ACM Template

UESTC_Jungle

Last build at October 24, 2018

Contents

1 Datastructure

1.1 Fenwick

```
/* Fenwick Tree (Binary Indexed Tree), by Abreto <m@abreto.net>. */
#include <cstring>

using namespace std;

template <class T = int, int MAXN = 100001>
struct fenwick {
  static inline int lowbit(int x) {
    return (x&(-x));
  }
  int N;
  T f[MAXN]; /* 1=based. */
  fenwick(void):N(MAXN) {
    init();
  }
  fenwick(int n):N(n) {
    init();
  }
  void init(void) {
    memset(f,0,sizeof(f));
  }
  void upd(int i, T dx) {
    while(i <= N) {
      f[i] += dx;
      i += lowbit(i);
    }
  }
  T sum(int i) {
    T ret = 0;
    while(i) {
      ret += f[i];
      i -= lowbit(i);
    }
    return ret;
  }
};
```

```
/* Fenwick Tree (Binary Indexed Tree), by Abreto <m@abreto.net>. */

#define MAXN 100001
#define LOWBIT(x)   ((x)&(-(x)))

int N;
int fen[MAXN];

void update(int i, int dx) {
  while(i <= N) {
    fen[i] += dx;
    i += LOWBIT(i);
  }
}

int sum(int i) {
  int s = 0;
  while(i > 0) {
    s += fen[i];
    i -= LOWBIT(i);
  }
  return s;
}
```

1.2 BST in pb_ds

```
/* Red-Black tree via pb_ds. */
#include<bits/stdc++.h>
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
using namespace std;
template <typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

int main() {
  ordered_set<int>  s;
  s.insert(1);
  s.insert(3);
  cout << s.order_of_key(2) << endl; // the number of elements in the s less than 2
  cout << *s.find_by_order(0) << endl; // print the 0-th smallest number in s(0-based
      )
}
```

1.3 Segment Tree

```
/* Segment tree (Interval tree, range tree), by Abreto <m@abreto.net>. */

template <int STMAX = 1000000>
struct segment_tree {
  struct node_t {
    static inline node_t merge(node_t n1, node_t n2) {
      node_t ans;
      ans.l = n1.l;
      ans.r = n2.r;
      /* merge n1 and n2 to ans. */
      return ans;
    }

    /* Data field */
    int l,r;
  } nodes[(STMAX+1)<<2];

  struct lazy_t {
    int marked; /* Optional */
    /* lazy mark. */

    lazy_t(void) {
      clear();
    }
    void clear(void) {
      marked=0;
    }
  } marks[(STMAX+1)<<2];

  inline void maintain_leaf(int o, int idx) {
    nodes[o].l = nodes[o].r = idx;
    /* Operations to single elements ... */
  }
  inline void maintain(int o) {
    nodes[o] = node_t::merge(nodes[o<<1], nodes[o<<1|1]);
  }

  /* Usage: build(1,1,n); */
  void build(int o, int l, int r) { /* [l,r] */
```

```
    if( r <= l ) {
      maintain_leaf(o, l);
    } else {
      int mid = l+r>>1;
      build(o<<1, l, mid);
      build(o<<1|1, mid+1, r);
      maintain(o);
    }
  }

  /* Modify all elements in [l,r] */
  void mark(lazy_t act, int o) {
    /* do something .. */
    marks[o].marked = 1;
  }

  /* Pass cached updates. */
  void pushdown(int o) {
    if( marks[o].marked ) {
      mark(marks[o], o<<1);
      mark(marks[o], o<<1|1);
      marks[o].clear();
    }
  }

  /* Do act on all elements in [L,R] */
  void upd(int L, int R, lazy_t act, int o, int l, int r) {
    if( L <= l && r <= R ) {
      mark(act, o);
    } else if (L <= R) {
      int mid = (l+r)>>1;
      pushdown(o);
      if( L <= mid ) upd(L, R, act, o<<1, l, mid);
      if( R > mid ) upd(L, R, act, o<<1|1, mid+1, r);
      maintain(o);
    }
  }

  node_t qry(int L, int R, int o, int l, int r) {
    if(L <= l && r <= R)
      return nodes[o];
    else if (L <= R) {
      int mid = (l+r)>>1;
      pushdown(o);
      if(R <= mid) return qry(L,R,o<<1,l,mid);
      if(L > mid) return qry(L,R,o<<1|1,mid+1,r);
      return node_t::merge(qry(L,R,o<<1,l,mid),qry(L,R,o<<1|1,mid+1,r));
    }
  }

  int N;

  segment_tree(void):N(STMAX) {}
  segment_tree(int n):N(n) {}
  void build(int n) {
    N = n;
    build(1,1,N);
  }
  void update(int L, int R, lazy_t act) {
    upd(L,R,act,1,1,N);
  }
  node_t query(int L, int R) {
    return qry(L,R,1,1,N);
  }
```

```
};
```

```
/* Segment tree (Interval tree, range tree), by Abreto <m@abreto.net>. */

#define MAXN    1000001

typedef struct {
  int l,r;
  /* Data field */
} node_t;

node_t merge(node_t n1, node_t n2) {
  node_t ans;
  ans.l = n1.l;
  ans.r = n2.r;
  /* merge n1 and n2 to ans. */
  return ans;
}

typedef struct {
  int marked; /* Optional */
  /* lazy mark. */
} lazy_t;

int A[MAXN];
node_t nodes[MAXN<<2];
lazy_t marks[MAXN<<2];

void maintain_leaf(int o, int idx) {
  nodes[o].l = nodes[o].r = idx;
  /* Operations to single elements ... */
}
void maintain(int o) {
  nodes[o] = merge(nodes[o<<1], nodes[o<<1|1]);
}

/* Usage: build(1,1,n); */
void build(int o, int l, int r) { /* [l,r] */
  if( r <= l ) {
    maintain_leaf(o, l);
  } else {
    int mid = l+r>>1;
    build(o<<1, l, mid);
    build(o<<1|1, mid+1, r);
    maintain(o);
  }
  marks[o].marked = 0;
}

/* Modify all elements in [l,r] */
void mark(lazy_t act, int o) {
  /* do something .. */
  marks[o].marked = 1;
}

/* Pass cached updates. */
void pushdown(int o) {
  if( marks[o].marked ) {
    mark(marks[o], o<<1);
    mark(marks[o], o<<1|1);
    marks[o].marked = 0;
  }
}

```

```
/* **DISCARDED** */
/* Set A[p]=v. Usage: modify(p, v, 1, 1, n);
void modify(int p, int v, int o, int l, int r)
{
    if( r - l < 2 )
    {
        maintain_leaf(o, v);
    } else {
        int mid = (l+r)/2;
        pushdown(o);
        if( p <= mid ) modify(p, v, o*2, l, mid);
        else modify(p, v, o*2+1, mid, r);
        maintain(o);
    }
}*/

/* Do act on all elements in [L,R] */
void update(int L, int R, lazy_t act, int o, int l, int r) {
  if( L <= l && r <= R ) {
    mark(act, o);
  } else if (L <= R) {
    int mid = (l+r)>>1;
    pushdown(o);
    if( L <= mid ) update(L, R, act, o<<1, l, mid);
    if( R > mid ) update(L, R, act, o<<1|1, mid+1, r);
    maintain(o);
  }
}
```

1.4 Sparse Table

```
/* RMQ with Sparse Table, by Abreto <m@abreto.net>. */

int min(int a, int b) {
  return (a<b)?a:b;
}

#define MAXN    100001
#define MAXLOG  32

int N;
int A[MAXN];    /* indexed from 0. */
int st[MAXN][MAXLOG];

void st_init() {
  int i = 0, j = 0, t = 0;
  for(i = 0; i < N; ++i) st[i][0] = A[i];
  for(j = 1; (t=(1<<j)) <= N; ++j)
    for(i = 0; (i+t-1) < N; ++i)
      st[i][j] = min(st[i][j-1], st[i+(t>>1)][j-1]);
  /* st(i,j) = min(st(i,j-1), st(i+2^(j-1),j-1)). */
}

int st_query(int l, int r) {
  int k = 0;
  while((1<<(k+1)) <= (r-l+1)) k++;
  return min(st[l][k], st[r-(1<<k)+1][k]);
}
```

1.5 Treap

```
#include <bits/stdc++.h>

using namespace std;

#define MAXN  (2560000)

int __treap_mem[MAXN];
void init_treap_mem(void) {
  for( int i = 1 ; i < MAXN ; i++ )
    __treap_mem[i-1] = i;
}
int alloc_address(void) {
  int ret = __treap_mem[0];
  __treap_mem[0] = __treap_mem[ret];
  return ret;
}
void free_address(int p) {
  __treap_mem[p] = __treap_mem[0];
  __treap_mem[0] = p;
}

typedef int key_t;
typedef int val_t;
struct treap {
  key_t x;
  val_t v;
  int r;    /* random priority */
  int eq, s;  /* number of equal ones, size of subtree (include root itself) */
  treap *fa;  /* point to its father */
  treap *ch[2]; /* 0 for left child, 1 for right child. */

  treap(void);
  inline void maintain(void); /* update s */
  inline void set_child(int d, treap *child);
  inline int which(void); /* determine which child this is of its father */
  inline int cmp(key_t ox); /* determine which child to insert ox */
  treap *rotate(void);  /* rotate this to its father, return this */
} treap_nodes[MAXN];

treap *new_treap(key_t x, val_t v, treap *f) {
  treap *ret = treap_nodes + alloc_address();
  ret->x = x;
  ret->v = v;
  ret->eq = ret->s = 1;
  ret->fa = f;
  ret->ch[0] = ret->ch[1] = NULL;
}
void free_treap(treap *p) {
  free_address( p - treap_nodes );
}
void treap_clear(void) {
  init_treap_mem();
}

treap::treap(void) {
  r = rand();
  eq = s = 0;
  fa = ch[0] = ch[1] = NULL;
}
inline void treap::maintain(void) {
  s = eq;
  for( int i = 0 ; i < 2 ; i++ )
    if( ch[i] )
      s += ch[i]->s;
```

```
}
inline void treap::set_child(int d, treap *child) {
  ch[d] = child;
  maintain();
  if( child ) child->fa = this;
}
inline int treap::which(void) {
  if( NULL == fa ) return -1; /* this is not a child */
  else return ( fa->ch[1] == this );
}
inline int treap::cmp(key_t ox) {
  if( ox == x ) return -1;  /* equal */
  else return ( ox > x );   /* left less, right more */
}
treap *treap::rotate(void) {
  if ( NULL == fa ) return this;  /* no father, already global root. */
  int d = which();
  fa->set_child(d, ch[d^1]);
  set_child(d^1, fa);
  return this;
}

// --- deprecated |
typedef int T;

struct node {
  T v;  /* value of this node */
  int r;  /* random priority */
  int eq; /* the number of equal things */
  int s;  /* the size of subtree rooted at this */
  node *ch[2];  /* 0 for left child, 1 for right child. */
  node(void) {
    r = rand();
    ch[0] = ch[1] = NULL;
  }
  /* return where to insert x */
  int cmp(T x) {
    if(v == x) return -1;
    else return (x < v) ? 0 : 1;
  }
  /* return 1 if this node is prior to other */
  int pri(node *o) {
    return (r > (o->r));
  }
  /* maintain the s field */
  void maintain(void) {
    s = eq;
    if(NULL != ch[0]) s += ch[0]->s;
    if(NULL != ch[1]) s += ch[1]->s;
  }
};

/* move o to ch[d] of o->ch[d^1] */
void rotate(node *&o, int d) {
  node *k = o->ch[d^1];
  o->ch[d^1] = k->ch[d];
  o->maintain();
  k->ch[d] = o;
  k->maintain();
  o = k;
}
```

1.6 Leftist Heap

```
/* HDU 1512 Monkey King（左偏树模板题） */
#include<iostream>
#include<cstdio>
using namespace std;
const int maxn = 100000+5;

int n, m;

struct Heap {
  int l,r,fa,val,dis;
} t[maxn];


int finds(int x) {
  return t[x].fa == -1? x:t[x].fa = finds(t[x].fa);
}

int merge(int x, int y) {
  if(x == 0)  return y;   //如果为0的话，就说明是空子树，根节点当然就是另一节点了
  if(y == 0)  return x;
  if(t[y].val>t[x].val)  swap(x,y);  //始终往右子树进行插入
  t[x].r = merge(t[x].r,y);
  t[t[x].r].fa = x;
  if(t[t[x].l].dis < t[t[x].r].dis) swap(t[x].l,t[x].r);   //是否需要左右子树的对换，这样是为了右子树尽量短
  if(t[x].r == 0)  t[x].dis = 0;   //距离的重新分配
  else t[x].dis = t[t[x].r].dis + 1;
  return x;
}

int pop(int &root) {
  int l = t[root].l;
  int r = t[root].r;
  t[root].l = t[root].r = t[root].dis = 0;
  t[root].fa = -1;
  t[l].fa = t[r].fa = -1;  //删除root根节点
  return merge(l,r);       //这样一来相当于分裂成了两棵子树，重新进行合并，最后返回值为合并后的根节点
}

int push(int x, int y) {
  return merge(x,y);
}

int main() {
  //freopen("in.txt","r",stdin);
  while(~scanf("%d",&n)) {
    for(int i=1; i<=n; i++) {
      t[i].l=t[i].r=t[i].dis=0;
      t[i].fa=-1;
      scanf("%d",&t[i].val);
    }
    scanf("%d",&m);
    while(m--) {
      int a,b;
      scanf("%d%d",&a,&b);
      int x=finds(a);
      int y=finds(b);
      if(x!=y) {
        t[x].val/=2;
        int xx = push( pop(x),x);
        t[y].val/=2;
```

```
61          int yy = push( pop(y),y);
62          printf("%d\n",t[merge(xx,yy)].val);
63        } else puts("-1");
64      }
65    }
66    return 0;
67  }
```

1.7 Splay

```
1   /* splay, by Abreto<m@abreto.net>. */
2
3   #ifndef NULL
4   #define NULL 0
5   #endif
6
7   struct node {
8     node *f, *ch[2];
9     int sz;
10    node(node *fa = NULL, node *lc = NULL, node *rc = NULL) {
11      f = fa;
12      ch[0] = lc;
13      ch[1] = rc;
14      maintain();
15    }
16    inline int szof(const int d) const {
17      return ch[d] ? ch[d]->sz : 0;
18    }
19    inline void maintain(void) {
20      sz = szof(0) + szof(1) + 1;
21    }
22    inline int which(void) {
23      if (NULL == f) return 0;
24      return (f->ch[1] == this);  /* f[which()] == this */
25    }
26    inline node *setf(node *fa, int d = 0) {
27      f = fa;
28      if (f) {
29        f->ch[d] = this;
30        f->maintain();
31      }
32      return f;
33    }
34    inline node *setc(node *son, int d = 0) {
35      ch[d] = son;
36      if (son) son->f = this;
37      maintain();
38      return this;
39    }
40    /* rotate this to its fater, return this. */
41    inline node *rotate(void) {
42      if (f != NULL) {
43        node *ff = f->f;
44        int d = which(), fd = f->which();
45        setc(f->setc(ch[d ^ 1], d), d ^ 1);
46        setf(ff, fd);
47      }
48      return this;
49    }
50    /* splay this to child of target */
51    inline node *splay(node * const target = NULL) {
52      while (f != target) {
53        if (target != f->f) {
```

```
      ( (which() == f->which()) ? f : this )->rotate();
    }
    rotate();
  }
  return this;
 }
 /* 0-based rank */
 inline node *get_k_th(unsigned k) {
  node *p = this;
  int rank;
  while (k != (rank = (p->szof(0)))) {
   if (k < rank) {
    p = p->ch[0];
   } else {
    k -= (rank + 1);
    p = p->ch[1];
   }
  }
  return p->splay(f);
 }
};
```

```
/* HDU 3487 - Play with Chain, by Abreto<m@abreto.net>. */
#include <bits/stdc++.h>

using namespace std;

#define MAXN    300300

int n, m;

#define LC(p)   ch[p][0]
#define RC(p)   ch[p][1]
#define TARGET(p) LC(RC(p))

int nodes;
int val[MAXN], ch[MAXN][2], fa[MAXN], sz[MAXN];
int rev[MAXN];

inline int new_node(int v, int f) {
 int p = (++nodes);
 val[p] = v;
 fa[p] = f;
 ch[p][0] = ch[p][1] = rev[p] = 0;
 sz[p] = 1;
 return p;
}
inline void maintain(int p) {
 if (p) {
  sz[p] = sz[LC(p)] + sz[RC(p)] + 1;
 }
}
inline void make_child(int f, int d, int p) { /* make p the d-th ch of f */
 ch[f][d] = p;
 if(p) fa[p] = f;
}
inline void myrev(int p) {
 if (p) {
  rev[p] ^= 1;
  swap(LC(p), RC(p));
 }
}
inline void pushdown(int p) {
 if(p && rev[p]) {
```

```
43      if(LC(p)) myrev(LC(p));
44      if(RC(p)) myrev(RC(p));
45      rev[p] = 0;
46    }
47  }
48  int build(int f = 0, int l = 0, int r = n+1) {
49    if(r < l) return 0;
50    if(l == r) return new_node(l, f);
51    int mid = l+r>>1;
52    int p = new_node(mid, f);
53    LC(p) = build(p, l, mid-1);
54    RC(p) = build(p, mid+1, r);
55    maintain(p);
56    return p;
57  }
58  inline int which(int p) {  /* return 1 if p is a right child or 0 if p is a left
        child. */
59    return (RC(fa[p]) == p);
60  }
61  inline int rotate(int p) { /* rotate p to its father. [!] make sure p is not global
        root. */
62    int f = fa[p], ff = fa[f];
63    if(0 == f) return p;  /* p is global root */
64    pushdown(f);
65    pushdown(p);
66    int d = which(p), df = which(f);
67    make_child(f, d, ch[p][d^1]);
68    make_child(p, d^1, f);
69    maintain(f);
70    maintain(p);
71    fa[p] = ff;
72    if(ff) ch[ff][df] = p;
73    return p;
74  }
75  inline int splay(int p, int fr) { /* splay p to the son of fr, return p. */
76    pushdown(p);
77    while(fa[p] != fr) {
78      int f = fa[p], dp = which(p);
79      if(fa[f] == fr) {
80        return rotate(p);
81      } else {
82        int df = which(f);
83        if(dp == df) {
84          rotate(f);
85        } else {
86          rotate(p);
87        }
88        rotate(p);
89      }
90    }
91    return p;
92  }
93  inline int get_k_th(int root, int k) {
94    int p = root;
95    int rank;
96    while(k != (rank = (sz[LC(p)] + 1))) {
97      pushdown(p);
98      if(k < rank) p = LC(p);
99      else {
100       k -= rank;
101       p = RC(p);
102     }
103   }
104   return splay(p, fa[root]);
```

```
105 }
106 inline int merge(int left, int right) {
107   pushdown(left);
108   if(RC(left)) left = get_k_th(left, sz[left]);
109   RC(left) = right;
110   maintain(left);
111   fa[right] = left;
112   return left;
113 }
114 inline int split(int root, int d) { /* split ch[root][d], return the root of splited
        out. */
115   pushdown(root);
116   int child = ch[root][d];
117   ch[root][d] = 0;
118   maintain(root);
119   fa[child] = 0;
120   return child;
121 }
122 inline int concat(int root, int d, int p) { /* make p be ch[root][d], return root */
123   pushdown(root);
124   ch[root][d] = p;
125   fa[p] = root;
126   maintain(root);
127   return root;
128 }
129
130 void myclear(void) {
131   nodes = 0;
132 }
133
134 int ans[MAXN];
135 void inorder(int p, int &pos) {
136   if(0 == p) return;
137   pushdown(p);
138   inorder(LC(p), pos);
139   if( (0 < val[p]) && (val[p] < n+1) ) ans[pos++] = val[p];
140   inorder(RC(p), pos);
141 }
142
143 void handle() {
144   int i;
145   int root;
146   myclear();
147   root = build(0);
148   while(m--) {
149     char command[8];
150     int a, b, c;
151     int tar;
152     scanf("%s%d%d", command, &a, &b);
153     if('C' == command[0]) {
154       scanf("%d", &c);
155       root = get_k_th(root, a);
156       RC(root) = get_k_th(RC(root), b-a+2);
157       tar = split(RC(root), 0);
158       maintain(root);
159       root = get_k_th(root, c+1);
160       RC(root) = get_k_th(RC(root), 1);
161       RC(root) = concat(RC(root), 0, tar);
162       maintain(root);
163     } else {
164       root = get_k_th(root, a);
165       RC(root) = get_k_th(RC(root), b-a+2);
166       myrev(TARGET(root));
167     }
```

```
168   }
169   int pos = 0;
170   inorder(root, pos);
171   for(i = 0; i < n; i++) printf("%s%d", i ? " ":"", ans[i]);
172   puts("");
173 }
174
175 int main(void) {
176   while( scanf("%d%d", &n, &m) && (n > 0) && (m > 0) )
177     handle();
178   return 0;
179 }
```

1.8 Persistent Segment Tree

1. 首先, 给你一颗值为横坐标的线段树, 每个节点上存着该值出现了多少次, 这样的一颗线段树你会求区间 k 大值吧. 二分即可.

2. 然后, 假设区间是数组 $arr[n]$, 区间长度是 n, 那么给你 n 颗线段树, 第 i 颗线段树是第 $i-1$ 颗线段树插入 $arr[i]$ 得到.

3. 如果你有了这 n 颗线段树, 想求区间 $[l,r]$ 中的第 k 大值, 那么你需要在第 r 颗和第 $l-1$ 颗线段树的差线段树上作二分, 就可以求得区间第 k 大值.

4. 差线段树很好理解, 比如你有一个部分和数组 sum, $sum[r]-sum[l-1]$ 就是部分和的差, 代表区间 $[l,r]$ 的和, 差线段树同理.

5. 现在, 可持久化线段树出现为你解决最后一个问题, 空间问题. 内存很小, 不能够存下 n 颗线段树. 但是, 第 2 条中提到, 由于第 i 颗线段是是第 $i-1$ 颗线段是插入仅一个值得到的, 两颗线段树的区别不大, 仅有 $\log(n)$ 个节点发生了改变, 我们仅仅需要记录这 $\log(n)$ 的数据就可以记录这个增量, 这就是可持久化线段树.

2 Dynamic Programming

2.1 LIS $O(n\log n)$

```
1
2  int top = 0;
3  for( int i=1; i<=n; i++ ) {
4    if( ap[i] > dp[top] ) { // 如果大于 "模拟栈" 的栈顶元素直接 入栈 长度加 1
5      top++;
6      dp[top] = ap[i];
7      continue;
8    }
9    int m = ap[i];
10   // lower_bound 前闭后开 返回不小于 m 的最小值的位置
11   pos = lower_bound(dp,dp+top,m)-dp; // 注意减去dp
12   if( dp[pos] > ap[i])
13     dp[pos] = ap[i];
14 }
```

2.2 LCS $O(n\log n)$

总的来说，就是把 LCS 转化成 LIS，然后用 LIS 的 $\mathcal{O}(N\log N)$ 算法来求解。
实现如下：（引用）
假设有两个序列 $s_1[1\ldots6]=abcadc$, $s_2[1\ldots7]=cabedab$.
记录 s_1 中每个元素在 s_2 中出现的位置, 再将位置按降序排列, 则上面的例子可表示为：
$loc(a)=\{6,2\}$, $loc(b)=\{7,3\}$, $loc(c)=\{1\}$, $loc(d)=\{5\}$. (倒着扫一遍 s_2 即可把位置扔进 vector).

将 s_1 中每个元素的位置按 s_1 中元素的顺序排列成一个序列 $s_3 = \{6, 2, 7, 3, 1, 6, 2, 5, 1\}$.
在对 s_3 求 LIS 得到的值即为求 LCS 的答案。

2.3 Improved by quadrilateral inequality

```
/*
 * 四边形不等式
 *
 * 如果 dp(i,j) 满足 dp(i,j)<=dp(i,j+1)<=dp(i+1,j+1)
 * 那么决策 s(i,j) 满足 s(i,j)<=s(i,j+1)<=s(i+1,j+1)
 * 可以变形为:
 *      s(i-1,j) <= s(i,j) <= s(i,j+1)  // dp方向：i增j减
 *  或
 *      s(i,j-1) <= s(i,j) <= s(i+1,j)  // dp方向：区间长度L增
 */
#include <bits/stdc++.h>

using namespace std;

#define MAXN    1024
#define inf     (0x3fffffff)

int n, m;
int v[MAXN];
int s[MAXN];
int w[MAXN][MAXN];
int dp[MAXN][MAXN];
int c[MAXN][MAXN];

int wa(void) {
  int i, j, k;
  for(i = 1; i <= n; ++i) {
    scanf("%d", v+i);
    s[i] = v[i] + s[i-1];
  }
  for(i = 1; i <= n; ++i) {
    w[i][i] = 0;
    for(j = i+1; j <= n; ++j)
      w[i][j] = w[i][j-1] + v[j] * (s[j-1] - s[i-1]);
  }
  /* doing dp */
  for(i = 1; i <= n; ++i) {
    dp[i][0] = w[1][i];
    c[i][0] = 1;
    c[i][i] = i-1;
    for(j = i-1; j > 0; j--) {
      dp[i][j] = inf;
      for(k = c[i-1][j]; k <= c[i][j+1]; ++k)
        if(dp[k][j-1]+w[k+1][i] <= dp[i][j]) {
          dp[i][j] = dp[k][j-1] + w[k+1][i];
          c[i][j] = k;
        }
    }
  }
  /* dp done */
  return dp[n][m];
}

int main(void) {
  while(EOF != scanf("%d%d", &n, &m) && n && m) {
    printf("%d\n", wa());
  }
  return 0;
```

```
}
```

2.4 Improved by Slope

```
/* type 1: */
/* bzoj 1010 */
#include <bits/stdc++.h>

using namespace std;
typedef long double ll;
#define MAXN    50050
#define eps     (1e-8)

int N;
ll L;
ll S[MAXN];
ll f[MAXN];
ll dp[MAXN];

inline ll k(int j) {
  return (-2.0) * (f[j] + L);
}
inline ll b(int j) {
  return dp[j] + f[j]*f[j] + 2ll*f[j]*L;
}
inline ll g(int j, int i) {
  return k(j) * f[i] + b(j);
}

/* check if l1 & l3 <= l2 */
inline int check(int l1, int l2, int l3) {
  /*ll left = b(l3)*k(l1)+b(l1)*k(l2)+b(l2)*k(l3);
  ll right = b(l1)*k(l3)+b(l3)*k(l2)+b(l2)*k(l1);*/
  ll left = b(l3)*k(l1)-b(l1)*k(l3);
  ll right = k(l2)*(b(l3)-b(l1))+b(l2)*(k(l1)-k(l3));
  return (left <= right);
}

int Q[MAXN], ql, qr;

int main(void) {
  int i;
  scanf("%d%Lf", &N, &L);
  L += 1.0;
  for(i = 1; i <= N; ++i) {
    scanf("%Lf", S+i);
    S[i] += S[i-1];
    f[i] = S[i] + (double)i;
  }
  Q[qr++] = 0;
  for(i = 1; i <= N; ++i) {
    /* <!-- STARED */
    for(; ql+1 < qr && g(Q[ql],i) >= g(Q[ql+1],i); ql++);
    dp[i] = g(Q[ql], i) + f[i]*f[i] + L*L; //printf("%d: %lld,%lld\n", i, dp[i], dp[i
        ]-f[i]*f[i]);
    for(; ql+1 < qr && check(Q[qr-2], Q[qr-1], i); qr--);
    Q[qr++] = i;
    /* --> */
  }
  printf("%lld\n", (long long int)round(dp[N]));
  return 0;
}
```

2.5 Steiner Tree

令 $f[i][sta]$ 表示 i 号节点，与其他节点的连通性为 sta 时的最小代价，这里 sta 是一个二进制数，在它二进制下的每一位中，0 表示不连通，1 表示联通
状态转移：

- 由子集转移而来

$$f[i][sta] = \min_{s \subseteq sta}\{f[i][s] + f[i][sta \setminus s] - val[i]\}$$

- 由不含该节点的状态转移而来

$$f[i][j] = \min\{f[k][j] + val[i]\}$$

```
#include<cstdio>
#include<queue>
#include<cstring>
using namespace std;
const int limit = 1050;
const int INF = 1e9;
inline int read() {
  char c = getchar();
  int x = 0, f = 1;
  while(c < '0' || c > '9') {
    if(c == '-') f = -1;
    c = getchar();
  }
  while(c >= '0' && c <= '9') {
    x = x * 10 + c - '0';
    c = getchar();
  }
  return x * f;
}
#define MP(i,j) make_pair(i,j)
#define se second
#define fi first
#define Pair pair<int,int>
int N, M, tot = 0;
int a[12][12], f[12][12][limit];
int xx[5] = {-1, +1, 0, 0};
int yy[5] = {0, 0, -1, +1};
int vis[12][12];
struct PRE {
  int x, y, S;
} Pre[12][12][limit];
queue<Pair>q;
void SPFA(int cur) {
  while(q.size() != 0) {
    Pair p = q.front();
    q.pop();
    vis[p.fi][p.se] = 0;
    for(int i = 0; i <4; i++) {
      int wx = p.fi + xx[i], wy = p.se + yy[i];
      if(wx < 1 || wx > N || wy < 1 || wy > M) continue;
      if(f[wx][wy][cur] > f[p.fi][p.se][cur] + a[wx][wy]) {
        f[wx][wy][cur] = f[p.fi][p.se][cur] + a[wx][wy];
        Pre[wx][wy][cur] = (PRE) {
          p.fi, p.se, cur
        };
        if(!vis[wx][wy])
          vis[wx][wy] = 1, q.push(MP(wx,wy));
      }
    }
```

```
  }
}
void dfs(int x, int y, int now) {
  vis[x][y] = 1;
  PRE tmp = Pre[x][y][now];
  if(tmp.x == 0 && tmp.y == 0) return;
  dfs(tmp.x, tmp.y, tmp.S);
  if(tmp.x == x && tmp.y == y) dfs(tmp.x, tmp.y, now - tmp.S);
}
int main() {
  //freopen("a.in", "r", stdin);
  N = read();
  M = read();
  memset(f, 0x3f, sizeof(f));
  for(int i = 1; i <= N; i++)
    for(int j = 1; j <= M; j++) {
      a[i][j] = read();
      if(a[i][j] == 0)
        f[i][j][1 << tot] = 0, tot++;
    }
  int limit = (1 << tot) - 1;
  for(int sta = 0; sta <= limit; sta++) {
    for(int i = 1; i<= N; i++)
      for(int j = 1; j <= M; j++) {
        for(int s = sta; s; s = (s - 1) & sta) {
          if(f[i][j][s] + f[i][j][sta - s] - a[i][j] < f[i][j][sta])
            f[i][j][sta] = f[i][j][s] + f[i][j][sta - s] - a[i][j],
            Pre[i][j][sta] = (PRE) {
            i,j,s
          };
        }
        if(f[i][j][sta] < INF) q.push(MP(i,j)), vis[i][j] = 1;
      }
    SPFA(sta);
  }
  int ansx, ansy, flag = 0;
  for(int i = 1; i <= N && !flag; i++)
    for(int j = 1; j <= M; j++)
      if(!a[i][j]) {
        ansx = i, ansy = j;
        flag = 1;
        break;
      }
  printf("%d\n",f[ansx][ansy][limit]);
  memset(vis, 0, sizeof(vis));
  dfs(ansx, ansy, limit);
  for(int i = 1; i <= N; i++, puts("")) {
    for(int j = 1; j <= M; j++) {
      if(a[i][j] == 0) putchar('x');
      else if(vis[i][j]) putchar('o');
      else putchar('_');
    }
  }
  return 0;
}
```

3 Geometry

3.1 2D

3.1.1 Point

```
/* 2D Point Class, by Abreto<m@abreto.net> */
#include <cmath>

/**
 * Define ABG2d_USE_LL if you want to use long long int for cordnates.
 */

namespace ab_geometry_2d {

using namespace std;

typedef double ab_float;

const ab_float pi = acos(-1.);

#ifdef ABG2d_USE_LL
typedef long long int T;
#else
typedef ab_float T;
const ab_float eps = 1e-8;
#endif

inline T myabs(T x) {
  if(x < 0) return (-x);
  return x;
}

inline int sgn(T x) {
  /* no difference'' in fact */
#ifdef ABG2d_USE_LL
  if (0 == x) return 0;
#else
  if (myabs(x) < eps) return 0;
#endif
  return (x > 0) ? 1 : -1;
}

inline T sqr(T x) {
  return (x * x);
}

struct point {
  T x,y;
  point(void):x(T()),y(T()) {}
  point(T xx, T yy):x(xx),y(yy) {}
  inline T norm2(void) {
    return sqr(x) + sqr(y);
  }
  inline ab_float norm(void) {
    return sqrt((ab_float)(norm2()));
  }
  inline point rotate(const ab_float &cost, const ab_float &sint) {} // TODO:
  inline point operator-(void) const {
    return point(-x,-y);
  }
  inline point operator+(const point& b) const {
    return point(x+b.x,y+b.y);
  }
  inline point operator-(const point& b) const {
    return point(x-b.x,y-b.y);
  }
  inline point operator->*(const point &b) const {
    return (b-(*this));
  }
```

```
65    inline T operator*(const point& b) const {
66      return ((x)*(b.x))+((y)*(b.y)); /* inner product */
67    }
68    inline T operator^(const point& b) const {
69      return ((x)*(b.y))-((b.x)*(y)); /* outter product */
70    }
71    inline point& operator+=(const point& b) {
72      point tmp=(*this)+b;
73      (*this)=tmp;
74      return (*this);
75    }
76    inline point& operator-=(const point& b) {
77      point tmp=(*this)-b;
78      (*this)=tmp;
79      return (*this);
80    }
81    inline bool operator==(const point& b) const {
82      return (0==sgn(x-b.x))&&(0==sgn(y-b.y));
83    }
84    inline bool operator!=(const point& b) const {
85      return !((*this)==b);
86    }
87    inline point operator<<(const ab_float& theta) const {
88      ab_float ct = cos(theta), st = sin(theta);  /* rotate counter-clockwise in radian
          */
89      return point(ct*x - st*y, st*x + ct*y);
90    }
91  };
92
93  typedef point vec;
94
95
96  }   // namespace ab_geometry_2d
```

3.1.2 Circle

Base

```
1   /* 2D Circle Base Class, by Abreto<m@abreto.net>. */
2
3   /* requirement: point.cc */
4   #include "point.cc"
5
6   #include <utility>
7
8   namespace ab_geometry_2d {
9
10  using namespace std;
11
12  struct circle {
13    point o;
14    T r;
15    circle(void) : r(T()) {}
16    circle(point center, T radius) : o(center), r(radius) {}
17
18    inline ab_float arclen(ab_float theta) {
19      return theta * r;
20    }
21    inline ab_float circumference(void) {
22      return 2. * pi * r;
23    }
24    inline ab_float area(void) {
25      return pi * r * r;
```

```
26    }
27
28    /* bool contain(const circle &C, const bool including_touch = false) const
29    {
30        T dis2 = (o->*(C.o)).norm2();
31        T raw_diff = r - C.r;
32        if ( -1 == sgn(raw_diff) ) return false;
33        T dr2 = sqr(raw_diff);
34        return (dis2 < dr2) || (including_touch && (dis2 == dr2));
35    }
36    inline bool in(const circle &C, const bool including_touch = false) const
37    {
38      return C.contain(*this, including_touch);
39    } */
40    enum relation_t {
41      same = 0x00000,
42      contain = 0x00001,
43      intouch = 0x00010,
44      intersect = 0x00100,
45      outtouch = 0x01000,
46      separate = 0x10000,
47      unknow_relation = 0xfffff
48    };
49    relation_t with(const circle &C) const {
50      T dis2 = (o->*(C.o)).norm2();
51      T dr2 = sqr(r - C.r), rs2 = sqr(r + C.r);
52      if ( 0 == sgn(dis2) && 0 == sgn(dr2) ) return same;
53      if ( -1 == sgn(dis2 - dr2) ) return contain;
54      if ( 0 == sgn(dis2 - dr2) ) return intouch;
55      if ( -1 == sgn(dr2 - dis2) && -1 == sgn(dis2 - rs2) ) return intersect;
56      if ( 0 == sgn(dis2 - rs2) ) return outtouch;
57      if ( -1 == sgn(rs2 - dis2) ) return separate;
58      return unknow_relation;
59    }
60
61    enum point_relation_t {
62      in = 0x001,
63      on = 0x010,
64      out = 0x100,
65      unknow_point_relation = 0xfff
66    };
67    point_relation_t with(const point &P) const {
68      T dis2 = (o->*P).norm2();
69      T r2 = sqr(r);
70      int type = sgn(dis2 - r2);
71      if (-1 == type) return in;
72      if ( 0 == type) return on;
73      if (+1 == type) return out;
74      return unknow_point_relation;
75    }
76
77    ab_float central_angle(const point &A, const point &B, const bool reflex = false)
          const {
78      T dot = (A * B);
79      if (0 == sgn(dot)) return 1. * (A != B) * pi;
80      ab_float angle = ((ab_float)(dot)) / r / r;
81      if ( reflex ) angle = 2. * pi - angle;
82      return angle;
83    }
84
85    /* be sure (*this) intersect with C */
86    pair<point,point> crosspoint(const circle &C) const {
87      ab_float d = (o ->* (C.o)).norm();
88      // TODO:
```

```
  }
};

}
```

k 次圆交

```
//china no.1
#pragma comment(linker, "/STACK:1024000000,1024000000")
#include <vector>
#include <iostream>
#include <string>
#include <map>
#include <stack>
#include <cstring>
#include <queue>
#include <list>
#include <stdio.h>
#include <set>
#include <algorithm>
#include <cstdlib>
#include <cmath>
#include <iomanip>
#include <cctype>
#include <sstream>
#include <functional>
#include <stdlib.h>
#include <time.h>
#include <bitset>
using namespace std;

#define pi acos(-1)
#define PI acos(-1)
#define endl '\n'
#define srand() srand(time(0));
#define me(x,y) memset(x,y,sizeof(x));
#define foreach(it,a) for(__typeof((a).begin()) it=(a).begin();it!=(a).end();it++)
#define close() ios::sync_with_stdio(0); cin.tie(0);
#define FOR(x,n,i) for(int i=x;i<=n;i++)
#define FOr(x,n,i) for(int i=x;i<n;i++)
#define W while
#define sgn(x) ((x) < 0 ? -1 : (x) > 0)
#define bug printf("**********\n");
#define db double
typedef long long LL;
const int INF=0x3f3f3f3f;
const LL LINF=0x3f3f3f3f3f3f3f3fLL;
const int dx[]= {-1,0,1,0,1,-1,-1,1};
const int dy[]= {0,1,0,-1,-1,1,-1,1};
const int maxn=1e3+10;
const int maxx=1e6+100;
const double EPS=1e-8;
const double eps=1e-8;
const int mod=10000007;
template<class T>inline T min(T a,T b,T c) {
  return min(min(a,b),c);
}
template<class T>inline T max(T a,T b,T c) {
  return max(max(a,b),c);
}
template<class T>inline T min(T a,T b,T c,T d) {
  return min(min(a,b),min(c,d));
}
template<class T>inline T max(T a,T b,T c,T d) {
  return max(max(a,b),max(c,d));
```

```
}
template <class T>
inline bool scan_d(T &ret) {
  char c;
  int sgn;
  if (c = getchar(), c == EOF) {
    return 0;
  }
  while (c != '-' && (c < '0' || c > '9')) {
    c = getchar();
  }
  sgn = (c == '-') ? -1 : 1;
  ret = (c == '-') ? 0 : (c - '0');
  while (c = getchar(), c >= '0' && c <= '9') {
    ret = ret * 10 + (c - '0');
  }
  ret *= sgn;
  return 1;
}

inline bool scan_lf(double &num) {
  char in;
  double Dec=0.1;
  bool IsN=false,IsD=false;
  in=getchar();
  if(in==EOF) return false;
  while(in!='-'&&in!='.'&&(in<'0'||in>'9'))in=getchar();
  if(in=='-') {
    IsN=true;
    num=0;
  } else if(in=='.') {
    IsD=true;
    num=0;
  } else num=in-'0';
  if(!IsD) {
    while(in=getchar(),in>='0'&&in<='9') {
      num*=10;
      num+=in-'0';
    }
  }
  if(in!='.') {
    if(IsN) num=-num;
    return true;
  } else {
    while(in=getchar(),in>='0'&&in<='9') {
      num+=Dec*(in-'0');
      Dec*=0.1;
    }
  }
  if(IsN) num=-num;
  return true;
}

void Out(LL a) {
  if(a < 0) {
    putchar('-');
    a = -a;
  }
  if(a >= 10) Out(a / 10);
  putchar(a % 10 + '0');
}
void print(LL a) {
  Out(a),puts("");
}
```

```
123 //freopen( "in.txt" , "r" , stdin );
124 //freopen( "data.txt" , "w" , stdout );
125 //cerr << "run time is " << clock() << endl;
126 /*struct Point
127 {
128     double x, y;
129     Point(const Point& rhs): x(rhs.x), y(rhs.y) { } //拷贝构造函数
130     Point(double x = 0, double y = 0) : x(x), y(y) { }
131     inline void input()
132     {
133         scanf("%lf%lf",&x,&y);
134     }
135     inline void print()
136     {
137         printf("%.6lf %.6lf\n",x,y);
138     }
139 };*/
140 db sqr(db x) {
141   return x*x;
142 }
143 int dcmp(double x) {
144   if(fabs(x) < EPS) return 0;
145   else return x < 0 ? -1 : 1;
146 }
147 struct Circle {
148   double x, y, r, angle;
149   int d;
150   Circle() {}
151   Circle(double xx, double yy, double ang = 0, int t = 0) {
152     x = xx;
153     y = yy;
154     angle = ang;
155     d = t;
156   }
157   void get() {
158     scanf("%lf%lf%lf", &x, &y, &r);
159     d = 1;
160   }
161 };
162 Circle cir[maxn],tp[maxn*2];
163 double area[maxn];
164 double dis(Circle a,Circle b) {
165   return sqrt(sqr(a.x - b.x) + sqr(a.y - b.y));
166 }
167 double cross(Circle p0,Circle p1,Circle p2) {
168   return (p1.x - p0.x) * (p2.y - p0.y) - (p1.y - p0.y) * (p2.x - p0.x);
169 }
170 //圆相交
171 int CirCrossCir(Circle p1, double r1,Circle p2, double r2,Circle &cp1,Circle &cp2) {
172   double mx = p2.x - p1.x, sx = p2.x + p1.x, mx2 = mx * mx;
173   double my = p2.y - p1.y, sy = p2.y + p1.y, my2 = my * my;
174   double sq = mx2 + my2, d = -(sq - sqr(r1 - r2)) * (sq - sqr(r1 + r2));
175   if (d + eps < 0) return 0;
176   if (d < eps) d = 0;
177   else d = sqrt(d);
178   double x = mx * ((r1 + r2) * (r1 - r2) + mx * sx) + sx * my2;
179   double y = my * ((r1 + r2) * (r1 - r2) + my * sy) + sy * mx2;
180   double dx = mx * d, dy = my * d;
181   sq *= 2;
182   cp1.x = (x - dy) / sq;
183   cp1.y = (y + dx) / sq;
184   cp2.x = (x + dy) / sq;
185   cp2.y = (y - dx) / sq;
186   if (d > eps) return 2;
```

```
187   else return 1;
188 }
189 bool circmp(const Circle& u, const Circle& v) {
190   return dcmp(u.r - v.r) < 0;
191 }
192 bool cmp(const Circle& u, const Circle& v) {
193   if (dcmp(u.angle - v.angle)) return u.angle < v.angle;
194   return u.d > v.d;
195 }
196 //0.5*r*r*(K-sin(K))
197 double calc(Circle cir,Circle cp1,Circle cp2) {
198   double ans = (cp2.angle - cp1.angle) * sqr(cir.r)
199                - cross(cir, cp1, cp2) + cross(Circle(0, 0), cp1, cp2);
200   return ans / 2;
201 }
202
203 void CirUnion(Circle cir[], int n) {
204   Circle cp1, cp2;
205   sort(cir, cir + n, circmp);
206   for (int i = 0; i < n; ++i)
207     for (int j = i + 1; j < n; ++j)
208       if (dcmp(dis(cir[i], cir[j]) + cir[i].r - cir[j].r) <= 0)
209         cir[i].d++;
210   for (int i = 0; i < n; ++i) {
211     int tn = 0, cnt = 0;
212     for (int j = 0; j < n; ++j) {
213       if (i == j) continue;
214       if (CirCrossCir(cir[i], cir[i].r, cir[j], cir[j].r,
215                       cp2, cp1) < 2) continue;
216       cp1.angle = atan2(cp1.y - cir[i].y, cp1.x - cir[i].x);
217       cp2.angle = atan2(cp2.y - cir[i].y, cp2.x - cir[i].x);
218       cp1.d = 1;
219       tp[tn++] = cp1;
220       cp2.d = -1;
221       tp[tn++] = cp2;
222       if (dcmp(cp1.angle - cp2.angle) > 0) cnt++;
223     }
224     tp[tn++] = Circle(cir[i].x - cir[i].r, cir[i].y, pi, -cnt);
225     tp[tn++] = Circle(cir[i].x - cir[i].r, cir[i].y, -pi, cnt);
226     sort(tp, tp + tn, cmp);
227     int p, s = cir[i].d + tp[0].d;
228     for (int j = 1; j < tn; ++j) {
229       p = s;
230       s += tp[j].d;
231       area[p] += calc(cir[i], tp[j - 1], tp[j]);
232     }
233   }
234 }
235 int n;
236 void solve() {
237   for(int i=0; i<n; i++)
238     cir[i].get();
239   me(area,0);
240   CirUnion(cir,n);
241   for(int i=1; i<=n; i++) {
242     area[i]-=area[i+1];
243     printf("[%d] = %.3f\n", i, area[i]);
244   }
245 }
246 int main() {
247   while(scanf("%d",&n)!=EOF)
248     solve();
249 }
```

universe

```
1
2  Point CircumCenter(Point a,Point b,Point c) { //三角形的外心
3    Point cp;
4    double a1 = b.x-a.x,b1 = b.y-a.y,c1 = (a1*a1 + b1*b1)/2;
5    double a2 = c.x-a.x,b2 = c.y-a.y,c2 = (a2*a2 + b2*b2)/2;
6    double d = a1*b2 - a2*b1;
7    cp.x = a.x + (c1*b2-c2*b1)/d;
8    cp.y = a.y + (a1*c2-a2*c1)/d;
9    return cp;
10 }
```

3.1.3 Convex hull

```
1  /* 2D Convex Hull, by Abreto <m@abreto.net>. */
2  #include "2d_base.hh"
3  #include <cmath>
4  #include <algorithm>
5
6  using namespace std;
7
8  point O;
9
10 bool comp_angle(point_t a, point_t b) {
11   double t = (a-O).X(b-O);
12   if(fe(t,0.0)) return fl((b-O).mag2(),(a-O).mag2());
13   else return fl(0.0,t);
14 }
15
16 void convex_hull_graham(vp& convex, vp src) {
17   int i = 0, top = 0;
18   O = src[0];
19   for(auto pt : src)
20     if( pt.x < O.x || (pt.x == O.x && pt.y < O.y))
21       O = pt;
22   sort(src.begin(), src.end(), comp_angle);
23   convex.push_back(src[0]);
24   convex.push_back(src[1]);
25   top = 1;
26   for(i = 2; i < src.size(); ++i) {
27     while(top>1 && fle((convex[top]-convex[top-1]).X(src[i]-convex[top]),0.0)) {
28       convex.pop_back();
29       --top;
30     }
31     convex.push_back(src[i]);
32     ++top;
33   }
34 }
```

3.1.4 Intersect Area

```
1  #include <cstdio>
2  #include <cmath>
3  #include <algorithm>
4
5  using namespace std;
6
7  //#define inf 1000000000000
8  #define M 8
9  #define LL long long
10 #define eps 1e-12
```

```
#define PI acos(-1.0)
using namespace std;
struct node {
  double x,y;
  node() {}
  node(double xx,double yy) {
    x=xx;
    y=yy;
  }
  node operator -(node s) {
    return node(x-s.x,y-s.y);
  }
  node operator +(node s) {
    return node(x+s.x,y+s.y);
  }
  double operator *(node s) {
    return x*s.x+y*s.y;
  }
  double operator ^(node s) {
    return x*s.y-y*s.x;
  }
};
double max(double a,double b) {
  return a>b?a:b;
}
double min(double a,double b) {
  return a<b?a:b;
}
double len(node a) {
  return sqrt(a*a);
}
double dis(node a,node b) { //两点之间的距离
  return len(b-a);
}
double cross(node a,node b,node c) { //叉乘
  return (b-a)^(c-a);
}
double dot(node a,node b,node c) { //点成
  return (b-a)*(c-a);
}
int judge(node a,node b,node c) { //判断c是否在ab线段上（前提是c在直线ab上）
  if(c.x>=min(a.x,b.x)
      &&c.x<=max(a.x,b.x)
      &&c.y>=min(a.y,b.y)
      &&c.y<=max(a.y,b.y))
    return 1;
  return 0;
}
double area(node b,node c,double r) {
  node a(0.0,0.0);
  if(dis(b,c)<eps)
    return 0.0;
  double h=fabs(cross(a,b,c))/dis(b,c);
  if(dis(a,b)>r-eps&&dis(a,c)>r-eps) { //两个端点都在圆的外面则分为两种情况
    double angle=acos(dot(a,b,c)/dis(a,b)/dis(a,c));
    if(h>r-eps) {
      return 0.5*r*r*angle;
    } else if(dot(b,a,c)>0&&dot(c,a,b)>0) {
      double angle1=2*acos(h/r);
      return 0.5*r*r*fabs(angle-angle1)+0.5*r*r*sin(angle1);
    } else {
      return 0.5*r*r*angle;
    }
  } else if(dis(a,b)<r+eps&&dis(a,c)<r+eps) { //两个端点都在圆内的情况
```

```
75      return 0.5*fabs(cross(a,b,c));
76    } else { //一个端点在圆上一个端点在圆内的情况
77      if(dis(a,b)>dis(a,c)) { //默认b在圆内
78        swap(b,c);
79      }
80      if(fabs(dis(a,b))<eps) { //ab距离为0直接返回0
81        return 0.0;
82      }
83      if(dot(b,a,c)<eps) {
84        double angle1=acos(h/dis(a,b));
85        double angle2=acos(h/r)-angle1;
86        double angle3=acos(h/dis(a,c))-acos(h/r);
87        return 0.5*dis(a,b)*r*sin(angle2)+0.5*r*r*angle3;
88
89      } else {
90        double angle1=acos(h/dis(a,b));
91        double angle2=acos(h/r);
92        double angle3=acos(h/dis(a,c))-angle2;
93        return 0.5*r*dis(a,b)*sin(angle1+angle2)+0.5*r*r*angle3;
94      }
95    }
96  }
97
98  node A, B, C;
99  int R;
100
101 bool compar(node &p1, node &p2) {
102   return (p1^p2)>eps;
103 }
104
105 double f(double x, double y) {
106   node O(x,y);
107   node p[8];
108   p[0] = A-O;
109   p[1] = B-O;
110   p[2] = C-O;
111   sort(p, p+3, compar);
112   p[3] = p[0];
113   O=node(0,0);
114   double sum=0;
115   /* <!-- 求面积交部分 */
116   for(int i=0; i<3; i++) { /* 按顺或逆时针顺序最后取绝对值就好 */
117     int j=i+1;
118     double s=area(p[i],p[j],(double)R);
119     if(cross(O,p[i],p[j])>0)
120       sum+=s;
121     else
122       sum-=s;
123   }
124   if(sum < -eps) sum = -sum;
125   /* --> */
126   return sum;
127 }
128
129 double trifind(double x, double y1, double y2) {
130   double l = y1, r = y2;
131   while(r-l>eps) {
132     double mid = (l+r)/2.0;
133     double mmid = (mid+r)/2.0;
134     if( f(x,mmid) > f(x,mid)+eps )
135       l = mid;
136     else
137       r = mmid;
138   }
```

```
  return f(x,l);
}

double findmin(double x1, double x2, double y1, double y2) {
  double l = x1, r = x2;
  while(r-l>eps) {
    double mid = (l+r)/2.0;
    double mmid = (mid+r)/2.0;
    if( trifind(mmid,y1,y2) > trifind(mid,y1,y2)+eps )
      l = mid;
    else
      r = mmid;
  }
  return trifind(l,y1,y2);
}

double ans(int a, int b, int c, int r) {
  A = node(0,0);
  B = node((double)c,0);
  R = r;
  double da = a, db = b, dc = c;
  double cosa = (db*db+dc*dc-da*da)/(2.0*db*dc);
  double alpha = acos(cosa);
  C = node(db*cosa, db*sin(alpha));
  return findmin(0.0, c, 0.0, db*sin(alpha));
}

int main(void) {
  int a = 0, b = 0, c = 0, r = 0;
  while(EOF != scanf("%d%d%d%d",&a,&b,&c,&r) && (a||b||c||r))
    printf("%.8lf\n", ans(a,b,c,r));
  return 0;
}
```

3.1.5 Universe

```
#include <bits/stdc++.h>
using namespace std;

struct Point {
  double x, y;
  Point(double x = 0, double y = 0) : x(x), y(y) {}
};

typedef Point Vector;

Vector operator + (Vector A, Vector B) {
  return Vector(A.x + B.x, A.y + B.y);
}
Vector operator - (Vector A, Vector B) {
  return Vector(A.x - B.x, A.y - B.y);
}
Vector operator * (Vector A, double p) {
  return Vector(A.x*p, A.x*p);
}
Vector operator / (Vector A, double p) {
  return Vector(A.x/p, A.x/p);
}

bool operator < (const Point& a, const Point b) {
  return a.x < b.x || (a.x == b.x && a.y < b.y);
}

```

```
28 const double EPS = 1e-10;
29
30 int dcmp(double x) {
31   if(fabs(x) < EPS) return 0;
32   else return x < 0 ? -1 : 1;
33 }
34
35 bool operator == (const Point& a, const Point& b) {
36   return dcmp(a.x-b.x) == 0 && dcmp(a.y-b.y);
37 }
38
39 //向量a的极角
40 double Angle(const Vector& v) {
41   return atan2(v.y, v.x);//\share\CodeBlocks\templates\wizard\console\cpp
42 }
43
44 //向量点积
45 double Dot(Vector A, Vector B) {
46   return A.x*B.x + A.y*B.y;
47 }
48
49 //向量长度\share\CodeBlocks\templates\wizard\console\cpp
50 double Length(Vector A) {
51   return sqrt(Dot(A, A));
52 }
53
54 //向量夹角
55 double Angle(Vector A, Vector B) {
56   return acos(Dot(A, B) / Length(A) / Length(B));
57 }
58
59 //向量叉积
60 double Cross(Vector A, Vector B) {
61   return A.x*B.y - A.y*B.x;
62 }
63
64 //三角形有向面积的二倍
65 double Area2(Point A, Point B, Point C) {
66   return Cross(B-A, C-A);
67 }
68
69 //向量逆时针旋转rad度(弧度)
70 Vector Rotate(Vector A, double rad) {
71   return Vector(A.x*cos(rad)-A.y*sin(rad), A.x*sin(rad)+A.y*cos(rad));
72 }
73
74 //计算向量A的单位法向量。左转90°，把长度归一。调用前确保A不是零向量。
75 Vector Normal(Vector A) {
76   double L = Length(A);
77   return Vector(-A.y/L, A.x/L);
78 }
79
80 /*********************************************************************
81 使用复数类实现点及向量的简单操作
82
83 #include <complex>
84 typedef complex<double> Point;
85 typedef Point Vector;
86
87 double Dot(Vector A, Vector B) { return real(conj(A)*B)}
88 double Cross(Vector A, Vector B) { return imag(conj(A)*B);}
89 Vector Rotate(Vector A, double rad) { return A*exp(Point(0, rad)); }
90
91 *********************************************************************/
```

```
92
93  /*******************************************************************************
94  * 用直线上的一点p0和方向向量v表示一条指向。直线上的所有点P满足P = P0+t*v;
95  * 如果知道直线上的两个点则方向向量为B-A, 所以参数方程为A+(B-A)*t;
96  * 当t 无限制时, 该参数方程表示直线。
97  * 当t > 0时, 该参数方程表示射线。
98  * 当 0 < t < 1时, 该参数方程表示线段。
99  *******************************************************************************/
100
101 //直线交点,须确保两直线有唯一交点。
102 Point GetLineIntersection(Point P, Vector v, Point Q, Vector w) {
103   Vector u = P - Q;
104   double t = Cross(w, u)/Cross(v, w);
105   return P+v*t;
106 }
107
108 //点到直线距离
109 double DistanceToLine(Point P, Point A, Point B) {
110   Vector v1 = B - A, v2 = P - A;
111   return fabs(Cross(v1, v2) / Length(v1)); //不取绝对值, 得到的是有向距离
112 }
113
114 //点到线段的距离
115 double DistanceToSegmentS(Point P, Point A, Point B) {
116   if(A == B) return Length(P-A);
117   Vector v1 = B-A, v2 = P-A, v3 = P-B;
118   if(dcmp(Dot(v1, v2)) < 0) return Length(v2);
119   else if(dcmp(Dot(v1, v3)) > 0) return Length(v3);
120   else return fabs(Cross(v1, v2)) / Length(v1);
121 }
122
123 //点在直线上的投影
124 Point GetLineProjection(Point P, Point A, Point B) {
125   Vector v = B - A;
126   return A+v*(Dot(v, P-A)/Dot(v, v));
127 }
128
129 //线段相交判定, 交点不在一条线段的端点
130 bool SegmentProperIntersection(Point a1, Point a2, Point b1, Point b2) {
131   double c1 = Cross(a2-a1, b1-a1), c2 = Cross(a2-a1, b2-a1);
132   double c3 = Cross(b2-b1, a1-b1), c4 = Cross(b2-b1, a2-b1);
133   return dcmp(c1)*dcmp(c2) < 0 && dcmp(c3)*dcmp(c4) < 0;
134 }
135
136 //判断点是否在点段上, 不包含端点
137 bool OnSegment(Point P, Point a1, Point a2) {
138   return dcmp(Cross(a1-P, a2-P) == 0 && dcmp((Dot(a1-P, a2-P)) < 0));
139 }
140
141 //计算凸多边形面积
142 double ConvexPolygonArea(Point *p, int n) {
143   double area = 0;
144   for(int i = 1; i < n-1; i++)
145     area += Cross(p[i] - p[0], p[i+1] - p[0]);
146   return area/2;
147 }
148
149 //计算多边形的有向面积
150 double PolygonArea(Point *p, int n) {
151   double area = 0;
152   for(int i = 1; i < n-1; i++)
153     area += Cross(p[i] - p[0], p[i+1] - p[0]);
154   return area/2;
155 }
```

```
156
157 /*************************************************************************
158 * Morley定理：三角形每个内角的三等分线，相交成的三角形是等边三角形。
159 * 欧拉定理：设平面图的定点数，边数和面数分别为V,E,F。则V+F-E = 2;
160 *************************************************************************/
161
162 struct Circle {
163   Point c;
164   double r;
165
166   Circle(Point c, double r) : c(c), r(r) {}
167   //通过圆心角确定圆上坐标
168   Point point(double a) {
169     return Point(c.x + cos(a)*r, c.y + sin(a)*r);
170   }
171 };
172
173 struct Line {
174   Point p;
175   Vector v;
176   double ang;
177   Line() {}
178   Line(Point p, Vector v) : p(p), v(v) {}
179   bool operator < (const Line& L) const {
180     return ang < L.ang;
181   }
182 };
183
184 //直线和圆的交点，返回交点个数，结果存在sol中。
185 //该代码没有清空sol。
186 int getLineCircleIntersecion(Line L, Circle C, double& t1, double& t2, vector<Point>&
        sol) {
187   double a = L.v.x, b = L.p.x - C.c.x, c = L.v.y, d = L.p.y - C.c.y;
188   double e = a*a + c*c, f = 2*(a*b + c*d), g = b*b + d*d - C.r*C.r;
189   double delta = f*f - 4*e*g;
190   if(dcmp(delta) < 0) return 0; //相离
191   if(dcmp(delta) == 0) {        //相切
192     t1 = t2 = -f / (2*e);
193     sol.push_back(C.point(t1));
194     return 1;
195   }
196   //相交
197   t1 = (-f - sqrt(delta)) / (2*e);
198   sol.push_back(C.point(t1));
199   t2 = (-f + sqrt(delta)) / (2*e);
200   sol.push_back(C.point(t2));
201   return 2;
202 }
203
204 //两圆相交
205 int getCircleCircleIntersection(Circle C1, Circle C2, vector<Point>& sol) {
206   double d = Length(C1.c - C2.c);
207   if(dcmp(d) == 0) {
208     if(dcmp(C1.r - C2.r == 0)) return -1;    //两圆完全重合
209     return 0;                                //同心圆，半径不一样
210   }
211   if(dcmp(C1.r + C2.r - d) < 0) return 0;
212   if(dcmp(fabs(C1.r - C2.r) == 0)) return -1;
213
214   double a = Angle(C2.c - C1.c);               //向量C1C2的极角
215   double da = acos((C1.r*C1.r + d*d - C2.r*C2.r) / (2*C1.r*d));
216   //C1C2到C1P1的角
217   Point p1 = C1.point(a-da), p2 = C1.point(a+da);
218   sol.push_back(p1);
```

```
219   if(p1 == p2) return 1;
220   sol.push_back(p2);
221   return 2;
222 }
223
224 const double PI = acos(-1);
225 //过定点做圆的切线
226 //过点p做圆C的切线，返回切线个数。v[i]表示第i条切线
227 int getTangents(Point p, Circle C, Vector* v) {
228   Vector u = C.c - p;
229   double dist = Length(u);
230   if(dist < C.r) return 0;
231   else if(dcmp(dist - C.r) == 0) {
232     v[0] = Rotate(u, PI/2);
233     return 1;
234   } else {
235     double ang = asin(C.r / dist);
236     v[0] = Rotate(u, -ang);
237     v[1] = Rotate(u, +ang);
238     return 2;
239   }
240 }
241
242 //两圆的公切线
243 //返回切线的个数，-1表示有无数条公切线。
244 //a[i], b[i] 表示第i条切线在圆A，圆B上的切点
245 int getTangents(Circle A, Circle B, Point *a, Point *b) {
246   int cnt = 0;
247   if(A.r < B.r) {
248     swap(A, B);
249     swap(a, b);
250   }
251   int d2 = (A.c.x - B.c.x)*(A.c.x - B.c.x) + (A.c.y - B.c.y)*(A.c.y - B.c.y);
252   int rdiff = A.r - B.r;
253   int rsum = A.r + B.r;
254   if(d2 < rdiff*rdiff) return 0;   //内含
255   double base = atan2(B.c.y - A.c.y, B.c.x - A.c.x);
256   if(d2 == 0 && A.r == B.r) return -1;   //无限多条切线
257   if(d2 == rdiff*rdiff) {         //内切一条切线
258     a[cnt] = A.point(base);
259     b[cnt] = B.point(base);
260     cnt++;
261     return 1;
262   }
263   //有外共切线
264   double ang = acos((A.r-B.r) / sqrt(d2));
265   a[cnt] = A.point(base+ang);
266   b[cnt] = B.point(base+ang);
267   cnt++;
268   a[cnt] = A.point(base-ang);
269   b[cnt] = B.point(base-ang);
270   cnt++;
271   if(d2 == rsum*rsum) {  //一条公切线
272     a[cnt] = A.point(base);
273     b[cnt] = B.point(PI+base);
274     cnt++;
275   } else if(d2 > rsum*rsum) {   //两条公切线
276     double ang = acos((A.r + B.r) / sqrt(d2));
277     a[cnt] = A.point(base+ang);
278     b[cnt] = B.point(PI+base+ang);
279     cnt++;
280     a[cnt] = A.point(base-ang);
281     b[cnt] = B.point(PI+base-ang);
282     cnt++;
```

```
283   }
284   return cnt;
285 }
286
287 typedef vector<Point> Polygon;
288
289 //点在多边形内的判定
290 int isPointInPolygon(Point p, Polygon poly) {
291   int wn = 0;
292   int n = poly.size();
293   for(int i = 0; i < n; i++) {
294     if(OnSegment(p, poly[i], poly[(i+1)%n])) return -1; //在边界上
295     int k = dcmp(Cross(poly[(i+1)%n]-poly[i], p-poly[i]));
296     int d1 = dcmp(poly[i].y - p.y);
297     int d2 = dcmp(poly[(i+1)%n].y - p.y);
298     if(k > 0 && d1 <= 0 && d2 > 0) wn++;
299     if(k < 0 && d2 <= 0 && d1 > 0) wn++;
300   }
301   if(wn != 0) return 1;       //内部
302   return 0;                   //外部
303 }
304
305 //凸包
306 /************************************************************
307 * 输入点数组p, 个数为p, 输出点数组ch。 返回凸包顶点数
308 * 不希望凸包的边上有输入点，把两个<= 改成 <
309 * 高精度要求时建议用dcmp比较
310 * 输入点不能有重复点。函数执行完以后输入点的顺序被破坏
311 ************************************************************/
312 int ConvexHull(Point *p, int n, Point* ch) {
313   sort(p, p+n);      //先比较x坐标，再比较y坐标
314   int m = 0;
315   for(int i = 0; i < n; i++) {
316     while(m > 1 && Cross(ch[m-1] - ch[m-2], p[i]-ch[m-2]) <= 0) m--;
317     ch[m++] = p[i];
318   }
319   int k = m;
320   for(int i = n-2; i >= 0; i++) {
321     while(m > k && Cross(ch[m-1] - ch[m-2], p[i]-ch[m-2]) <= 0) m--;
322     ch[m++] = p[i];
323   }
324   if(n > 1) m--;
325   return m;
326 }
327
328 //用有向直线A->B切割多边形poly, 返回“左侧”。 如果退化，可能会返回一个单点或者线段
329 //复杂度O(n2);
330 Polygon CutPolygon(Polygon poly, Point A, Point B) {
331   Polygon newpoly;
332   int n = poly.size();
333   for(int i = 0; i < n; i++) {
334     Point C = poly[i];
335     Point D = poly[(i+1)%n];
336     if(dcmp(Cross(B-A, C-A)) >= 0) newpoly.push_back(C);
337     if(dcmp(Cross(B-A, C-D)) != 0) {
338       Point ip = GetLineIntersection(A, B-A, C, D-C);
339       if(OnSegment(ip, C, D)) newpoly.push_back(ip);
340     }
341   }
342   return newpoly;
343 }
344
345 //半平面交
346
```

```
347 //点p再有向直线L的左边。（线上不算）
348 bool Onleft(Line L, Point p) {
349   return Cross(L.v, p-L.p) > 0;
350 }
351
352 //两直线交点，假定交点唯一存在
353 Point GetIntersection(Line a, Line b) {
354   Vector u = a.p - b.p;
355   double t = Cross(b.v, u) / Cross(a.v, b.v);
356   return a.p+a.v*t;
357 }
358
359 int HalfplaneIntersection(Line* L, int n, Point* poly) {
360   sort(L, L+n);                //按极角排序
361
362   int first, last;             //双端队列的第一个元素和最后一个元素
363   Point *p = new Point[n];     //p[i]为q[i]和q[i+1]的交点
364   Line *q = new Line[n];       //双端队列
365   q[first = last = 0] = L[0]; //队列初始化为只有一个半平面L[0]
366   for(int i = 0; i < n; i++) {
367     while(first < last && !Onleft(L[i], p[last-1])) last--;
368     while(first < last && !Onleft(L[i], p[first])) first++;
369     q[++last] = L[i];
370     if(fabs(Cross(q[last].v, q[last-1].v)) < EPS) {
371       last--;
372       if(Onleft(q[last], L[i].p)) q[last] = L[i];
373     }
374     if(first < last) p[last-1] = GetIntersection(q[last-1], q[last]);
375   }
376   while(first < last && !Onleft(q[first], p[last-1])) last--;
377   //删除无用平面
378   if(last-first <= 1) return 0;   //空集
379   p[last] = GetIntersection(q[last], q[first]);
380
381   //从deque复制到输出中
382   int m = 0;
383   for(int i = first; i <= last; i++) poly[m++] = p[i];
384   return m;
385 }
```

4 Graph

4.1 Tree

4.1.1 Universe

```
 1
 2 /* find root(重心) */
 3
 4 void findroot(int u, int fa) {
 5   int i;
 6   size[u] = 1;
 7   f[u] = 0;
 8   for (i = last[u]; i; i = e[i][2]) {
 9     if (!vis[e[i][0]] && e[i][0] != fa) {
10       findroot(e[i][0], u);
11       size[u] += size[e[i][0]];
12       if (f[u] < size[e[i][0]])
13         f[u] = size[e[i][0]];
14     }
15   }
16   if (f[u] < ALL - size[u])
```

```
17      f[u] = ALL - size[u];
18    if (f[u] < f[root]) root = u;
19  }
20
21  /* ---- da ---- */
22
23  int dep[MAXN+1];
24  int ancestor[MAXN+1][MAXLGN];
25  int minw[MAXN+1][MAXLGN];
26
27  void dfs(int u, int fa) {
28    ancestor[u][0] = fa;
29    dep[u] = dep[fa] + 1;
30    for(int e = u[front]; e; e = E[e].n) {
31      int v = E[e].v, w = E[e].w;
32      if(v != fa) {
33        minw[v][0] = w;
34        dfs(v, u);
35      }
36    }
37  }
38
39  void init_system(void) {
40    int i = 0, w = 0;
41    int t = 0;
42    dep[0] = -1;
43    dfs(1,0);
44    for(w = 1; (t=(1<<w)) < N; ++w)
45      for(i = 1; i <= N; ++i) if( dep[i] >= t ) {
46          ancestor[i][w] = ancestor[ancestor[i][w-1]][w-1];
47          minw[i][w] = min(minw[i][w-1], minw[ancestor[i][w-1]][w-1]);
48        }
49  }
50
51  int query(int a, int b) {
52    if(dep[a] < dep[b]) return query(b,a);
53    else {  /* now dep[s] > dep[t] */
54      int i = 0;
55      int maxbit = MAXLGN-1;
56      int ret = INF;
57      //while((1<<maxbit) <= dep[a]) maxbit++;
58      /* first up a to same dep with b. */
59      for(i = maxbit; i >= 0; i--)
60        if(dep[a] - (1<<i) >= dep[b]) {
61          ret = min(ret, minw[a][i]);
62          a = ancestor[a][i];
63        }
64      if(a == b) return ret;
65      for(i = maxbit; i >= 0; i--)
66        if(dep[a] - (1<<i) >= 0 && ancestor[a][i] != ancestor[b][i]) {
67          ret = min(ret, min(minw[a][i], minw[b][i]));
68          a = ancestor[a][i];
69          b = ancestor[b][i];
70        }
71      ret = min(ret, min(minw[a][0], minw[b][0]));
72      return ret;
73    }
74  }
```

4.1.2 Point Divide and Conquer

Version 1

```
/* Tree::Point divide and conquer, by Abreto<m@abreto.net>. */
#include <bits/stdc++.h>

using namespace std;
typedef long long int ll;

#define MAXN    (100001)
#define MAXV    (MAXN+1)
#define MAXE    (MAXN<<1)
struct edge {
  int v;
  edge *n;
  edge(void):v(0),n(NULL) {}
  edge(int vv,edge *nn):v(vv),n(nn) {}
};
int nE;
edge E[MAXE];
edge *front[MAXV];
int label[MAXV];    /* 0 for '(', 1 for ')' */
void add_edge(int u, int v) {
  int ne = ++nE;
  E[ne] = edge(v, u[front]);
  u[front] = &(E[ne]);
}

int n;
ll ans;

char del[MAXV];
namespace findroot {
int ALL;
int nfind;
int vis[MAXV];
int size[MAXV];
int f[MAXV];
int root;
void __find(int u, int fa) {
  vis[u] = nfind;
  size[u] = 1;
  f[u] = 0;
  for(edge *e=u[front]; e; e = e->n) {
    int v = e->v;
    if((!del[v]) && (vis[v] != nfind) && (v != fa)) {
      __find(v, u);
      size[u] += size[v];
      if(f[u] < size[v])  f[u] = size[v];
    }
  }
  if(f[u] < ALL - size[u])  f[u] = ALL - size[u];
  if(f[u] < f[root])  root = u;
}
int find(int u, int all) {
  ++nfind;
  ALL = all;
  f[root = 0] = MAXV;
  __find(u,0);
  return root;
}
}

namespace workspaces {
int maxdep;
int dep[MAXV];
ll cntin[MAXV], cntout[MAXV];
```

```
65  int in[2][MAXV];    /* 0 for '(', 1 for ')' */
66  int out[2][MAXV];
67  void getdeep(int u, int fa) {
68    dep[u] = dep[fa] + 1;
69    if(dep[u] > maxdep) maxdep = dep[u];
70    for(edge *e = u[front]; e; e = e->n)
71      if((!del[e->v]) && (fa != e->v))
72        getdeep(e->v, u);
73  }
74  void dfs(int u, int fa) {
75    {
76      /* out from root */
77      out[0][u] = out[0][fa];
78      out[1][u] = out[1][fa];
79      if(0 == label[u]) { /* meet '(' */
80        out[0][u]++;
81      } else {    /* meet ')' */
82        if(out[0][u]) out[0][u]--;
83        else out[1][u]++;
84      }
85      if(out[0][u] == 0)
86        cntout[out[1][u]]++;
87    }
88    {
89      /* in to root */
90      in[0][u] = in[0][fa];
91      in[1][u] = in[1][fa];
92      if(0 == label[u]) { /* meet '(' */
93        if(in[1][u]) in[1][u]--;
94        else in[0][u]++;
95      } else {    /* meet ')' */
96        in[1][u]++;
97      }
98      if(0 == in[1][u])
99        cntin[in[0][u]]++;
100   }
101   /* do something */
102   for(edge *e = u[front]; e; e = e->n) {
103     int v = e->v;
104     if((!del[v]) && (v != fa)) {
105       dfs(v, u);
106     }
107   }
108 }
109 inline void init_maxdep(void) {
110   maxdep = 0;
111 }
112 inline void update_maxdep(int u) {
113   dep[u] = 1;
114   if(dep[u] > maxdep) maxdep = dep[u];
115   for(edge *e = u[front]; e; e = e->n)
116     if((!del[e->v]))
117       getdeep(e->v, u);
118 }
119 inline void clear(void) {
120   for(int i = 0; i <= maxdep+1; ++i)
121     cntin[i] = cntout[i] = 0;
122 }
123 inline void work(int u) {
124   in[0][u] = in[1][u] = out[0][u] = out[1][u] = 0;
125   in[label[u]][u] = out[label[u]][u] = 1;
126   if(out[0][u] == 0) cntout[out[1][u]]++;
127   if(0 == in[1][u]) cntin[in[0][u]]++;
128   /* update in and out if neccessary */
```

```
129   for(edge *e = u[front]; e; e = e->n)
130     if(!(del[e->v]))
131       dfs(e->v, u);
132 }
133 };
134 
135 ll count(int u, int p) {
136   ll ret = 0;
137   workspace::init_maxdep();
138   workspace::update_maxdep(u);
139   workspace::clear();
140   if(-1 == p) {
141     for(edge *e = u[front]; e; e = e->n)
142       if((!(del[e->v])))
143         workspace::work(e->v);
144     p = label[u];
145     /* single end */
146     if(0 == p) ret = workspace::cntout[1];
147     else ret = workspace::cntin[1];
148   } else {
149     workspace::work(u);
150   }
151   if(0 == p) { /* p is '(' */
152     for(int i = 0; i < workspace::maxdep; ++i)  /* concatenation */
153       ret += workspace::cntin[i] * workspace::cntout[i+1];
154   } else {    /* p is ')' */
155     for(int i = 0; i < workspace::maxdep; ++i)  /* concatenation */
156       ret += workspace::cntin[i+1] * workspace::cntout[i];
157   }
158   return ret;
159 }
160 
161 void handle(int u) {
162   del[u] = 1; /* delete current root. */
163   ans += count(u, -1);
164   /* do something */
165   for(edge *e = u[front]; e; e = e->n) {
166     int v = e->v;
167     if(!del[v]) {
168       ans -= count(v, label[u]);
169       /* do something */
170       int r = findroot::find(v, findroot::size[v]);
171       handle(r);
172     }
173   }
174 }
175 
176 void proc(void) {
177   int r = findroot::find(1,n);
178   handle(r);
179 }
180 
181 char ls[MAXV+1];
182 int main(void) {
183   int i = 0;
184   scanf("%d", &n);
185   scanf("%s", ls);
186   for(i = 0; i < n; ++i)
187     label[i+1] = ls[i] - '(';
188   for(i = 1; i < n; ++i) {
189     int ai, bi;
190     scanf("%d␣%d", &ai, &bi);
191     add_edge(ai, bi);
192     add_edge(bi, ai);
```

```
193   }
194   proc();
195   printf("%lld\n", ans);
196   return 0;
197 }
```

Version 2

```
1  /* 2016 ACM/ICPC Asia Regional Dalian. Problem , by Abreto<m@abreto.net>. */
2  #include <bits/stdc++.h>
3
4  using namespace std;
5  typedef long long int ll;
6
7  /* offset in [1,k] */
8  #define GET(i,offset)   (((i)>>((offset)-1))&1)
9  #define SET(i,offset)   ((i)|(1<<((offset)-1)))
10 #define REV(i,offset)   ((i)^(1<<((offset)-1)))
11
12 #define MAXN    (50005)
13 #define MAXV    (MAXN+1)
14 #define MAXE    (MAXN<<1)
15 struct edge {
16   int v;
17   edge *n;
18   edge(void):v(0),n(NULL) {}
19   edge(int vv,edge *nn):v(vv),n(nn) {}
20 };
21 int nE;
22 edge E[MAXE];
23 edge *front[MAXV];
24 int label[MAXV];    /* each kind */
25 void add_edge(int u, int v) {
26   int ne = ++nE;
27   E[ne] = edge(v, u[front]);
28   u[front] = &(E[ne]);
29 }
30
31 int n, k;
32 ll ans;
33 int all_kind;
34
35 int ndel;
36 int del[MAXV];
37 namespace findroot {
38 int ALL;
39 ll nfind;
40 ll vis[MAXV];
41 int size[MAXV];
42 int f[MAXV];
43 int root;
44 void __find(int u, int fa) {
45   vis[u] = nfind;
46   size[u] = 1;
47   f[u] = 0;
48   for(edge *e=u[front]; e; e = e->n) {
49     int v = e->v;
50     if((del[v] != ndel) && (vis[v] != nfind) && (v != fa)) {
51       __find(v, u);
52       size[u] += size[v];
53       if(f[u] < size[v])  f[u] = size[v];
54     }
55   }
56   if(f[u] < ALL - size[u])  f[u] = ALL - size[u];
57   if(f[u] < f[root])  root = u;
```

```
}
int find(int u, int all) {
  ++nfind;
  ALL = all;
  f[root = 0] = MAXV;
  __find(u,0);
  return root;
}
}

namespace workspace {
ll cnt[1024];
int dp[MAXV];
void dfs(int u, int fa) {
  dp[u] = dp[fa] | label[u];
  cnt[dp[u]] ++;
  /* dig into children */
  for(edge *e = u[front]; e; e = e->n) {
    int v = e->v;
    if((del[v] != ndel) && (v != fa)) {
      dfs(v, u);
    }
  }
}
inline void clear(void) {
  for(int i = 1; i <= all_kind; ++i)
    cnt[i] = 0;
}
inline void work(int u) {
  dp[u] = label[u];
  cnt[dp[u]] ++;
  for(edge *e = u[front]; e; e = e->n)
    if((del[e->v] != ndel))
      dfs(e->v, u);
}
inline void show(void) {
  for(int i = 0; i <= all_kind; ++i)
    printf("cnt[%d] = %lld\n", i, cnt[i]);
  for(int i = 1; i <= n; ++i)
    printf("dp[%d] = %d\n", i, dp[i]);
}
};


ll count(int u, int p) {
  ll ret = 0;
  workspace::clear();
  //printf("%d,%d  :\n", u, p);
  if(-1 == p) {
    for(edge *e = u[front]; e; e = e->n)
      if(((del[e->v]) != ndel))
        workspace::work(e->v);
    p = label[u];
    /* single end */
    for(int i = 1; i <= all_kind; i++)
      if(all_kind == (i|p))
        ret += (workspace::cnt[i]<<1);
  } else {
    workspace::work(u);
  }
  //workspace::show();
  for(int i = 1; i <= all_kind; ++i)
    if( workspace::cnt[i] > 0 )
      for(int j = 1; j <= all_kind; ++j)
```

```
122         if(all_kind == (i|p|j))
123           ret += workspace::cnt[i] * workspace::cnt[j];
124   //printf("%lld\n", ret);
125   return ret;
126 }
127
128 void handle(int u) {
129   //printf("proccessing %d\n", u);
130   del[u] = ndel; /* delete current root. */
131   ans += count(u, -1);
132   /* do something */
133   for(edge *e = u[front]; e; e = e->n) {
134     int v = e->v;
135     if(del[v] != ndel) {
136       ans -= count(v, label[u]);
137       /* do something */
138       int r = findroot::find(v, findroot::size[v]);
139       handle(r);
140     }
141   }
142 }
143
144 void proc(void) {
145   int r = findroot::find(1,n);
146   handle(r);
147 }
148
149 void clear(void) {
150   int i;
151   ans = 0;
152   nE = 0;
153   for(i = 0; i <= n; ++i) {
154     front[i] = NULL;
155   }
156   //findroot::nfind = 0;
157   ndel++;
158 }
159
160 void mozhu(void) {
161   int i = 0;
162   int li;
163   for(i = 1; i <= n; ++i) {
164     scanf("%d", &li);
165     label[i] = 1<<(li-1);
166   }
167   for(i = 1; i < n; ++i) {
168     int ai, bi;
169     scanf("%d %d", &ai, &bi);
170     add_edge(ai, bi);
171     add_edge(bi, ai);
172   }
173   all_kind = (1<<k)-1;
174   proc();
175   if(1 == k) ans += n;
176   printf("%lld\n", ans);
177 }
178
179 int main(void) {
180   while( EOF != scanf("%d%d", &n, &k) ) {
181     clear();
182     mozhu();
183   }
184   return 0;
185 }
```

4.1.3 Hevay chain decompostion

```
/* bzoj 1036 */
/* 树链剖分 */
#include <bits/stdc++.h>

using namespace std;

#define MAXN    30030
#define MAXM    (MAXN<<1)
struct edge {
  int v;
  edge *n;
  edge(void) {}
  edge(int vv, edge *nn):v(vv),n(nn) {}
};
typedef edge *ep;
int nE;
edge E[MAXM];
ep front[MAXN];
void add_edge(int u, int v) {
  int ne = ++nE;
  E[ne] = edge(v, u[front]);
  u[front] = &(E[ne]);
}

int n;
int fa[MAXN], son[MAXN], sz[MAXN], dep[MAXN];
int top[MAXN];
int id[MAXN];
int tot;

void calc(int u, int uf) {
  dep[u] = dep[uf] + 1;
  fa[u] = uf;
  sz[u] = 1;
  son[u] = -1;
  for(ep e = u[front]; e; e = e->n) {
    if(e->v != uf) {
      calc(e->v, u);
      sz[u] += sz[e->v];
      if( -1 == son[u] || sz[son[u]] < sz[e->v] )
        son[u] = e->v;
    }
  }
}
void link(int u, int f) {
  id[u] = (++tot);
  top[u] = f;
  if(son[u] > 0) {
    link(son[u], f);
  }
  for(ep e = u[front]; e; e = e->n) {
    if(e->v != fa[u] && e->v != son[u]) {
      link(e->v, e->v);
    }
  }
}

/* 其实是树链剖分 */
void make_link_cut_tree(void) {
  calc(1, 0);
  link(1, 1);
}
```

```
63
64 int w[MAXN];
65 int sum[MAXN<<2], mx[MAXN<<2];
66
67 void maintain(int o, int l, int r) {
68   sum[o] = sum[o<<1] + sum[o<<1|1];
69   mx[o] = max(mx[o<<1], mx[o<<1|1]);
70 }
71 void build(int o = 1, int l = 1, int r = n) {
72   if(r == l) {
73     sum[o] = w[l];
74     mx[o] = w[l];
75   } else {
76     int mid = l+r>>1;
77     build(o<<1, l, mid);
78     build(o<<1|1, mid+1, r);
79     maintain(o, l, r);
80   }
81 }
82 void update(int p, int x, int o = 1, int l = 1, int r = n) {
83   if(p <= l && r <= p) {
84     sum[o] = x;
85     mx[o] = x;
86   } else {
87     int mid = l+r>>1;
88     if(p <= mid) update(p,x,o<<1,l,mid);
89     else update(p,x,o<<1|1,mid+1,r);
90     maintain(o,l,r);
91   }
92 }
93 int qs(int L, int R, int o = 1, int l = 1, int r = n) {
94   if(R < l || r < L) return 0;
95   else if (L <= l && r <= R) {
96     return sum[o];
97   } else {
98     int mid = l+r>>1;
99     return qs(L,R,o<<1,l,mid)+qs(L,R,o<<1|1,mid+1,r);
100  }
101 }
102 int qm(int L, int R, int o = 1, int l = 1, int r = n) {
103   if(L <= l && r <= R) {
104     return mx[o];
105   } else {
106     int mid = l+r>>1;
107     if(R <= mid) return qm(L, R, o<<1, l, mid);
108     else if ( L > mid ) return qm(L, R, o<<1|1, mid+1, r);
109     else return max(qm(L, R, o<<1, l, mid),qm(L, R, o<<1|1, mid+1, r));
110   }
111 }
112
113 void change(int u, int t) {
114   update(id[u], t);
115 }
116 int qmax(int u, int v) {
117   int ret = -1000000000;
118   while(top[u] != top[v]) {
119     if( dep[top[u]] > dep[top[v]] ) {
120       /* jump u */
121       ret = max(ret, qm(id[top[u]], id[u]));
122       u = fa[top[u]];
123     } else {
124       ret = max(ret, qm(id[top[v]], id[v]));
125       v = fa[top[v]];
126     }
```

```
127   }
128   ret = max(ret, qm(min(id[u],id[v]),max(id[u],id[v])));
129   return ret;
130 }
131 int qsum(int u, int v) {
132   int ret = 0;
133   while(top[u] != top[v]) {
134     if( dep[top[u]] > dep[top[v]] ) {
135       /* jump u */
136       ret += qs(id[top[u]], id[u]);
137       u = fa[top[u]];
138     } else {
139       ret += qs(id[top[v]], id[v]);
140       v = fa[top[v]];
141     }
142   }
143   ret += qs(min(id[u],id[v]),max(id[u],id[v]));
144   return ret;
145 }
146
147 int main(void) {
148   int i;
149   scanf("%d", &n);
150   for(i = 1; i < n; ++i) {
151     int a, b;
152     scanf("%d%d", &a, &b);
153     add_edge(a, b);
154     add_edge(b, a);
155   }
156   make_link_cut_tree();
157   for(i = 1; i <= n; ++i) {
158     scanf("%d", &(w[id[i]]));
159   }
160   build();
161   scanf("%d", &i);
162   while(i--) {
163     char command[8];
164     int a, b;
165     scanf("%s␣%d␣%d", command, &a, &b);
166     if('C' == command[0]) change(a, b);
167     else if ('M' == command[1]) printf("%d\n", qmax(a, b));
168     else if ('S' == command[1]) printf("%d\n", qsum(a, b));
169   }
170   return 0;
171 }
```

4.2 2-SAT

```
1  #include <bits/stdc++.h>
2
3  using namespace std;
4
5  namespace two_sat {
6  const int maxn = 100000;
7  const int maxm = 1000000;
8  struct edge {
9    int v;
10   edge *n;
11   edge(void):v(0),n(NULL) {}
12   edge(int vv, edge *nn):v(vv),n(nn) {}
13 };
14 typedef edge *ep;
15 int n;
```

```
int nE;
edge E[maxm];
ep front[maxn];
void add_edge(int u, int v) {
  int ne = ++nE;
  E[ne] = edge(v, u[front]);
  u[front] = &(E[ne]);
}
/* (x = xval or y = yval), indexed from 0 */
void add_clause(int x, int xv, int y, int yv) {
  x = x*2 + xv;
  y = y*2 + yv;
  add_edge(x^1, y);
  add_edge(y^1, x);
}

char mark[maxn<<1];
int S[maxn<<1], c;
void init(int N) {
  n = N;
  for(int i = 0; i < n*2; ++i) {
    i[front] = NULL;
    i[mark] = 0;
  }
  nE = 0;
}

int dfs(int x) {
  if(mark[x^1]) return 0;
  if(mark[x]) return 1;
  mark[x] = 1;
  S[c++] = x;
  for(ep e = x[front]; e; e = e->n)
    if(!dfs(e->v)) return 0;
  return 1;
}

int solve(void) {
  for(int i = 0; i < n*2; i += 2)
    if(!mark[i] && !mark[i+1]) {
      c = 0;
      if(!dfs(i)) {
        while(c > 0) mark[S[--c]] = 0;
        if(!dfs(i+1)) return 0;
      }
    }
  return 1;
}
}
```

4.3 Cut Edge and Point

```
Finding cut edges
The code below works properly because the lemma above (first lemma):
  h[root] = 0
            par[v] = -1
                     dfs (v):
                     d[v] = h[v]
                            color[v] = gray
                                     for u in adj[v]:
                                         if color[u] == white
                                           then par[u] = v and dfs(u) and d[v] = min(
                                               d[v], d[u])
```

```
11                                          if d[u] > h[v]
12                                            then the edge v–u is a cut edge
13                                            else if u != par[v])
14          then d[v] = min(d[v], h[u])
15                      color[v] = black
16                                  In this code, h[v] = height of vertex v in the DFS
                                       tree and d[v] = min(h[w] where there is at least
                                       vertex u in subtree of v in the DFS tree where
                                       there is an edge between u and w).
17
18                                   Finding cut vertices
19                                   The code below works properly because the lemma
                                        above (first lemma):
20                                   h[root] = 0
21                                            par[v] = –1
22                                                dfs (v):
23                                                d[v] = h[v]
24                                                    color[v] = gray
25                                              for u in adj[v]:
26                                                        if color[u] == white
27                                                          then par[u] = v and dfs(
                                                               u) and d[v] = min(d[v
                                                               ], d[u])
28                                                             if d[u] >= h[v]
                                                                  and (v != root
                                                                  or
                                                                  number_of_children
                                                                  (v) > 1)
29                                                              then the edge v
                                                                   is a cut
                                                                   vertex
30                                                              else if u != par
                                                                   [v])
31                  then d[v] = min(d[v], h[u])
32                              color[v] = black
33                                        In this code, h[v] = height of vertex v in
                                             the DFS tree and d[v] = min(h[w] where
                                             there is at least vertex u in subtree of v
                                             in the DFS tree where there is an edge
                                             between u and w).
```

4.4 Euler Path

```
1  /* Euler path, by Abreto<m@abreto.net>. */
2  #define MAXV    (1024)
3  #define MAXE    (MAXV*MAXV)
4
5  typedef struct {
6    int id;
7    int nxt;
8    int del;
9  } egde_t;
10 int front[MAXV];
11 egde_t edg[MAXE];
12 int d[MAXV];
13 int ind[MAXV], outd[MAXV];
14 int nedges;
15 void add_edge(int u, int v) {
16   int newedge = ++nedges;
17   edg[newedge].id = v;
18   edg[newedge].nxt = u[front];
19   edg[newedge].del = 0;
20   u[front] = newedge;
```

```
  outd[u]++;
  ind[v]++;
  d[u]++;
  d[v]++;
}
void del_edge(int u, int v) {
  int e = 0;
  for(e=u[front]; e; e=edg[e].nxt)
    if(edg[e].id==v) {
      edg[e].del = 1;
      outd[u]--;
      ind[v]--;
      d[u]--;
      d[v]--;
      return;
    }
}

int path[MAXV];
int l;

void add2path(int u) {
  path[l++] = u;
}

/* Directed graph */
void euler(int x) {
  if(outd[x]) {
    int e = 0;
    for(e=x[front]; e; e=edg[e].nxt)
      if(!edg[e].del) {
        int v = edg[e].id;
        del_edge(x,v);
        euler(v);
      }
  }
  add2path(x);
}

/* Undirected graph */
void euler(int x) {
  if(d[x]) {
    int e = 0;
    for(e=x[front]; e; e=edg[e].nxt)
      if(!edg[e].del) {
        int v = edg[e].id;
        del_edge(x,v);
        del_edge(v,x);
        euler(v);
      }
  }
  add2path(x);
}
```

4.5 Shortest Path

4.5.1 Dijkstra

```
/* Shortest Path Dijstra, by Abreto<m@abreto.net>. */
#include <cstdio>
#include <set>
#include <utility>

```

```
using namespace std;
typedef set< pair<int,int> > spii;

#define MAXN    512
#define MAXV    (MAXN*MAXN)

struct egde_t {
  int id;
  int nxt;
};
int front[MAXV];
egde_t edg[MAXV<<3];
int nedges;
void add_edge(int u, int v) {
  int newedge = ++nedges;
  edg[newedge].id = v;
  edg[newedge].nxt = u[front];
  u[front] = newedge;
}

int d[MAXV];
int vis[MAXN];
int solid[MAXV];

int dijstra(int s, int t) {
  int v = s[front];
  spii q;
  q.insert(make_pair(0, s));
  while(!q.empty()) {
    auto it = q.begin();
    int u = it->second;
    int v = u[front];
    q.erase(it);
    solid[u] = 1;
    if(u == t) break;
    while(v) {
      int w = edg[v].id;
      if(!solid[w]) {
        if( (0==d[w]) || (d[u] + 1 < d[w]) ) {
          q.erase(make_pair(d[w],w));
          d[w] = d[u] + 1;
          q.insert(make_pair(d[w],w));
        }
      }
      v = edg[v].nxt;
    }
  }
  return d[t];
}
```

4.5.2 Shortest Path Fast Algorithm

```
/* Shortest Path Fast Algorithm, by Abreto<m@abreto.net>. */
#include <cstdio>
#include <cstring>
#include <queue>
#include <utility>

using namespace std;

#define MAXN    128

struct edge {
```

```
12    int v;
13    int w;
14    int n;
15  };
16  edge edg[MAXN<<1];
17  int nedg;
18  int indegree[MAXN];
19  int front[MAXN];
20  int find_edge(int u, int v) {
21    int e = u[front];
22    while(e) {
23      if(edg[e].v == v) return e;
24      e = edg[e].n;
25    }
26    return 0;
27  }
28  void add_edge(int u, int v, int w) {
29    int e = find_edge(u,v);
30    if(0==e) {
31      int newnode = ++nedg;
32      edg[newnode].v = v;
33      edg[newnode].w = w;
34      edg[newnode].n = u[front];
35      u[front] = newnode;
36      indegree[v]++;
37    } else {
38      edg[e].w = (w < edg[e].w)?w:(edg[e].w);
39    }
40  }
41
42  int n;
43
44  char inq[MAXN];
45  int vis[MAXN];
46  int d[MAXN];
47  int spfa(int s) { /* return 1 if fuhuan exists. */
48    queue<int> q;
49    memset(inq, 0, sizeof(inq));
50    memset(d, -1, sizeof(d));
51    memset(vis, 0, sizeof(vis));
52    d[s] = 0;
53    inq[s] = 1;
54    q.push(s);
55    while(!q.empty()) {
56      int u = q.front();
57      q.pop();
58      printf("proc␣%d..\n", u);
59      inq[u] = 0;
60      if(vis[u]++ > n)
61        return 1;
62      for(int e = front[u]; e; e = edg[e].n) {
63        int v = edg[e].v, w = edg[e].w;
64        if( -1==d[v] || d[u] + w < d[v] ) {
65          d[v] = d[u] + w;
66          if(!inq[v]) {
67            inq[v] = 1;
68            q.push(v);
69          }
70        }
71      }
72    }
73    return 0;
74  }
```

4.5.3 K-th shortest path

```
/**
 * poj
 * Problem#2449
 * Accepted
 * Time: 250ms
 * Memory: 9252k
 */
#include <iostream>
#include <fstream>
#include <sstream>
#include <algorithm>
#include <cstdio>
#include <cstdlib>
#include <cstring>
#include <ctime>
#include <cctype>
#include <cmath>
#include <vector>
#include <queue>
#include <stack>
#include <map>
#include <set>
#include <bitset>
using namespace std;
typedef bool boolean;

typedef class Edge {
public:
  int end;
  int next;
  int w;

  Edge(int end = 0, int next = -1, int w = 0):end(end), next(next), w(w) {        }
} Edge;

const int N = 1e3, M = 1e5;

typedef class MapManager {
public:
  int cnt;
  int h[N + 5];
  Edge edge[M + 5];

  MapManager() {        }
  MapManager(int n):cnt(-1) {
//            h = new int[(n + 1)];
//            edge = new Edge[(m + 1)];
    memset(h, -1, sizeof(int) * (n + 1));
  }

  inline void addEdge(int u, int v, int w) {
    edge[++cnt] = (Edge(v, h[u], w));
//            h[u] = (signed)edge.size() - 1;
    h[u] = cnt;
  }

  inline int start(int node) {
    return h[node];
  }

  Edge& operator [] (int pos) {
    return edge[pos];
```

```
63    }
64  } MapManager;
65  #define m_endpos -1
66
67  int n, m;
68  MapManager g;
69  MapManager rg;
70  int s, t, k;
71  int ds[N + 5];
72
73  inline void init() {
74    scanf("%d%d", &n, &m);
75    memset(g.h, -1, sizeof(int) * (n + 1));
76    memset(rg.h, -1, sizeof(int) * (n + 1));
77    for(int i = 1, u, v, w; i <= m; i++) {
78      scanf("%d%d%d", &u, &v, &w);
79      g.addEdge(u, v, w);
80      rg.addEdge(v, u, w);
81    }
82    scanf("%d%d%d", &s, &t, &k);
83  //    ds = new int[(n + 1)];
84  }
85
86  #define g rg
87  #define f ds
88  #define que que1
89  boolean vis[N + 5];
90  queue<int> que;
91  boolean spfa(int s, int t) {
92    memset(f, 0x7f, sizeof(int) * (n + 1));
93    memset(vis, false, sizeof(boolean) * (n + 1));
94    que.push(s);
95    f[s] = 0;
96    while(!que.empty()) {
97      int e = que.front();
98      que.pop();
99      vis[e] = false;
100     for(int i = g.start(e); i != m_endpos; i = g[i].next) {
101       int& eu = g[i].end;
102 //            cout << e << " " << eu << " " << i <<endl;
103       if(f[e] + g[i].w < f[eu]) {
104         f[eu] = f[e] + g[i].w;
105         if(!vis[eu]) {
106           que.push(eu);
107           vis[eu] = true;
108         }
109       }
110     }
111   }
112   return (f[t] != 0x7f7f7f7f);
113 }
114 #undef g
115 #undef f
116 #undef que
117
118 typedef class Status {
119 public:
120   int node;
121   int dis;
122   int priority;
123
124   Status(int node = 0, int dis = 0):node(node), dis(dis), priority(h()) {        }
125
126   int h() {
```

```
127     return dis + ds[node];
128   }
129
130   boolean operator < (Status b) const {
131     return priority > b.priority;
132   }
133 } Status;
134
135 int label[N + 5];
136 priority_queue<Status> que;
137 int bfs(int s, int t) {
138   if(s == t)    k++;
139 //    label = new int[(n + 1)];
140   memset(label, 0, sizeof(int) * (n + 1));
141   que.push(Status(s, 0));
142   while(!que.empty()) {
143     Status e = que.top();
144     que.pop();
145     label[e.node]++;
146     if(e.node == t && label[e.node] == k)
147       return e.dis;
148     for(int i = g.start(e.node); i != m_endpos; i = g[i].next) {
149       if(label[g[i].end] < k)
150         que.push(Status(g[i].end, e.dis + g[i].w));
151     }
152   }
153   return -1;
154 }
155
156 inline void solve() {
157   if(!spfa(t, s)) {
158     puts("-1");
159     return;
160   }
161   printf("%d", bfs(s, t));
162 }
163
164 int main() {
165   init();
166   solve();
167   return 0;
168 }
```

4.6 Maxflow

```
1  /* Max Flow Problem, by Abreto<m@abreto.net> */
2
3  #include <bits/stdc++.h>
4  using namespace std;
5
6  #define MAXV    (100000)
7  #define MAXE    (1000000)
8  struct edge {
9    static int N;
10   int v, w;
11   edge *n;
12   edge(void):v(0),w(0),n(NULL) {}
13   edge(int vv, int ww, edge *nn):v(vv),w(ww),n(nn) {}
14 };
15 int nE;
16 edge E[MAXE];
17 edge *front[MAXV];
18 void add_edge(int u, int v, int w) {
```

```
19   int ne = ++nE;
20   E[ne] = edge(v, w, u[front]);
21   u[front] = &(E[ne]);
22 }
23 edge *find_edge(int u, int v) {
24   for(edge *e = u[front]; e != NULL; e = e->n)
25     if(e->v == v)
26       return e;
27   return NULL;
28 }
29 void grant_e(int u, int v, int w) {
30   edge *e = find_edge(u, v);
31   if(NULL == e) add_edge(u,v,w);
32   else e->w += w;
33 }
34
35 int vis[MAXV];
36 int path[MAXV];
37 int dfs(int u, int t) {
38   vis[u] = 1;
39   if(u == t) return 1;
40   for(edge *e = u[front]; e != NULL; e = e->n) {
41     int v = e->v;
42     if(!vis[v] && e->w && dfs(v,t)) {
43       path[u] = v;
44       return 1;
45     }
46   }
47   return 0;
48 }
49 int find_path(int s, int t) {
50   memset(vis, 0, sizeof(vis));
51   return dfs(s,t);
52 }
53 int max_flow(int s, int t) {
54   int flow = 0;
55   while(find_path(s,t)) {
56     int i = 0;
57     int minf = find_edge(s,path[s])->w;
58     for(i = path[s]; i != t; i = path[i])
59       minf = min(minf, find_edge(i,path[i])->w);
60     for(i = s; i != t; i = path[i]) {
61       grant_e(i, path[i], -minf);
62       grant_e(path[i], i, minf);
63     }
64     flow += minf;
65   }
66   return flow;
67 }
68
69 /* Dinic */
70 #define N 1000
71 #define INF 100000000
72
73 struct Edge {
74   int from,to,cap,flow;
75   Edge(int u,int v,int c,int f):from(u),to(v),cap(c),flow(f) {}
76 };
77
78 struct Dinic {
79   int n,m,s,t;//结点数，边数（包括反向弧），源点编号，汇点编号
80   vector<Edge>edges;//边表，dges[e]和dges[e^1]互为反向弧
81   vector<int>G[N];//邻接表，G[i][j]表示结点i的第j条边在e数组中的编号
82   bool vis[N]; //BFS的使用
```

```
 83    int d[N]; //从起点到i的距离
 84    int cur[N]; //当前弧下标
 85
 86    void addedge(int from,int to,int cap) {
 87      edges.push_back(Edge(from,to,cap,0));
 88      edges.push_back(Edge(to,from,0,0));
 89      int  m=edges.size();
 90      G[from].push_back(m-2);
 91      G[to].push_back(m-1);
 92    }
 93
 94    bool bfs() {
 95      memset(vis,0,sizeof(vis));
 96      queue<int>Q;
 97      Q.push(s);
 98      d[s]=0;
 99      vis[s]=1;
100      while(!Q.empty()) {
101        int x=Q.front();
102        Q.pop();
103        for(int i=0; i<G[x].size(); i++) {
104          Edge&e=edges[G[x][i]];
105          if(!vis[e.to]&&e.cap>e.flow) { //只考虑残量网络中的弧
106            vis[e.to]=1;
107            d[e.to]=d[x]+1;
108            Q.push(e.to);
109          }
110        }
111
112      }
113      return vis[t];
114    }
115
116    int dfs(int x,int a) { //x表示当前结点，a表示目前为止的最小残量
117      if(x==t||a==0)return a;//a等于0时及时退出，此时相当于断路了
118      int flow=0,f;
119      for(int&i=cur[x]; i<G[x].size(); i++) { //从上次考虑的弧开始，注意要使用引用，同
           时修改cur[x]
120        Edge&e=edges[G[x][i]];//e是一条边
121        if(d[x]+1==d[e.to]&&(f=dfs(e.to,min(a,e.cap-e.flow)))>0) {
122          e.flow+=f;
123          edges[G[x][i]^1].flow-=f;
124          flow+=f;
125          a-=f;
126          if(!a)break;//a等于0及时退出，当a!=0,说明当前节点还存在另一个曾广路分支。
127
128        }
129      }
130      return flow;
131    }
132
133    int Maxflow(int s,int t) { //主过程
134      this->s=s,this->t=t;
135      int flow=0;
136      while(bfs()) { //不停地用bfs构造分层网络，然后用dfs沿着阻塞流增广
137        memset(cur,0,sizeof(cur));
138        flow+=dfs(s,INF);
139      }
140      return flow;
141    }
142  };
143
144  /* ISAP */
145  struct Edge {
```

```
146   int from,to,cap,flow;
147 };
148 const int maxn=650;
149 const int INF=0x3f3f3f3f;
150 struct ISAP {
151   int n,m,s,t;//结点数，边数（包括反向弧），源点编号，汇点编号
152   vector<Edge>edges;
153   vector<int>G[maxn];
154   bool vis[maxn];
155   int d[maxn];
156   int cur[maxn];
157   int p[maxn];
158   int num[maxn];
159   void AddEdge(int from,int to,int cap) {
160     edges.push_back((Edge) {
161       from,to,cap,0
162     });
163     edges.push_back((Edge) {
164       to,from,0,0
165     });
166     m=edges.size();
167     G[from].push_back(m-2);
168     G[to].push_back(m-1);
169   }
170   bool RevBFS() {
171     memset(vis,0,sizeof(vis));
172     queue<int>Q;
173     Q.push(t);
174     d[t]=0;
175     vis[t]=1;
176     while(!Q.empty()) {
177       int x=Q.front();
178       Q.pop();
179       for(int i=0; i<G[x].size(); i++) {
180         Edge &e =edges[G[x][i]^1];
181         if(!vis[e.from]&&e.cap>e.flow) {
182           vis[e.from]=1;
183           d[e.from]=d[x]+1;
184           Q.push(e.from);
185         }
186       }
187     }
188     return vis[s];
189   }
190   int Augment() {
191     int x=t, a=INF;
192     while(x!=s) {
193       Edge &e = edges[p[x]];
194       a= min(a,e.cap-e.flow);
195       x=edges[p[x]].from;
196     }
197     x=t;
198     while(x!=s) {
199       edges[p[x]].flow+=a;
200       edges[p[x]^1].flow-=a;
201       x=edges[p[x]].from;
202     }
203     return a;
204   }
205   int Maxflow(int s,int t,int n) {
206     this->s=s,this->t=t,this->n=n;
207     int flow=0;
208     RevBFS();
209     memset(num,0,sizeof(num));
```

```
210     for(int i=0; i<n; i++) {
211       num[d[i]]++;
212     }
213     int x=s;
214     memset(cur,0,sizeof(cur));
215     while(d[s]<n) {
216       if(x==t) {
217         flow+=Augment();
218         x=s;
219       }
220       int ok=0;
221       for(int i=cur[x]; i<G[x].size(); i++) {
222         Edge &e =edges[G[x][i]];
223         if(e.cap>e.flow&&d[x]==d[e.to]+1) {
224           ok=1;
225           p[e.to]=G[x][i];
226           cur[x]=i;
227           x=e.to;
228           break;
229         }
230       }
231       if(!ok) {
232         int m=n-1;
233         for(int i=0; i<G[x].size(); i++) {
234           Edge &e =edges[G[x][i]];
235           if(e.cap>e.flow)
236             m=min(m,d[e.to]);
237         }
238         if(--num[d[x]]==0)
239           break;
240         num[d[x]=m+1]++;
241         cur[x]=0;
242         if(x!=s)
243           x=edges[p[x]].from;
244       }
245     }
246     return flow;
247   }
248 };
249 int main() {
250   int n,m,a,b,c,res;
251   while(scanf("%d%d",&m,&n)!=EOF) {
252     ISAP tmp;
253     for(int i=0; i<m; i++) {
254       scanf("%d%d%d",&a,&b,&c);
255       tmp.AddEdge(a,b,c);
256     }
257     res=tmp.Maxflow(1,n,n);
258     printf("%d\n",res);
259   }
260   return 0;
261 }
```

4.7 Strongly Connected Component

```
1 /* Kosaraju */
2 #define MAXN    10010
3 #define MAXM    100010
4 struct edge {
5   int v;
6   edge *n;
7   edge(void):v(0),n(NULL) {}
8   edge(int vv, edge *nn):v(vv),n(nn) {}
```

```
 9 };
10 int nE;
11 edge E[MAXM<<1];
12 edge *ori[MAXN];
13 edge *inv[MAXN];
14 void add_edge(edge *front[], int u, int v) {
15   int ne = ++nE;
16   E[ne] = edge(v, u[front]);
17   u[front] = &(E[ne]);
18 }
19 void connect(int u, int v) {
20   add_edge(ori, u, v);
21   add_edge(inv, v, u);
22 }
23
24 int vis[MAXN];
25 int vst[MAXN];
26 void first_dfs(int u, int &sig) {
27   vis[u] = 1;
28   for(edge *e = u[ori]; e; e = e->n)
29     if(!vis[e->v])
30       first_dfs(e->v, sig);
31   vst[++sig] = u;
32 }
33 int mark[MAXN];
34 void second_dfs(int u, int sig) {
35   vis[u] = 1;
36   mark[u] = sig;
37   for(edge *e = u[inv]; e; e = e->n)
38     if(!vis[e->v])
39       second_dfs(e->v, sig);
40 }
41
42 int N, M;
43
44 int kosaraju(void) {
45   int i;
46   int sig = 0;
47   for(i = 0; i <= N; ++i) vis[i] = 0;
48   for(i = 1; i <= N; ++i) {
49     if(!vis[i])
50       first_dfs(i, sig);
51   }
52   sig = 1;
53   for(i = 0; i <= N; ++i) vis[i] = 0;
54   for(i = N; i > 0; --i) {
55     if(!vis[vst[i]])
56       second_dfs(vst[i], sig++);
57   }
58   for(i = 1; i <= N; ++i)
59     if(mark[i] != 1)
60       return 0;
61   return 1;
62 }
63
64
65 void clear(void) {
66   nE = 0;
67   for(int i = 0; i <= N; ++i) {
68     ori[i] = inv[i] = NULL;
69   }
70 }
71
72 /* Tarjan */
```

```
#define MAXN    10010
#define MAXM    100010
struct edge {
  int v;
  edge *n;
  edge(void):v(0),n(NULL) {}
  edge(int vv, edge *nn):v(vv),n(nn) {}
};
typedef edge *ep;
int nE;
edge E[MAXM];
edge *front[MAXN];
void add_edge(int u, int v) {
  int ne = ++nE;
  E[ne] = edge(v, u[front]);
  u[front] = &(E[ne]);
}

int mark[MAXN];
int dfn[MAXN], low[MAXN];
int stk[MAXN];
int stk_top;

void tardfs(int u, int stamp, int &scc) {
  mark[u] = 1;
  dfn[u] = low[u] = stamp;
  stk[stk_top++] = u;
  for(ep e = u[front]; e; e = e->n) {
    if(0 == mark[e->v]) tardfs(e->v, ++stamp, scc);
    if(1 == mark[e->v]) low[u] = min(low[u], low[e->v]);
  }
  if(dfn[u] == low[u]) {
    ++scc;
    do {
      low[stk[stk_top-1]] = scc;
      mark[stk[stk_top-1]] = 2;
    } while(stk[(stk_top--)-1] != u);
  }
}

int tarjan(int n) {
  int scc = 0, lay = 1;
  for(int i = 1; i <= n; ++i)
    if(0 == mark[i])
      tardfs(i, lay, scc);
  return scc;
}

int N, M;

void clear(void) {
  nE = 0;
  for(int i = 0; i <= N; ++i) {
    i[front] = NULL;
    mark[i] = low[i] = 0;
  }
  stk_top = 0;
}

/* Garbow */
#define MAXN    10010
#define MAXM    100010

struct edge {
```

```
137   int v;
138   edge *n;
139   edge(void):v(0),n(NULL) {}
140   edge(int vv, edge *nn):v(vv),n(nn) {}
141 };
142 typedef edge *ep;
143
144 int nE;
145 edge E[MAXM];
146 edge *front[MAXN];
147 void add_edge(int u, int v) {
148   int ne = ++nE;
149   E[ne] = edge(v, u[front]);
150   u[front] = &(E[ne]);
151 }
152
153 int stk1[MAXN], stk1t;
154 int stk2[MAXN], stk2t;
155 int low[MAXN], belg[MAXN];
156
157 void garbowdfs(int u, int lay, int &scc) {
158   stk1[++stk1t] = u;
159   stk2[++stk2t] = u;
160   low[u] = ++lay;
161   for(ep e=u[front]; e; e = e->n) {
162     if(!low[e->v]) garbowdfs(e->v, lay, scc);
163     else if (0 == belg[e->v])
164       while(low[stk2[stk2t]] > low[e->v])
165         --stk2t;
166   }
167   if(stk2[stk2t] == u) {
168     stk2t--;
169     scc++;
170     do {
171       belg[stk1[stk1t]] = scc;
172     } while(stk1[stk1t--] != u);
173   }
174 }
175
176 int grabow(int n) {
177   int i;
178   int scc = 0, lay = 0;
179   for(i = 0; i <= n; ++i) {
180     belg[i] = low[i] = 0;
181   }
182   for(i = 1; i <= n; ++i)
183     if(0 == low[i])
184       garbowdfs(i, lay, scc);
185   return scc;
186 }
187
188 int N, M;
189
190 void clear(void) {
191   nE = 0;
192   for(int i = 0; i <= N; ++i) {
193     front[i] = NULL;
194   }
195 }
```

4.8 Perfect elimination ordering

求弦图的最大团数/最小色数的时候，只要在完美消除序列上从后往前贪心染色即可。

而求最大独立集/最小团覆盖的时候，只要在完美消除序列上从前往后贪心取点即可。

```
/**
 * BZOJ 1006
 * [HNOI2008] 神奇的国度
 * 最大势法求完美消除序列
 * by Abreto<m@abreto.net>.
 **/
#include <cassert>
#include <cstdio>
#include <vector>
#include <bitset>
#include <algorithm>

using namespace std;
typedef vector<int> vi;
typedef vi::iterator vii;
#define pb push_back
#define MAXN 10100
#define MAXM 1000100

int n;
vi g[MAXN];
int ans;

struct node_t {
  int v;
  node_t *nxt;
} node[MAXM << 2];
int used;
node_t *new_node(void) {
  return node + (used ++);
}

node_t *f[MAXN];  /* head */
void lkto(int pos, int item) {
  node_t *t = new_node();
  t->v = item;
  t->nxt = f[pos];
  f[pos] = t;
}

int usedby[MAXN];
int color[MAXN];
bitset<MAXN> added;
int label[MAXN], max_label;
void mcs(void) {
  for (int i = 1; i <= n; i++) lkto(0, i);
  for (int i = n; i > 0; i--) {
    node_t *cur = f[max_label];
    assert(cur != NULL);
    while (added.test(cur->v)) {  /* already added */
      cur = cur->nxt;
      while (NULL == cur)
        cur = f[ --max_label ];
    }
    f[ max_label ] = cur->nxt;
    while (max_label && NULL == f[max_label]) max_label--;
    int u = cur->v;
    added.set(u);
    /* the i-th is u */
    for (vii it = g[u].begin(); it != g[u].end(); it++) {
      int v = *it;
      if (!added.test(v)) {
```

```
63          label[v] ++;
64          max_label = max(max_label, label[v]);
65          lkto(label[v], v);
66        }
67
68        usedby[color[v]] = i;
69      }
70      for (int j = 1; j <= n; j++)
71        if (usedby[j] != i) {
72          color[u] = j;
73          break;
74        }
75      ans = max(ans, color[u]);
76    }
77  }
78
79  int main(void) {
80    int m;
81    scanf("%d%d", &n, &m);
82    while (m--) {
83      int ai, bi;
84      scanf("%d%d", &ai, &bi);
85      g[ai].pb(bi);
86      g[bi].pb(ai);
87    }
88    mcs();
89    printf("%d\n", ans);
90    return 0;
91  }
```

5 Math

5.1 Euler Function

```
1  /* Euler function phi(x), by Abreto<m@abreto.net>. */
2
3  #define MAXX    3000000
4
5  int phi[MAXX];
6  void get_euler(void) {
7    int i = 0, j = 0;
8    phi[1] = 1;
9    for(i = 2; i < MAXX; ++i)
10     if(!phi[i])
11       for(j = i; j < MAXX; j += i) {
12         if(!phi[j]) phi[j] = j;
13         phi[j] = phi[j]/i * (i-1);
14       }
15 }
```

5.2 Möbius Function

```
1  void sieve() {
2    fill(isPrime, isPrime + maxn, 1);
3    mu[1] = 1, num = 0;
4    for (int i = 2; i < maxn; ++i) {
5      if (isPrime[i]) primes[num++] = i, mu[i] = -1;
6      static int d;
7      for (int j = 0; j < num && (d = i * primes[j]) < maxn; ++j) {
8        isPrime[d] = false;
9        if (i % primes[j] == 0) {
```

```
10          mu[d] = 0;
11          break;
12        } else mu[d] = -mu[i];
13      }
14    }
15  }
```

5.3 Number Theory Inverse

```
1  #include <bits/stdc++.h>
2  using namespace std;
3
4  const int n=10000000;      /*  */
5  const long long mod=1e9+7;  /*  prime required. */
6
7  long long fact[n],fiv[n],inv[n];
8
9  int main() {
10   fact[0]=fact[1]=1;
11   fiv[0]=fiv[1]=1;
12   inv[1]=1;
13   for (int i=2; i<n; i++) {
14     fact[i]=fact[i-1]*i%mod;
15     inv[i]=(mod-mod/i)*inv[mod%i]%mod;
16     fiv[i]=inv[i]*fiv[i-1]%mod;
17   }
18   for (int i=1; i<n; i++) {
19     if (fact[i]*fiv[i]%mod!=1)  printf("fact wrong: %d\n",i);
20     if (inv[i]*i%mod!=1)        printf("intv wrong: %d\n",i);
21   }
22   cout<<"complete"<<endl;
23   return 0;
24 }
```

5.4 Chinese Remainder Theorem

$$x \equiv a_i \pmod{m_i}$$

```
1  /* Chinese Remainder Theorem, by Abreto<m@abreto.net>. */
2  #include "euler.c"
3
4  #define MAXN    64
5
6  typedef long long int ll;
7
8  ll quickpow(ll a, ll b, ll mod) {
9    ll ret = 1, base = a;
10   while(b > 0) {
11     if(b & 1) ret = (ret * base) % mod;
12     base = (base * base) % mod;
13     b >>= 1;
14   }
15   return ret;
16 }
17
18 ll N;
19 ll a[MAXN], m[MAXN]; /* a and m is indexed from 0. */
20 ll x, M;
21
22 void naive_crt(void) {
```

```
  int i = 0;
  ll Mi[MAXN], nMi[MAXN];
  ll t[MAXN];

  M = 1;
  for(i = 0; i < N; ++i)
    M *= a[i];
  for(i = 0; i < N; ++i)
    Mi[i] = M / a[i];
  get_euler();
  for(i = 0; i < N; ++i)
    nMi[i] = quickpow(Mi[i], phi[a[i]]-1, a[i]);
  for(i = 0; i < N; ++i)
    t[i] = ((a[i] * Mi[i]) % M) * nMi[i] % M;
  for(i = 0; i < N; ++i)
    x = (x + t[i]) % M;
}
```

5.5 Linear congruences

```
#include <cstdio>
#include <cassert>
#include <cstdlib>

using namespace std;

class mod_equ_resolver {
  typedef long long int ll;
  ll a, m;
  inline void gurantee(void) {
    if ( a < 0 ) {
      ll k = ( -a ) / m;
      a += ( k + 1ll ) * m;
      a = ( a + m ) % m;
    } else {
      a %= m;
    }
    // printf("x = %lld (mod %lld)\n",a, m);
  }
public:
  mod_equ_resolver(void) {
    a = 0ll;
    m = 1ll;
  }
  ll exgcd(ll m, ll n, ll &x, ll &y) {
    if ( 0 == n ) {
      x = 1;
      y = 0;
      return m;
    }
    ll g = exgcd( n, m % n, x, y );
    ll t = x;
    x = y;
    y = t - m / n * y;
    return g;
  }
  int onemore(ll a2, ll m2) {
    ll x, y;
    ll g = exgcd( m, m2, x, y );
    assert(x*m+y*m2==g);
    a2 = (a2 + m2) % m2;
    if ( abs( a2 - a ) % g ) return -1;
    ll newm = m / g * m2;
```

```
    ll newa = a + ( a2 - a ) / g * x * m;
    a = newa;
    m = newm;
    gurantee();
    return 0;
  }
  ll resolve(void) {
    return a;
  }
};
```

Usage: For

$$\begin{cases} x \equiv a_1 \mod m_1 \\ x \equiv a_2 \mod m_2 \\ \vdots \\ x \equiv a_k \mod m_k \end{cases}$$

run

```
mod_equ_resolver solver;
for (int i = 1; i <= k; i++)
    solver.onemore(a[i], m[i]);
```

then the solution is

$$x \equiv solver.a \mod solver.m$$

5.6 FFT

```
#include <cmath>
using namespace std;
namespace fft {
#define eps (1e-9)
template < typename T = double >
struct dbl {
  T x;
  dbl(void):x(0.0) {}
  template <typename U>
  dbl(U a):x((T)a) {}
  inline char sgn(void) {
    return ((x>=-eps)&&(x<=eps))?(0):((x>eps)?(1):(-1));
  }
  inline T tabs(void) {
    return ((x>=-eps)&&(x<=eps))?(0.0):((x>eps)?(x):(-x));
  }
  inline dbl abs(void) {
    return dbl(tabs());
  }
  template <typename U> inline dbl &operator=(const U b) {
    x=(T)b;
    return (*this);
  }
  inline T *operator&(void) {
    return &x;
  }
  inline dbl operator-(void) const {
    return dbl(-x);
  }
  inline dbl operator+(const dbl &b) const {
    return dbl(x+b.x);
  }
  inline dbl operator-(const dbl &b) const {
    return dbl(x-b.x);
  }
```

```
36    inline dbl operator*(const dbl &b) const {
37      return dbl(x*b.x);
38    }
39    inline dbl operator/(const dbl &b) const {
40      return dbl(x/b.x);
41    }
42    template <typename U> inline dbl operator^(const U &b) const {
43      T ret=1.0,base=x;
44      while(b) {
45        if(b&1)ret*=base;
46        base*=base;
47        b>>=1;
48      }
49      return dbl(ret);
50    }
51    inline dbl operator+=(const dbl &b) {
52      return dbl(x+=b.x);
53    }
54    inline dbl operator-=(const dbl &b) {
55      return dbl(x-=b.x);
56    }
57    inline dbl operator*=(const dbl &b) {
58      return dbl(x*=b.x);
59    }
60    inline dbl operator/=(const dbl &b) {
61      return dbl(x/=b.x);
62    }
63    template <typename U> inline dbl operator^=(const U &b) {
64      dbl tmp=(*this)^b;
65      *this=tmp;
66      return tmp;
67    }
68    inline bool operator==(const dbl &b) const {
69      return (0 == ((*this)-b).sgn());
70    }
71    inline bool operator!=(const dbl &b) const {
72      return (0 != ((*this)-b).sgn());
73    }
74    inline bool operator<(const dbl &b) const {
75      return (-1 == ((*this)-b).sgn());
76    }
77    inline bool operator<=(const dbl &b) const {
78      return (((*this)==b) || ((*this)<b));
79    }
80    inline bool operator>(const dbl &b) const {
81      return (b < (*this));
82    }
83    inline bool operator>=(const dbl &b) const {
84      return (((*this)==b) || ((*this)>b));
85    }
86    template <typename U> inline operator U() const {
87      return (U)x;
88    }
89    inline char operator[](unsigned n) {
90      if(n >= 0) {
91        long long int ret=x;
92        while(n--) {
93          ret/=10;
94        }
95        return (ret%10);
96      } else {
97        T ret=x;
98        n=-n;
99        while(n--)ret*=10.0;
```

```
100       return ((long long int)ret)%10;
101     }
102   }
103 };
104 template <typename T>
105 struct Complex {
106   T x,y;  /* x + iy */
107   Complex(void):x(T()),y(T()) {}
108   Complex(T xx):x(xx) {}
109   Complex(T xx,T yy):x(xx),y(yy) {}
110   inline Complex operator-(void) const {
111     return Complex(-x,-y);
112   }
113   inline Complex operator+(const Complex& b) const {
114     return Complex(x+b.x,y+b.y);
115   }
116   inline Complex operator-(const Complex& b) const {
117     return Complex(x-b.x,y-b.y);
118   }
119   inline Complex operator*(const Complex& b) const {
120     return Complex(x*b.x-y*b.y,x*b.y+y*b.x);
121   }
122   inline Complex operator/(const Complex& b) const {
123     T bo=b.x*b.x+b.y*b.y;
124     return Complex((x*b.x+y*b.y)/bo,(y*b.x-x*b.y)/bo);
125   }
126   inline Complex& operator+=(const Complex& b) {
127     Complex tmp=(*this)+b;
128     (*this)=tmp;
129     return (*this);
130   }
131   inline Complex& operator-=(const Complex& b) {
132     Complex tmp=(*this)-b;
133     (*this)=tmp;
134     return (*this);
135   }
136   inline Complex& operator*=(const Complex& b) {
137     Complex tmp=(*this)*b;
138     (*this)=tmp;
139     return (*this);
140   }
141   inline Complex& operator/=(const Complex& b) {
142     Complex tmp=(*this)/b;
143     (*this)=tmp;
144     return (*this);
145   }
146   inline friend Complex operator+(const T& a, const Complex& b) {
147     return Complex(a)+b;
148   }
149   inline friend Complex operator-(const T& a, const Complex& b) {
150     return Complex(a)-b;
151   }
152   inline friend Complex operator*(const T& a, const Complex& b) {
153     return Complex(a)*b;
154   }
155   inline friend Complex operator/(const T& a, const Complex& b) {
156     return Complex(a)/b;
157   }
158 };
159 typedef dbl<> Double;
160 typedef Complex<Double> ComplexD;
161 typedef long long int ll;
162 const int maxn = 2000000; /* !! */
163 const Double pi(acos(-1.0));
```

```
164
165 void build(ComplexD _P[], ComplexD P[], int n, int m, int curr, int &cnt) {
166   if(m == n) {
167     _P[curr] = P[cnt++];
168   } else {
169     build(_P, P, n, m*2, curr, cnt);
170     build(_P, P, n, m*2, curr+m, cnt);
171   }
172 }
173
174 void FFT(ComplexD P[], int n, int oper) { /* n should be 2^k. */
175   static ComplexD _P[maxn];
176   int cnt = 0;
177   build(_P, P, n, 1, 0, cnt);
178   copy(_P, _P+n, P);
179   for(int d = 0; (1<<d)<n; ++d) {
180     int m = 1<<d;
181     int m2 = m*2;
182     Double p0 = pi / m * oper;
183     ComplexD unit_p0(cos(p0.x), sin(p0.x));
184     for(int i = 0; i < n; i += m2) {
185       ComplexD unit(1,0);
186       for(int j = 0; j < m; ++j) {
187         ComplexD &P1 = P[i+j+m], &P2 = P[i+j];
188         ComplexD t = unit * P1;
189         P1 = P2 - t;
190         P2 = P2 + t;
191         unit *= unit_p0;
192       }
193     }
194   }
195   if(-1 == oper) {
196     for(int i = 0; i < n; ++i)
197       P[i] /= Double(n);
198   }
199 }
200 }
```

5.7 NTT

```
1  #include<bits/stdc++.h>
2  #define ll long long
3  const int N=262144;
4  const ll MOD=5000000001507329LL;//998244353 1004535809
5  using namespace std;
6  int n,m;
7  ll a[N],b[N],x[N],y[N];
8  ll wn[25];
9  ll Mul(ll x,ll y) { //乘法超ll用快速乘，主函数也需要用
10   ll ans=(x*y-(ll)((long double)x/MOD*y+1e-8)*MOD);
11   return ans<0?ans+MOD:ans;
12 }
13 ll Qpow(ll a,ll b,ll M) {
14   ll ans=1;
15   a%=M;
16   while(b) {
17     if(b&1) ans=Mul(ans,a);
18     a=Mul(a,a);
19     b>>=1;
20   }
21   return ans;
22 }
23 void Getwn() { //主函数预处理getwn()
```

```
  for(int i=0; i<25; i++) {
    wn[i]=Qpow(3,(MOD-1)/(1<<i),MOD);
  }
}
void NTT(ll *x,int n,int rev) {
  int i,j,k,ds;
  ll w,u,v;
  for(i=1,j=n>>1,k=n>>1; i<n-1; i++,k=n>>1) {
    if(i<j) swap(x[i],x[j]);
    while(j>=k) j-=k,k>>=1;
    if(j<k) j+=k;
  }
  for(i=2,ds=1; i<=n; i<<=1,ds++) {
    for(j=0; j<n; j+=i) {
      w=1;
      for(k=j; k<j+i/2; k++) {
        u=x[k];
        v=Mul(w,x[k+i/2]);
        x[k]=(u+v)%MOD;
        x[k+i/2]=(u-v+MOD)%MOD;
        w=Mul(w,wn[ds]);
      }
    }
  }
  if(rev==-1) {
    for(i=1; i<n/2; i++) swap(x[i],x[n-i]);
    w=Qpow(n,MOD-2,MOD);
    for(i=0; i<n; i++) x[i]=Mul(x[i],w);
  }
}
int main() {
  Getwn();
  while(~scanf("%d%d",&n,&m)) {
    for(int i=0; i<n; i++)scanf("%lld",&a[i]);
    for(int i=0; i<m; i++)scanf("%lld",&b[i]);
    int len=1,s=n+m;
    while(len<s)len<<=1;
    for(int i=n; i<len; i++)a[i]=0;
    for(int i=m; i<len; i++)b[i]=0;
    NTT(a,len,1);
    NTT(b,len,1);
    for(int i=0; i<len; i++)a[i]=Mul(a[i],b[i]);
    NTT(a,len,-1);
    for(int i=0; i<=s; i++)printf("%lld ",a[i]);
    puts("");
  }
}

// ------------------------------------------------------------------------------
#include<cstdio>
#include<iostream>
#include<cstring>
#include<cmath>
#include<complex>
using namespace std;
typedef long long LL;
const LL MOD=998244353,g=3,gi=332748118;
const LL N=1000005;
LL n,m;
LL a[N],b[N];
LL pow (LL x,LL y) {
  if (y==1) return x;
  LL lalal=pow(x,y>>1);
  lalal=lalal*lalal%MOD;
```

```
  if (y&1) lalal=lalal*x%MOD;
  return lalal;
}
void ntt (LL *a,LL n,LL o) {
  if (n==1) return ;
  LL k=(n>>1);
  LL w=1,wn=pow(o==1?g:gi,(MOD-1)/n),a0[k],a1[k];
  for (LL u=0; u<k; u++) {
    LL i=u*2;
    a0[u]=a[i];
    a1[u]=a[i+1];
  }
  ntt(a0,k,o);
  ntt(a1,k,o);
  for (LL u=0; u<k; u++) {
    a[u]=a0[u]+w*a1[u]%MOD;
    a[u]=(a[u]%MOD+MOD)%MOD;
    a[u+k]=a0[u]-w*a1[u];
    a[u+k]=(a[u+k]%MOD+MOD)%MOD;
    w=w*wn%MOD;
  }
}
void ntt(LL *a,LL n,LL op) {
  for (LL u=0; u<n; u++) bin[u]=(bin[u>>1]>>1)|((u&1)*(n>>1));
  for (LL u=0; u<n; u++) if (u<bin[u]) swap(a[u],a[bin[u]]);
  for (LL u=1; u<n; u<<=1) {
    LL wn=pow(op==1?g:gi,(MOD-1)/(u<<1)),w,t;
    for (LL i=0; i<n; i=i+(u<<1)) {
      w=1;
      for (LL k=0; k<u; k++) {
        t=w*a[u+i+k]%MOD;
        a[u+i+k]=(a[i+k]-t+MOD)%MOD;
        a[i+k]=(a[i+k]+t)%MOD;
        w=w*wn%MOD;
      }
    }
  }
  if(op==-1) {
    LL Inv=pow(n,MOD-2);
    for(LL i=0; i<n; i++) a[i]=a[i]*Inv%MOD;
  }
}

int main() {
  scanf("%I64d%I64d",&n,&m);
  for (LL u=0; u<=n; u++) scanf("%I64d",&a[u]);
  for (LL u=0; u<=m; u++) scanf("%I64d",&b[u]);
  m=m+n;
  n=1;
  while (n<=m) n<<=1;
  ntt(a,n,1);
  ntt(b,n,1);
  for (LL u=0; u<=n; u++) a[u]*=b[u];
  ntt(a,n,-1);
  LL inv=pow(n,MOD-2);
  for (LL u=0; u<=m; u++)   printf("%I64d␣",a[u]*inv%MOD);
  return 0;
}
```

5.8 Fast Walsh-Hadamard transform

- 异或

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\} + \mathcal{F}\{A_1\}, \mathcal{F}\{A_0\} - \mathcal{F}\{A_1\}]$$

$$\mathcal{F}^{-1}\{A\} = \left[\mathcal{F}^{-1}\{\frac{A_0 + A_1}{2}\}, \mathcal{F}^{-1}\{\frac{A_0 - A_1}{2}\}\right]$$

- 按位与

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\} + \mathcal{F}\{A_1\}, \mathcal{F}\{A_1\}]$$
$$\mathcal{F}^{-1}\{A\} = \left[\mathcal{F}^{-1}\{A_0\} - \mathcal{F}^{-1}\{A_1\}, \mathcal{F}^{-1}\{A_1\}\right]$$

- 按位或

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\}, \mathcal{F}\{A_1\} + \mathcal{F}\{A_0\}]$$
$$\mathcal{F}^{-1}\{A\} = \left[\mathcal{F}^{-1}\{A_0\}, \mathcal{F}^{-1}\{A_1\} - \mathcal{F}^{-1}\{A_0\}\right]$$

```
void FWT(int a[],int n) {
  for(int d=1; d<n; d<<=1)
    for(int m=d<<1,i=0; i<n; i+=m)
      for(int j=0; j<d; j++) {
        int x=a[i+j],y=a[i+j+d];
        a[i+j]=(x+y)%mod,a[i+j+d]=(x-y+mod)%mod;
        //xor:a[i+j]=x+y,a[i+j+d]=(x-y+mod)%mod;
        //and:a[i+j]=x+y;
        //or:a[i+j+d]=x+y;
      }
}

void UFWT(int a[],int n) {
  for(int d=1; d<n; d<<=1)
    for(int m=d<<1,i=0; i<n; i+=m)
      for(int j=0; j<d; j++) {
        int x=a[i+j],y=a[i+j+d];
        a[i+j]=1LL*(x+y)*rev%mod,a[i+j+d]=(1LL*(x-y)*rev%mod+mod)%mod;
        //xor:a[i+j]=(x+y)/2,a[i+j+d]=(x-y)/2;
        //and:a[i+j]=x-y;
        //or:a[i+j+d]=y-x;
      }
}
void solve(int a[],int b[],int n) {
  FWT(a,n);
  FWT(b,n);
  for(int i=0; i<n; i++) a[i]=1LL*a[i]*b[i]%mod;
  UFWT(a,n);
}
```

5.9 Lucas

```
/* Lucas, by Abreto<m@abreto.net>. */

struct __lucas {
  static const int maxp = 100000;
  typedef long long int ll;
  int p;
  int f[maxp]; // fiv[maxp], inv[maxp];
  inline int mul(const int a, const int b) {
    ll z = 1ll * a * b;
    z -= z / p * p;
    return z;
  }
  int qow(int a, int x) {
    int ret = 1;
    while (x) {
      if (1 & x) ret = mul(ret, a);
      a = mul(a, a);
```

```
18        x >>= 1;
19      }
20      return ret;
21    }
22    void init(int np) {
23      p = np;
24      // return; // uncomment this line if use binom()
25      f[0] = f[1] = 1;
26      // fiv[0] = fiv[1] = 1;
27      // inv[1] = 1;
28      for (int i = 2; i < p; i++) {
29        f[i] = mul(f[i - 1], i);
30        // inv[i] = mul(p - p / i, inv[p % i]);
31        // fiv[i] = mul(fiv[i - 1], inv[i]);
32      }
33    }
34    int C(int n, int k) {
35      if (n < k) return 0;
36      return mul(f[n], qow(mul(f[k], f[n - k]), p - 2));
37    }
38    /** use following if get TLE { */
39    int binom(int n, int k) {
40      if (n < k) return 0;
41      if (k > n - k) k = n - k;
42      int a = 1, b = 1;
43      while (k) {
44        a = mul(a, n);
45        b = mul(b, k);
46        n--;
47        k--;
48      }
49      return mul(a, qow(b, p - 2));
50    }
51    /** } --- */
52    int operator()(int n, int k) {
53      if (0 == k) return 1;
54      if (n < p && k < p) return C(n, k);
55      return mul(C(n % p, k % p), (*this)(n / p, k / p));
56    }
57  } lucas;
```

5.10 Linear Programming

```
1  /* 线性规划 */
2  #include<bits/stdc++.h>
3
4  using namespace std;
5  const int Maxn=110,Maxm=59;
6  class Simplex {
7    /*
8        功能:
9        接受有n个约束，m个基本变量的方程组a[0~n][0~m]
10       a[0][]存放需要最大化的目标函数，a[][0]存放常数
11       Base[]存放基本变量的id,初始为1~m
12       Rest[]存放松弛变量的id,初始为m+1~m+n
13       返回此线性规划的最小值ans
14       要求方案的话，Base[]中的变量值为0,Rest[]中的变量值为相应行的[0]
15       如果solve
16       返回1,说明运行正常ans是它的最大值
17       返回0,说明无可行解
18       返回-1,说明解没有最大值
19       测试:
20       m=2,n=3
```

```
21      double a[4][3]={
22      {0,1,3},
23      {8,-1,1},
24      {-3,1,1},
25      {2,1,-4}
26      };
27      solve=1,ans=64/3;
28      注意ac不了可能是eps的问题
29    */
30  public:
31    static const double Inf;
32    static const double eps;
33    int n,m;
34    double a[Maxn][Maxm];
35    int Base[Maxm],Rest[Maxn];
36    double val[Maxm];
37    double ans;
38    void pt() {
39      for(int i=0; i<=n; i++) {
40        for(int j=0; j<=m; j++)printf("%.2f ",a[i][j]);
41        puts("");
42      }
43    }
44    void pivot(int x,int y) { //将第x个非基本变量和第y个基本变量调换
45      swap(Rest[x],Base[y]);
46      double tmp=-1./a[x][y];
47      a[x][y]=-1.;
48      for(int j=0; j<=m; j++)a[x][j]*=tmp;
49      for(int i=0; i<=n; i++) {
50        if(i==x||fabs(a[i][y])<eps)continue;
51        tmp=a[i][y];
52        a[i][y]=0;
53        for(int j=0; j<=m; j++)a[i][j]+=tmp*a[x][j];
54      }
55    }
56    bool opt() {
57      while(1) {
58        int csi=0;
59        for(int i=1; i<=m; i++)if(a[0][i]>eps&&(!csi||Base[i]<Base[csi]))csi=i;
60        if(!csi)break;
61        int csj=0;
62        double cur;
63        for(int j=1; j<=n; j++) {
64          if(a[j][csi]>-eps)continue;
65          double tmp=-a[j][0]/a[j][csi];
66          if(!csj||tmp+eps<cur||(fabs(tmp-cur)<eps&&Rest[j]<Rest[csj]))csj=j,cur=tmp;
67        }
68        if(!csj)return 0;
69        pivot(csj,csi);
70      }
71      ans=a[0][0];
72      return 1;
73    }
74    bool init() {
75      ans=0;
76      for(int i=1; i<=m; i++)Base[i]=i;
77      for(int i=1; i<=n; i++)Rest[i]=m+i;
78      int cs=1;
79      for(int i=2; i<=n; i++)if(a[i][0]<a[cs][0])cs=i;
80      if(a[cs][0]>=-eps)return 1;
81      static double tmp[Maxm];
82      for(int i=0; i<=m; i++)tmp[i]=a[0][i],a[0][i]=0;
83      for(int i=1; i<=n; i++)a[i][m+1]=1.;
84      a[0][m+1]=-1.;
```

```
    Base[m+1]=m+n+1;
    pivot(cs,++m);
    opt();
    m--;
    if(a[0][0]<-eps)return 0;
    cs=-1;
    for(int i=1; i<=n; i++) {
      if(Rest[i]>m+n) {
        cs=i;
        break;
      }
    }
    if(cs>=1) {
      int nxt=-1;
      m++;
      for(int i=1; i<=m; i++)if(a[cs][i]>eps||a[cs][i]<-eps) {
          nxt=i;
          break;
        }
      pivot(cs,nxt);
      m--;
    }
    for(int i=1; i<=m; i++) {
      if(Base[i]>m+n) {
        swap(Base[i],Base[m+1]);
        for(int j=0; j<=n; j++)a[j][i]=a[j][m+1];
        break;
      }
    }
    for(int i=1; i<=m; i++)a[0][i]=0;
    a[0][0]=tmp[0];
    for(int i=1; i<=m; i++)if(Base[i]<=m)a[0][i]=tmp[Base[i]];
    for(int i=1; i<=n; i++) {
      if(Rest[i]<=m) {
        for(int j=0; j<=m; j++)a[0][j]+=tmp[Rest[i]]*a[i][j];
      }
    }
    return 1;
  }
  void getval() {
    for(int i=1; i<=m; i++)val[i]=0;
    for(int i=1; i<=n; i++)if(Rest[i]<=m)val[Rest[i]]=a[i][0];
    //for(int i=1;i<=m;i++)printf("%.2f ",val[i]);puts("");
  }
  int solve() {
    if(!init())return 0;
    if(!opt())return -1;
    getval();
    return 1;
  }
} solver;
const double Simplex:: Inf=1e80;
const double Simplex:: eps=1e-8;
int main() {
  int m,n,type;
  scanf("%d%d%d",&m,&n,&type);
  solver.a[0][0]=0;
  for(int i=1; i<=m; i++)scanf("%lf",&solver.a[0][i]);
  for(int i=1; i<=n; i++) {
    for(int j=1; j<=m+1; j++) {
      if(j==m+1)scanf("%lf",&solver.a[i][0]);
      else {
        scanf("%lf",&solver.a[i][j]);
        solver.a[i][j]=-solver.a[i][j];
```

```
      }
    }
  }
  solver.m=m,solver.n=n;
  int rep=solver.solve();
  if(rep==0)puts("Infeasible");
  else if(rep==-1)puts("Unbounded");
  else {
    printf("%.12f\n",solver.ans);
    if(type==1) {
      for(int i=1; i<=m; i++)printf("%.12f%c",solver.val[i],i==m?'\n':' ');
    }
  }
}
```

5.11 Big Prime Test

```
#include <iostream>
#include <cstdlib>
using namespace std;
typedef long long LL;
LL minfactor, p[11] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29};
LL gcd(LL a, LL b) {
  return b ? gcd(b, a % b) : a;
}
LL qmult(LL a, LL b, LL mod) {  // 快速乘模
  LL sum = 0;
  while (b) {
    if (b & 1) {
      sum += a;
      if (sum >= mod) sum -= mod;  // 此处无需用%, %运算比减法慢很多
    }
    b >>= 1, a <<= 1;
    if (a >= mod) a -= mod;
  }
  return sum;
}
LL qpow(LL a, LL b, LL mod) {  // 快速幂模
  LL res = 1;
  while (b) {
    if (b & 1) res = qmult(res, a, mod);
    b >>= 1;
    a = qmult(a, a, mod);
  }
  return res;
}
bool prime_test(LL n, LL a) {  // 对整数n, 底数a进行测试, 返回true表示通过测试
  LL p = qpow(a, n - 1, n);
  if (p != 1) return false;
  else {  // 二次探测
    LL s = n - 1;
    while (!(s & 1) && p == 1) {
      s >>= 1;
      p = qpow(a, s, n);
    }
    if (p == 1 || p == n - 1) return true;
    else return false;
  }
}
bool Miller_Rabin(LL n) {  // 对整数n进行Miller_Rabin素数测试, 返回true表示通过测试
  if (n <= 29) {    // if这一块其实可以不用
    for (int i = 0; i < 10; i++) {
      if (n == p[i]) return true;
```

```
47      }
48      return false;
49    }
50    for (int i = 0; i < 10; i++) {  // 利用前10个素数作为底数测试的正确率已经非常高
51      if (gcd(n, p[i]) == 1 && !prime_test(n, p[i])) return false;
52    }
53    return true;
54  }
55  LL randf(LL x, LL n, LL c) {  // 满足要求的产生伪随机数函数
56    return (qmult(x, x, n) + c) % n;
57  }
58  LL pollard_rho(LL n, LL c) {  // 查找n的因数，c为上面函数要用的随机数，c也可自己指定
        （但要有变化）
59    LL x = rand() % n, y = x, i = 1, k = 2, p;  // 随机生成随机数的初始值，也可自己指定
60    while (true) {
61      i++;
62      x = randf(x, n, c);
63      p = gcd(y - x + n, n);
64      if (p > 1 && p < n) return p;
65      if (y == x) return n;   // 判圈，返回n表示查找失败，要更新随机种子重新查找
66      if (i == k) {
67        y = x;  // 更新范围和记录的数
68        k <<= 1;
69      }
70    }
71  }
72  void find_factor(LL n) {  // 查找所有因数
73    if (Miller_Rabin(n)) {
74      minfactor = min(minfactor, n);
75      return ;
76    }
77    LL p = n;
78    while (p == n) p = pollard_rho(n, rand() % (n - 1) + 1);  // 查找失败则更新随机种子
          重新查找，直到找到因子
79    find_factor(p);     // 递归查找更小因子
80    find_factor(n / p);
81  }
82
83  int main() {
84    int t;
85    cin >> t;
86    while (t--) {
87      LL N;
88      cin >> N;
89      if (Miller_Rabin(N)) cout << "Prime" << endl;
90      else {
91        minfactor = N;
92        find_factor(N);
93        cout << minfactor << endl;
94      }
95    }
96    return 0;
97  }
```

5.11.1 Miller Rabin

```
1  /* Miller–Rabin Prime Test, by Abreto<m@abreto.net>. */
2
3  namespace miller_rabin {
4
5  typedef long long int ll;
6
7  inline ll add(const ll a, const ll b, const ll mod) {
```

```
 8   ll z = a + b;
 9   if (z >= mod) z -= mod;
10   return z;
11 }
12 inline ll mul(ll a, ll b, const ll mod) {
13   ll z = 0;
14   if (a >= mod) a %= mod;
15   if (b >= mod) b %= mod;
16   while (b) {
17     if (1 & b) z = add(z, a, mod);
18     a = add(a, a, mod);
19     b >>= 1;
20   }
21   return z;
22 }
23
24 ll qow(ll a, ll x, ll mod) {
25   ll ret = 1ll;
26   while (x) {
27     if (1 & x) ret = mul(ret, a, mod);
28     a = mul(a, a, mod);
29     x >>= 1;
30   }
31   return ret;
32 }
33
34 const int K = 5;
35 const int p[] = {
36   2, 3, 7, 61, 24251
37 };
38 const ll strong = 4685624825598lll;
39 /* 46 856 248 255 981 in (0, 1e16) */
40
41 bool mr(ll n, int k) {
42   ll d = n - 1;
43   int s = 0;
44   while (d > 1 && 0 == (d & 1)) {
45     s++;
46     d >>= 1;
47   }
48   for (int i = 0; i < k; i++) {
49     ll a = (i < K) ? p[i] : (1 + rand() % (n - 1));
50     ll x = qow(a, d, n);
51     for (int j = 0; j < s; j++) {
52       ll xp = mul(x, x, n);
53       if (1 == xp && x != 1 && x != n-1) return false;
54       x = xp;
55     }
56     if (x != 1) return false;
57   }
58   return true;
59 }
60
61 /* 2,3,5,7,11,13 */
62 const int pre[] = {3, 5, 7, 11, 13};
63 bool test(ll n, int k = 5) {
64   if (2 == n) return true;
65   if (0 == (n & 1)) return false;
66   if (strong == n) return false;
67   for (int i = 0; i < 5; i++) {
68     if (n == pre[i]) return true;
69     if (n == n / pre[i] * pre[i])
70       return false;
71   }
```

```
72   return mr(n, k);
73 }
74
75 }
```

5.11.2 Pollard's rho

```
1  /* Pollard's rho, by Abreto<m@abreto.net>. */
2
3  namespace pollards_rho {
4
5  typedef long long int ll;
6
7  inline ll add(const ll a, const ll b, ll mod) {
8    ll z = a + b;
9    if (z >= mod) z -= mod;
10   return z;
11 }
12 inline ll mul(ll a, ll b, ll mod) {
13   ll z = 0ll;
14   if (a >= mod) a -= a / mod * mod;
15   if (b >= mod) b -= b / mod * mod;
16   while (b) {
17     if (1 & b) z = add(z, a, mod);
18     a = add(a, a, mod);
19     b >>= 1;
20   }
21   return z;
22 }
23
24 ll gcd(ll m, ll n) {
25   return (0 == n) ? m : gcd(n, m % n);
26 }
27
28 ll find(ll n, int c = -1) {
29   ll x = rand() % n;
30   ll y = x, k = 2;
31   for (int i = 2; ; i++) {
32     x = add(mul(x, x, n), (n + c) % n, n);
33     ll d = gcd( y - x + n, n );  // change to abs(y - x) if get WA
34     if (1 != d && n != d) return d;
35     if (y == x) return n;
36     if (i == k) {
37       y = x;
38       k <<= 1;
39     }
40   }
41 }
42
43 /** usage:
44  * void find(ll n, int c = 107)
45  * {
46  *   if (1 == n) return;
47  *   if ( miller-rabin(n) )
48  *   {
49  *     n is a prime;
50  *     return;
51  *   }
52  *   ll p = n, k = c;
53  *   while (p >= n) p = pollards_rho(p, k--);
54  *   find(p, c);
55  *   find(n/p, c);
56  * }
```

```
 **/

}
```

5.12 Montgomery modular multiplication

```

/* -- Montgomery modular algorithm { -- */
struct Mod64 {
  typedef long long ll;
  typedef unsigned long long u64;
  typedef __int128_t i128;
  typedef __uint128_t u128;
  Mod64() :n_(0) {}
  Mod64(u64 n) :n_(init(n)) {}
  static u64 init(u64 w) {
    return reduce(u128(w) * r2);
  }
  static void set_mod(u64 m) {
    mod = m;
    assert(mod & 1);
    inv = m;
    for (int i = 0; i < 5; ++i) inv *= 2 - inv * m;
    r2 = -u128(m) % m;
  }
  static u64 reduce(u128 x) {
    u64 y = u64(x >> 64) - u64((u128(u64(x)*inv)*mod) >> 64);
    return ll(y)<0 ? y + mod : y;
  }
  Mod64& operator += (Mod64 rhs) {
    n_ += rhs.n_ - mod;
    if (ll(n_)<0) n_ += mod;
    return *this;
  }
  Mod64 operator + (Mod64 rhs) const {
    return Mod64(*this) += rhs;
  }
  Mod64& operator -= (Mod64 rhs) {
    n_ -= rhs.n_;
    if (ll(n_)<0) n_ += mod;
    return *this;
  }
  Mod64 operator - (Mod64 rhs) const {
    return Mod64(*this) -= rhs;
  }
  Mod64& operator *= (Mod64 rhs) {
    n_ = reduce(u128(n_)*rhs.n_);
    return *this;
  }
  Mod64 operator * (Mod64 rhs) const {
    return Mod64(*this) *= rhs;
  }
  u64 get() const {
    return reduce(n_);
  }
  static u64 mod, inv, r2;
  u64 n_;
};

Mod64::u64 Mod64::mod, Mod64::inv, Mod64::r2;
/* -- } Montgomery modular algorithm -- */

/**
```

```
 * usage:
 * First, Mod64::set_mod();
 * Mod64 a, b, c(init_val);
 * a = b * c;
 * printf("%llu\n", a.get());
 **/
```

5.13 Berlekamp Massey

```
/* Berlekamp Massey by HoldZhu. */
#include <cstdio>
#include <vector>

using namespace std;

namespace BerlekampMassey {
const int mod = 1e9 + 7;
int L, m, b, n;
vector<int> s, C, B;
void init() {
  s.clear();
  C.clear();
  B.clear();
  C.push_back(1);
  B.push_back(1);
  L = n = 0;
  m = b = 1;
}
int pow_mod(int a, int k) {
  int s = 1;
  while (k) {
    if (k & 1)
      s = 1ll * s * a % mod;
    a = 1ll * a * a % mod;
    k >>= 1;
  }
  return s;
}
void update(int d) {
  s.push_back(d);
  for (int i = 1; i <= L; ++i)
    d = (d + 1ll * C[i] * s[n - i] % mod) % mod;
  if (d == 0)
    ++m;
  else if (2 * L <= n) {
    vector<int> T = C;
    C.resize(n + 1 - L + 1);
    for (int i = L + 1; i <= n + 1 - L; ++i)
      C[i] = 0;
    for (int i = 0; i < B.size(); ++i)
      C[i + m] = (C[i + m] + mod - 1ll * d * pow_mod(b, mod - 2) % mod * B[i] % mod)
          % mod;
    L = n + 1 - L;
    B = T;
    b = d;
    m = 1;
  } else {
    for (int i = 0; i < B.size(); ++i)
      C[i + m] = (C[i + m] + mod - 1ll * d * pow_mod(b, mod - 2) % mod * B[i] % mod)
          % mod;
    ++m;
  }
  ++n;
```

```
}
void output() {
  printf("F(n)=");
  for (int i = 1; i < C.size(); ++i) {
    int output = (mod - C[i]) % mod;
    if (output > mod / 2)
      output -= mod;
    printf("%s%d*F(n-%d)", (output < 0 || i == 1) ? "" : "+", output, i);
  }
  puts("");
}
void output_code_for() {
  static const char *name = "dp";
  static const char *index = "i";
  static const char *upperbound = "maxn";
  puts("// Generated by Berlekamp-Massey algorithm");
  for (int i = 1; i < C.size(); ++i) {
    printf("%s[%d]=%d;\n", name, i - 1, s[i - 1]);
  }
  printf("for(int i=%d;i<%s;++i)\n", (int)C.size() - 1, upperbound);
  printf("  %s[%s]=((", name, index);
  for (int i = 1; i < C.size(); ++i) {
    int output = (mod - C[i]) % mod;
    if (output > mod / 2)
      output -= mod;
    printf("%s%d*%s[%s-%d]%%mod", (output < 0 || i == 1) ? "" : "+", output, name,
        index, i);
  }
  puts(")%mod+mod)%mod;");
}
void output_code_matrix() {
  // TODO
}
};

/** usage */
int usage() {
  // int arr[12] = {2, 24, 96, 416, 1536, 5504, 18944, 64000, 212992, 702464,
      2301952, 7512064};
  int arr[] = {3, 20, 119, 696, 4059, 23660, 137903, 803760, 4684659};
  BerlekampMassey::init();
  for (auto ai : arr) {
    BerlekampMassey::update(ai);
  }
  printf("Formule: ");
  BerlekampMassey::output();
  printf("Code: \n");
  BerlekampMassey::output_code_for();
  return 0;
}
```

5.14 Lindström-Gessel-Viennot lemma

对于一张无边权的 DAG 图，给定 n 个起点和对应的 n 个终点，这 n 条不相交路径的方案数为

$$\begin{vmatrix} e(a_1,b_1) & e(a_1,b_2) & \cdots & e(a_1,b_n) \\ e(a_2,b_1) & e(a_2,b_2) & \cdots & e(a_2,b_n) \\ \vdots & \vdots & \ddots & \vdots \\ e(a_n,b_1) & e(a_n,b_2) & \cdots & e(a_n,b_n) \end{vmatrix}$$ (该矩阵的行列式)

其中 $e(a,b)$ 为图上 a 到 b 的方案数.

6 String

6.1 Hash

```
/* Common hash for any substrings. */

typedef unsigned long long int llu;
#define MAXN 1000000
int n;
char s[MAXN];
llu H[MAXN], xP[MAXN], P = 99991ll;
void init(void) {
  int i = 0;
  xP[0] = 1ll;
  for(i = 1; i < MAXN; ++i) xP[i] = xP[i-1] * P;
  H[n] = 0;
  for(i = n-1; i >= 0; --i) H[i] = H[i+1]*P + s[i];
}
#define HASH(i,l)   (H[i] - H[i+l]*xP[l])
```

6.2 KMP

```
/* KMP, by Abreto<m@abreto.net>. */
#include <string.h>

/* !!NEED IMPROVING!! */

#define MAXL  (1000010)

char W[MAXL], T[MAXL];
int f[MAXL];
int lW, lT;

int count(void) {
  int cnt = 0;
  int i, j;
  lW = strlen(W);
  lT = strlen(T);
  // -- self-matching
  f[0] = j = -1;
  for(i = 1; i < lW; i++) {
    while( j >= 0 && W[j+1] != W[i] )
      j = f[j];
    if( W[j+1] == W[i] ) j++;
    f[i] = j;
  }
  // ---
  j = -1;
  for(i = 0; i < lT; i++) {
    while( j >= 0 && W[j+1] != T[i] )
      j = f[j];
    if( W[j+1] == T[i] ) j++;
    if( j == lW-1 ) {
      cnt++;
      j = f[j];
    }
  }
  return cnt;
}
```

6.3 exKMP

```
#include <bits/stdc++.h>
using namespace std;

namespace exkmp {

const int maxn = 1000100, maxm = 1000100;

int n, m;
char S[maxn], T[maxm];

/* the length of longest prefix between T[i..m-1] and T[0..m-1] */
int nxt[maxm];
/* the length of longest prefix between S[i..n-1] and T[0..m-1] */
int ex[maxn];

void getsize() {
  n = strlen(S);
  m = strlen(T);
}

void self(void) {
  int q = 1, p = 0;
  nxt[0] = m;
  while (1 + p < m && T[1 + p] == T[p]) p++;
  nxt[1] = p;
  for (int i = 2; i < m; i++) {
    int l = nxt[i - q];
    if (i + l - 1 < p) {
      nxt[i] = l;
    } else {
      int j = max(0, p - i + 1);
      while (i + j < m && T[i + j] == T[j]) j++;
      nxt[i] = j;
      p = i + j - 1;
      q = i;
    }
  }
}

void run(void) {
  int q = 0, p = 0;
  self();
  while (p < n && p < m && S[p] == T[p]) p++;
  ex[0] = p;
  p--;
  for (int i = 1; i < n; i++) {
    int l = nxt[i - q];
    if (i + l - 1 < p) {
      ex[i] = l;
    } else {
      int j = max(0, p - i + 1);
      while (i + j < n && S[i + j] == T[j]) j++;
      ex[i] = j;
      p = i + j - 1;
      q = i;
    }
  }
}

void inspect(void) {
  printf("S: ");
  for (int i = 0; i < n; i++) putchar(S[i]);
  puts("");
  printf("T: ");
```

```
65   for (int i = 0; i < m; i++) putchar(T[i]);
66   puts("");
67   printf("next:");
68   for (int i = 0; i < m; i++) printf(" %d", nxt[i]);
69   puts("");
70   printf("extend:");
71   for (int i = 0; i < n; i++) printf(" %d", ex[i]);
72   puts("");
73 }
74
75 } // exkmp
```

6.4 Suffix Array

```
1  /* Suffix Array, copied. */
2
3  #define MAXN    (200010)
4  namespace mzry_sa {
5  int wx[MAXN],wy[MAXN],*x,*y,wss[MAXN],wv[MAXN];
6
7  bool dacmp(int *r,int n,int a,int b,int l) {
8    return a+l<n && b+l<n && r[a]==r[b]&&r[a+l]==r[b+l];
9  }
10 void da(int str[],int sa[],int rank[],int height[],int n,int m) {
11   int *s = str;
12   int *x=wx,*y=wy,*t,p;
13   int i,j;
14   for(i=0; i<m; i++)wss[i]=0;
15   for(i=0; i<n; i++)wss[x[i]=s[i]]++;
16   for(i=1; i<m; i++)wss[i]+=wss[i-1];
17   for(i=n-1; i>=0; i--)sa[--wss[x[i]]]=i;
18   for(j=1,p=1; p<n && j<n; j*=2,m=p) {
19     for(i=n-j,p=0; i<n; i++)y[p++]=i;
20     for(i=0; i<n; i++)if(sa[i]-j>=0)y[p++]=sa[i]-j;
21     for(i=0; i<n; i++)wv[i]=x[y[i]];
22     for(i=0; i<m; i++)wss[i]=0;
23     for(i=0; i<n; i++)wss[wv[i]]++;
24     for(i=1; i<m; i++)wss[i]+=wss[i-1];
25     for(i=n-1; i>=0; i--)sa[--wss[wv[i]]]=y[i];
26     for(t=x,x=y,y=t,p=1,i=1,x[sa[0]]=0; i<n; i++)
27       x[sa[i]]=dacmp(y,n,sa[i-1],sa[i],j)?p-1:p++;
28   }
29   for(int i=0; i<n; i++) rank[sa[i]]=i;
30   for(int i=0,j=0,k=0; i<n; height[rank[i++]]=k)
31     if(rank[i]>0)
32       for(k?k--:0,j=sa[rank[i]-1];
33           i+k < n && j+k < n && str[i+k]==str[j+k];
34           k++);
35 }
36 }
37
38 /*
39 Suffix array O(n lg^2 n)
40 LCP table O(n)
41 */
42 #include <cstdio>
43 #include <algorithm>
44 #include <cstring>
45
46 using namespace std;
47
48 #define REP(i, n) for (int i = 0; i < (int)(n); ++i)
49
```

```
50 namespace SuffixArray {
51 const int MAXN = 1 << 21;
52 char * S;
53 int N, gap;
54 int sa[MAXN], pos[MAXN], tmp[MAXN], lcp[MAXN];
55
56 bool sufCmp(int i, int j) {
57   if (pos[i] != pos[j])
58     return pos[i] < pos[j];
59   i += gap;
60   j += gap;
61   return (i < N && j < N) ? pos[i] < pos[j] : i > j;
62 }
63
64 void buildSA() {
65   N = strlen(S);
66   REP(i, N) sa[i] = i, pos[i] = S[i];
67   for (gap = 1;; gap <<= 1) {
68     sort(sa, sa + N, sufCmp);
69     REP(i, N - 1) tmp[i + 1] = tmp[i] + sufCmp(sa[i], sa[i + 1]);
70     REP(i, N) pos[sa[i]] = tmp[i];
71     if (tmp[N - 1] == N - 1) break;
72   }
73 }
74
75 void buildLCP() {
76   for (int i = 0, k = 0; i < N; ++i) if (pos[i] != N - 1) {
77       for (int j = sa[pos[i] + 1]; S[i + k] == S[j + k];)
78         ++k;
79       lcp[pos[i]] = k;
80       if (k)--k;
81     }
82 }
83 } // end namespace SuffixArray
84
85 namespace HashSuffixArray {
86 const int
87 MAXN = 1 << 21;
88
89 typedef unsigned long long hash;
90
91 const hash BASE = 137;
92
93 int N;
94 char * S;
95 int sa[MAXN];
96 hash h[MAXN], hPow[MAXN];
97
98 #define getHash(lo, size) (h[lo] - h[(lo) + (size)] * hPow[size])
99
100 inline bool sufCmp(int i, int j) {
101   int lo = 1, hi = min(N - i, N - j);
102   while (lo <= hi) {
103     int mid = (lo + hi) >> 1;
104     if (getHash(i, mid) == getHash(j, mid))
105       lo = mid + 1;
106     else
107       hi = mid - 1;
108   }
109   return S[i + hi] < S[j + hi];
110 }
111
112 void buildSA() {
113   N = strlen(S);
```

```
  hPow[0] = 1;
  for (int i = 1; i <= N; ++i)
    hPow[i] = hPow[i - 1] * BASE;
  h[N] = 0;
  for (int i = N - 1; i >= 0; --i)
    h[i] = h[i + 1] * BASE + S[i], sa[i] = i;

  stable_sort(sa, sa + N, sufCmp);
}

} // end namespace HashSuffixArray

namespace lrj_sa {
const int MAXN = 1000;
char s[MAXN]; /* 原始字符数组（最后一个字符应必须是0，而前面的字符必须非0） */
int sa[MAXN], t[MAXN], t2[MAXN], c[MAXN], n;  /* n seems to be the length of s. */
/* every charactor is in [0,m-1] */
void build_sa(int m) {
  int i, *x = t, *y = t2;
  for(i = 0; i < m; ++i) c[i] = 0;
  for(i = 0; i < n; i++) c[x[i]=s[i]]++;
  for(i = 1; i < m; ++i) c[i] += c[i-1];
  for(i = n-1; i >= 0; --i) sa[--c[x[i]]] = i;
  for(int k = 1; k <= n; k <<= 1) {
    int p = 0;
    for(i = n-k; i < n; ++i) y[p++] = i;
    for(i = 0; i < n; ++i) if(sa[i] >= k) y[p++] = sa[i]-k;
    for(i = 0; i < m; i++) c[i] = 0;
    for(i = 0; i < n; i++) c[x[y[i]]]++;
    for(i = 0; i < m; ++i) c[i]+=c[i-1];
    for(i = n-1; i >= 0; --i) sa[--c[x[y[i]]]] = y[i];
    swap(x,y);
    p = 1;
    x[sa[0]] = 0;
    for(i = 1; i < n; ++i)
      x[sa[i]] = y[sa[i-1]]==y[sa[i]] && y[sa[i-1]+k]==y[sa[i]+k] ? p-1:p++;
    if(p >= n) break;
    m = p;
  }
}
int rank[MAXN], height[MAXN];
void get_height(void) {
  int i,j,k = 0;
  for(i = 0; i < n; ++i) rank[sa[i]] = i;
  for(i = 0; i < n; ++i) {
    if(k) k--;
    j = sa[rank[i]-1];
    while(s[i+k]==s[j+k]) k++;
    height[rank[i]] = k;
  }
}
} // end namespace lrj_sa
```

6.5 Aho-Corasick Automata

```
/* Aho-Corasick automaton algorithm, by Abreto<m@abreto.net>. */

#define MAXN  500500
#define NALPHA  26
#define FIRSTA  'a'

/* pointer version => { */
struct vtx {
```

```
 9   vtx *nxt[NALPHA];
10   vtx *fail;
11   int end;
12 } vtxs[MAXN];
13 int nvtxs;
14 void myclr(void) {
15   nvtxs = 0;
16 }
17 vtx *new_vtx(void) {
18   vtx *ret = vtxs+(nvtxs++);
19   for(int i = 0; i < NALPHA; i++)
20     ret->nxt[i] = NULL;
21   ret->fail = NULL;
22   ret->end = 0;
23   return ret;
24 }
25 void myins(vtx *root, char const *s) {
26   for( ; *s ; s++ ) {
27     int of = (*s) - FIRSTA;
28     if ( NULL == root->nxt[of] ) {
29       root->nxt[of] = new_vtx();
30     }
31     root = root->nxt[of];
32   }
33   root->end++;
34 }
35 void build_ac(vtx *root) {
36   queue<vtx *> q;
37   q.push(root);
38   while(!q.empty()) {
39     vtx *p = q.front();
40     q.pop();
41     for(int i = 0; i < NALPHA; i++) {
42       if( NULL == p->nxt[i] ) continue;
43       if( root == p ) p->nxt[i]->fail = root;
44       else {
45         vtx *t = p->fail;
46         while ( t && NULL == t->nxt[i] ) {
47           t = t->fail;
48         }
49         if (t) p->nxt[i]->fail = t->nxt[i];
50         else p->nxt[i]->fail = root;
51       }
52       /* version[1] { */
53       p->nxt[i]->end += p->nxt[i]->fail->end; /* update this sum, add its existing
             prefix to this. */
54       /* } */
55       q.push(p->nxt[i]);
56     }
57   }
58 }
59 int qry(vtx *root, char const *s) {
60   vtx *p = root;
61   int cnt = 0;
62   for( ; *s ; s++ ) {
63     int of = (*s) - FIRSTA;
64     while( p != root && NULL == p->nxt[of] ) {
65       p = p->fail;
66     }
67     if (p->nxt[of]) p = p->nxt[of];
68     cnt += p->end;  // correct when version[1] exists.
69     // if version[1] not exists, you need to add all ends from this vertex up.
70     //for( vtx *t = p ; t ; t = t->fail )
71     //  cnt += t->end;
```

```
  }
  return cnt;
}
/* } */

/* -- usage (of pointer version) -- */
#include <bits/stdc++.h>
using namespace std;

char S[1000100];
char pat[64];

int main(void) {
  int T, N;
  vtx *root = NULL;
  scanf("%d", &T);
  while(T--) {
    myclr();
    root = new_vtx();
    scanf("%s", S);
    scanf("%d", &N);
    while(N--) {
      scanf("%s", pat);
      myins(root, pat);
    }
    build_ac(root);
    printf("%d\n", qry(root, S));
  }
  return 0;
}
```

6.6 Manacher

```
char t[MAXL<<1];
int p[MAXL<<1];
int manacher(char *s) {
  int i;
  int sl = strlen(s);
  int pos = 0, mxr = 0;
  int ret = 0;
  t[0] = '^';
  for(i = 0; i < sl; ++i) {
    t[i*2+1] = '#';
    t[i*2+2] = s[i];
  }
  t[sl*2+1] = '#';
  t[sl*2+2] = '$';
  sl = sl*2+2;
  for(i = 1; i < sl; ++i) {
    if(i <= mxr) {
      p[i] = min(p[2*pos - i], mxr-i+1);
    } else {
      p[i] = 1;
    }
    while( t[i-p[i]] == t[i+p[i]] ) p[i]++;
    if( i + p[i] - 1 > mxr ) {
      mxr = i+p[i]-1;
      pos = i;
    }
    ret = max(ret, p[i]-1);
  }
  return ret;
}
```