w;
11 │   edge *n;
12 │   edge(void):v(0),w(0),n(NULL) {}
13 │   edge(int vv, int ww, edge *nn):v(vv),w(ww),n(nn) {}
14 │ };
15 │ int nE;
16 │ edge E[MAXE];
17 │ edge *front[MAXV];
18 │ void add_edge(int u, int v, int w) {
19 │   int ne = ++nE;
20 │   E[ne] = edge(v, w, u[front]);
21 │   u[front] = &(E[ne]);
22 │ }
23 │ edge *find_edge(int u, int v) {
24 │   for(edge *e = u[front]; e != NULL; e = e->n)
25 │     if(e->v == v)
26 │       return e;
27 │   return NULL;
28 │ }
29 │ void grant_e(int u, int v, int w) {
30 │   edge *e = find_edge(u, v);
31 │   if(NULL == e) add_edge(u,v,w);
32 │   else e->w += w;
33 │ }
34 │
35 │ int vis[MAXV];
36 │ int path[MAXV];
37 │ int dfs(int u, int t) {
38 │   vis[u] = 1;
39 │   if(u == t) return 1;
40 │   for(edge *e = u[front]; e != NULL; e = e->n) {
41 │     int v = e->v;
42 │     if(!vis[v] && e->w && dfs(v,t)) {
43 │       path[u] = v;
44 │       return 1;
45 │     }
46 │   }
47 │   return 0;
48 │ }
49 │ int find_path(int s, int t) {
50 │   memset(vis, 0, sizeof(vis));
51 │   return dfs(s,t);
52 │ }
53 │ int max_flow(int s, int t) {
54 │   int flow = 0;
55 │   while(find_path(s,t)) {
56 │     int i = 0;
57 │     int minf = find_edge(s,path[s])->w;
58 │     for(i = path[s]; i != t; i = path[i])
59 │       minf = min(minf, find_edge(i,path[i])->w);
60 │     for(i = s; i != t; i = path[i]) {
61 │       grant_e(i, path[i], -minf);
62 │       grant_e(path[i], i, minf);
63 │     }
64 │     flow += minf;
65 │   }
66 │   return flow;
67 │ }
68 │
69 │ /* Dinic */
70 │ #define N 1000
71 │ #define INF 100000000
72 │
```

```
73   struct Edge {
74     int from,to,cap,flow;
75     Edge(int u,int v,int c,int f):from(u),to(v),cap(c),flow(f) {}
76   };
77
78   struct Dinic {
79     int n,m,s,t;//结点数，边数（包括反向弧），源点编号，汇点编号
80     vector<Edge>edges;//边表，dges[e]和dges[e^1]互为反向弧
81     vector<int>G[N];//邻接表，G[i][j]表示结点i的第j条边在e数组中的编号
82     bool vis[N]; //BFS的使用
83     int d[N]; //从起点到i的距离
84     int cur[N]; //当前弧下标
85
86     void addedge(int from,int to,int cap) {
87       edges.push_back(Edge(from,to,cap,0));
88       edges.push_back(Edge(to,from,0,0));
89       int  m=edges.size();
90       G[from].push_back(m-2);
91       G[to].push_back(m-1);
92     }
93
94     bool bfs() {
95       memset(vis,0,sizeof(vis));
96       queue<int>Q;
97       Q.push(s);
98       d[s]=0;
99       vis[s]=1;
100      while(!Q.empty()) {
101        int x=Q.front();
102        Q.pop();
103        for(int i=0; i<G[x].size(); i++) {
104          Edge&e=edges[G[x][i]];
105          if(!vis[e.to]&&e.cap>e.flow) { //只考虑残量网络中的弧
106            vis[e.to]=1;
107            d[e.to]=d[x]+1;
108            Q.push(e.to);
109          }
110        }
111
112      }
113      return vis[t];
114    }
115
116    int dfs(int x,int a) { //x表示当前结点，a表示目前为止的最小残量
117      if(x==t||a==0)return a;//a等于0时及时退出，此时相当于断路了
118      int flow=0,f;
119      for(int&i=cur[x]; i<G[x].size(); i++) { //从上次考虑的弧开始，注意要使用引用，同
                时修改cur[x]
120        Edge&e=edges[G[x][i]];//e是一条边
121        if(d[x]+1==d[e.to]&&(f=dfs(e.to,min(a,e.cap-e.flow)))>0) {
122          e.flow+=f;
123          edges[G[x][i]^1].flow-=f;
124          flow+=f;
125          a-=f;
126          if(!a)break;//a等于0及时退出，当a!=0,说明当前节点还存在另一个曾广路分支。
127
128        }
129      }
130      return flow;
131    }
132
133    int Maxflow(int s,int t) { //主过程
134      this->s=s,this->t=t;
135      int flow=0;
```

```
136        while(bfs()) { //不停地用bfs构造分层网络，然后用dfs沿着阻塞流增广
137            memset(cur,0,sizeof(cur));
138            flow+=dfs(s,INF);
139        }
140        return flow;
141    }
142 };
143
144 /* ISAP */
145 struct Edge {
146    int from,to,cap,flow;
147 };
148 const int maxn=650;
149 const int INF=0x3f3f3f3f;
150 struct ISAP {
151    int n,m,s,t;//结点数，边数（包括反向弧），源点编号，汇点编号
152    vector<Edge>edges;
153    vector<int>G[maxn];
154    bool vis[maxn];
155    int d[maxn];
156    int cur[maxn];
157    int p[maxn];
158    int num[maxn];
159    void AddEdge(int from,int to,int cap) {
160        edges.push_back((Edge) {
161            from,to,cap,0
162        });
163        edges.push_back((Edge) {
164            to,from,0,0
165        });
166        m=edges.size();
167        G[from].push_back(m-2);
168        G[to].push_back(m-1);
169    }
170    bool RevBFS() {
171        memset(vis,0,sizeof(vis));
172        queue<int>Q;
173        Q.push(t);
174        d[t]=0;
175        vis[t]=1;
176        while(!Q.empty()) {
177            int x=Q.front();
178            Q.pop();
179            for(int i=0; i<G[x].size(); i++) {
180                Edge &e =edges[G[x][i]^1];
181                if(!vis[e.from]&&e.cap>e.flow) {
182                    vis[e.from]=1;
183                    d[e.from]=d[x]+1;
184                    Q.push(e.from);
185                }
186            }
187        }
188        return vis[s];
189    }
190    int Augment() {
191        int x=t, a=INF;
192        while(x!=s) {
193            Edge &e = edges[p[x]];
194            a= min(a,e.cap-e.flow);
195            x=edges[p[x]].from;
196        }
197        x=t;
198        while(x!=s) {
199            edges[p[x]].flow+=a;
```

```
200       edges[p[x]^1].flow-=a;
201       x=edges[p[x]].from;
202     }
203     return a;
204   }
205   int Maxflow(int s,int t,int n) {
206     this->s=s,this->t=t,this->n=n;
207     int flow=0;
208     RevBFS();
209     memset(num,0,sizeof(num));
210     for(int i=0; i<n; i++) {
211       num[d[i]]++;
212     }
213     int x=s;
214     memset(cur,0,sizeof(cur));
215     while(d[s]<n) {
216       if(x==t) {
217         flow+=Augment();
218         x=s;
219       }
220       int ok=0;
221       for(int i=cur[x]; i<G[x].size(); i++) {
222         Edge &e =edges[G[x][i]];
223         if(e.cap>e.flow&&d[x]==d[e.to]+1) {
224           ok=1;
225           p[e.to]=G[x][i];
226           cur[x]=i;
227           x=e.to;
228           break;
229         }
230       }
231       if(!ok) {
232         int m=n-1;
233         for(int i=0; i<G[x].size(); i++) {
234           Edge &e =edges[G[x][i]];
235           if(e.cap>e.flow)
236             m=min(m,d[e.to]);
237         }
238         if(--num[d[x]]==0)
239           break;
240         num[d[x]=m+1]++;
241         cur[x]=0;
242         if(x!=s)
243           x=edges[p[x]].from;
244       }
245     }
246     return flow;
247   }
248 };
249 int main() {
250   int n,m,a,b,c,res;
251   while(scanf("%d%d",&m,&n)!=EOF) {
252     ISAP tmp;
253     for(int i=0; i<m; i++) {
254       scanf("%d%d%d",&a,&b,&c);
255       tmp.AddEdge(a,b,c);
256     }
257     res=tmp.Maxflow(1,n,n);
258     printf("%d\n",res);
259   }
260   return 0;
261 }
```

4.7 Strongly Connected Component

```
1  /* Kosaraju */
2  #define MAXN    10010
3  #define MAXM    100010
4  struct edge {
5    int v;
6    edge *n;
7    edge(void):v(0),n(NULL) {}
8    edge(int vv, edge *nn):v(vv),n(nn) {}
9  };
10 int nE;
11 edge E[MAXM<<1];
12 edge *ori[MAXN];
13 edge *inv[MAXN];
14 void add_edge(edge *front[], int u, int v) {
15   int ne = ++nE;
16   E[ne] = edge(v, u[front]);
17   u[front] = &(E[ne]);
18 }
19 void connect(int u, int v) {
20   add_edge(ori, u, v);
21   add_edge(inv, v, u);
22 }
23
24 int vis[MAXN];
25 int vst[MAXN];
26 void first_dfs(int u, int &sig) {
27   vis[u] = 1;
28   for(edge *e = u[ori]; e; e = e→n)
29     if(!vis[e→v])
30       first_dfs(e→v, sig);
31   vst[++sig] = u;
32 }
33 int mark[MAXN];
34 void second_dfs(int u, int sig) {
35   vis[u] = 1;
36   mark[u] = sig;
37   for(edge *e = u[inv]; e; e = e→n)
38     if(!vis[e→v])
39       second_dfs(e→v, sig);
40 }
41
42 int N, M;
43
44 int kosaraju(void) {
45   int i;
46   int sig = 0;
47   for(i = 0; i <= N; ++i) vis[i] = 0;
48   for(i = 1; i <= N; ++i) {
49     if(!vis[i])
50       first_dfs(i, sig);
51   }
52   sig = 1;
53   for(i = 0; i <= N; ++i) vis[i] = 0;
54   for(i = N; i > 0; —i) {
55     if(!vis[vst[i]])
56       second_dfs(vst[i], sig++);
57   }
58   for(i = 1; i <= N; ++i)
59     if(mark[i] != 1)
60       return 0;
61   return 1;
62 }
```

```
63
64
65  void clear(void) {
66    nE = 0;
67    for(int i = 0; i <= N; ++i) {
68      ori[i] = inv[i] = NULL;
69    }
70  }
71
72  /* Tarjan */
73  #define MAXN     10010
74  #define MAXM     100010
75  struct edge {
76    int v;
77    edge *n;
78    edge(void):v(0),n(NULL) {}
79    edge(int vv, edge *nn):v(vv),n(nn) {}
80  };
81  typedef edge *ep;
82  int nE;
83  edge E[MAXM];
84  edge *front[MAXN];
85  void add_edge(int u, int v) {
86    int ne = ++nE;
87    E[ne] = edge(v, u[front]);
88    u[front] = &(E[ne]);
89  }
90
91  int mark[MAXN];
92  int dfn[MAXN], low[MAXN];
93  int stk[MAXN];
94  int stk_top;
95
96  void tardfs(int u, int stamp, int &scc) {
97    mark[u] = 1;
98    dfn[u] = low[u] = stamp;
99    stk[stk_top++] = u;
100   for(ep e = u[front]; e; e = e->n) {
101     if(0 == mark[e->v]) tardfs(e->v, ++stamp, scc);
102     if(1 == mark[e->v]) low[u] = min(low[u], low[e->v]);
103   }
104   if(dfn[u] == low[u]) {
105     ++scc;
106     do {
107       low[stk[stk_top-1]] = scc;
108       mark[stk[stk_top-1]] = 2;
109     } while(stk[(stk_top--)-1] != u);
110   }
111 }
112
113 int tarjan(int n) {
114   int scc = 0, lay = 1;
115   for(int i = 1; i <= n; ++i)
116     if(0 == mark[i])
117       tardfs(i, lay, scc);
118   return scc;
119 }
120
121 int N, M;
122
123 void clear(void) {
124   nE = 0;
125   for(int i = 0; i <= N; ++i) {
126     i[front] = NULL;
```

```
127        mark[i] = low[i] = 0;
128      }
129      stk_top = 0;
130   }
131
132   /* Garbow */
133   #define MAXN     10010
134   #define MAXM     100010
135
136   struct edge {
137      int v;
138      edge *n;
139      edge(void):v(0),n(NULL) {}
140      edge(int vv, edge *nn):v(vv),n(nn) {}
141   };
142   typedef edge *ep;
143
144   int nE;
145   edge E[MAXM];
146   edge *front[MAXN];
147   void add_edge(int u, int v) {
148      int ne = ++nE;
149      E[ne] = edge(v, u[front]);
150      u[front] = &(E[ne]);
151   }
152
153   int stk1[MAXN], stk1t;
154   int stk2[MAXN], stk2t;
155   int low[MAXN], belg[MAXN];
156
157   void garbowdfs(int u, int lay, int &scc) {
158      stk1[++stk1t] = u;
159      stk2[++stk2t] = u;
160      low[u] = ++lay;
161      for(ep e=u[front]; e; e = e→n) {
162         if(!low[e→v]) garbowdfs(e→v, lay, scc);
163         else if (0 == belg[e→v])
164            while(low[stk2[stk2t]] > low[e→v])
165               —stk2t;
166      }
167      if(stk2[stk2t] == u) {
168         stk2t—;
169         scc++;
170         do {
171            belg[stk1[stk1t]] = scc;
172         } while(stk1[stk1t—] != u);
173      }
174   }
175
176   int grabow(int n) {
177      int i;
178      int scc = 0, lay = 0;
179      for(i = 0; i <= n; ++i) {
180         belg[i] = low[i] = 0;
181      }
182      for(i = 1; i <= n; ++i)
183         if(0 == low[i])
184            garbowdfs(i, lay, scc);
185      return scc;
186   }
187
188   int N, M;
189
190   void clear(void) {
```

```
191    nE = 0;
192    for(int i = 0; i <= N; ++i) {
193      front[i] = NULL;
194    }
195  }
```

4.8　Perfect elimination ordering

求弦图的最大团数/最小色数的时候，只要在完美消除序列上从后往前贪心染色即可。
而求最大独立集/最小团覆盖的时候，只要在完美消除序列上从前往后贪心取点即可。

```
1   /**
2    * BZOJ 1006
3    * [HNOI2008] 神奇的国度
4    * 最大势法求完美消除序列
5    * by Abreto<m@abreto.net>.
6    **/
7   #include <cassert>
8   #include <cstdio>
9   #include <vector>
10  #include <bitset>
11  #include <algorithm>
12
13  using namespace std;
14  typedef vector<int> vi;
15  typedef vi::iterator vii;
16  #define pb push_back
17  #define MAXN 10100
18  #define MAXM 1000100
19
20  int n;
21  vi g[MAXN];
22  int ans;
23
24  struct node_t {
25    int v;
26    node_t *nxt;
27  } node[MAXM << 2];
28  int used;
29  node_t *new_node(void) {
30    return node + (used ++);
31  }
32
33  node_t *f[MAXN];  /* head */
34  void lkto(int pos, int item) {
35    node_t *t = new_node();
36    t->v = item;
37    t->nxt = f[pos];
38    f[pos] = t;
39  }
40
41  int usedby[MAXN];
42  int color[MAXN];
43  bitset<MAXN> added;
44  int label[MAXN], max_label;
45  void mcs(void) {
46    for (int i = 1; i <= n; i++) lkto(0, i);
47    for (int i = n; i > 0; i--) {
48      node_t *cur = f[max_label];
49      assert(cur != NULL);
50      while (added.test(cur->v)) {  /* already added */
51        cur = cur->nxt;
52        while (NULL == cur)
```

```
53        cur = f[ --max_label ];
54      }
55      f[ max_label ] = cur->nxt;
56      while (max_label && NULL == f[max_label]) max_label--;
57      int u = cur->v;
58      added.set(u);
59      /* the i-th is u */
60      for (vii it = g[u].begin(); it != g[u].end(); it++) {
61        int v = *it;
62        if (!added.test(v)) {
63          label[v] ++;
64          max_label = max(max_label, label[v]);
65          lkto(label[v], v);
66        }
67
68        usedby[color[v]] = i;
69      }
70      for (int j = 1; j <= n; j++)
71        if (usedby[j] != i) {
72          color[u] = j;
73          break;
74        }
75      ans = max(ans, color[u]);
76    }
77 }
78
79 int main(void) {
80   int m;
81   scanf("%d%d", &n, &m);
82   while (m--) {
83     int ai, bi;
84     scanf("%d%d", &ai, &bi);
85     g[ai].pb(bi);
86     g[bi].pb(ai);
87   }
88   mcs();
89   printf("%d\n", ans);
90   return 0;
91 }
```

5 Math

5.1 Euler Function

```
1 /* Euler function phi(x), by Abreto<m@abreto.net>. */
2
3 #define MAXX    3000000
4
5 int phi[MAXX];
6 void get_euler(void) {
7   int i = 0, j = 0;
8   phi[1] = 1;
9   for(i = 2; i < MAXX; ++i)
10    if(!phi[i])
11      for(j = i; j < MAXX; j += i) {
12        if(!phi[j]) phi[j] = j;
13        phi[j] = phi[j]/i * (i-1);
14      }
15 }
```

5.2 Möbius Function

```
1  void sieve() {
2    fill(isPrime, isPrime + maxn, 1);
3    mu[1] = 1, num = 0;
4    for (int i = 2; i < maxn; ++i) {
5      if (isPrime[i]) primes[num++] = i, mu[i] = −1;
6      static int d;
7      for (int j = 0; j < num && (d = i * primes[j]) < maxn; ++j) {
8        isPrime[d] = false;
9        if (i % primes[j] == 0) {
10         mu[d] = 0;
11         break;
12       } else mu[d] = −mu[i];
13     }
14   }
15 }
```

5.3 Number Theory Inverse

```
1  #include <bits/stdc++.h>
2  using namespace std;
3
4  const int n=10000000;      /* */
5  const long long mod=1e9+7;  /* prime required. */
6
7  long long fact[n],fiv[n],inv[n];
8
9  int main() {
10   fact[0]=fact[1]=1;
11   fiv[0]=fiv[1]=1;
12   inv[1]=1;
13   for (int i=2; i<n; i++) {
14     fact[i]=fact[i−1]*i%mod;
15     inv[i]=(mod−mod/i)*inv[mod%i]%mod;
16     fiv[i]=inv[i]*fiv[i−1]%mod;
17   }
18   for (int i=1; i<n; i++) {
19     if (fact[i]*fiv[i]%mod!=1)  printf("fact wrong: %d\n",i);
20     if (inv[i]*i%mod!=1)        printf("intv wrong: %d\n",i);
21   }
22   cout<<"complete"<<endl;
23   return 0;
24 }
```

5.4 Chinese Remainder Theorem

$$x \equiv a_i \pmod{m_i}$$

```
1  /* Chinese Remainder Theorem, by Abreto<m@abreto.net>. */
2  #include "euler.c"
3
4  #define MAXN    64
5
6  typedef long long int ll;
7
8  ll quickpow(ll a, ll b, ll mod) {
9    ll ret = 1, base = a;
10   while(b > 0) {
11     if(b & 1) ret = (ret * base) % mod;
12     base = (base * base) % mod;
13     b >>= 1;
```

```
14      }
15      return ret;
16   }
17
18   ll N;
19   ll a[MAXN], m[MAXN]; /* a and m is indexed from 0. */
20   ll x, M;
21
22   void naive_crt(void) {
23      int i = 0;
24      ll Mi[MAXN], nMi[MAXN];
25      ll t[MAXN];
26
27      M = 1;
28      for(i = 0; i < N; ++i)
29        M *= a[i];
30      for(i = 0; i < N; ++i)
31        Mi[i] = M / a[i];
32      get_euler();
33      for(i = 0; i < N; ++i)
34        nMi[i] = quickpow(Mi[i], phi[a[i]]-1, a[i]);
35      for(i = 0; i < N; ++i)
36        t[i] = ((a[i] * Mi[i]) % M) * nMi[i] % M;
37      for(i = 0; i < N; ++i)
38        x = (x + t[i]) % M;
39   }
```

5.5   Linear congruences

```
 1   #include <cstdio>
 2   #include <cassert>
 3   #include <cstdlib>
 4
 5   using namespace std;
 6
 7   class mod_equ_resolver {
 8      typedef long long int ll;
 9      ll a, m;
10      inline void gurantee(void) {
11        if ( a < 0 ) {
12          ll k = ( -a ) / m;
13          a += ( k + 1ll ) * m;
14          a = ( a + m ) % m;
15        } else {
16          a %= m;
17        }
18        // printf("x = %lld (mod %lld)\n",a, m);
19      }
20   public:
21      mod_equ_resolver(void) {
22        a = 0ll;
23        m = 1ll;
24      }
25      ll exgcd(ll m, ll n, ll &x, ll &y) {
26        if ( 0 == n ) {
27          x = 1;
28          y = 0;
29          return m;
30        }
31        ll g = exgcd( n, m % n, x, y );
32        ll t = x;
33        x = y;
34        y = t - m / n * y;
```

```
35        return g;
36    }
37    int onemore(ll a2, ll m2) {
38        ll x, y;
39        ll g = exgcd( m, m2, x, y );
40        assert(x*m+y*m2==g);
41        a2 = (a2 + m2) % m2;
42        if ( abs( a2 - a ) % g ) return -1;
43        ll newm = m / g * m2;
44        ll newa = a + ( a2 - a ) / g * x * m;
45        a = newa;
46        m = newm;
47        gurantee();
48        return 0;
49    }
50    ll resolve(void) {
51        return a;
52    }
53 };
```

Usage: For

$$
\begin{cases}
x \equiv a_1 \quad \mod m_1 \\
x \equiv a_2 \quad \mod m_2 \\
\vdots \\
x \equiv a_k \quad \mod m_k
\end{cases}
$$

run

```
1 mod_equ_resolver solver;
2 for (int i = 1; i <= k; i++)
3     solver.onemore(a[i], m[i]);
```

then the solution is

$$
x \equiv solver.a \quad \mod solver.m
$$

5.6 FFT

```
1  #include <cmath>
2  using namespace std;
3  namespace fft {
4  #define eps (1e-9)
5  template < typename T = double >
6  struct dbl {
7      T x;
8      dbl(void):x(0.0) {}
9      template <typename U>
10     dbl(U a):x((T)a) {}
11     inline char sgn(void) {
12         return ((x>=-eps)&&(x<=eps))?(0):((x>eps)?(1):(-1));
13     }
14     inline T tabs(void) {
15         return ((x>=-eps)&&(x<=eps))?(0.0):((x>eps)?(x):(-x));
16     }
17     inline dbl abs(void) {
18         return dbl(tabs());
19     }
20     template <typename U> inline dbl &operator=(const U b) {
21         x=(T)b;
22         return (*this);
23     }
24     inline T *operator&(void) {
25         return &x;
26     }
```

```
27    inline dbl operator-(void) const {
28      return dbl(-x);
29    }
30    inline dbl operator+(const dbl &b) const {
31      return dbl(x+b.x);
32    }
33    inline dbl operator-(const dbl &b) const {
34      return dbl(x-b.x);
35    }
36    inline dbl operator*(const dbl &b) const {
37      return dbl(x*b.x);
38    }
39    inline dbl operator/(const dbl &b) const {
40      return dbl(x/b.x);
41    }
42    template <typename U> inline dbl operator^(const U &b) const {
43      T ret=1.0,base=x;
44      while(b) {
45        if(b&1)ret*=base;
46        base*=base;
47        b>>=1;
48      }
49      return dbl(ret);
50    }
51    inline dbl operator+=(const dbl &b) {
52      return dbl(x+=b.x);
53    }
54    inline dbl operator-=(const dbl &b) {
55      return dbl(x-=b.x);
56    }
57    inline dbl operator*=(const dbl &b) {
58      return dbl(x*=b.x);
59    }
60    inline dbl operator/=(const dbl &b) {
61      return dbl(x/=b.x);
62    }
63    template <typename U> inline dbl operator^=(const U &b) {
64      dbl tmp=(*this)^b;
65      *this=tmp;
66      return tmp;
67    }
68    inline bool operator==(const dbl &b) const {
69      return (0 == ((*this)-b).sgn());
70    }
71    inline bool operator!=(const dbl &b) const {
72      return (0 != ((*this)-b).sgn());
73    }
74    inline bool operator<(const dbl &b) const {
75      return (-1 == ((*this)-b).sgn());
76    }
77    inline bool operator<=(const dbl &b) const {
78      return (((*this)==b) || ((*this)<b));
79    }
80    inline bool operator>(const dbl &b) const {
81      return (b < (*this));
82    }
83    inline bool operator>=(const dbl &b) const {
84      return (((*this)==b) || ((*this)>b));
85    }
86    template <typename U> inline operator U() const {
87      return (U)x;
88    }
89    inline char operator[](unsigned n) {
90      if(n >= 0) {
```

```
 91         long long int ret=x;
 92         while(n--) {
 93           ret/=10;
 94         }
 95         return (ret%10);
 96       } else {
 97         T ret=x;
 98         n=-n;
 99         while(n--)ret*=10.0;
100         return ((long long int)ret)%10;
101       }
102     }
103 };
104 template <typename T>
105 struct Complex {
106   T x,y;  /* x + iy */
107   Complex(void):x(T()),y(T()) {}
108   Complex(T xx):x(xx) {}
109   Complex(T xx,T yy):x(xx),y(yy) {}
110   inline Complex operator-(void) const {
111     return Complex(-x,-y);
112   }
113   inline Complex operator+(const Complex& b) const {
114     return Complex(x+b.x,y+b.y);
115   }
116   inline Complex operator-(const Complex& b) const {
117     return Complex(x-b.x,y-b.y);
118   }
119   inline Complex operator*(const Complex& b) const {
120     return Complex(x*b.x-y*b.y,x*b.y+y*b.x);
121   }
122   inline Complex operator/(const Complex& b) const {
123     T bo=b.x*b.x+b.y*b.y;
124     return Complex((x*b.x+y*b.y)/bo,(y*b.x-x*b.y)/bo);
125   }
126   inline Complex& operator+=(const Complex& b) {
127     Complex tmp=(*this)+b;
128     (*this)=tmp;
129     return (*this);
130   }
131   inline Complex& operator-=(const Complex& b) {
132     Complex tmp=(*this)-b;
133     (*this)=tmp;
134     return (*this);
135   }
136   inline Complex& operator*=(const Complex& b) {
137     Complex tmp=(*this)*b;
138     (*this)=tmp;
139     return (*this);
140   }
141   inline Complex& operator/=(const Complex& b) {
142     Complex tmp=(*this)/b;
143     (*this)=tmp;
144     return (*this);
145   }
146   inline friend Complex operator+(const T& a, const Complex& b) {
147     return Complex(a)+b;
148   }
149   inline friend Complex operator-(const T& a, const Complex& b) {
150     return Complex(a)-b;
151   }
152   inline friend Complex operator*(const T& a, const Complex& b) {
153     return Complex(a)*b;
154   }
```

```
155    inline friend Complex operator/(const T& a, const Complex& b) {
156      return Complex(a)/b;
157    }
158  };
159  typedef dbl<> Double;
160  typedef Complex<Double> ComplexD;
161  typedef long long int ll;
162  const int maxn = 2000000; /* !! */
163  const Double pi(acos(-1.0));
164
165  void build(ComplexD _P[], ComplexD P[], int n, int m, int curr, int &cnt) {
166    if(m == n) {
167      _P[curr] = P[cnt++];
168    } else {
169      build(_P, P, n, m*2, curr, cnt);
170      build(_P, P, n, m*2, curr+m, cnt);
171    }
172  }
173
174  void FFT(ComplexD P[], int n, int oper) { /* n should be 2^k. */
175    static ComplexD _P[maxn];
176    int cnt = 0;
177    build(_P, P, n, 1, 0, cnt);
178    copy(_P, _P+n, P);
179    for(int d = 0; (1<<d)<n; ++d) {
180      int m = 1<<d;
181      int m2 = m*2;
182      Double p0 = pi / m * oper;
183      ComplexD unit_p0(cos(p0.x), sin(p0.x));
184      for(int i = 0; i < n; i += m2) {
185        ComplexD unit(1,0);
186        for(int j = 0; j < m; ++j) {
187          ComplexD &P1 = P[i+j+m], &P2 = P[i+j];
188          ComplexD t = unit * P1;
189          P1 = P2 - t;
190          P2 = P2 + t;
191          unit *= unit_p0;
192        }
193      }
194    }
195    if(-1 == oper) {
196      for(int i = 0; i < n; ++i)
197        P[i] /= Double(n);
198    }
199  }
200  }
```

5.7 NTT

```
1   #include<bits/stdc++.h>
2   #define ll long long
3   const int N=262144;
4   const ll MOD=50000000001507329LL;//998244353 1004535809
5   using namespace std;
6   int n,m;
7   ll a[N],b[N],x[N],y[N];
8   ll wn[25];
9   ll Mul(ll x,ll y) { //乘法超ll用快速乘，主函数也需要用
10    ll ans=(x*y-(ll)((long double)x/MOD*y+1e-8)*MOD);
11    return ans<0?ans+MOD:ans;
12  }
13  ll Qpow(ll a,ll b,ll M) {
14    ll ans=1;
```

```
15      a%=M;
16      while(b) {
17        if(b&1) ans=Mul(ans,a);
18        a=Mul(a,a);
19        b>>=1;
20      }
21      return ans;
22  }
23  void Getwn() { //主函数预处理getwn()
24      for(int i=0; i<25; i++) {
25        wn[i]=Qpow(3,(MOD−1)/(1<<i),MOD);
26      }
27  }
28  void NTT(ll *x,int n,int rev) {
29      int i,j,k,ds;
30      ll w,u,v;
31      for(i=1,j=n>>1,k=n>>1; i<n−1; i++,k=n>>1) {
32        if(i<j) swap(x[i],x[j]);
33        while(j>=k) j−=k,k>>=1;
34        if(j<k) j+=k;
35      }
36      for(i=2,ds=1; i<=n; i<<=1,ds++) {
37        for(j=0; j<n; j+=i) {
38          w=1;
39          for(k=j; k<j+i/2; k++) {
40            u=x[k];
41            v=Mul(w,x[k+i/2]);
42            x[k]=(u+v)%MOD;
43            x[k+i/2]=(u−v+MOD)%MOD;
44            w=Mul(w,wn[ds]);
45          }
46        }
47      }
48      if(rev==−1) {
49        for(i=1; i<n/2; i++) swap(x[i],x[n−i]);
50        w=Qpow(n,MOD−2,MOD);
51        for(i=0; i<n; i++) x[i]=Mul(x[i],w);
52      }
53  }
54  int main() {
55      Getwn();
56      while(~scanf("%d%d",&n,&m)) {
57        for(int i=0; i<n; i++)scanf("%lld",&a[i]);
58        for(int i=0; i<m; i++)scanf("%lld",&b[i]);
59        int len=1,s=n+m;
60        while(len<s)len<<=1;
61        for(int i=n; i<len; i++)a[i]=0;
62        for(int i=m; i<len; i++)b[i]=0;
63        NTT(a,len,1);
64        NTT(b,len,1);
65        for(int i=0; i<len; i++)a[i]=Mul(a[i],b[i]);
66        NTT(a,len,−1);
67        for(int i=0; i<=s; i++)printf("%lld␣",a[i]);
68        puts("");
69      }
70  }
71
72  // ─────────────────────────────────────────
73  #include<cstdio>
74  #include<iostream>
75  #include<cstring>
76  #include<cmath>
77  #include<complex>
78  using namespace std;
```

```
 79 | typedef long long LL;
 80 | const LL MOD=998244353,g=3,gi=332748118;
 81 | const LL N=1000005;
 82 | LL n,m;
 83 | LL a[N],b[N];
 84 | LL pow (LL x,LL y) {
 85 |   if (y==1) return x;
 86 |   LL lalal=pow(x,y>>1);
 87 |   lalal=lalal*lalal%MOD;
 88 |   if (y&1) lalal=lalal*x%MOD;
 89 |   return lalal;
 90 | }
 91 | void ntt (LL *a,LL n,LL o) {
 92 |   if (n==1) return ;
 93 |   LL k=(n>>1);
 94 |   LL w=1,wn=pow(o==1?g:gi,(MOD-1)/n),a0[k],a1[k];
 95 |   for (LL u=0; u<k; u++) {
 96 |     LL i=u*2;
 97 |     a0[u]=a[i];
 98 |     a1[u]=a[i+1];
 99 |   }
100 |   ntt(a0,k,o);
101 |   ntt(a1,k,o);
102 |   for (LL u=0; u<k; u++) {
103 |     a[u]=a0[u]+w*a1[u]%MOD;
104 |     a[u]=(a[u]%MOD+MOD)%MOD;
105 |     a[u+k]=a0[u]-w*a1[u];
106 |     a[u+k]=(a[u+k]%MOD+MOD)%MOD;
107 |     w=w*wn%MOD;
108 |   }
109 | }
110 | void ntt(LL *a,LL n,LL op) {
111 |   for (LL u=0; u<n; u++) bin[u]=(bin[u>>1]>>1)|((u&1)*(n>>1));
112 |   for (LL u=0; u<n; u++) if (u<bin[u]) swap(a[u],a[bin[u]]);
113 |   for (LL u=1; u<n; u<<=1) {
114 |     LL wn=pow(op==1?g:gi,(MOD-1)/(u<<1)),w,t;
115 |     for (LL i=0; i<n; i=i+(u<<1)) {
116 |       w=1;
117 |       for (LL k=0; k<u; k++) {
118 |         t=w*a[u+i+k]%MOD;
119 |         a[u+i+k]=(a[i+k]-t+MOD)%MOD;
120 |         a[i+k]=(a[i+k]+t)%MOD;
121 |         w=w*wn%MOD;
122 |       }
123 |     }
124 |   }
125 |   if(op==-1) {
126 |     LL Inv=pow(n,MOD-2);
127 |     for(LL i=0; i<n; i++) a[i]=a[i]*Inv%MOD;
128 |   }
129 | }
130 |
131 | int main() {
132 |   scanf("%I64d%I64d",&n,&m);
133 |   for (LL u=0; u<=n; u++) scanf("%I64d",&a[u]);
134 |   for (LL u=0; u<=m; u++) scanf("%I64d",&b[u]);
135 |   m=m+n;
136 |   n=1;
137 |   while (n<=m) n<<=1;
138 |   ntt(a,n,1);
139 |   ntt(b,n,1);
140 |   for (LL u=0; u<=n; u++) a[u]*=b[u];
141 |   ntt(a,n,-1);
142 |   LL inv=pow(n,MOD-2);
```

```
143  for (LL u=0; u<=m; u++)   printf("%I64d ",a[u]*inv%MOD);
144  return 0;
145 }
```

5.8 Fast Walsh-Hadamard transform

- 异或

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\} + \mathcal{F}\{A_1\}, \mathcal{F}\{A_0\} - \mathcal{F}\{A_1\}]$$

$$\mathcal{F}^{-1}\{A\} = \left[\mathcal{F}^{-1}\{\frac{A_0+A_1}{2}\}, \mathcal{F}^{-1}\{\frac{A_0-A_1}{2}\}\right]$$

- 按位与

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\} + \mathcal{F}\{A_1\}, \mathcal{F}\{A_1\}]$$

$$\mathcal{F}^{-1}\{A\} = \left[\mathcal{F}^{-1}\{A_0\} - \mathcal{F}^{-1}\{A_1\}, \mathcal{F}^{-1}\{A_1\}\right]$$

- 按位或

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\}, \mathcal{F}\{A_1\} + \mathcal{F}\{A_0\}]$$

$$\mathcal{F}^{-1}\{A\} = \left[\mathcal{F}^{-1}\{A_0\}, \mathcal{F}^{-1}\{A_1\} - \mathcal{F}^{-1}\{A_0\}\right]$$

```
1  void FWT(int a[],int n) {
2    for(int d=1; d<n; d<<=1)
3      for(int m=d<<1,i=0; i<n; i+=m)
4        for(int j=0; j<d; j++) {
5          int x=a[i+j],y=a[i+j+d];
6          a[i+j]=(x+y)%mod,a[i+j+d]=(x-y+mod)%mod;
7          //xor:a[i+j]=x+y,a[i+j+d]=(x-y+mod)%mod;
8          //and:a[i+j]=x+y;
9          //or:a[i+j+d]=x+y;
10       }
11 }
12
13 void UFWT(int a[],int n) {
14   for(int d=1; d<n; d<<=1)
15     for(int m=d<<1,i=0; i<n; i+=m)
16       for(int j=0; j<d; j++) {
17         int x=a[i+j],y=a[i+j+d];
18         a[i+j]=1LL*(x+y)*rev%mod,a[i+j+d]=(1LL*(x-y)*rev%mod+mod)%mod;
19         //xor:a[i+j]=(x+y)/2,a[i+j+d]=(x-y)/2;
20         //and:a[i+j]=x-y;
21         //or:a[i+j+d]=y-x;
22       }
23 }
24 void solve(int a[],int b[],int n) {
25   FWT(a,n);
26   FWT(b,n);
27   for(int i=0; i<n; i++) a[i]=1LL*a[i]*b[i]%mod;
28   UFWT(a,n);
29 }
```

5.9 Lucas

```
1 /* Lucas, by Abreto<m@abreto.net>. */
2
3 struct __lucas {
4   static const int maxp = 100000;
5   typedef long long int ll;
6   int p;
7   int f[maxp]; // fiv[maxp], inv[maxp];
```

```
8     inline int mul(const int a, const int b) {
9       ll z = 1ll * a * b;
10      z -= z / p * p;
11      return z;
12    }
13    int qow(int a, int x) {
14      int ret = 1;
15      while (x) {
16        if (1 & x) ret = mul(ret, a);
17        a = mul(a, a);
18        x >>= 1;
19      }
20      return ret;
21    }
22    void init(int np) {
23      p = np;
24      // return; // uncomment this line if use binom()
25      f[0] = f[1] = 1;
26      // fiv[0] = fiv[1] = 1;
27      // inv[1] = 1;
28      for (int i = 2; i < p; i++) {
29        f[i] = mul(f[i - 1], i);
30        // inv[i] = mul(p - p / i, inv[p % i]);
31        // fiv[i] = mul(fiv[i - 1], inv[i]);
32      }
33    }
34    int C(int n, int k) {
35      if (n < k) return 0;
36      return mul(f[n], qow(mul(f[k], f[n - k]), p - 2));
37    }
38    /** use following if get TLE { */
39    int binom(int n, int k) {
40      if (n < k) return 0;
41      if (k > n - k) k = n - k;
42      int a = 1, b = 1;
43      while (k) {
44        a = mul(a, n);
45        b = mul(b, k);
46        n--;
47        k--;
48      }
49      return mul(a, qow(b, p - 2));
50    }
51    /** } --- */
52    int operator()(int n, int k) {
53      if (0 == k) return 1;
54      if (n < p && k < p) return C(n, k);
55      return mul(C(n % p, k % p), (*this)(n / p, k / p));
56    }
57 } lucas;
```

5.10 Linear Programming

```
1  /* 线性规划 */
2  #include<bits/stdc++.h>
3
4  using namespace std;
5  const int Maxn=110,Maxm=59;
6  class Simplex {
7    /*
8       功能：
9       接受有n个约束，m个基本变量的方程组a[0~n][0~m]
10      a[0][]存放需要最大化的目标函数，a[][0]存放常数
```

```
11        Base[]存放基本变量的id,初始为1~m
12        Rest[]存放松弛变量的id,初始为m+1~m+n
13        返回此线性规划的最小值ans
14        要求方案的话，Base[]中的变量值为0,Rest[]中的变量值为相应行的[0]
15        如果solve
16        返回1,说明运行正常ans是它的最大值
17        返回0,说明无可行解
18        返回−1,说明解没有最大值
19        测试:
20        m=2,n=3
21        double a[4][3]={
22        {0,1,3},
23        {8,−1,1},
24        {−3,1,1},
25        {2,1,−4}
26        };
27        solve=1,ans=64/3;
28        注意ac不了可能是eps的问题
29      */
30  public:
31      static const double Inf;
32      static const double eps;
33      int n,m;
34      double a[Maxn][Maxm];
35      int Base[Maxm],Rest[Maxn];
36      double val[Maxm];
37      double ans;
38      void pt() {
39        for(int i=0; i<=n; i++) {
40          for(int j=0; j<=m; j++)printf("%.2f ",a[i][j]);
41          puts("");
42        }
43      }
44      void pivot(int x,int y) { //将第x个非基本变量和第y个基本变量调换
45        swap(Rest[x],Base[y]);
46        double tmp=−1./a[x][y];
47        a[x][y]=−1.;
48        for(int j=0; j<=m; j++)a[x][j]*=tmp;
49        for(int i=0; i<=n; i++) {
50          if(i==x||fabs(a[i][y])<eps)continue;
51          tmp=a[i][y];
52          a[i][y]=0;
53          for(int j=0; j<=m; j++)a[i][j]+=tmp*a[x][j];
54        }
55      }
56      bool opt() {
57        while(1) {
58          int csi=0;
59          for(int i=1; i<=m; i++)if(a[0][i]>eps&&(!csi||Base[i]<Base[csi]))csi=i;
60          if(!csi)break;
61          int csj=0;
62          double cur;
63          for(int j=1; j<=n; j++) {
64            if(a[j][csi]>−eps)continue;
65            double tmp=−a[j][0]/a[j][csi];
66            if(!csj||tmp+eps<cur||(fabs(tmp−cur)<eps&&Rest[j]<Rest[csj]))csj=j,cur=tmp;
67          }
68          if(!csj)return 0;
69          pivot(csj,csi);
70        }
71        ans=a[0][0];
72        return 1;
73      }
74      bool init() {
```

```
75        ans=0;
76        for(int i=1; i<=m; i++)Base[i]=i;
77        for(int i=1; i<=n; i++)Rest[i]=m+i;
78        int cs=1;
79        for(int i=2; i<=n; i++)if(a[i][0]<a[cs][0])cs=i;
80        if(a[cs][0]>=-eps)return 1;
81        static double tmp[Maxm];
82        for(int i=0; i<=m; i++)tmp[i]=a[0][i],a[0][i]=0;
83        for(int i=1; i<=n; i++)a[i][m+1]=1.;
84        a[0][m+1]=-1.;
85        Base[m+1]=m+n+1;
86        pivot(cs,++m);
87        opt();
88        m--;
89        if(a[0][0]<-eps)return 0;
90        cs=-1;
91        for(int i=1; i<=n; i++) {
92          if(Rest[i]>m+n) {
93            cs=i;
94            break;
95          }
96        }
97        if(cs>=1) {
98          int nxt=-1;
99          m++;
100         for(int i=1; i<=m; i++)if(a[cs][i]>eps||a[cs][i]<-eps) {
101             nxt=i;
102             break;
103           }
104         pivot(cs,nxt);
105         m--;
106       }
107       for(int i=1; i<=m; i++) {
108         if(Base[i]>m+n) {
109           swap(Base[i],Base[m+1]);
110           for(int j=0; j<=n; j++)a[j][i]=a[j][m+1];
111           break;
112         }
113       }
114       for(int i=1; i<=m; i++)a[0][i]=0;
115       a[0][0]=tmp[0];
116       for(int i=1; i<=m; i++)if(Base[i]<=m)a[0][i]=tmp[Base[i]];
117       for(int i=1; i<=n; i++) {
118         if(Rest[i]<=m) {
119           for(int j=0; j<=m; j++)a[0][j]+=tmp[Rest[i]]*a[i][j];
120         }
121       }
122       return 1;
123     }
124     void getval() {
125       for(int i=1; i<=m; i++)val[i]=0;
126       for(int i=1; i<=n; i++)if(Rest[i]<=m)val[Rest[i]]=a[i][0];
127       //for(int i=1;i<=m;i++)printf("%.2f ",val[i]);puts("");
128     }
129     int solve() {
130       if(!init())return 0;
131       if(!opt())return -1;
132       getval();
133       return 1;
134     }
135 } solver;
136 const double Simplex:: Inf=1e80;
137 const double Simplex:: eps=1e-8;
138 int main() {
```

```
139    int m,n,type;
140    scanf("%d%d%d",&m,&n,&type);
141    solver.a[0][0]=0;
142    for(int i=1; i<=m; i++)scanf("%lf",&solver.a[0][i]);
143    for(int i=1; i<=n; i++) {
144      for(int j=1; j<=m+1; j++) {
145        if(j==m+1)scanf("%lf",&solver.a[i][0]);
146        else {
147          scanf("%lf",&solver.a[i][j]);
148          solver.a[i][j]=-solver.a[i][j];
149        }
150      }
151    }
152    solver.m=m,solver.n=n;
153    int rep=solver.solve();
154    if(rep==0)puts("Infeasible");
155    else if(rep==-1)puts("Unbounded");
156    else {
157      printf("%.12f\n",solver.ans);
158      if(type==1) {
159        for(int i=1; i<=m; i++)printf("%.12f%c",solver.val[i],i==m?'\n':'␣');
160      }
161    }
162  }
```

5.11 Big Prime Test

```
1  #include <iostream>
2  #include <cstdlib>
3  using namespace std;
4  typedef long long LL;
5  LL minfactor, p[11] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29};
6  LL gcd(LL a, LL b) {
7    return b ? gcd(b, a % b) : a;
8  }
9  LL qmult(LL a, LL b, LL mod) {  // 快速乘模
10    LL sum = 0;
11    while (b) {
12      if (b & 1) {
13        sum += a;
14        if (sum >= mod) sum -= mod;  // 此处无需用%, %运算比减法慢很多
15      }
16      b >>= 1, a <<= 1;
17      if (a >= mod) a -= mod;
18    }
19    return sum;
20  }
21  LL qpow(LL a, LL b, LL mod) {  // 快速幂模
22    LL res = 1;
23    while (b) {
24      if (b & 1) res = qmult(res, a, mod);
25      b >>= 1;
26      a = qmult(a, a, mod);
27    }
28    return res;
29  }
30  bool prime_test(LL n, LL a) {  // 对整数n, 底数a进行测试, 返回true表示通过测试
31    LL p = qpow(a, n - 1, n);
32    if (p != 1) return false;
33    else {  // 二次探测
34      LL s = n - 1;
35      while (!(s & 1) && p == 1) {
36        s >>= 1;
```

```
37          p = qpow(a, s, n);
38        }
39        if (p == 1 || p == n - 1) return true;
40        else return false;
41      }
42  }
43  bool Miller_Rabin(LL n) {  // 对整数n进行Miller_Rabin素数测试，返回true表示通过测试
44      if (n <= 29) {      // if这一块其实可以不用
45        for (int i = 0; i < 10; i++) {
46          if (n == p[i]) return true;
47        }
48        return false;
49      }
50      for (int i = 0; i < 10; i++) {  // 利用前10个素数作为底数测试的正确率已经非常高
51        if (gcd(n, p[i]) == 1 && !prime_test(n, p[i])) return false;
52      }
53      return true;
54  }
55  LL randf(LL x, LL n, LL c) {  // 满足要求的产生伪随机数函数
56      return (qmult(x, x, n) + c) % n;
57  }
58  LL pollard_rho(LL n, LL c) {  // 查找n的因数，c为上面函数要用的随机数，c也可自己指定
        （但要有变化）
59      LL x = rand() % n, y = x, i = 1, k = 2, p;  // 随机生成随机数的初始值，也可自己指定
60      while (true) {
61        i++;
62        x = randf(x, n, c);
63        p = gcd(y - x + n, n);
64        if (p > 1 && p < n) return p;
65        if (y == x) return n;   // 判圈，返回n表示查找失败，要更新随机种子重新查找
66        if (i == k) {
67          y = x;  // 更新范围和记录的数
68          k <<= 1;
69        }
70      }
71  }
72  void find_factor(LL n) {  // 查找所有因数
73      if (Miller_Rabin(n)) {
74        minfactor = min(minfactor, n);
75        return ;
76      }
77      LL p = n;
78      while (p == n) p = pollard_rho(n, rand() % (n - 1) + 1);  // 查找失败则更新随机种子
        重新查找，直到找到因子
79      find_factor(p);      // 递归查找更小因子
80      find_factor(n / p);
81  }
82
83  int main() {
84      int t;
85      cin >> t;
86      while (t--) {
87        LL N;
88        cin >> N;
89        if (Miller_Rabin(N)) cout << "Prime" << endl;
90        else {
91          minfactor = N;
92          find_factor(N);
93          cout << minfactor << endl;
94        }
95      }
96      return 0;
97  }
```

5.11.1 Miller Rabin

```
1  /* Miller-Rabin Prime Test, by Abreto<m@abreto.net>. */
2
3  namespace miller_rabin {
4
5  typedef long long int ll;
6
7  inline ll add(const ll a, const ll b, const ll mod) {
8      ll z = a + b;
9      if (z >= mod) z -= mod;
10     return z;
11  }
12  inline ll mul(ll a, ll b, const ll mod) {
13     ll z = 0;
14     if (a >= mod) a %= mod;
15     if (b >= mod) b %= mod;
16     while (b) {
17         if (1 & b) z = add(z, a, mod);
18         a = add(a, a, mod);
19         b >>= 1;
20     }
21     return z;
22  }
23
24  ll qow(ll a, ll x, ll mod) {
25     ll ret = 1ll;
26     while (x) {
27         if (1 & x) ret = mul(ret, a, mod);
28         a = mul(a, a, mod);
29         x >>= 1;
30     }
31     return ret;
32  }
33
34  const int K = 5;
35  const int p[] = {
36     2, 3, 7, 61, 24251
37  };
38  const ll strong = 4685624825598111;
39  /* 46 856 248 255 981 in (0, 1e16) */
40
41  bool mr(ll n, int k) {
42     ll d = n - 1;
43     int s = 0;
44     while (d > 1 && 0 == (d & 1)) {
45         s++;
46         d >>= 1;
47     }
48     for (int i = 0; i < k; i++) {
49         ll a = (i < K) ? p[i] : (1 + rand() % (n - 1));
50         ll x = qow(a, d, n);
51         for (int j = 0; j < s; j++) {
52             ll xp = mul(x, x, n);
53             if (1 == xp && x != 1 && x != n-1) return false;
54             x = xp;
55         }
56         if (x != 1) return false;
57     }
58     return true;
59  }
60
61  /* 2,3,5,7,11,13 */
62  const int pre[] = {3, 5, 7, 11, 13};
```

```
63 bool test(ll n, int k = 5) {
64   if (2 == n) return true;
65   if (0 == (n & 1)) return false;
66   if (strong == n) return false;
67   for (int i = 0; i < 5; i++) {
68     if (n == pre[i]) return true;
69     if (n == n / pre[i] * pre[i])
70       return false;
71   }
72   return mr(n, k);
73 }
74
75 }
```

5.11.2 Pollard's rho

```
 1 /* Pollard's rho, by Abreto<m@abreto.net>. */
 2
 3 namespace pollards_rho {
 4
 5 typedef long long int ll;
 6
 7 inline ll add(const ll a, const ll b, ll mod) {
 8   ll z = a + b;
 9   if (z >= mod) z -= mod;
10   return z;
11 }
12 inline ll mul(ll a, ll b, ll mod) {
13   ll z = 0ll;
14   if (a >= mod) a -= a / mod * mod;
15   if (b >= mod) b -= b / mod * mod;
16   while (b) {
17     if (1 & b) z = add(z, a, mod);
18     a = add(a, a, mod);
19     b >>= 1;
20   }
21   return z;
22 }
23
24 ll gcd(ll m, ll n) {
25   return (0 == n) ? m : gcd(n, m % n);
26 }
27
28 ll find(ll n, int c = -1) {
29   ll x = rand() % n;
30   ll y = x, k = 2;
31   for (int i = 2; ; i++) {
32     x = add(mul(x, x, n), (n + c) % n, n);
33     ll d = gcd( y - x + n, n );  // change to abs(y - x) if get WA
34     if (1 != d && n != d) return d;
35     if (y == x) return n;
36     if (i == k) {
37       y = x;
38       k <<= 1;
39     }
40   }
41 }
42
43 /** usage:
44  * void find(ll n, int c = 107)
45  * {
46  *   if (1 == n) return;
47  *   if ( miller-rabin(n) )
```

```
48  *    {
49  *       n is a prime;
50  *       return;
51  *    }
52  *    ll p = n, k = c;
53  *    while (p >= n) p = pollards_rho(p, k--);
54  *    find(p, c);
55  *    find(n/p, c);
56  * }
57  **/
58
59  }
```

5.12   Montgomery modular multiplication

```
1
2   /* -- Montgomery modular algorithm { -- */
3   struct Mod64 {
4     typedef long long ll;
5     typedef unsigned long long u64;
6     typedef __int128_t i128;
7     typedef __uint128_t u128;
8     Mod64() :n_(0) {}
9     Mod64(u64 n) :n_(init(n)) {}
10    static u64 init(u64 w) {
11      return reduce(u128(w) * r2);
12    }
13    static void set_mod(u64 m) {
14      mod = m;
15      assert(mod & 1);
16      inv = m;
17      for (int i = 0; i < 5; ++i) inv *= 2 - inv * m;
18      r2 = -u128(m) % m;
19    }
20    static u64 reduce(u128 x) {
21      u64 y = u64(x >> 64) - u64((u128(u64(x)*inv)*mod) >> 64);
22      return ll(y)<0 ? y + mod : y;
23    }
24    Mod64& operator += (Mod64 rhs) {
25      n_ += rhs.n_ - mod;
26      if (ll(n_)<0) n_ += mod;
27      return *this;
28    }
29    Mod64 operator + (Mod64 rhs) const {
30      return Mod64(*this) += rhs;
31    }
32    Mod64& operator -= (Mod64 rhs) {
33      n_ -= rhs.n_;
34      if (ll(n_)<0) n_ += mod;
35      return *this;
36    }
37    Mod64 operator - (Mod64 rhs) const {
38      return Mod64(*this) -= rhs;
39    }
40    Mod64& operator *= (Mod64 rhs) {
41      n_ = reduce(u128(n_)*rhs.n_);
42      return *this;
43    }
44    Mod64 operator * (Mod64 rhs) const {
45      return Mod64(*this) *= rhs;
46    }
47    u64 get() const {
48      return reduce(n_);
```

```
49        }
50      static u64 mod, inv, r2;
51      u64 n_;
52    };
53
54    Mod64::u64 Mod64::mod, Mod64::inv, Mod64::r2;
55    /* — } Montgomery modular algorithm — */
56
57    /**
58     * usage:
59     * First, Mod64::set_mod();
60     * Mod64 a, b, c(init_val);
61     * a = b * c;
62     * printf("%llu\n", a.get());
63     **/
```

5.13 Berlekamp Massey

```
 1    /* Berlekamp Massey by HoldZhu. */
 2    #include <cstdio>
 3    #include <vector>
 4
 5    using namespace std;
 6
 7    namespace BerlekampMassey {
 8    const int mod = 1e9 + 7;
 9    int L, m, b, n;
10    vector<int> s, C, B;
11    void init() {
12      s.clear();
13      C.clear();
14      B.clear();
15      C.push_back(1);
16      B.push_back(1);
17      L = n = 0;
18      m = b = 1;
19    }
20    int pow_mod(int a, int k) {
21      int s = 1;
22      while (k) {
23        if (k & 1)
24          s = 1ll * s * a % mod;
25        a = 1ll * a * a % mod;
26        k >>= 1;
27      }
28      return s;
29    }
30    void update(int d) {
31      s.push_back(d);
32      for (int i = 1; i <= L; ++i)
33        d = (d + 1ll * C[i] * s[n - i] % mod) % mod;
34      if (d == 0)
35        ++m;
36      else if (2 * L <= n) {
37        vector<int> T = C;
38        C.resize(n + 1 - L + 1);
39        for (int i = L + 1; i <= n + 1 - L; ++i)
40          C[i] = 0;
41        for (int i = 0; i < B.size(); ++i)
42          C[i + m] = (C[i + m] + mod - 1ll * d * pow_mod(b, mod - 2) % mod * B[i] % mod)
                 % mod;
43        L = n + 1 - L;
44        B = T;
```

```
45       b = d;
46       m = 1;
47     } else {
48       for (int i = 0; i < B.size(); ++i)
49         C[i + m] = (C[i + m] + mod − 1ll * d * pow_mod(b, mod − 2) % mod * B[i] % mod)
                 % mod;
50       ++m;
51     }
52     ++n;
53 }
54 void output() {
55     printf("F(n)=");
56     for (int i = 1; i < C.size(); ++i) {
57       int output = (mod − C[i]) % mod;
58       if (output > mod / 2)
59         output −= mod;
60       printf("%s%d*F(n−%d)", (output < 0 || i == 1) ? "" : "+", output, i);
61     }
62     puts("");
63 }
64 void output_code_for() {
65     static const char *name = "dp";
66     static const char *index = "i";
67     static const char *upperbound = "maxn";
68     puts("// Generated by Berlekamp−Massey algorithm");
69     for (int i = 1; i < C.size(); ++i) {
70       printf("%s[%d]=%d;\n", name, i − 1, s[i − 1]);
71     }
72     printf("for(int i=%d;i<%s;++i)\n", (int)C.size() − 1, upperbound);
73     printf("  %s[%s]=((", name, index);
74     for (int i = 1; i < C.size(); ++i) {
75       int output = (mod − C[i]) % mod;
76       if (output > mod / 2)
77         output −= mod;
78       printf("%s%d*%s[%s−%d]%%mod", (output < 0 || i == 1) ? "" : "+", output, name,
                 index, i);
79     }
80     puts(")%mod+mod)%mod;");
81 }
82 void output_code_matrix() {
83     // TODO
84 }
85 };
86
87 /** usage */
88 int usage() {
89   // int arr[12] = {2, 24, 96, 416, 1536, 5504, 18944, 64000, 212992, 702464,
           2301952, 7512064};
90   int arr[] = {3, 20, 119, 696, 4059, 23660, 137903, 803760, 4684659};
91   BerlekampMassey::init();
92   for (auto ai : arr) {
93     BerlekampMassey::update(ai);
94   }
95   printf("Formule: ");
96   BerlekampMassey::output();
97   printf("Code: \n");
98   BerlekampMassey::output_code_for();
99   return 0;
100 }
```

6 String

6.1 Hash

```
1  /* Common hash for any substrings. */
2
3  typedef unsigned long long int llu;
4  #define MAXN 1000000
5  int n;
6  char s[MAXN];
7  llu H[MAXN], xP[MAXN], P = 99991ll;
8  void init(void) {
9    int i = 0;
10   xP[0] = 1ll;
11   for(i = 1; i < MAXN; ++i) xP[i] = xP[i−1] * P;
12   H[n] = 0;
13   for(i = n−1; i >= 0; −−i) H[i] = H[i+1]*P + s[i];
14 }
15 #define HASH(i,l)   (H[i] − H[i+l]*xP[l])
```

6.2 KMP

```
1  /* KMP, by Abreto<m@abreto.net>. */
2  #include <string.h>
3
4  /* !!NEED IMPROVING!! */
5
6  #define MAXL  (1000010)
7
8  char W[MAXL], T[MAXL];
9  int f[MAXL];
10 int lW, lT;
11
12 int count(void) {
13   int cnt = 0;
14   int i, j;
15   lW = strlen(W);
16   lT = strlen(T);
17   // −− self−matching
18   f[0] = j = −1;
19   for(i = 1; i < lW; i++) {
20     while( j >= 0 && W[j+1] != W[i] )
21       j = f[j];
22     if( W[j+1] == W[i] ) j++;
23     f[i] = j;
24   }
25   // −−−
26   j = −1;
27   for(i = 0; i < lT; i++) {
28     while( j >= 0 && W[j+1] != T[i] )
29       j = f[j];
30     if( W[j+1] == T[i] ) j++;
31     if( j == lW−1 ) {
32       cnt++;
33       j = f[j];
34     }
35   }
36   return cnt;
37 }
```

6.3 exKMP

```
1  #include <bits/stdc++.h>
2  using namespace std;
3
4  namespace exkmp {
5
6  const int maxn = 1000100, maxm = 1000100;
7
8  int n, m;
9  char S[maxn], T[maxm];
10
11 /* the length of longest prefix between T[i..m-1] and T[0..m-1] */
12 int nxt[maxm];
13 /* the length of longest prefix between S[i..n-1] and T[0..m-1] */
14 int ex[maxn];
15
16 void getsize() {
17   n = strlen(S);
18   m = strlen(T);
19 }
20
21 void self(void) {
22   int q = 1, p = 0;
23   nxt[0] = m;
24   while (1 + p < m && T[1 + p] == T[p]) p++;
25   nxt[1] = p;
26   for (int i = 2; i < m; i++) {
27     int l = nxt[i - q];
28     if (i + l - 1 < p) {
29       nxt[i] = l;
30     } else {
31       int j = max(0, p - i + 1);
32       while (i + j < m && T[i + j] == T[j]) j++;
33       nxt[i] = j;
34       p = i + j - 1;
35       q = i;
36     }
37   }
38 }
39
40 void run(void) {
41   int q = 0, p = 0;
42   self();
43   while (p < n && p < m && S[p] == T[p]) p++;
44   ex[0] = p;
45   p--;
46   for (int i = 1; i < n; i++) {
47     int l = nxt[i - q];
48     if (i + l - 1 < p) {
49       ex[i] = l;
50     } else {
51       int j = max(0, p - i + 1);
52       while (i + j < n && S[i + j] == T[j]) j++;
53       ex[i] = j;
54       p = i + j - 1;
55       q = i;
56     }
57   }
58 }
59
60 void inspect(void) {
61   printf("S:␣");
62   for (int i = 0; i < n; i++) putchar(S[i]);
63   puts("");
64   printf("T:␣");
```

```
65    for (int i = 0; i < m; i++) putchar(T[i]);
66    puts("");
67    printf("next:");
68    for (int i = 0; i < m; i++) printf(" %d", nxt[i]);
69    puts("");
70    printf("extend:");
71    for (int i = 0; i < n; i++) printf(" %d", ex[i]);
72    puts("");
73  }
74
75  } // exkmp
```

6.4 Suffix Array

```
 1  /* Suffix Array, copied. */
 2
 3  #define MAXN    (200010)
 4  namespace mzry_sa {
 5  int wx[MAXN],wy[MAXN],*x,*y,wss[MAXN],wv[MAXN];
 6
 7  bool dacmp(int *r,int n,int a,int b,int l) {
 8      return a+l<n && b+l<n && r[a]==r[b]&&r[a+l]==r[b+l];
 9  }
10  void da(int str[],int sa[],int rank[],int height[],int n,int m) {
11      int *s = str;
12      int *x=wx,*y=wy,*t,p;
13      int i,j;
14      for(i=0; i<m; i++)wss[i]=0;
15      for(i=0; i<n; i++)wss[x[i]=s[i]]++;
16      for(i=1; i<m; i++)wss[i]+=wss[i-1];
17      for(i=n-1; i>=0; i--)sa[--wss[x[i]]]=i;
18      for(j=1,p=1; p<n && j<n; j*=2,m=p) {
19        for(i=n-j,p=0; i<n; i++)y[p++]=i;
20        for(i=0; i<n; i++)if(sa[i]-j>=0)y[p++]=sa[i]-j;
21        for(i=0; i<n; i++)wv[i]=x[y[i]];
22        for(i=0; i<m; i++)wss[i]=0;
23        for(i=0; i<n; i++)wss[wv[i]]++;
24        for(i=1; i<m; i++)wss[i]+=wss[i-1];
25        for(i=n-1; i>=0; i--)sa[--wss[wv[i]]]=y[i];
26        for(t=x,x=y,y=t,p=1,i=1,x[sa[0]]=0; i<n; i++)
27          x[sa[i]]=dacmp(y,n,sa[i-1],sa[i],j)?p-1:p++;
28      }
29      for(int i=0; i<n; i++) rank[sa[i]]=i;
30      for(int i=0,j=0,k=0; i<n; height[rank[i++]]=k)
31        if(rank[i]>0)
32          for(k?k--:0,j=sa[rank[i]-1];
33              i+k < n && j+k < n && str[i+k]==str[j+k];
34              k++);
35  }
36  }
37
38  /*
39  Suffix array O(n lg^2 n)
40  LCP table O(n)
41  */
42  #include <cstdio>
43  #include <algorithm>
44  #include <cstring>
45
46  using namespace std;
47
48  #define REP(i, n) for (int i = 0; i < (int)(n); ++i)
49
```

```
50  namespace SuffixArray {
51  const int MAXN = 1 << 21;
52  char * S;
53  int N, gap;
54  int sa[MAXN], pos[MAXN], tmp[MAXN], lcp[MAXN];
55
56  bool sufCmp(int i, int j) {
57    if (pos[i] != pos[j])
58      return pos[i] < pos[j];
59    i += gap;
60    j += gap;
61    return (i < N && j < N) ? pos[i] < pos[j] : i > j;
62  }
63
64  void buildSA() {
65    N = strlen(S);
66    REP(i, N) sa[i] = i, pos[i] = S[i];
67    for (gap = 1;; gap <<= 1) {
68      sort(sa, sa + N, sufCmp);
69      REP(i, N - 1) tmp[i + 1] = tmp[i] + sufCmp(sa[i], sa[i + 1]);
70      REP(i, N) pos[sa[i]] = tmp[i];
71      if (tmp[N - 1] == N - 1) break;
72    }
73  }
74
75  void buildLCP() {
76    for (int i = 0, k = 0; i < N; ++i) if (pos[i] != N - 1) {
77        for (int j = sa[pos[i] + 1]; S[i + k] == S[j + k];)
78          ++k;
79        lcp[pos[i]] = k;
80        if (k)--k;
81      }
82  }
83  } // end namespace SuffixArray
84
85  namespace HashSuffixArray {
86  const int
87  MAXN = 1 << 21;
88
89  typedef unsigned long long hash;
90
91  const hash BASE = 137;
92
93  int N;
94  char * S;
95  int sa[MAXN];
96  hash h[MAXN], hPow[MAXN];
97
98  #define getHash(lo, size) (h[lo] - h[(lo) + (size)] * hPow[size])
99
100 inline bool sufCmp(int i, int j) {
101   int lo = 1, hi = min(N - i, N - j);
102   while (lo <= hi) {
103     int mid = (lo + hi) >> 1;
104     if (getHash(i, mid) == getHash(j, mid))
105       lo = mid + 1;
106     else
107       hi = mid - 1;
108   }
109   return S[i + hi] < S[j + hi];
110 }
111
112 void buildSA() {
113   N = strlen(S);
```

```
114    hPow[0] = 1;
115    for (int i = 1; i <= N; ++i)
116      hPow[i] = hPow[i − 1] * BASE;
117    h[N] = 0;
118    for (int i = N − 1; i >= 0; −−i)
119      h[i] = h[i + 1] * BASE + S[i], sa[i] = i;
120
121    stable_sort(sa, sa + N, sufCmp);
122  }
123
124  } // end namespace HashSuffixArray
125
126  namespace lrj_sa {
127  const int MAXN = 1000;
128  char s[MAXN]; /* 原始字符数组（最后一个字符应必须是0，而前面的字符必须非0）*/
129  int sa[MAXN], t[MAXN], t2[MAXN], c[MAXN], n;  /* n seems to be the length of s. */
130  /* every charactor is in [0,m−1] */
131  void build_sa(int m) {
132    int i, *x = t, *y = t2;
133    for(i = 0; i < m; ++i) c[i] = 0;
134    for(i = 0; i < n; i++) c[x[i]=s[i]]++;
135    for(i = 1; i < m; ++i) c[i] += c[i−1];
136    for(i = n−1; i >= 0; −−i) sa[−−c[x[i]]] = i;
137    for(int k = 1; k <= n; k <<= 1) {
138      int p = 0;
139      for(i = n−k; i < n; ++i) y[p++] = i;
140      for(i = 0; i < n; ++i) if(sa[i] >= k) y[p++] = sa[i]−k;
141      for(i = 0; i < m; i++) c[i] = 0;
142      for(i = 0; i < n; i++) c[x[y[i]]]++;
143      for(i = 0; i < m; ++i) c[i]+=c[i−1];
144      for(i = n−1; i >= 0; −−i) sa[−−c[x[y[i]]]] = y[i];
145      swap(x,y);
146      p = 1;
147      x[sa[0]] = 0;
148      for(i = 1; i < n; ++i)
149        x[sa[i]] = y[sa[i−1]]==y[sa[i]] && y[sa[i−1]+k]==y[sa[i]+k] ? p−1:p++;
150      if(p >= n) break;
151      m = p;
152    }
153  }
154  int rank[MAXN], height[MAXN];
155  void get_height(void) {
156    int i,j,k = 0;
157    for(i = 0; i < n; ++i) rank[sa[i]] = i;
158    for(i = 0; i < n; ++i) {
159      if(k) k−−;
160      j = sa[rank[i]−1];
161      while(s[i+k]==s[j+k]) k++;
162      height[rank[i]] = k;
163    }
164  }
165  } // end namespace lrj_sa
```

6.5 Aho-Corasick Automata

```
1  /* Aho−Corasick automaton algorithm, by Abreto<m@abreto.net>. */
2
3  #define MAXN  500500
4  #define NALPHA  26
5  #define FIRSTA  ’a’
6
7  /* pointer version => { */
8  struct vtx {
```

```
 9      vtx *nxt[NALPHA];
10      vtx *fail;
11      int end;
12    } vtxs[MAXN];
13    int nvtxs;
14    void myclr(void) {
15      nvtxs = 0;
16    }
17    vtx *new_vtx(void) {
18      vtx *ret = vtxs+(nvtxs++);
19      for(int i = 0; i < NALPHA; i++)
20        ret→nxt[i] = NULL;
21      ret→fail = NULL;
22      ret→end = 0;
23      return ret;
24    }
25    void myins(vtx *root, char const *s) {
26      for( ; *s ; s++ ) {
27        int of = (*s) − FIRSTA;
28        if ( NULL == root→nxt[of] ) {
29          root→nxt[of] = new_vtx();
30        }
31        root = root→nxt[of];
32      }
33      root→end++;
34    }
35    void build_ac(vtx *root) {
36      queue<vtx *> q;
37      q.push(root);
38      while(!q.empty()) {
39        vtx *p = q.front();
40        q.pop();
41        for(int i = 0; i < NALPHA; i++) {
42          if( NULL == p→nxt[i] ) continue;
43          if( root == p ) p→nxt[i]→fail = root;
44          else {
45            vtx *t = p→fail;
46            while ( t && NULL == t→nxt[i] ) {
47              t = t→fail;
48            }
49            if (t) p→nxt[i]→fail = t→nxt[i];
50            else p→nxt[i]→fail = root;
51          }
52          /* version[1] { */
53          p→nxt[i]→end += p→nxt[i]→fail→end; /* update this sum, add its existing
                 prefix to this. */
54          /* } */
55          q.push(p→nxt[i]);
56        }
57      }
58    }
59    int qry(vtx *root, char const *s) {
60      vtx *p = root;
61      int cnt = 0;
62      for( ; *s ; s++ ) {
63        int of = (*s) − FIRSTA;
64        while( p != root && NULL == p→nxt[of] ) {
65          p = p→fail;
66        }
67        if (p→nxt[of]) p = p→nxt[of];
68        cnt += p→end;  // correct when version[1] exists.
69        // if version[1] not exists, you need to add all ends from this vertex up.
70        //for( vtx *t = p ; t ; t = t→fail )
71        //   cnt += t→end;
```

```
 72 |   }
 73 |   return cnt;
 74 | }
 75 | /* } */
 76 |
 77 | /* — usage (of pointer version) — */
 78 | #include <bits/stdc++.h>
 79 | using namespace std;
 80 |
 81 | char S[1000100];
 82 | char pat[64];
 83 |
 84 | int main(void) {
 85 |   int T, N;
 86 |   vtx *root = NULL;
 87 |   scanf("%d", &T);
 88 |   while(T--) {
 89 |     myclr();
 90 |     root = new_vtx();
 91 |     scanf("%s", S);
 92 |     scanf("%d", &N);
 93 |     while(N--) {
 94 |       scanf("%s", pat);
 95 |       myins(root, pat);
 96 |     }
 97 |     build_ac(root);
 98 |     printf("%d\n", qry(root, S));
 99 |   }
100 |   return 0;
101 | }
```

6.6 Manacher

```
 1 | char t[MAXL<<1];
 2 | int p[MAXL<<1];
 3 | int manacher(char *s) {
 4 |   int i;
 5 |   int sl = strlen(s);
 6 |   int pos = 0, mxr = 0;
 7 |   int ret = 0;
 8 |   t[0] = '^';
 9 |   for(i = 0; i < sl; ++i) {
10 |     t[i*2+1] = '#';
11 |     t[i*2+2] = s[i];
12 |   }
13 |   t[sl*2+1] = '#';
14 |   t[sl*2+2] = '$';
15 |   sl = sl*2+2;
16 |   for(i = 1; i < sl; ++i) {
17 |     if(i <= mxr) {
18 |       p[i] = min(p[2*pos - i], mxr-i+1);
19 |     } else {
20 |       p[i] = 1;
21 |     }
22 |     while( t[i-p[i]] == t[i+p[i]] ) p[i]++;
23 |     if( i + p[i] - 1 > mxr ) {
24 |       mxr = i+p[i]-1;
25 |       pos = i;
26 |     }
27 |     ret = max(ret, p[i]-1);
28 |   }
29 |   return ret;
30 | }
```