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# ACM TEMPLATE



UESTC\_Jungle

Last build at October 25, 2018

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## I Datastructure

### I.I Fenwick

```

1  /* Fenwick Tree (Binary Indexed Tree), by Abreto <m@abreto.net>. */
2  #include <cstring>
3
4  using namespace std;
5
6  template <class T = int, int MAXN = 100001>
7  struct fenwick {
8      static inline int lowbit(int x) {
9          return (x&(-x));
10     }
11     int N;
12     T f[MAXN]; /* 1-based. */
13     fenwick(void):N(MAXN) {
14         init();
15     }
16     fenwick(int n):N(n) {
17         init();
18     }
19     void init(void) {
20         memset(f,0,sizeof(f));
21     }
22     void upd(int i, T dx) {
23         while(i <= N) {
24             f[i] += dx;
25             i += lowbit(i);
26         }
27     }
28     T sum(int i) {
29         T ret = 0;
30         while(i) {
31             ret += f[i];
32             i -= lowbit(i);
33         }
34         return ret;
35     }
36 };

```

```

1  /* Fenwick Tree (Binary Indexed Tree), by Abreto <m@abreto.net>. */
2
3  #define MAXN 100001
4  #define LOWBIT(x) ((x)&(-(x)))
5
6  int N;
7  int fen[MAXN];
8
9  void update(int i, int dx) {
10     while(i <= N) {
11         fen[i] += dx;
12         i += LOWBIT(i);
13     }
14 }
15
16 int sum(int i) {
17     int s = 0;
18     while(i > 0) {
19         s += fen[i];
20         i -= LOWBIT(i);
21     }
22     return s;
23 }

```

## 1.2 BST in pb\_ds

```

1  /* Red-Black tree via pb_ds. */
2  #include<bits/stdc++.h>
3  #include<ext/pb_ds/assoc_container.hpp>
4  #include<ext/pb_ds/tree_policy.hpp>
5  using namespace __gnu_pbds;
6  using namespace std;
7  template <typename T>
8  using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
9
10 int main() {
11     ordered_set<int> s;
12     s.insert(1);
13     s.insert(3);
14     cout << s.order_of_key(2) << endl; // the number of elements in the s less than 2
15     cout << *s.find_by_order(0) << endl; // print the 0-th smallest number in s(0-based)
16 }

```

## 1.3 Segment Tree

```

1  /* Segment tree (Interval tree, range tree), by Abreto <m@abreto.net>. */
2
3  template <int STMAX = 1000000>
4  struct segment_tree {
5      struct node_t {
6          static inline node_t merge(node_t n1, node_t n2) {
7              node_t ans;
8              ans.l = n1.l;
9              ans.r = n2.r;
10             /* merge n1 and n2 to ans. */
11             return ans;
12         }
13
14         /* Data field */
15         int l,r;
16     } nodes[(STMAX+1)<<2];
17
18     struct lazy_t {
19         int marked; /* Optional */
20         /* lazy mark. */
21
22         lazy_t(void) {
23             clear();
24         }
25         void clear(void) {
26             marked=0;
27         }
28     } marks[(STMAX+1)<<2];
29
30     inline void maintain_leaf(int o, int idx) {
31         nodes[o].l = nodes[o].r = idx;
32         /* Operations to single elements ... */
33     }
34     inline void maintain(int o) {
35         nodes[o] = node_t::merge(nodes[o<<1], nodes[o<<1|1]);
36     }
37
38     /* Usage: build(1,1,n); */
39     void build(int o, int l, int r) { /* [l,r] */

```

```

40     if( r <= l ) {
41         maintain_leaf(o, l);
42     } else {
43         int mid = l+r>>1;
44         build(o<<1, l, mid);
45         build(o<<1|1, mid+1, r);
46         maintain(o);
47     }
48 }
49
50 /* Modify all elements in [l,r] */
51 void mark(lazy_t act, int o) {
52     /* do something .. */
53     marks[o].marked = 1;
54 }
55
56 /* Pass cached updates. */
57 void pushdown(int o) {
58     if( marks[o].marked ) {
59         mark(marks[o], o<<1);
60         mark(marks[o], o<<1|1);
61         marks[o].clear();
62     }
63 }
64
65 /* Do act on all elements in [L,R] */
66 void upd(int L, int R, lazy_t act, int o, int l, int r) {
67     if( L <= l && r <= R ) {
68         mark(act, o);
69     } else if (L <= R) {
70         int mid = (l+r)>>1;
71         pushdown(o);
72         if( L <= mid ) upd(L, R, act, o<<1, l, mid);
73         if( R > mid ) upd(L, R, act, o<<1|1, mid+1, r);
74         maintain(o);
75     }
76 }
77
78 node_t qry(int L, int R, int o, int l, int r) {
79     if(L <= l && r <= R)
80         return nodes[o];
81     else if (L <= R) {
82         int mid = (l+r)>>1;
83         pushdown(o);
84         if(R <= mid) return qry(L,R,o<<1,l,mid);
85         if(L > mid) return qry(L,R,o<<1|1,mid+1,r);
86         return node_t::merge(qry(L,R,o<<1,l,mid),qry(L,R,o<<1|1,mid+1,r));
87     }
88 }
89
90 int N;
91
92 segment_tree(void):N(STMAX) {}
93 segment_tree(int n):N(n) {}
94 void build(int n) {
95     N = n;
96     build(1,1,N);
97 }
98 void update(int L, int R, lazy_t act) {
99     upd(L,R,act,1,1,N);
100 }
101 node_t query(int L, int R) {
102     return qry(L,R,1,1,N);
103 }

```

```

104 |};

1  /* Segment tree (Interval tree, range tree), by Abreto <m@abreto.net>. */
2
3  #define MAXN    1000001
4
5  typedef struct {
6      int l,r;
7      /* Data field */
8  } node_t;
9
10 node_t merge(node_t n1, node_t n2) {
11     node_t ans;
12     ans.l = n1.l;
13     ans.r = n2.r;
14     /* merge n1 and n2 to ans. */
15     return ans;
16 }
17
18 typedef struct {
19     int marked; /* Optional */
20     /* lazy mark. */
21 } lazy_t;
22
23 int A[MAXN];
24 node_t nodes[MAXN<<2];
25 lazy_t marks[MAXN<<2];
26
27 void maintain_leaf(int o, int idx) {
28     nodes[o].l = nodes[o].r = idx;
29     /* Operations to single elements ... */
30 }
31 void maintain(int o) {
32     nodes[o] = merge(nodes[o<<1], nodes[o<<1|1]);
33 }
34
35 /* Usage: build(1,1,n); */
36 void build(int o, int l, int r) { /* [l,r] */
37     if( r <= l ) {
38         maintain_leaf(o, l);
39     } else {
40         int mid = l+r>>1;
41         build(o<<1, l, mid);
42         build(o<<1|1, mid+1, r);
43         maintain(o);
44     }
45     marks[o].marked = 0;
46 }
47
48 /* Modify all elements in [l,r] */
49 void mark(lazy_t act, int o) {
50     /* do something .. */
51     marks[o].marked = 1;
52 }
53
54 /* Pass cached updates. */
55 void pushdown(int o) {
56     if( marks[o].marked ) {
57         mark(marks[o], o<<1);
58         mark(marks[o], o<<1|1);
59         marks[o].marked = 0;
60     }
61 }
62

```

```

63 /* **DISCARDED** */
64 /* Set A[p]=v. Usage: modify(p, v, 1, 1, n);
65 void modify(int p, int v, int o, int l, int r)
66 {
67     if( r - l < 2 )
68     {
69         maintain_leaf(o, v);
70     } else {
71         int mid = (l+r)/2;
72         pushdown(o);
73         if( p <= mid ) modify(p, v, o*2, l, mid);
74         else modify(p, v, o*2+1, mid, r);
75         maintain(o);
76     }
77 }*/
78
79 /* Do act on all elements in [L,R] */
80 void update(int L, int R, lazy_t act, int o, int l, int r) {
81     if( L <= l && r <= R ) {
82         mark(act, o);
83     } else if (L <= R) {
84         int mid = (l+r)>>1;
85         pushdown(o);
86         if( L <= mid ) update(L, R, act, o<<1, l, mid);
87         if( R > mid ) update(L, R, act, o<<1|1, mid+1, r);
88         maintain(o);
89     }
90 }

```

#### 1.4 Sparse Table

```

1 /* RMQ with Sparse Table, by Abreto <m@abreto.net>. */
2
3 int min(int a, int b) {
4     return (a<b)?a:b;
5 }
6
7 #define MAXN    100001
8 #define MAXLOG  32
9
10 int N;
11 int A[MAXN];    /* indexed from 0. */
12 int st[MAXN][MAXLOG];
13
14 void st_init() {
15     int i = 0, j = 0, t = 0;
16     for(i = 0; i < N; ++i) st[i][0] = A[i];
17     for(j = 1; (t=(1<<j)) <= N; ++j)
18         for(i = 0; (i+t-1) < N; ++i)
19             st[i][j] = min(st[i][j-1], st[i+(t>>1)][j-1]);
20     /* st(i,j) = min(st(i,j-1), st(i+2^(j-1),j-1)). */
21 }
22
23 int st_query(int l, int r) {
24     int k = 0;
25     while((1<<(k+1)) <= (r-l+1)) k++;
26     return min(st[l][k], st[r-(1<<k)+1][k]);
27 }

```

#### 1.5 Treap



```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define MAXN (2560000)
6
7 int __treap_mem[MAXN];
8 void init_treap_mem(void) {
9     for( int i = 1 ; i < MAXN ; i++ )
10         __treap_mem[i-1] = i;
11 }
12 int alloc_address(void) {
13     int ret = __treap_mem[0];
14     __treap_mem[0] = __treap_mem[ret];
15     return ret;
16 }
17 void free_address(int p) {
18     __treap_mem[p] = __treap_mem[0];
19     __treap_mem[0] = p;
20 }
21
22 typedef int key_t;
23 typedef int val_t;
24 struct treap {
25     key_t x;
26     val_t v;
27     int r; /* random priority */
28     int eq, s; /* number of equal ones, size of subtree (include root itself) */
29     treap *fa; /* point to its father */
30     treap *ch[2]; /* 0 for left child, 1 for right child. */
31
32     treap(void);
33     inline void maintain(void); /* update s */
34     inline void set_child(int d, treap *child);
35     inline int which(void); /* determine which child this is of its father */
36     inline int cmp(key_t ox); /* determine which child to insert ox */
37     treap *rotate(void); /* rotate this to its father, return this */
38 } treap_nodes[MAXN];
39
40 treap *new_treap(key_t x, val_t v, treap *f) {
41     treap *ret = treap_nodes + alloc_address();
42     ret->x = x;
43     ret->v = v;
44     ret->eq = ret->s = 1;
45     ret->fa = f;
46     ret->ch[0] = ret->ch[1] = NULL;
47 }
48 void free_treap(treap *p) {
49     free_address( p - treap_nodes );
50 }
51 void treap_clear(void) {
52     init_treap_mem();
53 }
54
55 treap::treap(void) {
56     r = rand();
57     eq = s = 0;
58     fa = ch[0] = ch[1] = NULL;
59 }
60 inline void treap::maintain(void) {
61     s = eq;
62     for( int i = 0 ; i < 2 ; i++ )
63         if( ch[i] )
64             s += ch[i]->s;

```

```

65 }
66 inline void treap::set_child(int d, treap *child) {
67     ch[d] = child;
68     maintain();
69     if( child ) child->fa = this;
70 }
71 inline int treap::which(void) {
72     if( NULL == fa ) return -1; /* this is not a child */
73     else return ( fa->ch[1] == this );
74 }
75 inline int treap::cmp(key_t ox) {
76     if( ox == x ) return -1; /* equal */
77     else return ( ox > x ); /* left less, right more */
78 }
79 treap *treap::rotate(void) {
80     if ( NULL == fa ) return this; /* no father, already global root. */
81     int d = which();
82     fa->set_child(d, ch[d^1]);
83     set_child(d^1, fa);
84     return this;
85 }
86
87 // — deprecated |
88 typedef int T;
89
90 struct node {
91     T v; /* value of this node */
92     int r; /* random priority */
93     int eq; /* the number of equal things */
94     int s; /* the size of subtree rooted at this */
95     node *ch[2]; /* 0 for left child, 1 for right child. */
96     node(void) {
97         r = rand();
98         ch[0] = ch[1] = NULL;
99     }
100     /* return where to insert x */
101     int cmp(T x) {
102         if(v == x) return -1;
103         else return (x < v) ? 0 : 1;
104     }
105     /* return 1 if this node is prior to other */
106     int pri(node *o) {
107         return (r > (o->r));
108     }
109     /* maintain the s field */
110     void maintain(void) {
111         s = eq;
112         if(NULL != ch[0]) s += ch[0]->s;
113         if(NULL != ch[1]) s += ch[1]->s;
114     }
115 };
116
117 /* move o to ch[d] of o->ch[d^1] */
118 void rotate(node *&o, int d) {
119     node *k = o->ch[d^1];
120     o->ch[d^1] = k->ch[d];
121     o->maintain();
122     k->ch[d] = o;
123     k->maintain();
124     o = k;
125 }

```

## 1.6 Leftist Heap

```

1  /* HDU 1512 Monkey King (左偏树模板题) */
2  #include<iostream>
3  #include<cstdio>
4  using namespace std;
5  const int maxn = 100000+5;
6
7  int n, m;
8
9  struct Heap {
10     int l,r,fa,val,dis;
11 } t[maxn];
12
13
14 int finds(int x) {
15     return t[x].fa == -1? x:t[x].fa = finds(t[x].fa);
16 }
17
18 int merge(int x, int y) {
19     if(x == 0) return y;    //如果为0的话，就说明是空子树，根节点当然就是另一节点了
20     if(y == 0) return x;
21     if(t[y].val>t[x].val) swap(x,y);    //始终往右子树进行插入
22     t[x].r = merge(t[x].r,y);
23     t[t[x].r].fa = x;
24     if(t[t[x].l].dis < t[t[x].r].dis) swap(t[x].l,t[x].r);    //是否需要左右子树的对换，
        这样是为了右子树尽量短
25     if(t[x].r == 0) t[x].dis = 0;    //距离的重新分配
26     else t[x].dis = t[t[x].r].dis + 1;
27     return x;
28 }
29
30 int pop(int &root) {
31     int l = t[root].l;
32     int r = t[root].r;
33     t[root].l = t[root].r = t[root].dis = 0;
34     t[root].fa = -1;
35     t[l].fa = t[r].fa = -1;    //删除root根节点
36     return merge(l,r);    //这样一来相当于分裂成了两棵子树，重新进行合并，最后返回值
        为合并后的根节点
37 }
38
39 int push(int x, int y) {
40     return merge(x,y);
41 }
42
43 int main() {
44     //freopen("in.txt","r",stdin);
45     while(~scanf("%d",&n)) {
46         for(int i=1; i<=n; i++) {
47             t[i].l=t[i].r=t[i].dis=0;
48             t[i].fa=-1;
49             scanf("%d",&t[i].val);
50         }
51         scanf("%d",&m);
52         while(m--) {
53             int a,b;
54             scanf("%d%d",&a,&b);
55             int x=finds(a);
56             int y=finds(b);
57             if(x!=y) {
58                 t[x].val/=2;
59                 int xx = push( pop(x),x);

```

```

60         t[y].val/=2;
61         int yy = push( pop(y),y);
62         printf("%d\n",t[merge(xx,yy)].val);
63     } else puts("-1");
64 }
65 }
66 return 0;
67 }

```

## 1.7 Splay

```

1  /* splay, by Abreto<m@abreto.net>. */
2
3  #ifndef NULL
4  #define NULL 0
5  #endif
6
7  struct node {
8      node *f, *ch[2];
9      int sz;
10     node(node *fa = NULL, node *lc = NULL, node *rc = NULL) {
11         f = fa;
12         ch[0] = lc;
13         ch[1] = rc;
14         maintain();
15     }
16     inline int szof(const int d) const {
17         return ch[d] ? ch[d]->sz : 0;
18     }
19     inline void maintain(void) {
20         sz = szof(0) + szof(1) + 1;
21     }
22     inline int which(void) {
23         if (NULL == f) return 0;
24         return (f->ch[1] == this); /* f[which()] == this */
25     }
26     inline node *setf(node *fa, int d = 0) {
27         f = fa;
28         if (f) {
29             f->ch[d] = this;
30             f->maintain();
31         }
32         return f;
33     }
34     inline node *setc(node *son, int d = 0) {
35         ch[d] = son;
36         if (son) son->f = this;
37         maintain();
38         return this;
39     }
40     /* rotate this to its fater, return this. */
41     inline node *rotate(void) {
42         if (f != NULL) {
43             node *ff = f->f;
44             int d = which(), fd = f->which();
45             setc(f->setc(ch[d ^ 1], d), d ^ 1);
46             setf(ff, fd);
47         }
48         return this;
49     }
50     /* splay this to child of target */
51     inline node *splay(node * const target = NULL) {

```

```

52     while (f != target) {
53         if (target != f->f) {
54             ( (which() == f->which()) ? f : this )->rotate();
55         }
56         rotate();
57     }
58     return this;
59 }
60 /* 0-based rank */
61 inline node *get_k_th(unsigned k) {
62     node *p = this;
63     int rank;
64     while (k != (rank = (p->szof(0)))) {
65         if (k < rank) {
66             p = p->ch[0];
67         } else {
68             k -= (rank + 1);
69             p = p->ch[1];
70         }
71     }
72     return p->splay(f);
73 }
74 };

```

```

1  /* HDU 3487 – Play with Chain, by Abreto<m@abreto.net>. */
2  #include <bits/stdc++.h>
3
4  using namespace std;
5
6  #define MAXN    300300
7
8  int n, m;
9
10 #define LC(p)    ch[p][0]
11 #define RC(p)    ch[p][1]
12 #define TARGET(p) LC(RC(p))
13
14 int nodes;
15 int val[MAXN], ch[MAXN][2], fa[MAXN], sz[MAXN];
16 int rev[MAXN];
17
18 inline int new_node(int v, int f) {
19     int p = (++nodes);
20     val[p] = v;
21     fa[p] = f;
22     ch[p][0] = ch[p][1] = rev[p] = 0;
23     sz[p] = 1;
24     return p;
25 }
26 inline void maintain(int p) {
27     if (p) {
28         sz[p] = sz[LC(p)] + sz[RC(p)] + 1;
29     }
30 }
31 inline void make_child(int f, int d, int p) { /* make p the d-th ch of f */
32     ch[f][d] = p;
33     if (p) fa[p] = f;
34 }
35 inline void myrev(int p) {
36     if (p) {
37         rev[p] ^= 1;
38         swap(LC(p), RC(p));
39     }
40 }

```

```

41 inline void pushdown(int p) {
42     if(p && rev[p]) {
43         if(LC(p)) myrev(LC(p));
44         if(RC(p)) myrev(RC(p));
45         rev[p] = 0;
46     }
47 }
48 int build(int f = 0, int l = 0, int r = n+1) {
49     if(r < l) return 0;
50     if(l == r) return new_node(l, f);
51     int mid = l+r>>1;
52     int p = new_node(mid, f);
53     LC(p) = build(p, l, mid-1);
54     RC(p) = build(p, mid+1, r);
55     maintain(p);
56     return p;
57 }
58 inline int which(int p) { /* return 1 if p is a right child or 0 if p is a left
    child. */
59     return (RC(fa[p]) == p);
60 }
61 inline int rotate(int p) { /* rotate p to its father. [!] make sure p is not global
    root. */
62     int f = fa[p], ff = fa[f];
63     if(0 == f) return p; /* p is global root */
64     pushdown(f);
65     pushdown(p);
66     int d = which(p), df = which(f);
67     make_child(f, d, ch[p][d^1]);
68     make_child(p, d^1, f);
69     maintain(f);
70     maintain(p);
71     fa[p] = ff;
72     if(ff) ch[ff][df] = p;
73     return p;
74 }
75 inline int splay(int p, int fr) { /* splay p to the son of fr, return p. */
76     pushdown(p);
77     while(fa[p] != fr) {
78         int f = fa[p], dp = which(p);
79         if(fa[f] == fr) {
80             return rotate(p);
81         } else {
82             int df = which(f);
83             if(dp == df) {
84                 rotate(f);
85             } else {
86                 rotate(p);
87             }
88             rotate(p);
89         }
90     }
91     return p;
92 }
93 inline int get_k_th(int root, int k) {
94     int p = root;
95     int rank;
96     while(k != (rank = (sz[LC(p)] + 1))) {
97         pushdown(p);
98         if(k < rank) p = LC(p);
99         else {
100             k -= rank;
101             p = RC(p);
102         }

```

```

103     }
104     return splay(p, fa[root]);
105 }
106 inline int merge(int left, int right) {
107     pushdown(left);
108     if(RC(left)) left = get_k_th(left, sz[left]);
109     RC(left) = right;
110     maintain(left);
111     fa[right] = left;
112     return left;
113 }
114 inline int split(int root, int d) { /* split ch[root][d], return the root of splitted
    out. */
115     pushdown(root);
116     int child = ch[root][d];
117     ch[root][d] = 0;
118     maintain(root);
119     fa[child] = 0;
120     return child;
121 }
122 inline int concat(int root, int d, int p) { /* make p be ch[root][d], return root */
123     pushdown(root);
124     ch[root][d] = p;
125     fa[p] = root;
126     maintain(root);
127     return root;
128 }
129
130 void myclear(void) {
131     nodes = 0;
132 }
133
134 int ans[MAXN];
135 void inorder(int p, int &pos) {
136     if(0 == p) return;
137     pushdown(p);
138     inorder(LC(p), pos);
139     if( (0 < val[p]) && (val[p] < n+1) ) ans[pos++] = val[p];
140     inorder(RC(p), pos);
141 }
142
143 void handle() {
144     int i;
145     int root;
146     myclear();
147     root = build(0);
148     while(m--) {
149         char command[8];
150         int a, b, c;
151         int tar;
152         scanf("%s%d%d", command, &a, &b);
153         if('C' == command[0]) {
154             scanf("%d", &c);
155             root = get_k_th(root, a);
156             RC(root) = get_k_th(RC(root), b-a+2);
157             tar = split(RC(root), 0);
158             maintain(root);
159             root = get_k_th(root, c+1);
160             RC(root) = get_k_th(RC(root), 1);
161             RC(root) = concat(RC(root), 0, tar);
162             maintain(root);
163         } else {
164             root = get_k_th(root, a);
165             RC(root) = get_k_th(RC(root), b-a+2);

```

```

166     myrev(TARGET(root));
167 }
168 }
169 int pos = 0;
170 inorder(root, pos);
171 for(i = 0; i < n; i++) printf("%s%d", i ? " ": "", ans[i]);
172 puts("");
173 }
174
175 int main(void) {
176     while( scanf("%d%d", &n, &m) && (n > 0) && (m > 0) )
177         handle();
178     return 0;
179 }

```

## 1.8 Persistent Segment Tree

1. 首先, 给你一颗值为横坐标的线段树, 每个节点上存着该值出现了多少次, 这样的一颗线段树你会求区间  $k$  大值吧. 二分即可.
2. 然后, 假设区间是数组  $arr[n]$ , 区间长度是  $n$ , 那么给你  $n$  颗线段树, 第  $i$  颗线段树是第  $i - 1$  颗线段树插入  $arr[i]$  得到.
3. 如果你有了这  $n$  颗线段树, 想求区间  $[l, r]$  中的第  $k$  大值, 那么你需要在第  $r$  颗和第  $l - 1$  颗线段树的差线段树上作二分, 就可以求得区间第  $k$  大值.
4. 差线段树很好理解, 比如你有一个部分和数组  $sum$ ,  $sum[r] - sum[l - 1]$  就是部分和的差, 代表区间  $[l, r]$  的和, 差线段树同理.
5. 现在, 可持久化线段树出现为你解决最后一个问题, 空间问题. 内存很小, 不能够存下  $n$  颗线段树. 但是, 第 2 条中提到, 由于第  $i$  颗线段是第  $i - 1$  颗线段是插入仅一个值得到的, 两颗线段树的区别不大, 仅有  $\log(n)$  个节点发生了改变, 我们仅仅需要记录这  $\log(n)$  的数据就可以记录这个增量, 这就是可持久化线段树.

## 2 Dynamic Programming

### 2.1 LIS $O(n \log n)$

```

1
2 int top = 0;
3 for( int i=1; i<=n; i++ ) {
4     if( ap[i] > dp[top] ) { // 如果大于 "模拟栈" 的栈顶元素直接 入栈 长度加 1
5         top++;
6         dp[top] = ap[i];
7         continue;
8     }
9     int m = ap[i];
10    // lower_bound 前闭后开 返回不小于 m 的最小值的位置
11    pos = lower_bound(dp, dp+top, m)-dp; // 注意减去 dp
12    if( dp[pos] > ap[i] )
13        dp[pos] = ap[i];
14 }

```

### 2.2 LCS $O(n \log n)$

总的来说, 就是把 LCS 转化成 LIS, 然后用 LIS 的  $O(N \log N)$  算法来求解。

实现如下: (引用)

假设有两个序列  $s_1[1 \dots 6] = abcadac$ ,  $s_2[1 \dots 7] = cabedab$ .



记录  $s_1$  中每个元素在  $s_2$  中出现的位置, 再将位置按降序排列, 则上面的例子可表示为:  
 $loc(a) = \{6, 2\}, loc(b) = \{7, 3\}, loc(c) = \{1\}, loc(d) = \{5\}$ . (倒着扫一遍  $s_2$  即可把位置扔进 vector).  
 将  $s_1$  中每个元素的位置按  $s_1$  中元素的顺序排列成一个序列  $s_3 = \{6, 2, 7, 3, 1, 6, 2, 5, 1\}$ .  
 在对  $s_3$  求 LIS 得到的值即为求 LCS 的答案。

### 2.3 Improved by quadrilateral inequality

```

1  /*
2  * 四边形不等式
3  *
4  * 如果 dp(i,j) 满足 dp(i,j) <= dp(i,j+1) <= dp(i+1,j+1)
5  * 那么决策 s(i,j) 满足 s(i,j) <= s(i,j+1) <= s(i+1,j+1)
6  * 可以变形为:
7  *      s(i-1,j) <= s(i,j) <= s(i,j+1)  // dp方向: i增j减
8  * 或
9  *      s(i,j-1) <= s(i,j) <= s(i+1,j)  // dp方向: 区间长度L增
10 */
11 #include <bits/stdc++.h>
12
13 using namespace std;
14
15 #define MAXN    1024
16 #define inf     (0x3fffffff)
17
18 int n, m;
19 int v[MAXN];
20 int s[MAXN];
21 int w[MAXN][MAXN];
22 int dp[MAXN][MAXN];
23 int c[MAXN][MAXN];
24
25 int wa(void) {
26     int i, j, k;
27     for(i = 1; i <= n; ++i) {
28         scanf("%d", v+i);
29         s[i] = v[i] + s[i-1];
30     }
31     for(i = 1; i <= n; ++i) {
32         w[i][i] = 0;
33         for(j = i+1; j <= n; ++j)
34             w[i][j] = w[i][j-1] + v[j] * (s[j-1] - s[i-1]);
35     }
36     /* doing dp */
37     for(i = 1; i <= n; ++i) {
38         dp[i][0] = w[1][i];
39         c[i][0] = 1;
40         c[i][i] = i-1;
41         for(j = i-1; j > 0; j--) {
42             dp[i][j] = inf;
43             for(k = c[i-1][j]; k <= c[i][j+1]; ++k)
44                 if(dp[k][j-1] + w[k+1][i] <= dp[i][j]) {
45                     dp[i][j] = dp[k][j-1] + w[k+1][i];
46                     c[i][j] = k;
47                 }
48         }
49     }
50     /* dp done */
51     return dp[n][m];
52 }
53
54 int main(void) {
55     while(EOF != scanf("%d%d", &n, &m) && n && m) {

```

```

56     printf("%d\n", wa());
57 }
58 return 0;
59 }

```

## 2.4 Improved by Slope

```

1  /* type 1: */
2  /* bzoj 1010 */
3  #include <bits/stdc++.h>
4
5  using namespace std;
6  typedef long double ll;
7  #define MAXN    50050
8  #define eps     (1e-8)
9
10 int N;
11 ll L;
12 ll S[MAXN];
13 ll f[MAXN];
14 ll dp[MAXN];
15
16 inline ll k(int j) {
17     return (-2.0) * (f[j] + L);
18 }
19 inline ll b(int j) {
20     return dp[j] + f[j]*f[j] + 2ll*f[j]*L;
21 }
22 inline ll g(int j, int i) {
23     return k(j) * f[i] + b(j);
24 }
25
26 /* check if l1 & l3 <= l2 */
27 inline int check(int l1, int l2, int l3) {
28     /*ll left = b(l3)*k(l1)+b(l1)*k(l2)+b(l2)*k(l3);
29     ll right = b(l1)*k(l3)+b(l3)*k(l2)+b(l2)*k(l1);*/
30     ll left = b(l3)*k(l1)-b(l1)*k(l3);
31     ll right = k(l2)*(b(l3)-b(l1))+b(l2)*(k(l1)-k(l3));
32     return (left <= right);
33 }
34
35 int Q[MAXN], ql, qr;
36
37 int main(void) {
38     int i;
39     scanf("%d%Lf", &N, &L);
40     L += 1.0;
41     for(i = 1; i <= N; ++i) {
42         scanf("%Lf", S+i);
43         S[i] += S[i-1];
44         f[i] = S[i] + (double)i;
45     }
46     Q[qr++] = 0;
47     for(i = 1; i <= N; ++i) {
48         /* <!-- STARED */
49         for(; ql+1 < qr && g(Q[ql],i) >= g(Q[ql+1],i); ql++);
50         dp[i] = g(Q[ql], i) + f[i]*f[i] + L*L; //printf("%d: %lld,%lld\n", i, dp[i], dp[i]
                    -f[i]*f[i]);
51         for(; ql+1 < qr && check(Q[qr-2], Q[qr-1], i); qr--);
52         Q[qr++] = i;
53         /* --> */
54     }

```

```

55 | printf("%lld\n", (long long int)round(dp[N]));
56 | return 0;
57 | }

```

## 2.5 Steiner Tree

令  $f[i][sta]$  表示  $i$  号节点，与其他节点的连通性为  $sta$  时的最小代价，这里  $sta$  是一个二进制数，在它二进制下的每一位中，0 表示不连通，1 表示联通  
状态转移：

- 由子集转移而来

$$f[i][sta] = \min_{s \subseteq sta} \{f[i][s] + f[i][sta \setminus s] - val[i]\}$$

- 由不含该节点的状态转移而来

$$f[i][j] = \min \{f[k][j] + val[i]\}$$

流程：

```

1 | 枚举状态集S
2 | {
3 |     枚举S的子集s
4 |     {
5 |         更新f[S][1~n]
6 |     }
7 |     将 f[S][x]<inf 的x入队
8 |     spfa(S)
9 | }

```

代码：

```

1 | int f[1<<M][N];
2 | queue<int> q;
3 | bool in[N];
4 |
5 | void spfa(int S) {
6 |     while (!q.empty()) {
7 |         int now=q.front();
8 |         q.pop();
9 |         in[now]=0;
10 |         for (int i=st[now]; i; i=way[i].nxt) {
11 |             int y=way[i].y;
12 |             if (f[S][y]>f[S][now]+val[y]) {
13 |                 f[S][y]=f[S][now]+val[y];
14 |                 if (!in[y]) q.push(y),in[y]=1;
15 |             }
16 |         }
17 |     }
18 | }
19 |
20 | void work() {
21 |     int cnt=0;
22 |     memset(f,0x7f,sizeof(f));
23 |
24 |     for (int i=1; i<=n; i++)
25 |         if (!val[i]) f[1<<cnt][i]=0,cnt++;
26 |     for (int S=1; S<(1<<cnt); S++) {
27 |         for (s=(S-1)&S; s; s=(s-1)&S)
28 |             for (int i=1; i<=n; i++)
29 |                 f[S][i]=min(f[S][i],f[s][i]+f[S^s][i]-val[i]);
30 |         for (int i=1; i<=n; i++)
31 |             if (f[S][i]<INF&&!in[i])

```

```

32     q.push(i),in[i]=1;
33     spfa(S);
34 }
35
36 int ans=INF;
37 for (int i=1; i<=n; i++) ans=min(ans,f[(1<<cnt)-1][i]);
38 printf("%d\n",ans);
39 }
40 // _____
41 // 作者: Coco_T_
42 // 来源: CSDN
43 // 原文: https://blog.csdn.net/wu\_tongtong/article/details/78992913
44 // 版权声明: 本文为博主原创文章, 转载请附上博文链接!

```

### 3 Geometry

#### 3.1 2D

##### 3.1.1 Point

```

1  /* 2D Point Class, by Abreto<m@abreto.net> */
2  #include <cmath>
3
4  /**
5   * Define ABG2d_USE_LL if you want to use long long int for cordnates.
6   */
7
8  namespace ab_geometry_2d {
9
10 using namespace std;
11
12 typedef double ab_float;
13
14 const ab_float pi = acos(-1.);
15
16 #ifdef ABG2d_USE_LL
17 typedef long long int T;
18 #else
19 typedef ab_float T;
20 const ab_float eps = 1e-8;
21 #endif
22
23 inline T myabs(T x) {
24     if(x < 0) return (-x);
25     return x;
26 }
27
28 inline int sgn(T x) {
29     /* no difference'' in fact */
30 #ifdef ABG2d_USE_LL
31     if (0 == x) return 0;
32 #else
33     if (myabs(x) < eps) return 0;
34 #endif
35     return (x > 0) ? 1 : -1;
36 }
37
38 inline T sqr(T x) {
39     return (x * x);
40 }
41
42 struct point {

```

```

43 T x,y;
44 point(void):x(T()),y(T()) {}
45 point(T xx, T yy):x(xx),y(yy) {}
46 inline T norm2(void) {
47     return sqr(x) + sqr(y);
48 }
49 inline ab_float norm(void) {
50     return sqrt((ab_float)(norm2()));
51 }
52 inline point rotate(const ab_float &cost, const ab_float &sint) {} // TODO:
53 inline point operator-(void) const {
54     return point(-x,-y);
55 }
56 inline point operator+(const point& b) const {
57     return point(x+b.x,y+b.y);
58 }
59 inline point operator-(const point& b) const {
60     return point(x-b.x,y-b.y);
61 }
62 inline point operator->*(const point &b) const {
63     return (b-(*this));
64 }
65 inline T operator*(const point& b) const {
66     return ((x)*(b.x))+((y)*(b.y)); /* inner product */
67 }
68 inline T operator^(const point& b) const {
69     return ((x)*(b.y))-((b.x)*(y)); /* outter product */
70 }
71 inline point& operator+=(const point& b) {
72     point tmp=(*this)+b;
73     (*this)=tmp;
74     return (*this);
75 }
76 inline point& operator-=(const point& b) {
77     point tmp=(*this)-b;
78     (*this)=tmp;
79     return (*this);
80 }
81 inline bool operator==(const point& b) const {
82     return (0==sgn(x-b.x))&&(0==sgn(y-b.y));
83 }
84 inline bool operator!=(const point& b) const {
85     return !((*this)==b);
86 }
87 inline point operator<<(const ab_float& theta) const {
88     ab_float ct = cos(theta), st = sin(theta); /* rotate counter-clockwise in radian
89     */
90     return point(ct*x - st*y, st*x + ct*y);
91 };
92
93 typedef point vec;
94
95
96 } // namespace ab_geometry_2d

```

### 3.1.2 Circle

#### Base

```

1 /* 2D Circle Base Class, by Abreto<m@abreto.net>. */
2
3 /* requirement: point.cc */

```

```

4 #include "point.cc"
5
6 #include <utility>
7
8 namespace ab_geometry_2d {
9
10 using namespace std;
11
12 struct circle {
13     point o;
14     T r;
15     circle(void) : r(T()) {}
16     circle(point center, T radius) : o(center), r(radius) {}
17
18     inline ab_float arclen(ab_float theta) {
19         return theta * r;
20     }
21     inline ab_float circumference(void) {
22         return 2. * pi * r;
23     }
24     inline ab_float area(void) {
25         return pi * r * r;
26     }
27
28     /* bool contain(const circle &C, const bool including_touch = false) const
29     {
30         T dis2 = (o->*(C.o)).norm2();
31         T raw_diff = r - C.r;
32         if ( -1 == sgn(raw_diff) ) return false;
33         T dr2 = sqr(raw_diff);
34         return (dis2 < dr2) || (including_touch && (dis2 == dr2));
35     }
36     inline bool in(const circle &C, const bool including_touch = false) const
37     {
38         return C.contain(*this, including_touch);
39     } */
40     enum relation_t {
41         same = 0x000000,
42         contain = 0x000001,
43         intouch = 0x000010,
44         intersect = 0x000100,
45         outtouch = 0x001000,
46         separate = 0x010000,
47         unknow_relation = 0xffffffff
48     };
49     relation_t with(const circle &C) const {
50         T dis2 = (o->*(C.o)).norm2();
51         T dr2 = sqr(r - C.r), rs2 = sqr(r + C.r);
52         if ( 0 == sgn(dis2) && 0 == sgn(dr2) ) return same;
53         if ( -1 == sgn(dis2 - dr2) ) return contain;
54         if ( 0 == sgn(dis2 - dr2) ) return intouch;
55         if ( -1 == sgn(dr2 - dis2) && -1 == sgn(dis2 - rs2) ) return intersect;
56         if ( 0 == sgn(dis2 - rs2) ) return outtouch;
57         if ( -1 == sgn(rs2 - dis2) ) return separate;
58         return unknow_relation;
59     }
60
61     enum point_relation_t {
62         in = 0x001,
63         on = 0x010,
64         out = 0x100,
65         unknow_point_relation = 0xffff
66     };
67     point_relation_t with(const point &P) const {

```

```

68     T dis2 = (o->*P).norm2();
69     T r2 = sqr(r);
70     int type = sgn(dis2 - r2);
71     if (-1 == type) return in;
72     if ( 0 == type) return on;
73     if (+1 == type) return out;
74     return unknow_point_relation;
75 }
76
77 ab_float central_angle(const point &A, const point &B, const bool reflex = false)
78     const {
79     T dot = (A * B);
80     if (0 == sgn(dot)) return 1. * (A != B) * pi;
81     ab_float angle = ((ab_float)(dot)) / r / r;
82     if ( reflex ) angle = 2. * pi - angle;
83     return angle;
84 }
85 /* be sure (*this) intersect with C */
86 pair<point,point> crosspoint(const circle &C) const {
87     ab_float d = (o ->* (C.o)).norm();
88     // TODO:
89 }
90 };
91
92 }

```

### k 次圆交

```

1 //china no.1
2 #pragma comment(linker, "/STACK:1024000000,1024000000")
3 #include <vector>
4 #include <iostream>
5 #include <string>
6 #include <map>
7 #include <stack>
8 #include <cstring>
9 #include <queue>
10 #include <list>
11 #include <stdio.h>
12 #include <set>
13 #include <algorithm>
14 #include <cstdlib>
15 #include <cmath>
16 #include <iomanip>
17 #include <cctype>
18 #include <sstream>
19 #include <functional>
20 #include <stdlib.h>
21 #include <time.h>
22 #include <bitset>
23 using namespace std;
24
25 #define pi acos(-1)
26 #define PI acos(-1)
27 #define endl '\n'
28 #define srand() srand(time(0));
29 #define me(x,y) memset(x,y,sizeof(x));
30 #define foreach(it,a) for(__typeof((a).begin()) it=(a).begin();it!=(a).end();it++)
31 #define close() ios::sync_with_stdio(0); cin.tie(0);
32 #define FOR(x,n,i) for(int i=x;i<=n;i++)
33 #define FOr(x,n,i) for(int i=x;i<n;i++)
34 #define W while
35 #define sgn(x) ((x) < 0 ? -1 : (x) > 0)
36 #define bug printf("*****\n");

```

```

37 #define db double
38 typedef long long LL;
39 const int INF=0x3f3f3f3f;
40 const LL LINF=0x3f3f3f3f3f3f3f3fLL;
41 const int dx[]={-1,0,1,0,1,-1,-1,1};
42 const int dy[]={0,1,0,-1,-1,1,-1,1};
43 const int maxn=1e3+10;
44 const int maxx=1e6+100;
45 const double EPS=1e-8;
46 const double eps=1e-8;
47 const int mod=100000007;
48 template<class T>inline T min(T a,T b,T c) {
49     return min(min(a,b),c);
50 }
51 template<class T>inline T max(T a,T b,T c) {
52     return max(max(a,b),c);
53 }
54 template<class T>inline T min(T a,T b,T c,T d) {
55     return min(min(a,b),min(c,d));
56 }
57 template<class T>inline T max(T a,T b,T c,T d) {
58     return max(max(a,b),max(c,d));
59 }
60 template <class T>
61 inline bool scan_d(T &ret) {
62     char c;
63     int sgn;
64     if (c = getchar(), c == EOF) {
65         return 0;
66     }
67     while (c != '-' && (c < '0' || c > '9')) {
68         c = getchar();
69     }
70     sgn = (c == '-') ? -1 : 1;
71     ret = (c == '-') ? 0 : (c - '0');
72     while (c = getchar(), c >= '0' && c <= '9') {
73         ret = ret * 10 + (c - '0');
74     }
75     ret *= sgn;
76     return 1;
77 }
78
79 inline bool scan_lf(double &num) {
80     char in;
81     double Dec=0.1;
82     bool IsN=false,IsD=false;
83     in=getchar();
84     if(in==EOF) return false;
85     while(in!='-'&&in!='.'&&(in<'0' || in>'9'))in=getchar();
86     if(in=='-') {
87         IsN=true;
88         num=0;
89     } else if(in=='.') {
90         IsD=true;
91         num=0;
92     } else num=in-'0';
93     if(!IsD) {
94         while(in=getchar(),in>='0'&&in<='9') {
95             num*=10;
96             num+=in-'0';
97         }
98     }
99     if(in!='.') {
100         if(IsN) num=-num;

```



```

101     return true;
102 } else {
103     while(in=getchar(),in>='0'&&in<='9') {
104         num+=Dec*(in-'0');
105         Dec*=0.1;
106     }
107 }
108 if(IsN) num=-num;
109 return true;
110 }
111
112 void Out(LL a) {
113     if(a < 0) {
114         putchar('-');
115         a = -a;
116     }
117     if(a >= 10) Out(a / 10);
118     putchar(a % 10 + '0');
119 }
120 void print(LL a) {
121     Out(a),puts("");
122 }
123 //freopen( "in.txt" , "r" , stdin );
124 //freopen( "data.txt" , "w" , stdout );
125 //cerr << "run time is " << clock() << endl;
126 /*struct Point
127 {
128     double x, y;
129     Point(const Point& rhs): x(rhs.x), y(rhs.y) { } //拷贝构造函数
130     Point(double x = 0, double y = 0) : x(x), y(y) { }
131     inline void input()
132     {
133         scanf("%lf%lf",&x,&y);
134     }
135     inline void print()
136     {
137         printf("%.6lf %.6lf\n",x,y);
138     }
139 };*/
140 db sqr(db x) {
141     return x*x;
142 }
143 int dcmp(double x) {
144     if(fabs(x) < EPS) return 0;
145     else return x < 0 ? -1 : 1;
146 }
147 struct Circle {
148     double x, y, r, angle;
149     int d;
150     Circle() {}
151     Circle(double xx, double yy, double ang = 0, int t = 0) {
152         x = xx;
153         y = yy;
154         angle = ang;
155         d = t;
156     }
157     void get() {
158         scanf("%lf%lf%lf", &x, &y, &r);
159         d = 1;
160     }
161 };
162 Circle cir[maxn],tp[maxn*2];
163 double area[maxn];
164 double dis(Circle a,Circle b) {

```

```

165     return sqrt(sqr(a.x - b.x) + sqr(a.y - b.y));
166 }
167 double cross(Circle p0, Circle p1, Circle p2) {
168     return (p1.x - p0.x) * (p2.y - p0.y) - (p1.y - p0.y) * (p2.x - p0.x);
169 }
170 //圆相交
171 int CirCrossCir(Circle p1, double r1, Circle p2, double r2, Circle &cp1, Circle &cp2) {
172     double mx = p2.x - p1.x, sx = p2.x + p1.x, mx2 = mx * mx;
173     double my = p2.y - p1.y, sy = p2.y + p1.y, my2 = my * my;
174     double sq = mx2 + my2, d = -(sq - sqr(r1 - r2)) * (sq - sqr(r1 + r2));
175     if (d + eps < 0) return 0;
176     if (d < eps) d = 0;
177     else d = sqrt(d);
178     double x = mx * ((r1 + r2) * (r1 - r2) + mx * sx) + sx * my2;
179     double y = my * ((r1 + r2) * (r1 - r2) + my * sy) + sy * mx2;
180     double dx = mx * d, dy = my * d;
181     sq *= 2;
182     cp1.x = (x - dy) / sq;
183     cp1.y = (y + dx) / sq;
184     cp2.x = (x + dy) / sq;
185     cp2.y = (y - dx) / sq;
186     if (d > eps) return 2;
187     else return 1;
188 }
189 bool circmp(const Circle& u, const Circle& v) {
190     return dcmp(u.r - v.r) < 0;
191 }
192 bool cmp(const Circle& u, const Circle& v) {
193     if (dcmp(u.angle - v.angle)) return u.angle < v.angle;
194     return u.d > v.d;
195 }
196 //0.5*r*r*(K-sin(K))
197 double calc(Circle cir, Circle cp1, Circle cp2) {
198     double ans = (cp2.angle - cp1.angle) * sqr(cir.r)
199                 - cross(cir, cp1, cp2) + cross(Circle(0, 0), cp1, cp2);
200     return ans / 2;
201 }
202
203 void CirUnion(Circle cir[], int n) {
204     Circle cp1, cp2;
205     sort(cir, cir + n, circmp);
206     for (int i = 0; i < n; ++i)
207         for (int j = i + 1; j < n; ++j)
208             if (dcmp(dis(cir[i], cir[j]) + cir[i].r - cir[j].r) <= 0)
209                 cir[i].d++;
210     for (int i = 0; i < n; ++i) {
211         int tn = 0, cnt = 0;
212         for (int j = 0; j < n; ++j) {
213             if (i == j) continue;
214             if (CirCrossCir(cir[i], cir[i].r, cir[j], cir[j].r,
215                             cp2, cp1) < 2) continue;
216             cp1.angle = atan2(cp1.y - cir[i].y, cp1.x - cir[i].x);
217             cp2.angle = atan2(cp2.y - cir[i].y, cp2.x - cir[i].x);
218             cp1.d = 1;
219             tp[tn++] = cp1;
220             cp2.d = -1;
221             tp[tn++] = cp2;
222             if (dcmp(cp1.angle - cp2.angle) > 0) cnt++;
223         }
224         tp[tn++] = Circle(cir[i].x - cir[i].r, cir[i].y, pi, -cnt);
225         tp[tn++] = Circle(cir[i].x + cir[i].r, cir[i].y, -pi, cnt);
226         sort(tp, tp + tn, cmp);
227         int p, s = cir[i].d + tp[0].d;
228         for (int j = 1; j < tn; ++j) {

```

```

229     p = s;
230     s += tp[j].d;
231     area[p] += calc(cir[i], tp[j - 1], tp[j]);
232 }
233 }
234 }
235 int n;
236 void solve() {
237     for(int i=0; i<n; i++)
238         cir[i].get();
239     me(area,0);
240     CirUnion(cir,n);
241     for(int i=1; i<=n; i++) {
242         area[i]-=area[i+1];
243         printf("[%d]_=%.3f\n", i, area[i]);
244     }
245 }
246 int main() {
247     while(scanf("%d",&n)!=EOF)
248         solve();
249 }

```

### universe

```

1
2 Point CircumCenter(Point a,Point b,Point c) { //三角形的外心
3     Point cp;
4     double a1 = b.x-a.x,b1 = b.y-a.y,c1 = (a1*a1 + b1*b1)/2;
5     double a2 = c.x-a.x,b2 = c.y-a.y,c2 = (a2*a2 + b2*b2)/2;
6     double d = a1*b2 - a2*b1;
7     cp.x = a.x + (c1*b2-c2*b1)/d;
8     cp.y = a.y + (a1*c2-a2*c1)/d;
9     return cp;
10 }

```

### 3.1.3 Convex hull

```

1 /* 2D Convex Hull, by Abreto <m@abreto.net>. */
2 #include "2d_base.hh"
3 #include <cmath>
4 #include <algorithm>
5
6 using namespace std;
7
8 point O;
9
10 bool comp_angle(point_t a, point_t b) {
11     double t = (a-O).X(b-O);
12     if(fe(t,0.0)) return fl((b-O).mag2(),(a-O).mag2());
13     else return fl(0.0,t);
14 }
15
16 void convex_hull_graham(vp& convex, vp src) {
17     int i = 0, top = 0;
18     O = src[0];
19     for(auto pt : src)
20         if( pt.x < O.x || (pt.x == O.x && pt.y < O.y))
21             O = pt;
22     sort(src.begin(), src.end(), comp_angle);
23     convex.push_back(src[0]);
24     convex.push_back(src[1]);
25     top = 1;
26     for(i = 2; i < src.size(); ++i) {

```

```

27     while(top>1 && fle((convex[top]-convex[top-1]).X(src[i]-convex[top]),0.0)) {
28         convex.pop_back();
29         --top;
30     }
31     convex.push_back(src[i]);
32     ++top;
33 }
34 }

```

### 3.1.4 Intersect Area

```

1  #include <cstdio>
2  #include <cmath>
3  #include <algorithm>
4
5  using namespace std;
6
7  // #define inf 1000000000000000
8  #define M 8
9  #define LL long long
10 #define eps 1e-12
11 #define PI acos(-1.0)
12 using namespace std;
13 struct node {
14     double x,y;
15     node() {}
16     node(double xx,double yy) {
17         x=xx;
18         y=yy;
19     }
20     node operator -(node s) {
21         return node(x-s.x,y-s.y);
22     }
23     node operator +(node s) {
24         return node(x+s.x,y+s.y);
25     }
26     double operator *(node s) {
27         return x*s.x+y*s.y;
28     }
29     double operator ^(node s) {
30         return x*s.y-y*s.x;
31     }
32 };
33 double max(double a,double b) {
34     return a>b?a:b;
35 }
36 double min(double a,double b) {
37     return a<b?a:b;
38 }
39 double len(node a) {
40     return sqrt(a*a);
41 }
42 double dis(node a,node b) { //两点之间的距离
43     return len(b-a);
44 }
45 double cross(node a,node b,node c) { //叉乘
46     return (b-a)^(c-a);
47 }
48 double dot(node a,node b,node c) { //点积
49     return (b-a)*(c-a);
50 }
51 int judge(node a,node b,node c) { //判断c是否在ab线段上 (前提是c在直线ab上)
52     if(c.x>=min(a.x,b.x)

```

```

53     && c.x <= max(a.x, b.x)
54     && c.y >= min(a.y, b.y)
55     && c.y <= max(a.y, b.y))
56     return 1;
57     return 0;
58 }
59 double area(node b, node c, double r) {
60     node a(0.0, 0.0);
61     if(dis(b, c) < eps)
62         return 0.0;
63     double h = fabs(cross(a, b, c)) / dis(b, c);
64     if(dis(a, b) > r - eps && dis(a, c) > r - eps) { // 两个端点都在圆的外面则分为两种情况
65         double angle = acos(dot(a, b, c) / dis(a, b) / dis(a, c));
66         if(h > r - eps) {
67             return 0.5 * r * r * angle;
68         } else if(dot(b, a, c) > 0 && dot(c, a, b) > 0) {
69             double angle1 = 2 * acos(h / r);
70             return 0.5 * r * r * fabs(angle - angle1) + 0.5 * r * r * sin(angle1);
71         } else {
72             return 0.5 * r * r * angle;
73         }
74     } else if(dis(a, b) < r + eps && dis(a, c) < r + eps) { // 两个端点都在圆内的情况
75         return 0.5 * fabs(cross(a, b, c));
76     } else { // 一个端点在圆上一个端点在圆内的情况
77         if(dis(a, b) > dis(a, c)) { // 默认b在圆内
78             swap(b, c);
79         }
80         if(fabs(dis(a, b)) < eps) { // ab距离为0直接返回0
81             return 0.0;
82         }
83         if(dot(b, a, c) < eps) {
84             double angle1 = acos(h / dis(a, b));
85             double angle2 = acos(h / r) - angle1;
86             double angle3 = acos(h / dis(a, c)) - acos(h / r);
87             return 0.5 * dis(a, b) * r * sin(angle2) + 0.5 * r * r * angle3;
88         } else {
89             double angle1 = acos(h / dis(a, b));
90             double angle2 = acos(h / r);
91             double angle3 = acos(h / dis(a, c)) - angle2;
92             return 0.5 * r * dis(a, b) * sin(angle1 + angle2) + 0.5 * r * r * angle3;
93         }
94     }
95 }
96 }
97
98 node A, B, C;
99 int R;
100
101 bool compar(node &p1, node &p2) {
102     return (p1 ^ p2) > eps;
103 }
104
105 double f(double x, double y) {
106     node O(x, y);
107     node p[8];
108     p[0] = A - O;
109     p[1] = B - O;
110     p[2] = C - O;
111     sort(p, p + 3, compar);
112     p[3] = p[0];
113     O = node(0, 0);
114     double sum = 0;
115     /* <!-- 求面积交部分 */
116     for(int i = 0; i < 3; i++) { /* 按顺或逆时针顺序最后取绝对值就好 */

```

```

117     int j=i+1;
118     double s=area(p[i],p[j],(double)R);
119     if(cross(0,p[i],p[j])>0)
120         sum+=s;
121     else
122         sum-=s;
123 }
124 if(sum < -eps) sum = -sum;
125 /* --> */
126 return sum;
127 }
128
129 double trifind(double x, double y1, double y2) {
130     double l = y1, r = y2;
131     while(r-l>eps) {
132         double mid = (l+r)/2.0;
133         double mmid = (mid+r)/2.0;
134         if( f(x,mmid) > f(x,mid)+eps )
135             l = mid;
136         else
137             r = mmid;
138     }
139     return f(x,l);
140 }
141
142 double findmin(double x1, double x2, double y1, double y2) {
143     double l = x1, r = x2;
144     while(r-l>eps) {
145         double mid = (l+r)/2.0;
146         double mmid = (mid+r)/2.0;
147         if( trifind(mmid,y1,y2) > trifind(mid,y1,y2)+eps )
148             l = mid;
149         else
150             r = mmid;
151     }
152     return trifind(l,y1,y2);
153 }
154
155 double ans(int a, int b, int c, int r) {
156     A = node(0,0);
157     B = node((double)c,0);
158     R = r;
159     double da = a, db = b, dc = c;
160     double cosa = (db*db+dc*dc-da*da)/(2.0*db*dc);
161     double alpha = acos(cosa);
162     C = node(db*cosa, db*sin(alpha));
163     return findmin(0.0, c, 0.0, db*sin(alpha));
164 }
165
166 int main(void) {
167     int a = 0, b = 0, c = 0, r = 0;
168     while(EOF != scanf("%d%d%d%d",&a,&b,&c,&r) && (a||b||c||r))
169         printf("%.8lf\n", ans(a,b,c,r));
170     return 0;
171 }

```

### 3.1.5 Universe

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 struct Point {
5     double x, y;

```

```

6   Point(double x = 0, double y = 0) : x(x), y(y) {}
7   };
8
9   typedef Point Vector;
10
11  Vector operator + (Vector A, Vector B) {
12      return Vector(A.x + B.x, A.y + B.y);
13  }
14  Vector operator - (Vector A, Vector B) {
15      return Vector(A.x - B.x, A.y - B.y);
16  }
17  Vector operator * (Vector A, double p) {
18      return Vector(A.x*p, A.y*p);
19  }
20  Vector operator / (Vector A, double p) {
21      return Vector(A.x/p, A.y/p);
22  }
23
24  bool operator < (const Point& a, const Point b) {
25      return a.x < b.x || (a.x == b.x && a.y < b.y);
26  }
27
28  const double EPS = 1e-10;
29
30  int dcmp(double x) {
31      if(fabs(x) < EPS) return 0;
32      else return x < 0 ? -1 : 1;
33  }
34
35  bool operator == (const Point& a, const Point& b) {
36      return dcmp(a.x-b.x) == 0 && dcmp(a.y-b.y);
37  }
38
39  //向量a的极角
40  double Angle(const Vector& v) {
41      return atan2(v.y, v.x); // \share\CodeBlocks\templates\wizard\console\cpp
42  }
43
44  //向量点积
45  double Dot(Vector A, Vector B) {
46      return A.x*B.x + A.y*B.y;
47  }
48
49  //向量长度 \share\CodeBlocks\templates\wizard\console\cpp
50  double Length(Vector A) {
51      return sqrt(Dot(A, A));
52  }
53
54  //向量夹角
55  double Angle(Vector A, Vector B) {
56      return acos(Dot(A, B) / Length(A) / Length(B));
57  }
58
59  //向量叉积
60  double Cross(Vector A, Vector B) {
61      return A.x*B.y - A.y*B.x;
62  }
63
64  //三角形有向面积的二倍
65  double Area2(Point A, Point B, Point C) {
66      return Cross(B-A, C-A);
67  }
68
69  //向量逆时针旋转rad度(弧度)

```

```

70 Vector Rotate(Vector A, double rad) {
71     return Vector(A.x*cos(rad)-A.y*sin(rad), A.x*sin(rad)+A.y*cos(rad));
72 }
73
74 //计算向量A的单位法向量。左转90°, 把长度归一。调用前确保A不是零向量。
75 Vector Normal(Vector A) {
76     double L = Length(A);
77     return Vector(-A.y/L, A.x/L);
78 }
79
80 /*****
81 使用复数类实现点及向量的简单操作
82
83 #include <complex>
84 typedef complex<double> Point;
85 typedef Point Vector;
86
87 double Dot(Vector A, Vector B) { return real(conj(A)*B)}
88 double Cross(Vector A, Vector B) { return imag(conj(A)*B);}
89 Vector Rotate(Vector A, double rad) { return A*exp(Point(0, rad)); }
90
91 *****/
92
93 /*****
94 * 用直线上的一点p0和方向向量v表示一条指向。直线上的所有点P满足P = P0+t*v;
95 * 如果知道直线上的两个点则方向向量为B-A, 所以参数方程为A+(B-A)*t;
96 * 当t 无限制时, 该参数方程表示直线。
97 * 当t > 0时, 该参数方程表示射线。
98 * 当 0 < t < 1时, 该参数方程表示线段。
99 *****/
100
101 //直线交点, 须确保两直线有唯一交点。
102 Point GetLineIntersection(Point P, Vector v, Point Q, Vector w) {
103     Vector u = P - Q;
104     double t = Cross(w, u)/Cross(v, w);
105     return P+v*t;
106 }
107
108 //点到直线距离
109 double DistanceToLine(Point P, Point A, Point B) {
110     Vector v1 = B - A, v2 = P - A;
111     return fabs(Cross(v1, v2) / Length(v1)); //不取绝对值, 得到的是有向距离
112 }
113
114 //点到线段的距离
115 double DistanceToSegmentS(Point P, Point A, Point B) {
116     if(A == B) return Length(P-A);
117     Vector v1 = B-A, v2 = P-A, v3 = P-B;
118     if(dcmp(Dot(v1, v2)) < 0) return Length(v2);
119     else if(dcmp(Dot(v1, v3)) > 0) return Length(v3);
120     else return fabs(Cross(v1, v2)) / Length(v1);
121 }
122
123 //点在直线上的投影
124 Point GetLineProjection(Point P, Point A, Point B) {
125     Vector v = B - A;
126     return A+v*(Dot(v, P-A)/Dot(v, v));
127 }
128
129 //线段相交判定, 交点不在一条线段的端点
130 bool SegmentProperIntersection(Point a1, Point a2, Point b1, Point b2) {
131     double c1 = Cross(a2-a1, b1-a1), c2 = Cross(a2-a1, b2-a1);
132     double c3 = Cross(b2-b1, a1-b1), c4 = Cross(b2-b1, a2-b1);
133     return dcmp(c1)*dcmp(c2) < 0 && dcmp(c3)*dcmp(c4) < 0;

```



```

134 }
135
136 //判断点是否在点段上, 不包含端点
137 bool OnSegment(Point P, Point a1, Point a2) {
138     return dcmp(Cross(a1-P, a2-P) == 0 && dcmp((Dot(a1-P, a2-P)) < 0));
139 }
140
141 //计算凸多边形面积
142 double ConvexPolygonArea(Point *p, int n) {
143     double area = 0;
144     for(int i = 1; i < n-1; i++)
145         area += Cross(p[i] - p[0], p[i+1] - p[0]);
146     return area/2;
147 }
148
149 //计算多边形的有向面积
150 double PolygonArea(Point *p, int n) {
151     double area = 0;
152     for(int i = 1; i < n-1; i++)
153         area += Cross(p[i] - p[0], p[i+1] - p[0]);
154     return area/2;
155 }
156
157 /*****
158 * Morley定理: 三角形每个内角的三等分线, 相交成的三角形是等边三角形。
159 * 欧拉定理: 设平面图的定点数, 边数和面数分别为V,E,F。则V+F-E = 2;
160 *****/
161
162 struct Circle {
163     Point c;
164     double r;
165
166     Circle(Point c, double r) : c(c), r(r) {}
167     //通过圆心角确定圆上坐标
168     Point point(double a) {
169         return Point(c.x + cos(a)*r, c.y + sin(a)*r);
170     }
171 };
172
173 struct Line {
174     Point p;
175     Vector v;
176     double ang;
177     Line() {}
178     Line(Point p, Vector v) : p(p), v(v) {}
179     bool operator < (const Line& L) const {
180         return ang < L.ang;
181     }
182 };
183
184 //直线和圆的交点, 返回交点个数, 结果存在sol中。
185 //该代码没有清空sol。
186 int getLineCircleInterseccion(Line L, Circle C, double& t1, double& t2, vector<Point>& sol) {
187     double a = L.v.x, b = L.p.x - C.c.x, c = L.v.y, d = L.p.y - C.c.y;
188     double e = a*a + c*c, f = 2*(a*b + c*d), g = b*b + d*d - C.r*C.r;
189     double delta = f*f - 4*e*g;
190     if(dcmp(delta) < 0) return 0; //相离
191     if(dcmp(delta) == 0) { //相切
192         t1 = t2 = -f / (2*e);
193         sol.push_back(C.point(t1));
194         return 1;
195     }
196     //相交

```

```

197     t1 = (-f - sqrt(delta)) / (2*e);
198     sol.push_back(C.point(t1));
199     t2 = (-f + sqrt(delta)) / (2*e);
200     sol.push_back(C.point(t2));
201     return 2;
202 }
203
204 //两圆相交
205 int getCircleCircleIntersection(Circle C1, Circle C2, vector<Point>& sol) {
206     double d = Length(C1.c - C2.c);
207     if(dcmp(d) == 0) {
208         if(dcmp(C1.r - C2.r == 0)) return -1;    //两圆完全重合
209         return 0;                               //同心圆，半径不一样
210     }
211     if(dcmp(C1.r + C2.r - d) < 0) return 0;
212     if(dcmp(fabs(C1.r - C2.r) == 0)) return -1;
213
214     double a = Angle(C2.c - C1.c);              //向量C1C2的极角
215     double da = acos((C1.r*C1.r + d*d - C2.r*C2.r) / (2*C1.r*d));
216     //C1C2到C1P1的角
217     Point p1 = C1.point(a-da), p2 = C1.point(a+da);
218     sol.push_back(p1);
219     if(p1 == p2) return 1;
220     sol.push_back(p2);
221     return 2;
222 }
223
224 const double PI = acos(-1);
225 //过定点做圆的切线
226 //过点p做圆C的切线，返回切线个数。v[i]表示第i条切线
227 int getTangents(Point p, Circle C, Vector* v) {
228     Vector u = C.c - p;
229     double dist = Length(u);
230     if(dist < C.r) return 0;
231     else if(dcmp(dist - C.r) == 0) {
232         v[0] = Rotate(u, PI/2);
233         return 1;
234     } else {
235         double ang = asin(C.r / dist);
236         v[0] = Rotate(u, -ang);
237         v[1] = Rotate(u, +ang);
238         return 2;
239     }
240 }
241
242 //两圆的公切线
243 //返回切线的个数，-1表示有无数条公切线。
244 //a[i], b[i] 表示第i条切线在圆A，圆B上的切点
245 int getTangents(Circle A, Circle B, Point *a, Point *b) {
246     int cnt = 0;
247     if(A.r < B.r) {
248         swap(A, B);
249         swap(a, b);
250     }
251     int d2 = (A.c.x - B.c.x)*(A.c.x - B.c.x) + (A.c.y - B.c.y)*(A.c.y - B.c.y);
252     int rdiff = A.r - B.r;
253     int rsum = A.r + B.r;
254     if(d2 < rdiff*rdiff) return 0;    //内含
255     double base = atan2(B.c.y - A.c.y, B.c.x - A.c.x);
256     if(d2 == 0 && A.r == B.r) return -1;    //无限多条切线
257     if(d2 == rdiff*rdiff) {           //内切一条切线
258         a[cnt] = A.point(base);
259         b[cnt] = B.point(base);
260         cnt++;

```

```

261     return 1;
262 }
263 //有外共切线
264 double ang = acos((A.r-B.r) / sqrt(d2));
265 a[cnt] = A.point(base+ang);
266 b[cnt] = B.point(base+ang);
267 cnt++;
268 a[cnt] = A.point(base-ang);
269 b[cnt] = B.point(base-ang);
270 cnt++;
271 if(d2 == rsum*rsum) { //一条公切线
272     a[cnt] = A.point(base);
273     b[cnt] = B.point(PI+base);
274     cnt++;
275 } else if(d2 > rsum*rsum) { //两条公切线
276     double ang = acos((A.r + B.r) / sqrt(d2));
277     a[cnt] = A.point(base+ang);
278     b[cnt] = B.point(PI+base+ang);
279     cnt++;
280     a[cnt] = A.point(base-ang);
281     b[cnt] = B.point(PI+base-ang);
282     cnt++;
283 }
284 return cnt;
285 }
286
287 typedef vector<Point> Polygon;
288
289 //点 在多边形内的判定
290 int isPointInPolygon(Point p, Polygon poly) {
291     int wn = 0;
292     int n = poly.size();
293     for(int i = 0; i < n; i++) {
294         if(OnSegment(p, poly[i], poly[(i+1)%n])) return -1; //在边界上
295         int k = dcmp(Cross(poly[(i+1)%n]-poly[i], p-poly[i]));
296         int d1 = dcmp(poly[i].y - p.y);
297         int d2 = dcmp(poly[(i+1)%n].y - p.y);
298         if(k > 0 && d1 <= 0 && d2 > 0) wn++;
299         if(k < 0 && d2 <= 0 && d1 > 0) wn++;
300     }
301     if(wn != 0) return 1; //内部
302     return 0; //外部
303 }
304
305 //凸包
306 /*****
307 * 输入点数组p, 个数为p, 输出点数组ch。返回凸包顶点数
308 * 不希望凸包的边上有输入点, 把两个<= 改成 <
309 * 高精度要求时建议用dcmp比较
310 * 输入点不能有重复点。函数执行完以后输入点的顺序被破坏
311 *****/
312 int ConvexHull(Point *p, int n, Point* ch) {
313     sort(p, p+n); //先比较x坐标, 再比较y坐标
314     int m = 0;
315     for(int i = 0; i < n; i++) {
316         while(m > 1 && Cross(ch[m-1] - ch[m-2], p[i]-ch[m-2]) <= 0) m--;
317         ch[m++] = p[i];
318     }
319     int k = m;
320     for(int i = n-2; i >= 0; i--) {
321         while(m > k && Cross(ch[m-1] - ch[m-2], p[i]-ch[m-2]) <= 0) m--;
322         ch[m++] = p[i];
323     }
324     if(n > 1) m--;

```

```

325     return m;
326 }
327
328 //用有向直线A→B切割多边形poly, 返回“左侧”。 如果退化, 可能会返回一个单点或者线段
329 //复杂度O(n2);
330 Polygon CutPolygon(Polygon poly, Point A, Point B) {
331     Polygon newpoly;
332     int n = poly.size();
333     for(int i = 0; i < n; i++) {
334         Point C = poly[i];
335         Point D = poly[(i+1)%n];
336         if(dcmp(Cross(B-A, C-A)) >= 0) newpoly.push_back(C);
337         if(dcmp(Cross(B-A, C-D)) != 0) {
338             Point ip = GetLineIntersection(A, B-A, C, D-C);
339             if(OnSegment(ip, C, D)) newpoly.push_back(ip);
340         }
341     }
342     return newpoly;
343 }
344
345 //半平面交
346
347 //点p再有向直线L的左边。(线上不算)
348 bool Onleft(Line L, Point p) {
349     return Cross(L.v, p-L.p) > 0;
350 }
351
352 //两直线交点, 假定交点唯一存在
353 Point GetIntersection(Line a, Line b) {
354     Vector u = a.p - b.p;
355     double t = Cross(b.v, u) / Cross(a.v, b.v);
356     return a.p+a.v*t;
357 }
358
359 int HalfplaneIntersection(Line* L, int n, Point* poly) {
360     sort(L, L+n);           //按极角排序
361
362     int first, last;         //双端队列的第一个元素和最后一个元素
363     Point *p = new Point[n]; //p[i]为q[i]和q[i+1]的交点
364     Line *q = new Line[n];   //双端队列
365     q[first = last = 0] = L[0]; //队列初始化为只有一个半平面L[0]
366     for(int i = 0; i < n; i++) {
367         while(first < last && !Onleft(L[i], p[last-1])) last--;
368         while(first < last && !Onleft(L[i], p[first])) first++;
369         q[++last] = L[i];
370         if(fabs(Cross(q[last].v, q[last-1].v)) < EPS) {
371             last--;
372             if(Onleft(q[last], L[i].p)) q[last] = L[i];
373         }
374         if(first < last) p[last-1] = GetIntersection(q[last-1], q[last]);
375     }
376     while(first < last && !Onleft(q[first], p[last-1])) last--;
377     //删除无用平面
378     if(last-first <= 1) return 0; //空集
379     p[last] = GetIntersection(q[last], q[first]);
380
381     //从deque复制到输出中
382     int m = 0;
383     for(int i = first; i <= last; i++) poly[m++] = p[i];
384     return m;
385 }

```

## 4 Graph

### 4.1 Tree

#### 4.1.1 Universe

```

1  /* find root(重心) */
2
3
4  void findroot(int u, int fa) {
5      int i;
6      size[u] = 1;
7      f[u] = 0;
8      for (i = last[u]; i; i = e[i][2]) {
9          if (!vis[e[i][0]] && e[i][0] != fa) {
10             findroot(e[i][0], u);
11             size[u] += size[e[i][0]];
12             if (f[u] < size[e[i][0]])
13                 f[u] = size[e[i][0]];
14         }
15     }
16     if (f[u] < ALL - size[u])
17         f[u] = ALL - size[u];
18     if (f[u] < f[root]) root = u;
19 }
20
21 /* —— da —— */
22
23 int dep[MAXN+1];
24 int ancestor[MAXN+1][MAXLGN];
25 int minw[MAXN+1][MAXLGN];
26
27 void dfs(int u, int fa) {
28     ancestor[u][0] = fa;
29     dep[u] = dep[fa] + 1;
30     for(int e = u[front]; e; e = E[e].n) {
31         int v = E[e].v, w = E[e].w;
32         if(v != fa) {
33             minw[v][0] = w;
34             dfs(v, u);
35         }
36     }
37 }
38
39 void init_system(void) {
40     int i = 0, w = 0;
41     int t = 0;
42     dep[0] = -1;
43     dfs(1,0);
44     for(w = 1; (t=(1<<w)) < N; ++w)
45         for(i = 1; i <= N; ++i) if( dep[i] >= t ) {
46             ancestor[i][w] = ancestor[ancestor[i][w-1]][w-1];
47             minw[i][w] = min(minw[i][w-1], minw[ancestor[i][w-1]][w-1]);
48         }
49 }
50
51 int query(int a, int b) {
52     if(dep[a] < dep[b]) return query(b,a);
53     else { /* now dep[s] > dep[t] */
54         int i = 0;
55         int maxbit = MAXLGN-1;
56         int ret = INF;
57         //while((1<<maxbit) <= dep[a]) maxbit++;

```

```

58     /* first up a to same dep with b. */
59     for(i = maxbit; i >= 0; i--)
60         if(dep[a] - (1<<i) >= dep[b]) {
61             ret = min(ret, minw[a][i]);
62             a = ancestor[a][i];
63         }
64     if(a == b) return ret;
65     for(i = maxbit; i >= 0; i--)
66         if(dep[a] - (1<<i) >= 0 && ancestor[a][i] != ancestor[b][i]) {
67             ret = min(ret, min(minw[a][i], minw[b][i]));
68             a = ancestor[a][i];
69             b = ancestor[b][i];
70         }
71     ret = min(ret, min(minw[a][0], minw[b][0]));
72     return ret;
73 }
74 }

```

#### 4.1.2 Point Divide and Conquer

##### Version I

```

1  /* Tree::Point divide and conquer, by Abreto<m@abreto.net>. */
2  #include <bits/stdc++.h>
3
4  using namespace std;
5  typedef long long int ll;
6
7  #define MAXN    (100001)
8  #define MAXV    (MAXN+1)
9  #define MAXE    (MAXN<<1)
10 struct edge {
11     int v;
12     edge *n;
13     edge(void):v(0),n(NULL) {}
14     edge(int vv,edge *nn):v(vv),n(nn) {}
15 };
16 int nE;
17 edge E[MAXE];
18 edge *front[MAXV];
19 int label[MAXV]; /* 0 for '(', 1 for ')' */
20 void add_edge(int u, int v) {
21     int ne = ++nE;
22     E[ne] = edge(v, u[front]);
23     u[front] = &(E[ne]);
24 }
25
26 int n;
27 ll ans;
28
29 char del[MAXV];
30 namespace findroot {
31     int ALL;
32     int nfind;
33     int vis[MAXV];
34     int size[MAXV];
35     int f[MAXV];
36     int root;
37     void __find(int u, int fa) {
38         vis[u] = nfind;
39         size[u] = 1;
40         f[u] = 0;
41         for(edge *e=u[front]; e; e = e->n) {

```

```

42     int v = e->v;
43     if((!del[v]) && (vis[v] != nfind) && (v != fa)) {
44         __find(v, u);
45         size[u] += size[v];
46         if(f[u] < size[v]) f[u] = size[v];
47     }
48 }
49 if(f[u] < ALL - size[u]) f[u] = ALL - size[u];
50 if(f[u] < f[root]) root = u;
51 }
52 int find(int u, int all) {
53     ++nfind;
54     ALL = all;
55     f[root = 0] = MAXV;
56     __find(u, 0);
57     return root;
58 }
59 }
60
61 namespace workspaces {
62 int maxdep;
63 int dep[MAXV];
64 ll cntin[MAXV], cntout[MAXV];
65 int in[2][MAXV]; /* 0 for '(', 1 for ')' */
66 int out[2][MAXV];
67 void getdeep(int u, int fa) {
68     dep[u] = dep[fa] + 1;
69     if(dep[u] > maxdep) maxdep = dep[u];
70     for(edge *e = u[front]; e; e = e->n)
71         if((!del[e->v]) && (fa != e->v))
72             getdeep(e->v, u);
73 }
74 void dfs(int u, int fa) {
75     {
76         /* out from root */
77         out[0][u] = out[0][fa];
78         out[1][u] = out[1][fa];
79         if(0 == label[u]) { /* meet '(' */
80             out[0][u]++;
81         } else { /* meet ')' */
82             if(out[0][u]) out[0][u]--;
83             else out[1][u]++;
84         }
85         if(out[0][u] == 0)
86             cntout[out[1][u]]++;
87     }
88     {
89         /* in to root */
90         in[0][u] = in[0][fa];
91         in[1][u] = in[1][fa];
92         if(0 == label[u]) { /* meet '(' */
93             if(in[1][u]) in[1][u]--;
94             else in[0][u]++;
95         } else { /* meet ')' */
96             in[1][u]++;
97         }
98         if(0 == in[1][u])
99             cntin[in[0][u]]++;
100     }
101     /* do something */
102     for(edge *e = u[front]; e; e = e->n) {
103         int v = e->v;
104         if((!del[v]) && (v != fa)) {
105             dfs(v, u);

```

```

106     }
107 }
108 }
109 inline void init_maxdep(void) {
110     maxdep = 0;
111 }
112 inline void update_maxdep(int u) {
113     dep[u] = 1;
114     if(dep[u] > maxdep) maxdep = dep[u];
115     for(edge *e = u[front]; e; e = e->n)
116         if(!del[e->v])
117             getdeep(e->v, u);
118 }
119 inline void clear(void) {
120     for(int i = 0; i <= maxdep+1; ++i)
121         cntin[i] = cntout[i] = 0;
122 }
123 inline void work(int u) {
124     in[0][u] = in[1][u] = out[0][u] = out[1][u] = 0;
125     in[label[u]][u] = out[label[u]][u] = 1;
126     if(out[0][u] == 0) cntout[out[1][u]]++;
127     if(0 == in[1][u]) cntin[in[0][u]]++;
128     /* update in and out if neccessary */
129     for(edge *e = u[front]; e; e = e->n)
130         if(!del[e->v])
131             dfs(e->v, u);
132 }
133 };
134
135 ll count(int u, int p) {
136     ll ret = 0;
137     workspace::init_maxdep();
138     workspace::update_maxdep(u);
139     workspace::clear();
140     if(-1 == p) {
141         for(edge *e = u[front]; e; e = e->n)
142             if(!del[e->v])
143                 workspace::work(e->v);
144         p = label[u];
145         /* single end */
146         if(0 == p) ret = workspace::cntout[1];
147         else ret = workspace::cntin[1];
148     } else {
149         workspace::work(u);
150     }
151     if(0 == p) { /* p is '(' */
152         for(int i = 0; i < workspace::maxdep; ++i) /* concatenation */
153             ret += workspace::cntin[i] * workspace::cntout[i+1];
154     } else { /* p is ')' */
155         for(int i = 0; i < workspace::maxdep; ++i) /* concatenation */
156             ret += workspace::cntin[i+1] * workspace::cntout[i];
157     }
158     return ret;
159 }
160
161 void handle(int u) {
162     del[u] = 1; /* delete current root. */
163     ans += count(u, -1);
164     /* do something */
165     for(edge *e = u[front]; e; e = e->n) {
166         int v = e->v;
167         if(!del[v]) {
168             ans -= count(v, label[u]);
169             /* do something */

```



```

170     int r = findroot::find(v, findroot::size[v]);
171     handle(r);
172 }
173 }
174 }
175
176 void proc(void) {
177     int r = findroot::find(1,n);
178     handle(r);
179 }
180
181 char ls[MAXV+1];
182 int main(void) {
183     int i = 0;
184     scanf("%d", &n);
185     scanf("%s", ls);
186     for(i = 0; i < n; ++i)
187         label[i+1] = ls[i] - '(';
188     for(i = 1; i < n; ++i) {
189         int ai, bi;
190         scanf("%d_%d", &ai, &bi);
191         add_edge(ai, bi);
192         add_edge(bi, ai);
193     }
194     proc();
195     printf("%lld\n", ans);
196     return 0;
197 }

```

## Version 2

```

1  /* 2016 ACM/ICPC Asia Regional Dalian. Problem , by Abreto<m@abreto.net>. */
2  #include <bits/stdc++.h>
3
4  using namespace std;
5  typedef long long int ll;
6
7  /* offset in [1,k] */
8  #define GET(i,offset) (((i)>>((offset)-1))&1)
9  #define SET(i,offset) ((i)|(1<<((offset)-1)))
10 #define REV(i,offset) ((i)^(1<<((offset)-1)))
11
12 #define MAXN    (50005)
13 #define MAXV    (MAXN+1)
14 #define MAXE    (MAXN<<1)
15 struct edge {
16     int v;
17     edge *n;
18     edge(void):v(0),n(NULL) {}
19     edge(int vv,edge *nn):v(vv),n(nn) {}
20 };
21 int nE;
22 edge E[MAXE];
23 edge *front[MAXV];
24 int label[MAXV];    /* each kind */
25 void add_edge(int u, int v) {
26     int ne = ++nE;
27     E[ne] = edge(v, u[front]);
28     u[front] = &(E[ne]);
29 }
30
31 int n, k;
32 ll ans;
33 int all_kind;
34

```

```

35 int ndel;
36 int del[MAXV];
37 namespace findroot {
38 int ALL;
39 ll nfind;
40 ll vis[MAXV];
41 int size[MAXV];
42 int f[MAXV];
43 int root;
44 void __find(int u, int fa) {
45     vis[u] = nfind;
46     size[u] = 1;
47     f[u] = 0;
48     for(edge *e=u[front]; e; e = e->n) {
49         int v = e->v;
50         if((del[v] != ndel) && (vis[v] != nfind) && (v != fa)) {
51             __find(v, u);
52             size[u] += size[v];
53             if(f[u] < size[v]) f[u] = size[v];
54         }
55     }
56     if(f[u] < ALL - size[u]) f[u] = ALL - size[u];
57     if(f[u] < f[root]) root = u;
58 }
59 int find(int u, int all) {
60     ++nfind;
61     ALL = all;
62     f[root = 0] = MAXV;
63     __find(u, 0);
64     return root;
65 }
66 }
67
68 namespace workspace {
69 ll cnt[1024];
70 int dp[MAXV];
71 void dfs(int u, int fa) {
72     dp[u] = dp[fa] | label[u];
73     cnt[dp[u]] ++;
74     /* dig into children */
75     for(edge *e = u[front]; e; e = e->n) {
76         int v = e->v;
77         if((del[v] != ndel) && (v != fa)) {
78             dfs(v, u);
79         }
80     }
81 }
82 inline void clear(void) {
83     for(int i = 1; i <= all_kind; ++i)
84         cnt[i] = 0;
85 }
86 inline void work(int u) {
87     dp[u] = label[u];
88     cnt[dp[u]] ++;
89     for(edge *e = u[front]; e; e = e->n)
90         if((del[e->v] != ndel))
91             dfs(e->v, u);
92 }
93 inline void show(void) {
94     for(int i = 0; i <= all_kind; ++i)
95         printf("cnt[%d]_u=%lld\n", i, cnt[i]);
96     for(int i = 1; i <= n; ++i)
97         printf("dp[%d]_u=%d\n", i, dp[i]);
98 }

```

```

99 };
100
101
102 ll count(int u, int p) {
103     ll ret = 0;
104     workspace::clear();
105     //printf("%d,%d : \n", u, p);
106     if(-1 == p) {
107         for(edge *e = u[front]; e; e = e->n)
108             if(((del[e->v]) != ndel))
109                 workspace::work(e->v);
110         p = label[u];
111         /* single end */
112         for(int i = 1; i <= all_kind; i++)
113             if(all_kind == (i|p))
114                 ret += (workspace::cnt[i]<<1);
115     } else {
116         workspace::work(u);
117     }
118     //workspace::show();
119     for(int i = 1; i <= all_kind; ++i)
120         if( workspace::cnt[i] > 0 )
121             for(int j = 1; j <= all_kind; ++j)
122                 if(all_kind == (i|j))
123                     ret += workspace::cnt[i] * workspace::cnt[j];
124     //printf("%lld\n", ret);
125     return ret;
126 }
127
128 void handle(int u) {
129     //printf("proprocessing %d\n", u);
130     del[u] = ndel; /* delete current root. */
131     ans += count(u, -1);
132     /* do something */
133     for(edge *e = u[front]; e; e = e->n) {
134         int v = e->v;
135         if(del[v] != ndel) {
136             ans -= count(v, label[u]);
137             /* do something */
138             int r = findroot::find(v, findroot::size[v]);
139             handle(r);
140         }
141     }
142 }
143
144 void proc(void) {
145     int r = findroot::find(1,n);
146     handle(r);
147 }
148
149 void clear(void) {
150     int i;
151     ans = 0;
152     nE = 0;
153     for(i = 0; i <= n; ++i) {
154         front[i] = NULL;
155     }
156     //findroot::nfind = 0;
157     ndel++;
158 }
159
160 void mozhu(void) {
161     int i = 0;
162     int li;

```

```

163     for(i = 1; i <= n; ++i) {
164         scanf("%d", &li);
165         label[i] = 1<<(li-1);
166     }
167     for(i = 1; i < n; ++i) {
168         int ai, bi;
169         scanf("%d%d", &ai, &bi);
170         add_edge(ai, bi);
171         add_edge(bi, ai);
172     }
173     all_kind = (1<<k)-1;
174     proc();
175     if(1 == k) ans += n;
176     printf("%lld\n", ans);
177 }
178
179 int main(void) {
180     while( EOF != scanf("%d%d", &n, &k) ) {
181         clear();
182         mozhu();
183     }
184     return 0;
185 }

```

#### 4.1.3 Heavy chain decomposition

```

1  /* bzoj 1036 */
2  /* 树链剖分 */
3  #include <bits/stdc++.h>
4
5  using namespace std;
6
7  #define MAXN    30030
8  #define MAXM    (MAXN<<1)
9  struct edge {
10     int v;
11     edge *n;
12     edge(void) {}
13     edge(int vv, edge *nn):v(vv),n(nn) {}
14 };
15 typedef edge *ep;
16 int nE;
17 edge E[MAXM];
18 ep front[MAXN];
19 void add_edge(int u, int v) {
20     int ne = ++nE;
21     E[ne] = edge(v, u[front]);
22     u[front] = &(E[ne]);
23 }
24
25 int n;
26 int fa[MAXN], son[MAXN], sz[MAXN], dep[MAXN];
27 int top[MAXN];
28 int id[MAXN];
29 int tot;
30
31 void calc(int u, int uf) {
32     dep[u] = dep[uf] + 1;
33     fa[u] = uf;
34     sz[u] = 1;
35     son[u] = -1;
36     for(ep e = u[front]; e; e = e->n) {
37         if(e->v != uf) {

```

```

38     calc(e->v, u);
39     sz[u] += sz[e->v];
40     if( -1 == son[u] || sz[son[u]] < sz[e->v] )
41         son[u] = e->v;
42 }
43 }
44 }
45 void link(int u, int f) {
46     id[u] = (++tot);
47     top[u] = f;
48     if(son[u] > 0) {
49         link(son[u], f);
50     }
51     for(ep e = u[front]; e; e = e->n) {
52         if(e->v != fa[u] && e->v != son[u]) {
53             link(e->v, e->v);
54         }
55     }
56 }
57
58 /* 其实是树链剖分 */
59 void make_link_cut_tree(void) {
60     calc(1, 0);
61     link(1, 1);
62 }
63
64 int w[MAXN];
65 int sum[MAXN<<2], mx[MAXN<<2];
66
67 void maintain(int o, int l, int r) {
68     sum[o] = sum[o<<1] + sum[o<<1|1];
69     mx[o] = max(mx[o<<1], mx[o<<1|1]);
70 }
71 void build(int o = 1, int l = 1, int r = n) {
72     if(r == l) {
73         sum[o] = w[l];
74         mx[o] = w[l];
75     } else {
76         int mid = l+r>>1;
77         build(o<<1, l, mid);
78         build(o<<1|1, mid+1, r);
79         maintain(o, l, r);
80     }
81 }
82 void update(int p, int x, int o = 1, int l = 1, int r = n) {
83     if(p <= l && r <= p) {
84         sum[o] = x;
85         mx[o] = x;
86     } else {
87         int mid = l+r>>1;
88         if(p <= mid) update(p,x,o<<1,l,mid);
89         else update(p,x,o<<1|1,mid+1,r);
90         maintain(o,l,r);
91     }
92 }
93 int qs(int L, int R, int o = 1, int l = 1, int r = n) {
94     if(R < l || r < L) return 0;
95     else if (L <= l && r <= R) {
96         return sum[o];
97     } else {
98         int mid = l+r>>1;
99         return qs(L,R,o<<1,l,mid)+qs(L,R,o<<1|1,mid+1,r);
100     }
101 }

```

```

102 int qm(int L, int R, int o = 1, int l = 1, int r = n) {
103     if(L <= l && r <= R) {
104         return mx[o];
105     } else {
106         int mid = l+r>>1;
107         if(R <= mid) return qm(L, R, o<<1, l, mid);
108         else if ( L > mid ) return qm(L, R, o<<1|1, mid+1, r);
109         else return max(qm(L, R, o<<1, l, mid),qm(L, R, o<<1|1, mid+1, r));
110     }
111 }
112
113 void change(int u, int t) {
114     update(id[u], t);
115 }
116 int qmax(int u, int v) {
117     int ret = -1000000000;
118     while(top[u] != top[v]) {
119         if( dep[top[u]] > dep[top[v]] ) {
120             /* jump u */
121             ret = max(ret, qm(id[top[u]], id[u]));
122             u = fa[top[u]];
123         } else {
124             ret = max(ret, qm(id[top[v]], id[v]));
125             v = fa[top[v]];
126         }
127     }
128     ret = max(ret, qm(min(id[u],id[v]),max(id[u],id[v])));
129     return ret;
130 }
131 int qsum(int u, int v) {
132     int ret = 0;
133     while(top[u] != top[v]) {
134         if( dep[top[u]] > dep[top[v]] ) {
135             /* jump u */
136             ret += qs(id[top[u]], id[u]);
137             u = fa[top[u]];
138         } else {
139             ret += qs(id[top[v]], id[v]);
140             v = fa[top[v]];
141         }
142     }
143     ret += qs(min(id[u],id[v]),max(id[u],id[v]));
144     return ret;
145 }
146
147 int main(void) {
148     int i;
149     scanf("%d", &n);
150     for(i = 1; i < n; ++i) {
151         int a, b;
152         scanf("%d%d", &a, &b);
153         add_edge(a, b);
154         add_edge(b, a);
155     }
156     make_link_cut_tree();
157     for(i = 1; i <= n; ++i) {
158         scanf("%d", &(w[id[i]]));
159     }
160     build();
161     scanf("%d", &i);
162     while(i--) {
163         char command[8];
164         int a, b;
165         scanf("%s%d%d", command, &a, &b);

```

```

166     if('C' == command[0]) change(a, b);
167     else if ('M' == command[1]) printf("%d\n", qmax(a, b));
168     else if ('S' == command[1]) printf("%d\n", qsum(a, b));
169 }
170 return 0;
171 }

```

## 4.2 2-SAT

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 namespace two_sat {
6     const int maxn = 100000;
7     const int maxm = 1000000;
8     struct edge {
9         int v;
10        edge *n;
11        edge(void):v(0),n(NULL) {}
12        edge(int vv, edge *nn):v(vv),n(nn) {}
13    };
14    typedef edge *ep;
15    int n;
16    int nE;
17    edge E[maxm];
18    ep front[maxn];
19    void add_edge(int u, int v) {
20        int ne = ++nE;
21        E[ne] = edge(v, u[front]);
22        u[front] = &(E[ne]);
23    }
24    /* (x = xval or y = yval), indexed from 0 */
25    void add_clause(int x, int xv, int y, int yv) {
26        x = x*2 + xv;
27        y = y*2 + yv;
28        add_edge(x^1, y);
29        add_edge(y^1, x);
30    }
31
32    char mark[maxn<<1];
33    int S[maxn<<1], c;
34    void init(int N) {
35        n = N;
36        for(int i = 0; i < n*2; ++i) {
37            i[front] = NULL;
38            i[mark] = 0;
39        }
40        nE = 0;
41    }
42
43    int dfs(int x) {
44        if(mark[x^1]) return 0;
45        if(mark[x]) return 1;
46        mark[x] = 1;
47        S[c++] = x;
48        for(ep e = x[front]; e; e = e->n)
49            if(!dfs(e->v)) return 0;
50        return 1;
51    }
52
53    int solve(void) {

```

```

54 for(int i = 0; i < n*2; i += 2)
55     if(!mark[i] && !mark[i+1]) {
56         c = 0;
57         if(!dfs(i)) {
58             while(c > 0) mark[S--c] = 0;
59             if(!dfs(i+1)) return 0;
60         }
61     }
62     return 1;
63 }
64 }

```

### 4.3 Cut Edge and Point

```

1 Finding cut edges
2 The code below works properly because the lemma above (first lemma):
3   h[root] = 0
4   par[v] = -1
5   dfs (v):
6       d[v] = h[v]
7       color[v] = gray
8       for u in adj[v]:
9           if color[u] == white
10              then par[u] = v and dfs(u) and d[v] = min(
11                  d[v], d[u])
12                  if d[u] > h[v]
13                      then the edge v-u is a cut edge
14                      else if u != par[v])
15              then d[v] = min(d[v], h[u])
16              color[v] = black
17
18 In this code, h[v] = height of vertex v in the DFS
19 tree and d[v] = min(h[w] where there is at least
20 vertex u in subtree of v in the DFS tree where
21 there is an edge between u and w).
22
23 Finding cut vertices
24 The code below works properly because the lemma
25 above (first lemma):
26   h[root] = 0
27   par[v] = -1
28   dfs (v):
29       d[v] = h[v]
30       color[v] = gray
31       for u in adj[v]:
32           if color[u] == white
33              then par[u] = v and dfs(
34                  u) and d[v] = min(d[v]
35                      , d[u])
36                  if d[u] >= h[v]
37                      and (v != root
38                          or
39                          number_of_children
40                              (v) > 1)
41                      then the edge v
42                          is a cut
43                          vertex
44                      else if u != par
45                          [v])
46              then d[v] = min(d[v], h[u])
47              color[v] = black
48
49 In this code, h[v] = height of vertex v in
50 the DFS tree and d[v] = min(h[w] where

```



there is at least vertex  $u$  in subtree of  $v$  in the DFS tree where there is an edge between  $u$  and  $w$ ).

#### 4.4 Euler Path

```

1  /* Euler path, by Abreto<m@abreto.net>. */
2  #define MAXV    (1024)
3  #define MAXE    (MAXV*MAXV)
4
5  typedef struct {
6      int id;
7      int nxt;
8      int del;
9  } egde_t;
10 int front[MAXV];
11 egde_t edg[MAXE];
12 int d[MAXV];
13 int ind[MAXV], outd[MAXV];
14 int nedges;
15 void add_edge(int u, int v) {
16     int newedge = ++nedges;
17     edg[newedge].id = v;
18     edg[newedge].nxt = u[front];
19     edg[newedge].del = 0;
20     u[front] = newedge;
21     outd[u]++;
22     ind[v]++;
23     d[u]++;
24     d[v]++;
25 }
26 void del_edge(int u, int v) {
27     int e = 0;
28     for(e=u[front]; e; e=edg[e].nxt)
29         if(edg[e].id==v) {
30             edg[e].del = 1;
31             outd[u]--;
32             ind[v]--;
33             d[u]--;
34             d[v]--;
35             return;
36         }
37 }
38
39 int path[MAXV];
40 int l;
41
42 void add2path(int u) {
43     path[l++] = u;
44 }
45
46 /* Directed graph */
47 void euler(int x) {
48     if(outd[x]) {
49         int e = 0;
50         for(e=x[front]; e; e=edg[e].nxt)
51             if(!edg[e].del) {
52                 int v = edg[e].id;
53                 del_edge(x,v);
54                 euler(v);
55             }
56     }

```

```

57     add2path(x);
58 }
59
60 /* Undirected graph */
61 void euler(int x) {
62     if(d[x]) {
63         int e = 0;
64         for(e=x[front]; e; e=edg[e].nxt)
65             if(!edg[e].del) {
66                 int v = edg[e].id;
67                 del_edge(x,v);
68                 del_edge(v,x);
69                 euler(v);
70             }
71     }
72     add2path(x);
73 }

```

## 4.5 Shortest Path

### 4.5.1 Dijkstra

```

1  /* Shortest Path Dijkstra, by Abreto<m@abreto.net>. */
2  #include <cstdio>
3  #include <set>
4  #include <utility>
5
6  using namespace std;
7  typedef set< pair<int,int> > spii;
8
9  #define MAXN    512
10 #define MAXV    (MAXN*MAXN)
11
12 struct egde_t {
13     int id;
14     int nxt;
15 };
16 int front[MAXV];
17 egde_t edg[MAXV<<3];
18 int nedges;
19 void add_edge(int u, int v) {
20     int newedge = ++nedges;
21     edg[newedge].id = v;
22     edg[newedge].nxt = u[front];
23     u[front] = newedge;
24 }
25
26 int d[MAXV];
27 int vis[MAXN];
28 int solid[MAXV];
29
30 int dijkstra(int s, int t) {
31     int v = s[front];
32     spii q;
33     q.insert(make_pair(0, s));
34     while(!q.empty()) {
35         auto it = q.begin();
36         int u = it->second;
37         int v = u[front];
38         q.erase(it);
39         solid[u] = 1;
40         if(u == t) break;
41         while(v) {

```

```

42     int w = edg[v].id;
43     if(!solid[w]) {
44         if( (0==d[w]) || (d[u] + 1 < d[w]) ) {
45             q.erase(make_pair(d[w],w));
46             d[w] = d[u] + 1;
47             q.insert(make_pair(d[w],w));
48         }
49     }
50     v = edg[v].nxt;
51 }
52 }
53 return d[t];
54 }

```

#### 4.5.2 Shortest Path Fast Algorithm

```

1  /* Shortest Path Fast Algorithm, by Abreto<m@abreto.net>. */
2  #include <cstdio>
3  #include <cstring>
4  #include <queue>
5  #include <utility>
6
7  using namespace std;
8
9  #define MAXN    128
10
11 struct edge {
12     int v;
13     int w;
14     int n;
15 };
16 edge edg[MAXN<<1];
17 int nedg;
18 int indegree[MAXN];
19 int front[MAXN];
20 int find_edge(int u, int v) {
21     int e = u[front];
22     while(e) {
23         if(edg[e].v == v) return e;
24         e = edg[e].n;
25     }
26     return 0;
27 }
28 void add_edge(int u, int v, int w) {
29     int e = find_edge(u,v);
30     if(0==e) {
31         int newnode = ++nedg;
32         edg[newnode].v = v;
33         edg[newnode].w = w;
34         edg[newnode].n = u[front];
35         u[front] = newnode;
36         indegree[v]++;
37     } else {
38         edg[e].w = (w < edg[e].w)?w:(edg[e].w);
39     }
40 }
41
42 int n;
43
44 char inq[MAXN];
45 int vis[MAXN];
46 int d[MAXN];
47 int spfa(int s) { /* return 1 if fuhuan exists. */

```

```

48 queue<int> q;
49 memset(inq, 0, sizeof(inq));
50 memset(d, -1, sizeof(d));
51 memset(vis, 0, sizeof(vis));
52 d[s] = 0;
53 inq[s] = 1;
54 q.push(s);
55 while(!q.empty()) {
56     int u = q.front();
57     q.pop();
58     printf("proc_%d..\n", u);
59     inq[u] = 0;
60     if(vis[u]++ > n)
61         return 1;
62     for(int e = front[u]; e; e = edg[e].n) {
63         int v = edg[e].v, w = edg[e].w;
64         if( -1==d[v] || d[u] + w < d[v] ) {
65             d[v] = d[u] + w;
66             if(!inq[v]) {
67                 inq[v] = 1;
68                 q.push(v);
69             }
70         }
71     }
72 }
73 return 0;
74 }

```

#### 4.5.3 *K*-th shortest path

```

1 /**
2  * poj
3  * Problem#2449
4  * Accepted
5  * Time: 250ms
6  * Memory: 9252k
7  */
8 #include <iostream>
9 #include <fstream>
10 #include <sstream>
11 #include <algorithm>
12 #include <cstdio>
13 #include <cstdlib>
14 #include <cstring>
15 #include <ctime>
16 #include <cctype>
17 #include <cmath>
18 #include <vector>
19 #include <queue>
20 #include <stack>
21 #include <map>
22 #include <set>
23 #include <bitset>
24 using namespace std;
25 typedef bool boolean;
26
27 typedef class Edge {
28 public:
29     int end;
30     int next;
31     int w;
32
33     Edge(int end = 0, int next = -1, int w = 0):end(end), next(next), w(w) {

```

```

34 } Edge;
35
36 const int N = 1e3, M = 1e5;
37
38 typedef class MapManager {
39 public:
40     int cnt;
41     int h[N + 5];
42     Edge edge[M + 5];
43
44     MapManager() { }
45     MapManager(int n):cnt(-1) {
46 //         h = new int[(n + 1)];
47 //         edge = new Edge[(m + 1)];
48         memset(h, -1, sizeof(int) * (n + 1));
49     }
50
51     inline void addEdge(int u, int v, int w) {
52         edge[++cnt] = (Edge(v, h[u], w));
53 //         h[u] = (signed)edge.size() - 1;
54         h[u] = cnt;
55     }
56
57     inline int start(int node) {
58         return h[node];
59     }
60
61     Edge& operator [] (int pos) {
62         return edge[pos];
63     }
64 } MapManager;
65 #define m_endpos -1
66
67 int n, m;
68 MapManager g;
69 MapManager rg;
70 int s, t, k;
71 int ds[N + 5];
72
73 inline void init() {
74     scanf("%d%d", &n, &m);
75     memset(g.h, -1, sizeof(int) * (n + 1));
76     memset(rg.h, -1, sizeof(int) * (n + 1));
77     for(int i = 1, u, v, w; i <= m; i++) {
78         scanf("%d%d%d", &u, &v, &w);
79         g.addEdge(u, v, w);
80         rg.addEdge(v, u, w);
81     }
82     scanf("%d%d%d", &s, &t, &k);
83 //     ds = new int[(n + 1)];
84 }
85
86 #define g rg
87 #define f ds
88 #define que que1
89 boolean vis[N + 5];
90 queue<int> que;
91 boolean spfa(int s, int t) {
92     memset(f, 0x7f, sizeof(int) * (n + 1));
93     memset(vis, false, sizeof(boolean) * (n + 1));
94     que.push(s);
95     f[s] = 0;
96     while(!que.empty()) {
97         int e = que.front();

```

```

98     que.pop();
99     vis[e] = false;
100     for(int i = g.start(e); i != m_endpos; i = g[i].next) {
101         int& eu = g[i].end;
102         // cout << e << " " << eu << " " << i << endl;
103         if(f[e] + g[i].w < f[eu]) {
104             f[eu] = f[e] + g[i].w;
105             if(!vis[eu]) {
106                 que.push(eu);
107                 vis[eu] = true;
108             }
109         }
110     }
111 }
112 return (f[t] != 0x7f7f7f7f);
113 }
114 #undef g
115 #undef f
116 #undef que
117
118 typedef class Status {
119 public:
120     int node;
121     int dis;
122     int priority;
123
124     Status(int node = 0, int dis = 0):node(node), dis(dis), priority(h()) {
125
126     }
127     int h() {
128         return dis + ds[node];
129     }
130     boolean operator < (Status b) const {
131         return priority > b.priority;
132     }
133 } Status;
134
135 int label[N + 5];
136 priority_queue<Status> que;
137 int bfs(int s, int t) {
138     if(s == t) k++;
139     // label = new int[(n + 1)];
140     memset(label, 0, sizeof(int) * (n + 1));
141     que.push(Status(s, 0));
142     while(!que.empty()) {
143         Status e = que.top();
144         que.pop();
145         label[e.node]++;
146         if(e.node == t && label[e.node] == k)
147             return e.dis;
148         for(int i = g.start(e.node); i != m_endpos; i = g[i].next) {
149             if(label[g[i].end] < k)
150                 que.push(Status(g[i].end, e.dis + g[i].w));
151         }
152     }
153     return -1;
154 }
155
156 inline void solve() {
157     if(!spfa(t, s)) {
158         puts("-1");
159         return;
160     }
161     printf("%d", bfs(s, t));

```

```

162 }
163
164 int main() {
165     init();
166     solve();
167     return 0;
168 }

```

## 4.6 Maxflow

```

1  /* Max Flow Problem, by Abreto<m@abreto.net> */
2
3  #include <bits/stdc++.h>
4  using namespace std;
5
6  #define MAXV    (100000)
7  #define MAXE    (1000000)
8  struct edge {
9      static int N;
10     int v, w;
11     edge *n;
12     edge(void):v(0),w(0),n(NULL) {}
13     edge(int vv, int ww, edge *nn):v(vv),w(ww),n(nn) {}
14 };
15 int nE;
16 edge E[MAXE];
17 edge *front[MAXV];
18 void add_edge(int u, int v, int w) {
19     int ne = ++nE;
20     E[ne] = edge(v, w, u[front]);
21     u[front] = &(E[ne]);
22 }
23 edge *find_edge(int u, int v) {
24     for(edge *e = u[front]; e != NULL; e = e->n)
25         if(e->v == v)
26             return e;
27     return NULL;
28 }
29 void grant_e(int u, int v, int w) {
30     edge *e = find_edge(u, v);
31     if(NULL == e) add_edge(u,v,w);
32     else e->w += w;
33 }
34
35 int vis[MAXV];
36 int path[MAXV];
37 int dfs(int u, int t) {
38     vis[u] = 1;
39     if(u == t) return 1;
40     for(edge *e = u[front]; e != NULL; e = e->n) {
41         int v = e->v;
42         if(!vis[v] && e->w && dfs(v,t)) {
43             path[u] = v;
44             return 1;
45         }
46     }
47     return 0;
48 }
49 int find_path(int s, int t) {
50     memset(vis, 0, sizeof(vis));
51     return dfs(s,t);
52 }

```

```

53 int max_flow(int s, int t) {
54     int flow = 0;
55     while(find_path(s,t)) {
56         int i = 0;
57         int minf = find_edge(s,path[s])->w;
58         for(i = path[s]; i != t; i = path[i])
59             minf = min(minf, find_edge(i,path[i])->w);
60         for(i = s; i != t; i = path[i]) {
61             grant_e(i, path[i], -minf);
62             grant_e(path[i], i, minf);
63         }
64         flow += minf;
65     }
66     return flow;
67 }
68
69 /* Dinic */
70 #define N 1000
71 #define INF 1000000000
72
73 struct Edge {
74     int from,to,cap,flow;
75     Edge(int u,int v,int c,int f):from(u),to(v),cap(c),flow(f) {}
76 };
77
78 struct Dinic {
79     int n,m,s,t;//结点数, 边数 (包括反向弧), 源点编号, 汇点编号
80     vector<Edge>edges;//边表, dges[e]和dges[e^1]互为反向弧
81     vector<int>G[N];//邻接表, G[i][j]表示结点i的第j条边在e数组中的编号
82     bool vis[N]; //BFS的使用
83     int d[N]; //从起点到i的距离
84     int cur[N]; //当前弧下标
85
86     void addedge(int from,int to,int cap) {
87         edges.push_back(Edge(from,to,cap,0));
88         edges.push_back(Edge(to,from,0,0));
89         int m=edges.size();
90         G[from].push_back(m-2);
91         G[to].push_back(m-1);
92     }
93
94     bool bfs() {
95         memset(vis,0,sizeof(vis));
96         queue<int>Q;
97         Q.push(s);
98         d[s]=0;
99         vis[s]=1;
100         while(!Q.empty()) {
101             int x=Q.front();
102             Q.pop();
103             for(int i=0; i<G[x].size(); i++) {
104                 Edge&e=edges[G[x][i]];
105                 if(!vis[e.to]&&e.cap>e.flow) { //只考虑残量网络中的弧
106                     vis[e.to]=1;
107                     d[e.to]=d[x]+1;
108                     Q.push(e.to);
109                 }
110             }
111         }
112     }
113     return vis[t];
114 }
115
116 int dfs(int x,int a) { //x表示当前结点, a表示目前为止的最小残量

```



```

117     if(x==t||a==0)return a;//a等于0时及时退出，此时相当于断路了
118     int flow=0,f;
119     for(int&i=cur[x]; i<G[x].size(); i++) { //从上次考虑的弧开始，注意要使用引用，同
        时修改cur[x]
120         Edge&e=edges[G[x][i]);//e是一条边
121         if(d[x]+1==d[e.to]&&(f=dfs(e.to,min(a,e.cap-e.flow)))>0) {
122             e.flow+=f;
123             edges[G[x][i]^1].flow-=f;
124             flow+=f;
125             a-=f;
126             if(!a)break;//a等于0及时退出，当a!=0,说明当前节点还存在另一个曾广路分支。
127         }
128     }
129 }
130 return flow;
131 }
132
133 int Maxflow(int s,int t) { //主过程
134     this->s=s,this->t=t;
135     int flow=0;
136     while(bfs()) { //不停地用bfs构造分层网络，然后用dfs沿着阻塞流增广
137         memset(cur,0,sizeof(cur));
138         flow+=dfs(s,INF);
139     }
140     return flow;
141 }
142 };
143
144 /* ISAP */
145 struct Edge {
146     int from,to,cap,flow;
147 };
148 const int maxn=650;
149 const int INF=0x3f3f3f3f;
150 struct ISAP {
151     int n,m,s,t;//结点数，边数（包括反向弧），源点编号，汇点编号
152     vector<Edge>edges;
153     vector<int>G[maxn];
154     bool vis[maxn];
155     int d[maxn];
156     int cur[maxn];
157     int p[maxn];
158     int num[maxn];
159     void AddEdge(int from,int to,int cap) {
160         edges.push_back((Edge) {
161             from,to,cap,0
162         });
163         edges.push_back((Edge) {
164             to,from,0,0
165         });
166         m=edges.size();
167         G[from].push_back(m-2);
168         G[to].push_back(m-1);
169     }
170     bool RevBFS() {
171         memset(vis,0,sizeof(vis));
172         queue<int>Q;
173         Q.push(t);
174         d[t]=0;
175         vis[t]=1;
176         while(!Q.empty()) {
177             int x=Q.front();
178             Q.pop();
179             for(int i=0; i<G[x].size(); i++) {

```

```

180     Edge &e =edges[G[x][i]^1];
181     if(!vis[e.from]&&e.cap>e.flow) {
182         vis[e.from]=1;
183         d[e.from]=d[x]+1;
184         Q.push(e.from);
185     }
186 }
187 }
188 return vis[s];
189 }
190 int Augment() {
191     int x=t, a=INF;
192     while(x!=s) {
193         Edge &e = edges[p[x]];
194         a= min(a,e.cap-e.flow);
195         x=edges[p[x]].from;
196     }
197     x=t;
198     while(x!=s) {
199         edges[p[x]].flow+=a;
200         edges[p[x]^1].flow-=a;
201         x=edges[p[x]].from;
202     }
203     return a;
204 }
205 int Maxflow(int s,int t,int n) {
206     this->s=s,this->t=t,this->n=n;
207     int flow=0;
208     RevBFS();
209     memset(num,0,sizeof(num));
210     for(int i=0; i<n; i++) {
211         num[d[i]]++;
212     }
213     int x=s;
214     memset(cur,0,sizeof(cur));
215     while(d[s]<n) {
216         if(x==t) {
217             flow+=Augment();
218             x=s;
219         }
220         int ok=0;
221         for(int i=cur[x]; i<G[x].size(); i++) {
222             Edge &e =edges[G[x][i]];
223             if(e.cap>e.flow&&d[x]==d[e.to]+1) {
224                 ok=1;
225                 p[e.to]=G[x][i];
226                 cur[x]=i;
227                 x=e.to;
228                 break;
229             }
230         }
231         if(!ok) {
232             int m=n-1;
233             for(int i=0; i<G[x].size(); i++) {
234                 Edge &e =edges[G[x][i]];
235                 if(e.cap>e.flow)
236                     m=min(m,d[e.to]);
237             }
238             if(--num[d[x]]==0)
239                 break;
240             num[d[x]=m+1]++;
241             cur[x]=0;
242             if(x!=s)
243                 x=edges[p[x]].from;

```

```

244     }
245 }
246     return flow;
247 }
248 };
249 int main() {
250     int n,m,a,b,c,res;
251     while(scanf("%d%d",&m,&n)!=EOF) {
252         ISAP tmp;
253         for(int i=0; i<m; i++) {
254             scanf("%d%d%d",&a,&b,&c);
255             tmp.AddEdge(a,b,c);
256         }
257         res=tmp.Maxflow(1,n,n);
258         printf("%d\n",res);
259     }
260     return 0;
261 }

```

## 4.7 Strongly Connected Component

```

1  /* Kosaraju */
2  #define MAXN    10010
3  #define MAXM    100010
4  struct edge {
5      int v;
6      edge *n;
7      edge(void):v(0),n(NULL) {}
8      edge(int vv, edge *nn):v(vv),n(nn) {}
9  };
10 int nE;
11 edge E[MAXM<<1];
12 edge *ori[MAXN];
13 edge *inv[MAXN];
14 void add_edge(edge *front[], int u, int v) {
15     int ne = ++nE;
16     E[ne] = edge(v, u[front]);
17     u[front] = &(E[ne]);
18 }
19 void connect(int u, int v) {
20     add_edge(ori, u, v);
21     add_edge(inv, v, u);
22 }
23
24 int vis[MAXN];
25 int vst[MAXN];
26 void first_dfs(int u, int &sig) {
27     vis[u] = 1;
28     for(edge *e = u[ori]; e; e = e->n)
29         if(!vis[e->v])
30             first_dfs(e->v, sig);
31     vst[++sig] = u;
32 }
33 int mark[MAXN];
34 void second_dfs(int u, int sig) {
35     vis[u] = 1;
36     mark[u] = sig;
37     for(edge *e = u[inv]; e; e = e->n)
38         if(!vis[e->v])
39             second_dfs(e->v, sig);
40 }
41

```

```

42 int N, M;
43
44 int kosaraju(void) {
45     int i;
46     int sig = 0;
47     for(i = 0; i <= N; ++i) vis[i] = 0;
48     for(i = 1; i <= N; ++i) {
49         if(!vis[i])
50             first_dfs(i, sig);
51     }
52     sig = 1;
53     for(i = 0; i <= N; ++i) vis[i] = 0;
54     for(i = N; i > 0; --i) {
55         if(!vis[vst[i]])
56             second_dfs(vst[i], sig++);
57     }
58     for(i = 1; i <= N; ++i)
59         if(mark[i] != 1)
60             return 0;
61     return 1;
62 }
63
64
65 void clear(void) {
66     nE = 0;
67     for(int i = 0; i <= N; ++i) {
68         ori[i] = inv[i] = NULL;
69     }
70 }
71
72 /* Tarjan */
73 #define MAXN    10010
74 #define MAXM    100010
75 struct edge {
76     int v;
77     edge *n;
78     edge(void):v(0),n(NULL) {}
79     edge(int vv, edge *nn):v(vv),n(nn) {}
80 };
81 typedef edge *ep;
82 int nE;
83 edge E[MAXM];
84 edge *front[MAXN];
85 void add_edge(int u, int v) {
86     int ne = ++nE;
87     E[ne] = edge(v, u[front]);
88     u[front] = &(E[ne]);
89 }
90
91 int mark[MAXN];
92 int dfn[MAXN], low[MAXN];
93 int stk[MAXN];
94 int stk_top;
95
96 void tardfs(int u, int stamp, int &scc) {
97     mark[u] = 1;
98     dfn[u] = low[u] = stamp;
99     stk[stk_top++] = u;
100     for(ep e = u[front]; e; e = e->n) {
101         if(0 == mark[e->v]) tardfs(e->v, ++stamp, scc);
102         if(1 == mark[e->v]) low[u] = min(low[u], low[e->v]);
103     }
104     if(dfn[u] == low[u]) {
105         ++scc;

```

```

106     do {
107         low[stk[stk_top-1]] = scc;
108         mark[stk[stk_top-1]] = 2;
109     } while(stk[(stk_top-)-1] != u);
110 }
111 }
112
113 int tarjan(int n) {
114     int scc = 0, lay = 1;
115     for(int i = 1; i <= n; ++i)
116         if(0 == mark[i])
117             tardfs(i, lay, scc);
118     return scc;
119 }
120
121 int N, M;
122
123 void clear(void) {
124     nE = 0;
125     for(int i = 0; i <= N; ++i) {
126         i[front] = NULL;
127         mark[i] = low[i] = 0;
128     }
129     stk_top = 0;
130 }
131
132 /* Garbow */
133 #define MAXN    10010
134 #define MAXM    100010
135
136 struct edge {
137     int v;
138     edge *n;
139     edge(void):v(0),n(NULL) {}
140     edge(int vv, edge *nn):v(vv),n(nn) {}
141 };
142 typedef edge *ep;
143
144 int nE;
145 edge E[MAXM];
146 edge *front[MAXN];
147 void add_edge(int u, int v) {
148     int ne = ++nE;
149     E[ne] = edge(v, u[front]);
150     u[front] = &(E[ne]);
151 }
152
153 int stk1[MAXN], stk1t;
154 int stk2[MAXN], stk2t;
155 int low[MAXN], belg[MAXN];
156
157 void garbowdfs(int u, int lay, int &scc) {
158     stk1[++stk1t] = u;
159     stk2[++stk2t] = u;
160     low[u] = ++lay;
161     for(ep e=u[front]; e; e = e->n) {
162         if(!low[e->v]) garbowdfs(e->v, lay, scc);
163         else if (0 == belg[e->v])
164             while(low[stk2[stk2t]] > low[e->v])
165                 --stk2t;
166     }
167     if(stk2[stk2t] == u) {
168         stk2t--;
169         scc++;

```

```

170     do {
171         belg[stk1[stk1t]] = scc;
172     } while(stk1[stk1t--] != u);
173 }
174 }
175
176 int grabow(int n) {
177     int i;
178     int scc = 0, lay = 0;
179     for(i = 0; i <= n; ++i) {
180         belg[i] = low[i] = 0;
181     }
182     for(i = 1; i <= n; ++i)
183         if(0 == low[i])
184             garbowdfs(i, lay, scc);
185     return scc;
186 }
187
188 int N, M;
189
190 void clear(void) {
191     nE = 0;
192     for(int i = 0; i <= N; ++i) {
193         front[i] = NULL;
194     }
195 }

```

#### 4.8 Perfect elimination ordering

求弦图的最大团数/最小色数的时候，只要在完美消除序列上从后往前贪心染色即可。  
而求最大独立集/最小团覆盖的时候，只要在完美消除序列上从前往后贪心取点即可。

```

1  /**
2   * BZOJ 1006
3   * [HNOI2008] 神奇的国度
4   * 最大势法求完美消除序列
5   * by Abreto<m@abreto.net>.
6   */
7  #include <cassert>
8  #include <cstdio>
9  #include <vector>
10 #include <bitset>
11 #include <algorithm>
12
13 using namespace std;
14 typedef vector<int> vi;
15 typedef vi::iterator vii;
16 #define pb push_back
17 #define MAXN 10100
18 #define MAXM 1000100
19
20 int n;
21 vi g[MAXN];
22 int ans;
23
24 struct node_t {
25     int v;
26     node_t *nxt;
27 } node[MAXM << 2];
28 int used;
29 node_t *new_node(void) {
30     return node + (used ++);
31 }

```

```

32
33 node_t *f[MAXN]; /* head */
34 void lkto(int pos, int item) {
35     node_t *t = new_node();
36     t->v = item;
37     t->nxt = f[pos];
38     f[pos] = t;
39 }
40
41 int usedby[MAXN];
42 int color[MAXN];
43 bitset<MAXN> added;
44 int label[MAXN], max_label;
45 void mcs(void) {
46     for (int i = 1; i <= n; i++) lkto(0, i);
47     for (int i = n; i > 0; i--) {
48         node_t *cur = f[max_label];
49         assert(cur != NULL);
50         while (added.test(cur->v)) { /* already added */
51             cur = cur->nxt;
52             while (NULL == cur)
53                 cur = f[ --max_label ];
54         }
55         f[ max_label ] = cur->nxt;
56         while (max_label && NULL == f[max_label]) max_label--;
57         int u = cur->v;
58         added.set(u);
59         /* the i-th is u */
60         for (vii it = g[u].begin(); it != g[u].end(); it++) {
61             int v = *it;
62             if (!added.test(v)) {
63                 label[v] ++;
64                 max_label = max(max_label, label[v]);
65                 lkto(label[v], v);
66             }
67             usedby[color[v]] = i;
68         }
69         for (int j = 1; j <= n; j++)
70             if (usedby[j] != i) {
71                 color[u] = j;
72                 break;
73             }
74         ans = max(ans, color[u]);
75     }
76 }
77 }
78
79 int main(void) {
80     int m;
81     scanf("%d%d", &n, &m);
82     while (m--) {
83         int ai, bi;
84         scanf("%d%d", &ai, &bi);
85         g[ai].pb(bi);
86         g[bi].pb(ai);
87     }
88     mcs();
89     printf("%d\n", ans);
90     return 0;
91 }

```

## 5 Math

### 5.1 Euler Function

```

1  /* Euler function phi(x), by Abreto<m@abreto.net>. */
2
3  #define MAXX    3000000
4
5  int phi[MAXX];
6  void get_euler(void) {
7      int i = 0, j = 0;
8      phi[1] = 1;
9      for(i = 2; i < MAXX; ++i)
10         if(!phi[i])
11             for(j = i; j < MAXX; j += i) {
12                 if(!phi[j]) phi[j] = j;
13                 phi[j] = phi[j]/i * (i-1);
14             }
15 }
```

### 5.2 Möbius Function

```

1  void sieve() {
2      fill(isPrime, isPrime + maxn, 1);
3      mu[1] = 1, num = 0;
4      for (int i = 2; i < maxn; ++i) {
5          if (isPrime[i]) primes[num++] = i, mu[i] = -1;
6          static int d;
7          for (int j = 0; j < num && (d = i * primes[j]) < maxn; ++j) {
8              isPrime[d] = false;
9              if (i % primes[j] == 0) {
10                  mu[d] = 0;
11                  break;
12              } else mu[d] = -mu[i];
13          }
14      }
15 }
```

### 5.3 Number Theory Inverse

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  const int n=10000000;    /* */
5  const long long mod=1e9+7; /* prime required. */
6
7  long long fact[n],fiv[n],inv[n];
8
9  int main() {
10     fact[0]=fact[1]=1;
11     fiv[0]=fiv[1]=1;
12     inv[1]=1;
13     for (int i=2; i<n; i++) {
14         fact[i]=fact[i-1]*i%mod;
15         inv[i]=(mod-mod/i)*inv[mod%i]%mod;
16         fiv[i]=inv[i]*fiv[i-1]%mod;
17     }
18     for (int i=1; i<n; i++) {
19         if (fact[i]*fiv[i]%mod!=1) printf("fact_wrong:%d\n",i);
20         if (inv[i]*i%mod!=1)      printf("inv_wrong:%d\n",i);
21     }
```



```

22 | cout<<"complete"<<endl;
23 | return 0;
24 | }

```

## 5.4 Chinese Remainder Theorem

$$x \equiv a_i \pmod{m_i}$$

```

1 | /* Chinese Remainder Theorem, by Abreto<m@abreto.net>. */
2 | #include "euler.c"
3 |
4 | #define MAXN    64
5 |
6 | typedef long long int ll;
7 |
8 | ll quickpow(ll a, ll b, ll mod) {
9 |     ll ret = 1, base = a;
10 |    while(b > 0) {
11 |        if(b & 1) ret = (ret * base) % mod;
12 |        base = (base * base) % mod;
13 |        b >>= 1;
14 |    }
15 |    return ret;
16 | }
17 |
18 | ll N;
19 | ll a[MAXN], m[MAXN]; /* a and m is indexed from 0. */
20 | ll x, M;
21 |
22 | void naive_crt(void) {
23 |     int i = 0;
24 |     ll Mi[MAXN], nMi[MAXN];
25 |     ll t[MAXN];
26 |
27 |     M = 1;
28 |     for(i = 0; i < N; ++i)
29 |         M *= a[i];
30 |     for(i = 0; i < N; ++i)
31 |         Mi[i] = M / a[i];
32 |     get_euler();
33 |     for(i = 0; i < N; ++i)
34 |         nMi[i] = quickpow(Mi[i], phi[a[i]]-1, a[i]);
35 |     for(i = 0; i < N; ++i)
36 |         t[i] = ((a[i] * Mi[i]) % M) * nMi[i] % M;
37 |     for(i = 0; i < N; ++i)
38 |         x = (x + t[i]) % M;
39 | }

```

## 5.5 Linear congruences

```

1 | #include <cstdio>
2 | #include <cassert>
3 | #include <cstdlib>
4 |
5 | using namespace std;
6 |
7 | class mod_equ_resolver {
8 |     typedef long long int ll;
9 |     ll a, m;
10 |    inline void gurantee(void) {

```

```

11     if ( a < 0 ) {
12         ll k = ( -a ) / m;
13         a += ( k + 1ll ) * m;
14         a = ( a + m ) % m;
15     } else {
16         a %= m;
17     }
18     // printf("x = %lld (mod %lld)\n",a, m);
19 }
20 public:
21     mod_equ_resolver(void) {
22         a = 0ll;
23         m = 1ll;
24     }
25     ll exgcd(ll m, ll n, ll &x, ll &y) {
26         if ( 0 == n ) {
27             x = 1;
28             y = 0;
29             return m;
30         }
31         ll g = exgcd( n, m % n, x, y );
32         ll t = x;
33         x = y;
34         y = t - m / n * y;
35         return g;
36     }
37     int onemore(ll a2, ll m2) {
38         ll x, y;
39         ll g = exgcd( m, m2, x, y );
40         assert(x*m+y*m2==g);
41         a2 = (a2 + m2) % m2;
42         if ( abs( a2 - a ) % g ) return -1;
43         ll newm = m / g * m2;
44         ll newa = a + ( a2 - a ) / g * x * m;
45         a = newa;
46         m = newm;
47         gurantee();
48         return 0;
49     }
50     ll resolve(void) {
51         return a;
52     }
53 };

```

Usage: For

$$\begin{cases} x \equiv a_1 \pmod{m_1} \\ x \equiv a_2 \pmod{m_2} \\ \vdots \\ x \equiv a_k \pmod{m_k} \end{cases}$$

run

```

1 mod_equ_resolver solver;
2 for (int i = 1; i <= k; i++)
3     solver.onemore(a[i], m[i]);

```

then the solution is

$$x \equiv \text{solver.a} \pmod{\text{solver.m}}$$

## 5.6 FFT

```

1 #include <cmath>
2 using namespace std;

```

```

3 namespace fft {
4 #define eps (1e-9)
5 template < typename T = double >
6 struct dbl {
7     T x;
8     dbl(void):x(0.0) {}
9     template <typename U>
10    dbl(U a):x((T)a) {}
11    inline char sgn(void) {
12        return ((x>=eps)&&(x<=eps))?(0):((x>eps)?(1):(-1));
13    }
14    inline T tabs(void) {
15        return ((x>=eps)&&(x<=eps))?(0.0):((x>eps)?(x):(-x));
16    }
17    inline dbl abs(void) {
18        return dbl(tabs());
19    }
20    template <typename U> inline dbl &operator=(const U b) {
21        x=(T)b;
22        return (*this);
23    }
24    inline T *operator&(void) {
25        return &x;
26    }
27    inline dbl operator-(void) const {
28        return dbl(-x);
29    }
30    inline dbl operator+(const dbl &b) const {
31        return dbl(x+b.x);
32    }
33    inline dbl operator-(const dbl &b) const {
34        return dbl(x-b.x);
35    }
36    inline dbl operator*(const dbl &b) const {
37        return dbl(x*b.x);
38    }
39    inline dbl operator/(const dbl &b) const {
40        return dbl(x/b.x);
41    }
42    template <typename U> inline dbl operator^(const U &b) const {
43        T ret=1.0,base=x;
44        while(b) {
45            if(b&1)ret*=base;
46            base*=base;
47            b>>=1;
48        }
49        return dbl(ret);
50    }
51    inline dbl operator+=(const dbl &b) {
52        return dbl(x+=b.x);
53    }
54    inline dbl operator-=(const dbl &b) {
55        return dbl(x-=b.x);
56    }
57    inline dbl operator*=(const dbl &b) {
58        return dbl(x*=b.x);
59    }
60    inline dbl operator/=(const dbl &b) {
61        return dbl(x/=b.x);
62    }
63    template <typename U> inline dbl operator^=(const U &b) {
64        dbl tmp=(*this)^b;
65        *this=tmp;
66        return tmp;

```

```

67 }
68 inline bool operator==(const dbl &b) const {
69     return (0 == ((*this)-b).sgn());
70 }
71 inline bool operator!=(const dbl &b) const {
72     return (0 != ((*this)-b).sgn());
73 }
74 inline bool operator<(const dbl &b) const {
75     return (-1 == ((*this)-b).sgn());
76 }
77 inline bool operator<=(const dbl &b) const {
78     return (((*this)==b) || ((*this)<b));
79 }
80 inline bool operator>(const dbl &b) const {
81     return (b < (*this));
82 }
83 inline bool operator>=(const dbl &b) const {
84     return (((*this)==b) || ((*this)>b));
85 }
86 template <typename U> inline operator U() const {
87     return (U)x;
88 }
89 inline char operator[](unsigned n) {
90     if(n >= 0) {
91         long long int ret=x;
92         while(n--) {
93             ret/=10;
94         }
95         return (ret%10);
96     } else {
97         T ret=x;
98         n=-n;
99         while(n--)ret*=10.0;
100         return ((long long int)ret)%10;
101     }
102 }
103 };
104 template <typename T>
105 struct Complex {
106     T x,y; /* x + iy */
107     Complex(void):x(T()),y(T()) {}
108     Complex(T xx):x(xx) {}
109     Complex(T xx,T yy):x(xx),y(yy) {}
110     inline Complex operator-(void) const {
111         return Complex(-x,-y);
112     }
113     inline Complex operator+(const Complex& b) const {
114         return Complex(x+b.x,y+b.y);
115     }
116     inline Complex operator-(const Complex& b) const {
117         return Complex(x-b.x,y-b.y);
118     }
119     inline Complex operator*(const Complex& b) const {
120         return Complex(x*b.x-y*b.y,x*b.y+y*b.x);
121     }
122     inline Complex operator/(const Complex& b) const {
123         T bo=b.x*b.x+b.y*b.y;
124         return Complex((x*b.x+y*b.y)/bo,(y*b.x-x*b.y)/bo);
125     }
126     inline Complex& operator+=(const Complex& b) {
127         Complex tmp=(*this)+b;
128         (*this)=tmp;
129         return (*this);
130     }

```

```

131 inline Complex& operator--(const Complex& b) {
132     Complex tmp=(*this)-b;
133     (*this)=tmp;
134     return (*this);
135 }
136 inline Complex& operator*=(const Complex& b) {
137     Complex tmp=(*this)*b;
138     (*this)=tmp;
139     return (*this);
140 }
141 inline Complex& operator/=(const Complex& b) {
142     Complex tmp=(*this)/b;
143     (*this)=tmp;
144     return (*this);
145 }
146 inline friend Complex operator+(const T& a, const Complex& b) {
147     return Complex(a)+b;
148 }
149 inline friend Complex operator-(const T& a, const Complex& b) {
150     return Complex(a)-b;
151 }
152 inline friend Complex operator*(const T& a, const Complex& b) {
153     return Complex(a)*b;
154 }
155 inline friend Complex operator/(const T& a, const Complex& b) {
156     return Complex(a)/b;
157 }
158 };
159 typedef dbl<> Double;
160 typedef Complex<Double> ComplexD;
161 typedef long long int ll;
162 const int maxn = 2000000; /* !! */
163 const Double pi(acos(-1.0));
164
165 void build(ComplexD _P[], ComplexD P[], int n, int m, int curr, int &cnt) {
166     if(m == n) {
167         _P[curr] = P[cnt++];
168     } else {
169         build(_P, P, n, m*2, curr, cnt);
170         build(_P, P, n, m*2, curr+m, cnt);
171     }
172 }
173
174 void FFT(ComplexD P[], int n, int oper) { /* n should be 2^k. */
175     static ComplexD _P[maxn];
176     int cnt = 0;
177     build(_P, P, n, 1, 0, cnt);
178     copy(_P, _P+n, P);
179     for(int d = 0; (1<<d)<n; ++d) {
180         int m = 1<<d;
181         int m2 = m*2;
182         Double p0 = pi / m * oper;
183         ComplexD unit_p0(cos(p0.x), sin(p0.x));
184         for(int i = 0; i < n; i += m2) {
185             ComplexD unit(1,0);
186             for(int j = 0; j < m; ++j) {
187                 ComplexD &P1 = P[i+j+m], &P2 = P[i+j];
188                 ComplexD t = unit * P1;
189                 P1 = P2 - t;
190                 P2 = P2 + t;
191                 unit *= unit_p0;
192             }
193         }
194     }

```

```

195     if(-1 == oper) {
196         for(int i = 0; i < n; ++i)
197             P[i] /= Double(n);
198     }
199 }
200 }

```

## 5.7 NTT

```

1  #include<bits/stdc++.h>
2  #define ll long long
3  const int N=262144;
4  const ll MOD=500000000001507329LL;//998244353 1004535809
5  using namespace std;
6  int n,m;
7  ll a[N],b[N],x[N],y[N];
8  ll wn[25];
9  ll Mul(ll x,ll y) { //乘法超ll用快速乘，主函数也需要用
10     ll ans=(x*y-(ll)((long double)x/MOD*y+1e-8)*MOD);
11     return ans<0?ans+MOD:ans;
12 }
13 ll Qpow(ll a,ll b,ll M) {
14     ll ans=1;
15     a%=M;
16     while(b) {
17         if(b&1) ans=Mul(ans,a);
18         a=Mul(a,a);
19         b>>=1;
20     }
21     return ans;
22 }
23 void Getwn() { //主函数预处理getwn()
24     for(int i=0; i<25; i++) {
25         wn[i]=Qpow(3,(MOD-1)/(1<<i),MOD);
26     }
27 }
28 void NTT(ll *x,int n,int rev) {
29     int i,j,k,ds;
30     ll w,u,v;
31     for(i=1,j=n>>1,k=n>>1; i<n-1; i++,k=n>>1) {
32         if(i<j) swap(x[i],x[j]);
33         while(j>=k) j-=k,k>>=1;
34         if(j<k) j+=k;
35     }
36     for(i=2,ds=1; i<=n; i<=<=1,ds++) {
37         for(j=0; j<n; j+=i) {
38             w=1;
39             for(k=j; k<j+i/2; k++) {
40                 u=x[k];
41                 v=Mul(w,x[k+i/2]);
42                 x[k]=(u+v)%MOD;
43                 x[k+i/2]=(u-v+MOD)%MOD;
44                 w=Mul(w,wn[ds]);
45             }
46         }
47     }
48     if(rev==1) {
49         for(i=1; i<n/2; i++) swap(x[i],x[n-i]);
50         w=Qpow(n,MOD-2,MOD);
51         for(i=0; i<n; i++) x[i]=Mul(x[i],w);
52     }
53 }

```

```

54 int main() {
55     Getwn();
56     while(~scanf("%d%d",&n,&m)) {
57         for(int i=0; i<n; i++)scanf("%lld",&a[i]);
58         for(int i=0; i<m; i++)scanf("%lld",&b[i]);
59         int len=1,s=n+m;
60         while(len<s)len<<=1;
61         for(int i=n; i<len; i++)a[i]=0;
62         for(int i=m; i<len; i++)b[i]=0;
63         NTT(a,len,1);
64         NTT(b,len,1);
65         for(int i=0; i<len; i++)a[i]=Mul(a[i],b[i]);
66         NTT(a,len,-1);
67         for(int i=0; i<=s; i++)printf("%lld_",a[i]);
68         puts("");
69     }
70 }
71
72 // -----
73 #include<cstdio>
74 #include<iostream>
75 #include<cstring>
76 #include<cmath>
77 #include<complex>
78 using namespace std;
79 typedef long long LL;
80 const LL MOD=998244353,g=3,gi=332748118;
81 const LL N=1000005;
82 LL n,m;
83 LL a[N],b[N];
84 LL pow(LL x,LL y) {
85     if (y==1) return x;
86     LL lalal=pow(x,y>>1);
87     lalal=lalal*lalal%MOD;
88     if (y&1) lalal=lalal*x%MOD;
89     return lalal;
90 }
91 void ntt (LL *a,LL n,LL o) {
92     if (n==1) return ;
93     LL k=(n>>1);
94     LL w=1,wn=pow(o==1?g:gi,(MOD-1)/n),a0[k],a1[k];
95     for (LL u=0; u<k; u++) {
96         LL i=u*2;
97         a0[u]=a[i];
98         a1[u]=a[i+1];
99     }
100     ntt(a0,k,o);
101     ntt(a1,k,o);
102     for (LL u=0; u<k; u++) {
103         a[u]=a0[u]+w*a1[u]%MOD;
104         a[u]=(a[u]%MOD+MOD)%MOD;
105         a[u+k]=a0[u]-w*a1[u];
106         a[u+k]=(a[u+k]%MOD+MOD)%MOD;
107         w=w*wn%MOD;
108     }
109 }
110 void ntt(LL *a,LL n,LL op) {
111     for (LL u=0; u<n; u++) bin[u]=(bin[u>>1]>>1)|((u&1)*(n>>1));
112     for (LL u=0; u<n; u++) if (u<bin[u]) swap(a[u],a[bin[u]]);
113     for (LL u=1; u<n; u<<=1) {
114         LL wn=pow(op==1?g:gi,(MOD-1)/(u<<1)),w,t;
115         for (LL i=0; i<n; i=i+(u<<1)) {
116             w=1;
117             for (LL k=0; k<u; k++) {

```

```

118     t=w*a[u+i+k]%MOD;
119     a[u+i+k]=(a[i+k]-t+MOD)%MOD;
120     a[i+k]=(a[i+k]+t)%MOD;
121     w=w*wn%MOD;
122 }
123 }
124 }
125 if(op==1) {
126     LL Inv=pow(n,MOD-2);
127     for(LL i=0; i<n; i++) a[i]=a[i]*Inv%MOD;
128 }
129 }
130
131 int main() {
132     scanf("%I64d%I64d",&n,&m);
133     for (LL u=0; u<=n; u++) scanf("%I64d",&a[u]);
134     for (LL u=0; u<=m; u++) scanf("%I64d",&b[u]);
135     m=m+n;
136     n=1;
137     while (n<=m) n<<=1;
138     ntt(a,n,1);
139     ntt(b,n,1);
140     for (LL u=0; u<=n; u++) a[u]*=b[u];
141     ntt(a,n,-1);
142     LL inv=pow(n,MOD-2);
143     for (LL u=0; u<=m; u++) printf("%I64d",a[u]*inv%MOD);
144     return 0;
145 }

```

## 5.8 Fast Walsh–Hadamard transform

- 异或

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\} + \mathcal{F}\{A_1\}, \mathcal{F}\{A_0\} - \mathcal{F}\{A_1\}]$$

$$\mathcal{F}^{-1}\{A\} = \left[ \mathcal{F}^{-1}\left\{\frac{A_0 + A_1}{2}\right\}, \mathcal{F}^{-1}\left\{\frac{A_0 - A_1}{2}\right\} \right]$$

- 按位与

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\} + \mathcal{F}\{A_1\}, \mathcal{F}\{A_1\}]$$

$$\mathcal{F}^{-1}\{A\} = [\mathcal{F}^{-1}\{A_0\} - \mathcal{F}^{-1}\{A_1\}, \mathcal{F}^{-1}\{A_1\}]$$

- 按位或

$$\mathcal{F}\{A\} = [\mathcal{F}\{A_0\}, \mathcal{F}\{A_1\} + \mathcal{F}\{A_0\}]$$

$$\mathcal{F}^{-1}\{A\} = [\mathcal{F}^{-1}\{A_0\}, \mathcal{F}^{-1}\{A_1\} - \mathcal{F}^{-1}\{A_0\}]$$

```

1 void FWT(int a[],int n) {
2     for(int d=1; d<n; d<<=1)
3         for(int m=d<<1,i=0; i<n; i+=m)
4             for(int j=0; j<d; j++) {
5                 int x=a[i+j],y=a[i+j+d];
6                 a[i+j]=(x+y)%mod,a[i+j+d]=(x-y+mod)%mod;
7                 //xor:a[i+j]=x+y,a[i+j+d]=(x-y+mod)%mod;
8                 //and:a[i+j]=x+y;
9                 //or:a[i+j+d]=x+y;
10            }
11 }
12
13 void UFWT(int a[],int n) {
14     for(int d=1; d<n; d<<=1)
15         for(int m=d<<1,i=0; i<n; i+=m)

```



```

16     for(int j=0; j<d; j++) {
17         int x=a[i+j],y=a[i+j+d];
18         a[i+j]=1LL*(x+y)*rev%mod,a[i+j+d]=(1LL*(x-y)*rev%mod+mod)%mod;
19         //xor:a[i+j]=(x+y)/2,a[i+j+d]=(x-y)/2;
20         //and:a[i+j]=x-y;
21         //or:a[i+j+d]=y-x;
22     }
23 }
24 void solve(int a[],int b[],int n) {
25     FWT(a,n);
26     FWT(b,n);
27     for(int i=0; i<n; i++) a[i]=1LL*a[i]*b[i]%mod;
28     UFWT(a,n);
29 }

```

## 5.9 Lucas

```

1  /* Lucas, by Abreto<m@abreto.net>. */
2
3  struct __lucas {
4      static const int maxp = 1000000;
5      typedef long long int ll;
6      int p;
7      int f[maxp]; // fiv[maxp], inv[maxp];
8      inline int mul(const int a, const int b) {
9          ll z = 1ll * a * b;
10         z -= z / p * p;
11         return z;
12     }
13     int qow(int a, int x) {
14         int ret = 1;
15         while (x) {
16             if (1 & x) ret = mul(ret, a);
17             a = mul(a, a);
18             x >>= 1;
19         }
20         return ret;
21     }
22     void init(int np) {
23         p = np;
24         // return; // uncomment this line if use binom()
25         f[0] = f[1] = 1;
26         // fiv[0] = fiv[1] = 1;
27         // inv[1] = 1;
28         for (int i = 2; i < p; i++) {
29             f[i] = mul(f[i - 1], i);
30             // inv[i] = mul(p - p / i, inv[p % i]);
31             // fiv[i] = mul(fiv[i - 1], inv[i]);
32         }
33     }
34     int C(int n, int k) {
35         if (n < k) return 0;
36         return mul(f[n], qow(mul(f[k], f[n - k]), p - 2));
37     }
38     /** use following if get TLE { */
39     int binom(int n, int k) {
40         if (n < k) return 0;
41         if (k > n - k) k = n - k;
42         int a = 1, b = 1;
43         while (k) {
44             a = mul(a, n);
45             b = mul(b, k);

```

```

46     n--;
47     k--;
48 }
49 return mul(a, qow(b, p - 2));
50 }
51 /** } —— */
52 int operator()(int n, int k) {
53     if (0 == k) return 1;
54     if (n < p && k < p) return C(n, k);
55     return mul(C(n % p, k % p), (*this)(n / p, k / p));
56 }
57 } lucas;

```

## 5.10 Linear Programming

```

1  /* 线性规划 */
2  #include<bits/stdc++.h>
3
4  using namespace std;
5  const int Maxn=110,Maxm=59;
6  class Simplex {
7      /*
8      功能:
9      接受有n个约束, m个基本变量的方程组a[0~n][0~m]
10     a[0][]存放需要最大化的目标函数, a[][0]存放常数
11     Base[]存放基本变量的id,初始为1~m
12     Rest[]存放松弛变量的id,初始为m+1~m+n
13     返回此线性规划的最小值ans
14     要求方案的话, Base[]中的变量值为0,Rest[]中的变量值为相应行的[0]
15     如果solve
16     返回1,说明运行正常ans是它的最大值
17     返回0,说明无可行解
18     返回-1,说明解没有最大值
19     测试:
20     m=2,n=3
21     double a[4][3]={
22         {0,1,3},
23         {8,-1,1},
24         {-3,1,1},
25         {2,1,-4}
26     };
27     solve=1,ans=64/3;
28     注意ac不了可能是eps的问题
29     */
30 public:
31     static const double Inf;
32     static const double eps;
33     int n,m;
34     double a[Maxn][Maxm];
35     int Base[Maxm],Rest[Maxn];
36     double val[Maxm];
37     double ans;
38     void pt() {
39         for(int i=0; i<=n; i++) {
40             for(int j=0; j<=m; j++)printf("%.2f ",a[i][j]);
41             puts("");
42         }
43     }
44     void pivot(int x,int y) { //将第x个非基本变量和第y个基本变量调换
45         swap(Rest[x],Base[y]);
46         double tmp=-1./a[x][y];
47         a[x][y]=-1.;

```

```

48     for(int j=0; j<=m; j++)a[x][j]*=tmp;
49     for(int i=0; i<=n; i++) {
50         if(i==x||fabs(a[i][y])<eps)continue;
51         tmp=a[i][y];
52         a[i][y]=0;
53         for(int j=0; j<=m; j++)a[i][j]+=tmp*a[x][j];
54     }
55 }
56 bool opt() {
57     while(1) {
58         int csi=0;
59         for(int i=1; i<=m; i++)if(a[0][i]>eps&&(!csi||Base[i]<Base[csi]))csi=i;
60         if(!csi)break;
61         int csj=0;
62         double cur;
63         for(int j=1; j<=n; j++) {
64             if(a[j][csi]>=eps)continue;
65             double tmp=-a[j][0]/a[j][csi];
66             if(!csj||tmp+eps<cur||(fabs(tmp-cur)<eps&&Rest[j]<Rest[csj]))csj=j,cur=tmp;
67         }
68         if(!csj)return 0;
69         pivot(csj,csi);
70     }
71     ans=a[0][0];
72     return 1;
73 }
74 bool init() {
75     ans=0;
76     for(int i=1; i<=m; i++)Base[i]=i;
77     for(int i=1; i<=n; i++)Rest[i]=m+i;
78     int cs=1;
79     for(int i=2; i<=n; i++)if(a[i][0]<a[cs][0])cs=i;
80     if(a[cs][0]>=eps)return 1;
81     static double tmp[Maxm];
82     for(int i=0; i<=m; i++)tmp[i]=a[0][i],a[0][i]=0;
83     for(int i=1; i<=n; i++)a[i][m+1]=1.;
84     a[0][m+1]=-1.;
85     Base[m+1]=m+n+1;
86     pivot(cs,++m);
87     opt();
88     m--;
89     if(a[0][0]<=eps)return 0;
90     cs=-1;
91     for(int i=1; i<=n; i++) {
92         if(Rest[i]>m+n) {
93             cs=i;
94             break;
95         }
96     }
97     if(cs>=1) {
98         int nxt=-1;
99         m++;
100         for(int i=1; i<=m; i++)if(a[cs][i]>eps||a[cs][i]<=eps) {
101             nxt=i;
102             break;
103         }
104         pivot(cs,nxt);
105         m--;
106     }
107     for(int i=1; i<=m; i++) {
108         if(Base[i]>m+n) {
109             swap(Base[i],Base[m+1]);
110             for(int j=0; j<=n; j++)a[j][i]=a[j][m+1];
111             break;

```

```

112     }
113 }
114 for(int i=1; i<=m; i++)a[0][i]=0;
115 a[0][0]=tmp[0];
116 for(int i=1; i<=m; i++)if(Base[i]<=m)a[0][i]=tmp[Base[i]];
117 for(int i=1; i<=n; i++) {
118     if(Rest[i]<=m) {
119         for(int j=0; j<=m; j++)a[0][j]+=tmp[Rest[i]]*a[i][j];
120     }
121 }
122 return 1;
123 }
124 void getval() {
125     for(int i=1; i<=m; i++)val[i]=0;
126     for(int i=1; i<=n; i++)if(Rest[i]<=m)val[Rest[i]]=a[i][0];
127     //for(int i=1; i<=m; i++)printf("%.2f ",val[i]);puts("");
128 }
129 int solve() {
130     if(!init())return 0;
131     if(!opt())return -1;
132     getval();
133     return 1;
134 }
135 } solver;
136 const double Simplex:: Inf=1e80;
137 const double Simplex:: eps=1e-8;
138 int main() {
139     int m,n,type;
140     scanf("%d%d%d",&m,&n,&type);
141     solver.a[0][0]=0;
142     for(int i=1; i<=m; i++)scanf("%lf",&solver.a[0][i]);
143     for(int i=1; i<=n; i++) {
144         for(int j=1; j<=m+1; j++) {
145             if(j==m+1)scanf("%lf",&solver.a[i][0]);
146             else {
147                 scanf("%lf",&solver.a[i][j]);
148                 solver.a[i][j]=-solver.a[i][j];
149             }
150         }
151     }
152     solver.m=m,solver.n=n;
153     int rep=solver.solve();
154     if(rep==0)puts("Infeasible");
155     else if(rep==-1)puts("Unbounded");
156     else {
157         printf("%.12f\n",solver.ans);
158         if(type==1) {
159             for(int i=1; i<=m; i++)printf("%.12f%c",solver.val[i],i==m?'\\n':' ');
160         }
161     }
162 }

```

## 5.11 Big Prime Test

```

1 #include <iostream>
2 #include <cstdlib>
3 using namespace std;
4 typedef long long LL;
5 LL minfactor, p[11] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29};
6 LL gcd(LL a, LL b) {
7     return b ? gcd(b, a % b) : a;
8 }

```

```

9  LL qmult(LL a, LL b, LL mod) { // 快速乘模
10     LL sum = 0;
11     while (b) {
12         if (b & 1) {
13             sum += a;
14             if (sum >= mod) sum -= mod; // 此处无需用%, %运算比减法慢很多
15         }
16         b >>= 1, a <<= 1;
17         if (a >= mod) a -= mod;
18     }
19     return sum;
20 }
21 LL qpow(LL a, LL b, LL mod) { // 快速幂模
22     LL res = 1;
23     while (b) {
24         if (b & 1) res = qmult(res, a, mod);
25         b >>= 1;
26         a = qmult(a, a, mod);
27     }
28     return res;
29 }
30 bool prime_test(LL n, LL a) { // 对整数n, 底数a进行测试, 返回true表示通过测试
31     LL p = qpow(a, n - 1, n);
32     if (p != 1) return false;
33     else { // 二次探测
34         LL s = n - 1;
35         while (!(s & 1) && p == 1) {
36             s >>= 1;
37             p = qpow(a, s, n);
38         }
39         if (p == 1 || p == n - 1) return true;
40         else return false;
41     }
42 }
43 bool Miller_Rabin(LL n) { // 对整数n进行Miller_Rabin素数测试, 返回true表示通过测试
44     if (n <= 29) { // if这一块其实可以不用
45         for (int i = 0; i < 10; i++) {
46             if (n == p[i]) return true;
47         }
48         return false;
49     }
50     for (int i = 0; i < 10; i++) { // 利用前10个素数作为底数测试的正确率已经非常高
51         if (gcd(n, p[i]) == 1 && !prime_test(n, p[i])) return false;
52     }
53     return true;
54 }
55 LL randf(LL x, LL n, LL c) { // 满足要求的产生伪随机数函数
56     return (qmult(x, x, n) + c) % n;
57 }
58 LL pollard_rho(LL n, LL c) { // 查找n的因数, c为上面函数要用的随机数, c也可自己指定
    (但要有变化)
59     LL x = rand() % n, y = x, i = 1, k = 2, p; // 随机生成随机数的初始值, 也可自己指定
60     while (true) {
61         i++;
62         x = randf(x, n, c);
63         p = gcd(y - x + n, n);
64         if (p > 1 && p < n) return p;
65         if (y == x) return n; // 判圈, 返回n表示查找失败, 要更新随机种子重新查找
66         if (i == k) {
67             y = x; // 更新范围和记录的数
68             k <<= 1;
69         }
70     }
71 }

```

```

72 void find_factor(LL n) { // 查找所有因数
73     if (Miller_Rabin(n)) {
74         minfactor = min(minfactor, n);
75         return ;
76     }
77     LL p = n;
78     while (p == n) p = pollard_rho(n, rand() % (n - 1) + 1); // 查找失败则更新随机种子
79     // 重新查找, 直到找到因子
80     find_factor(p); // 递归查找更小因子
81     find_factor(n / p);
82 }
83 int main() {
84     int t;
85     cin >> t;
86     while (t--) {
87         LL N;
88         cin >> N;
89         if (Miller_Rabin(N)) cout << "Prime" << endl;
90         else {
91             minfactor = N;
92             find_factor(N);
93             cout << minfactor << endl;
94         }
95     }
96     return 0;
97 }

```

### 5.11.1 Miller Rabin

```

1  /* Miller-Rabin Prime Test, by Abreto<m@abreto.net>. */
2
3  namespace miller_rabin {
4
5  typedef long long int ll;
6
7  inline ll add(const ll a, const ll b, const ll mod) {
8      ll z = a + b;
9      if (z >= mod) z -= mod;
10     return z;
11 }
12 inline ll mul(ll a, ll b, const ll mod) {
13     ll z = 0;
14     if (a >= mod) a %= mod;
15     if (b >= mod) b %= mod;
16     while (b) {
17         if (1 & b) z = add(z, a, mod);
18         a = add(a, a, mod);
19         b >>= 1;
20     }
21     return z;
22 }
23
24 ll qow(ll a, ll x, ll mod) {
25     ll ret = 1ll;
26     while (x) {
27         if (1 & x) ret = mul(ret, a, mod);
28         a = mul(a, a, mod);
29         x >>= 1;
30     }
31     return ret;
32 }
33 }

```

```

34 const int K = 5;
35 const int p[] = {
36     2, 3, 7, 61, 24251
37 };
38 const ll strong = 4685624825598111;
39 /* 46 856 248 255 981 in (0, 1e16) */
40
41 bool mr(ll n, int k) {
42     ll d = n - 1;
43     int s = 0;
44     while (d > 1 && 0 == (d & 1)) {
45         s++;
46         d >>= 1;
47     }
48     for (int i = 0; i < k; i++) {
49         ll a = (i < K) ? p[i] : (1 + rand() % (n - 1));
50         ll x = qow(a, d, n);
51         for (int j = 0; j < s; j++) {
52             ll xp = mul(x, x, n);
53             if (1 == xp && x != 1 && x != n-1) return false;
54             x = xp;
55         }
56         if (x != 1) return false;
57     }
58     return true;
59 }
60
61 /* 2,3,5,7,11,13 */
62 const int pre[] = {3, 5, 7, 11, 13};
63 bool test(ll n, int k = 5) {
64     if (2 == n) return true;
65     if (0 == (n & 1)) return false;
66     if (strong == n) return false;
67     for (int i = 0; i < 5; i++) {
68         if (n == pre[i]) return true;
69         if (n == n / pre[i] * pre[i])
70             return false;
71     }
72     return mr(n, k);
73 }
74
75 }

```

### 5.11.2 Pollard's rho

```

1  /* Pollard's rho, by Abreto<m@abreto.net>. */
2
3  namespace pollards_rho {
4
5  typedef long long int ll;
6
7  inline ll add(const ll a, const ll b, ll mod) {
8      ll z = a + b;
9      if (z >= mod) z -= mod;
10     return z;
11 }
12 inline ll mul(ll a, ll b, ll mod) {
13     ll z = 0ll;
14     if (a >= mod) a -= a / mod * mod;
15     if (b >= mod) b -= b / mod * mod;
16     while (b) {
17         if (1 & b) z = add(z, a, mod);
18         a = add(a, a, mod);

```

```

19     b >= 1;
20 }
21 return z;
22 }
23
24 ll gcd(ll m, ll n) {
25     return (0 == n) ? m : gcd(n, m % n);
26 }
27
28 ll find(ll n, int c = -1) {
29     ll x = rand() % n;
30     ll y = x, k = 2;
31     for (int i = 2; ; i++) {
32         x = add(mul(x, x, n), (n + c) % n, n);
33         ll d = gcd(y - x + n, n); // change to abs(y - x) if get WA
34         if (1 != d && n != d) return d;
35         if (y == x) return n;
36         if (i == k) {
37             y = x;
38             k <<= 1;
39         }
40     }
41 }
42
43 /** usage:
44  * void find(ll n, int c = 107)
45  * {
46  *     if (1 == n) return;
47  *     if (miller-rabin(n) )
48  *     {
49  *         n is a prime;
50  *         return;
51  *     }
52  *     ll p = n, k = c;
53  *     while (p >= n) p = pollards_rho(p, k--);
54  *     find(p, c);
55  *     find(n/p, c);
56  * }
57  */
58
59 }

```

## 5.12 Montgomery modular multiplication

```

1  /* — Montgomery modular algorithm { — */
2  struct Mod64 {
3      typedef long long ll;
4      typedef unsigned long long u64;
5      typedef __int128_t i128;
6      typedef __uint128_t u128;
7      Mod64() :n_(0) {}
8      Mod64(u64 n) :n_(init(n)) {}
9      static u64 init(u64 w) {
10         return reduce(u128(w) * r2);
11     }
12 }
13 static void set_mod(u64 m) {
14     mod = m;
15     assert(mod & 1);
16     inv = m;
17     for (int i = 0; i < 5; ++i) inv *= 2 - inv * m;
18     r2 = -u128(m) % m;

```



```

19 }
20 static u64 reduce(u128 x) {
21     u64 y = u64(x >> 64) - u64((u128(u64(x)*inv)*mod) >> 64);
22     return ll(y)<0 ? y + mod : y;
23 }
24 Mod64& operator += (Mod64 rhs) {
25     n_ += rhs.n_ - mod;
26     if (ll(n_)<0) n_ += mod;
27     return *this;
28 }
29 Mod64 operator + (Mod64 rhs) const {
30     return Mod64(*this) += rhs;
31 }
32 Mod64& operator -= (Mod64 rhs) {
33     n_ -= rhs.n_;
34     if (ll(n_)<0) n_ += mod;
35     return *this;
36 }
37 Mod64 operator - (Mod64 rhs) const {
38     return Mod64(*this) -= rhs;
39 }
40 Mod64& operator *= (Mod64 rhs) {
41     n_ = reduce(u128(n_)*rhs.n_);
42     return *this;
43 }
44 Mod64 operator * (Mod64 rhs) const {
45     return Mod64(*this) *= rhs;
46 }
47 u64 get() const {
48     return reduce(n_);
49 }
50 static u64 mod, inv, r2;
51 u64 n_;
52 };
53
54 Mod64::u64 Mod64::mod, Mod64::inv, Mod64::r2;
55 /* — } Montgomery modular algorithm — */
56
57 /**
58  * usage:
59  * First, Mod64::set_mod();
60  * Mod64 a, b, c(init_val);
61  * a = b * c;
62  * printf("%llu\n", a.get());
63  */

```

### 5.13 Berlekamp Massey

```

1  /* Berlekamp Massey by HoldZhu. */
2  #include <cstdio>
3  #include <vector>
4
5  using namespace std;
6
7  namespace BerlekampMassey {
8      const int mod = 1e9 + 7;
9      int L, m, b, n;
10     vector<int> s, C, B;
11     void init() {
12         s.clear();
13         C.clear();
14         B.clear();

```

```

15 C.push_back(1);
16 B.push_back(1);
17 L = n = 0;
18 m = b = 1;
19 }
20 int pow_mod(int a, int k) {
21     int s = 1;
22     while (k) {
23         if (k & 1)
24             s = 1ll * s * a % mod;
25         a = 1ll * a * a % mod;
26         k >>= 1;
27     }
28     return s;
29 }
30 void update(int d) {
31     s.push_back(d);
32     for (int i = 1; i <= L; ++i)
33         d = (d + 1ll * C[i] * s[n - i] % mod) % mod;
34     if (d == 0)
35         ++m;
36     else if (2 * L <= n) {
37         vector<int> T = C;
38         C.resize(n + 1 - L + 1);
39         for (int i = L + 1; i <= n + 1 - L; ++i)
40             C[i] = 0;
41         for (int i = 0; i < B.size(); ++i)
42             C[i + m] = (C[i + m] + mod - 1ll * d * pow_mod(b, mod - 2) % mod * B[i] % mod)
43                 % mod;
44         L = n + 1 - L;
45         B = T;
46         b = d;
47         m = 1;
48     } else {
49         for (int i = 0; i < B.size(); ++i)
50             C[i + m] = (C[i + m] + mod - 1ll * d * pow_mod(b, mod - 2) % mod * B[i] % mod)
51                 % mod;
52         ++m;
53     }
54     ++n;
55 }
56 void output() {
57     printf("F(n)=");
58     for (int i = 1; i < C.size(); ++i) {
59         int output = (mod - C[i]) % mod;
60         if (output > mod / 2)
61             output -= mod;
62         printf("%s%d*F(n-%d)", (output < 0 || i == 1) ? "" : "+", output, i);
63     }
64     puts("");
65 }
66 void output_code_for() {
67     static const char *name = "dp";
68     static const char *index = "i";
69     static const char *upperbound = "maxn";
70     puts("//_Generated_by_Berlekamp-Massey_algorithm");
71     for (int i = 1; i < C.size(); ++i) {
72         printf("%s[%d]=%d;\n", name, i - 1, s[i - 1]);
73     }
74     printf("for(int i=%d;i<%s;++i)\n", (int)C.size() - 1, upperbound);
75     printf("_s[%s]=(", name, index);
76     for (int i = 1; i < C.size(); ++i) {
77         int output = (mod - C[i]) % mod;
78         if (output > mod / 2)

```

```

77     output -= mod;
78     printf("%s%d*s[%s-%d]%%mod", (output < 0 || i == 1) ? "" : "+", output, name,
        index, i);
79 }
80 puts(")%mod+mod)%mod;");
81 }
82 void output_code_matrix() {
83     // TODO
84 }
85 };
86
87 /** usage */
88 int usage() {
89     // int arr[12] = {2, 24, 96, 416, 1536, 5504, 18944, 64000, 212992, 702464,
        2301952, 7512064};
90     int arr[] = {3, 20, 119, 696, 4059, 23660, 137903, 803760, 4684659};
91     BerlekampMassey::init();
92     for (auto ai : arr) {
93         BerlekampMassey::update(ai);
94     }
95     printf("Formule:␣");
96     BerlekampMassey::output();
97     printf("Code:␣\n");
98     BerlekampMassey::output_code_for();
99     return 0;
100 }

```

## 5.14 Inclusion–exclusion principle

### 5.14.1 General form

若  $A_1, A_2, \dots, A_n$  为有限集, 则

$$\left| \bigcup_{i=1}^n A_i \right| = \sum_{\emptyset \neq J \subseteq \{1, 2, \dots, n\}} (-1)^{|J|-1} \left| \bigcap_{j \in J} A_j \right|$$

### 5.14.2 A generalization

若

$$g(A) = \sum_{S \subseteq A} f(S)$$

则

$$f(A) = \sum_{S \subseteq A} (-1)^{|A|-|S|} g(S)$$

更一般的, 如果  $S$  是多重集合 (multiset), 那么

$$f(A) = \sum_{S \subseteq A} \mu(A - S) g(S)$$

其中

- 当  $S$  是含有偶数个元素的集合 (没有重复元素) 时,  $\mu(S) = 1$
- 当  $S$  是含有奇数个元素的集合 (没有重复元素) 时,  $\mu(S) = -1$
- 当  $S$  含有重复元素时,  $\mu(S) = 0$ .

### 5.14.3 Applications

#### I. 乱序排列

如果集合  $A$  含有  $n$  个元素, 则乱序排列的数目为  $[n!/e]$ ,  $[x]$  表示最接近  $x$  的整数.

### 5.15 Lindström—Gessel—Viennot lemma

对于一张无边权的 DAG 图, 给定  $n$  个起点和对应的  $n$  个终点, 这  $n$  条不相交路径的方案数为

$$\begin{vmatrix} e(a_1, b_1) & e(a_1, b_2) & \cdots & e(a_1, b_n) \\ e(a_2, b_1) & e(a_2, b_2) & \cdots & e(a_2, b_n) \\ \vdots & \vdots & \ddots & \vdots \\ e(a_n, b_1) & e(a_n, b_2) & \cdots & e(a_n, b_n) \end{vmatrix} \quad (\text{该矩阵的行列式})$$

其中  $e(a, b)$  为图上  $a$  到  $b$  的方案数.

## 6 String

### 6.1 Hash

```

1 /* Common hash for any substrings. */
2
3 typedef unsigned long long int llu;
4 #define MAXN 1000000
5 int n;
6 char s[MAXN];
7 llu H[MAXN], xP[MAXN], P = 9999111;
8 void init(void) {
9     int i = 0;
10    xP[0] = 111;
11    for(i = 1; i < MAXN; ++i) xP[i] = xP[i-1] * P;
12    H[n] = 0;
13    for(i = n-1; i >= 0; --i) H[i] = H[i+1]*P + s[i];
14 }
15 #define HASH(i,l) (H[i] - H[i+l]*xP[l])

```

### 6.2 KMP

```

1 /* KMP, by Abreto<m@abreto.net>. */
2 #include <string.h>
3
4 /* !!NEED IMPROVING!! */
5
6 #define MAXL (1000010)
7
8 char W[MAXL], T[MAXL];
9 int f[MAXL];
10 int lW, lT;
11
12 int count(void) {
13     int cnt = 0;
14     int i, j;
15     lW = strlen(W);
16     lT = strlen(T);
17     // — self-matching
18     f[0] = j = -1;
19     for(i = 1; i < lW; i++) {
20         while( j >= 0 && W[j+1] != W[i] )

```

```

21     j = f[j];
22     if( W[j+1] == W[i] ) j++;
23     f[i] = j;
24 }
25 // ——
26 j = -1;
27 for(i = 0; i < lT; i++) {
28     while( j >= 0 && W[j+1] != T[i] )
29         j = f[j];
30     if( W[j+1] == T[i] ) j++;
31     if( j == lW-1 ) {
32         cnt++;
33         j = f[j];
34     }
35 }
36 return cnt;
37 }

```

### 6.3 exKMP

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  namespace exkmp {
5
6  const int maxn = 1000100, maxm = 1000100;
7
8  int n, m;
9  char S[maxn], T[maxm];
10
11  /* the length of longest prefix between T[i..m-1] and T[0..m-1] */
12  int nxt[maxm];
13  /* the length of longest prefix between S[i..n-1] and T[0..m-1] */
14  int ex[maxn];
15
16  void getsize() {
17     n = strlen(S);
18     m = strlen(T);
19 }
20
21  void self(void) {
22     int q = 1, p = 0;
23     nxt[0] = m;
24     while (1 + p < m && T[1 + p] == T[p]) p++;
25     nxt[1] = p;
26     for (int i = 2; i < m; i++) {
27         int l = nxt[i - q];
28         if (i + l - 1 < p) {
29             nxt[i] = l;
30         } else {
31             int j = max(0, p - i + 1);
32             while (i + j < m && T[i + j] == T[j]) j++;
33             nxt[i] = j;
34             p = i + j - 1;
35             q = i;
36         }
37     }
38 }
39
40  void run(void) {
41     int q = 0, p = 0;
42     self();

```

```

43 while (p < n && p < m && S[p] == T[p]) p++;
44 ex[0] = p;
45 p--;
46 for (int i = 1; i < n; i++) {
47     int l = nxt[i - q];
48     if (i + l - 1 < p) {
49         ex[i] = l;
50     } else {
51         int j = max(0, p - i + 1);
52         while (i + j < n && S[i + j] == T[j]) j++;
53         ex[i] = j;
54         p = i + j - 1;
55         q = i;
56     }
57 }
58 }
59
60 void inspect(void) {
61     printf("S:");
62     for (int i = 0; i < n; i++) putchar(S[i]);
63     puts("");
64     printf("T:");
65     for (int i = 0; i < m; i++) putchar(T[i]);
66     puts("");
67     printf("next:");
68     for (int i = 0; i < m; i++) printf(" %d", nxt[i]);
69     puts("");
70     printf("extend:");
71     for (int i = 0; i < n; i++) printf(" %d", ex[i]);
72     puts("");
73 }
74
75 } // exkmp

```

## 6.4 Suffix Array

```

1  /* Suffix Array, copied. */
2
3  #define MAXN    (200010)
4  namespace mzry_sa {
5  int wx[MAXN],wy[MAXN],*x,*y,wss[MAXN],wv[MAXN];
6
7  bool dacmp(int *r,int n,int a,int b,int l) {
8      return a+l<n && b+l<n && r[a]==r[b]&&r[a+l]==r[b+l];
9  }
10 void da(int str[],int sa[],int rank[],int height[],int n,int m) {
11     int *s = str;
12     int *x=wx,*y=wy,*t,p;
13     int i,j;
14     for(i=0; i<m; i++)wss[i]=0;
15     for(i=0; i<n; i++)wss[x[i]=s[i]]++;
16     for(i=1; i<m; i++)wss[i]+=wss[i-1];
17     for(i=n-1; i>=0; i--)sa[--wss[x[i]]]=i;
18     for(j=1,p=1; p<n && j<n; j*=2,m=p) {
19         for(i=n-j,p=0; i<n; i++)y[p++]=i;
20         for(i=0; i<n; i++)if(sa[i]-j>=0)y[p++]=sa[i]-j;
21         for(i=0; i<n; i++)wv[i]=x[y[i]];
22         for(i=0; i<m; i++)wss[i]=0;
23         for(i=0; i<n; i++)wss[wv[i]]++;
24         for(i=1; i<m; i++)wss[i]+=wss[i-1];
25         for(i=n-1; i>=0; i--)sa[--wss[wv[i]]]=y[i];
26         for(t=x,x=y,y=t,p=1,i=1,x[sa[0]]=0; i<n; i++)

```

```

27     x[sa[i]]=dacmp(y,n,sa[i-1],sa[i],j)?p-1:p++;
28 }
29 for(int i=0; i<n; i++) rank[sa[i]]=i;
30 for(int i=0,j=0,k=0; i<n; height[rank[i++]]=k)
31     if(rank[i]>0)
32         for(k?k--:0,j=sa[rank[i]-1];
33             i+k < n && j+k < n && str[i+k]==str[j+k];
34             k++);
35 }
36 }
37
38 /*
39 Suffix array O(n lg^2 n)
40 LCP table O(n)
41 */
42 #include <cstdio>
43 #include <algorithm>
44 #include <cstring>
45
46 using namespace std;
47
48 #define REP(i, n) for (int i = 0; i < (int)(n); ++i)
49
50 namespace SuffixArray {
51     const int MAXN = 1 << 21;
52     char * S;
53     int N, gap;
54     int sa[MAXN], pos[MAXN], tmp[MAXN], lcp[MAXN];
55
56     bool sufCmp(int i, int j) {
57         if (pos[i] != pos[j])
58             return pos[i] < pos[j];
59         i += gap;
60         j += gap;
61         return (i < N && j < N) ? pos[i] < pos[j] : i > j;
62     }
63
64     void buildSA() {
65         N = strlen(S);
66         REP(i, N) sa[i] = i, pos[i] = S[i];
67         for (gap = 1; gap <= N; gap <<= 1) {
68             sort(sa, sa + N, sufCmp);
69             REP(i, N - 1) tmp[i + 1] = tmp[i] + sufCmp(sa[i], sa[i + 1]);
70             REP(i, N) pos[sa[i]] = tmp[i];
71             if (tmp[N - 1] == N - 1) break;
72         }
73     }
74
75     void buildLCP() {
76         for (int i = 0, k = 0; i < N; ++i) if (pos[i] != N - 1) {
77             for (int j = sa[pos[i] + 1]; S[i + k] == S[j + k];)
78                 ++k;
79             lcp[pos[i]] = k;
80             if (k) --k;
81         }
82     }
83 } // end namespace SuffixArray
84
85 namespace HashSuffixArray {
86     const int
87     MAXN = 1 << 21;
88
89     typedef unsigned long long hash;
90

```

```

91 const hash BASE = 137;
92
93 int N;
94 char * S;
95 int sa[MAXN];
96 hash h[MAXN], hPow[MAXN];
97
98 #define getHash(lo, size) (h[lo] - h[(lo) + (size)] * hPow[size])
99
100 inline bool sufCmp(int i, int j) {
101     int lo = 1, hi = min(N - i, N - j);
102     while (lo <= hi) {
103         int mid = (lo + hi) >> 1;
104         if (getHash(i, mid) == getHash(j, mid))
105             lo = mid + 1;
106         else
107             hi = mid - 1;
108     }
109     return S[i + hi] < S[j + hi];
110 }
111
112 void buildSA() {
113     N = strlen(S);
114     hPow[0] = 1;
115     for (int i = 1; i <= N; ++i)
116         hPow[i] = hPow[i - 1] * BASE;
117     h[N] = 0;
118     for (int i = N - 1; i >= 0; --i)
119         h[i] = h[i + 1] * BASE + S[i], sa[i] = i;
120
121     stable_sort(sa, sa + N, sufCmp);
122 }
123
124 } // end namespace HashSuffixArray
125
126 namespace lrj_sa {
127     const int MAXN = 1000;
128     char s[MAXN]; /* 原始字符数组 (最后一个字符应必须是0, 而前面的字符必须非0) */
129     int sa[MAXN], t[MAXN], t2[MAXN], c[MAXN], n; /* n seems to be the length of s. */
130     /* every character is in [0,m-1] */
131     void build_sa(int m) {
132         int i, *x = t, *y = t2;
133         for(i = 0; i < m; ++i) c[i] = 0;
134         for(i = 0; i < n; i++) c[x[i]=s[i]]++;
135         for(i = 1; i < m; ++i) c[i] += c[i-1];
136         for(i = n-1; i >= 0; --i) sa[--c[x[i]]] = i;
137         for(int k = 1; k <= n; k <= 1) {
138             int p = 0;
139             for(i = n-k; i < n; ++i) y[p++] = i;
140             for(i = 0; i < n; ++i) if(sa[i] >= k) y[p++] = sa[i]-k;
141             for(i = 0; i < m; i++) c[i] = 0;
142             for(i = 0; i < n; i++) c[x[y[i]]]++;
143             for(i = 0; i < m; ++i) c[i] += c[i-1];
144             for(i = n-1; i >= 0; --i) sa[--c[x[y[i]]]] = y[i];
145             swap(x,y);
146             p = 1;
147             x[sa[0]] = 0;
148             for(i = 1; i < n; ++i)
149                 x[sa[i]] = y[sa[i-1]]==y[sa[i]] && y[sa[i-1]+k]==y[sa[i]+k] ? p-1:p++;
150             if(p >= n) break;
151             m = p;
152         }
153     }
154     int rank[MAXN], height[MAXN];

```



```

155 void get_height(void) {
156     int i,j,k = 0;
157     for(i = 0; i < n; ++i) rank[sa[i]] = i;
158     for(i = 0; i < n; ++i) {
159         if(k) k--;
160         j = sa[rank[i]-1];
161         while(s[i+k]==s[j+k]) k++;
162         height[rank[i]] = k;
163     }
164 }
165 } // end namespace lrj_sa

```

## 6.5 Aho-Corasick Automata

```

1  /* Aho-Corasick automaton algorithm, by Abreto<m@abreto.net>. */
2
3  #define MAXN 500500
4  #define NALPHA 26
5  #define FIRSTA 'a'
6
7  /* pointer version => { */
8  struct vtx {
9      vtx *nxt[NALPHA];
10     vtx *fail;
11     int end;
12 } vtxs[MAXN];
13 int nvtxs;
14 void myclr(void) {
15     nvtxs = 0;
16 }
17 vtx *new_vtx(void) {
18     vtx *ret = vtxs+(nvtxs++);
19     for(int i = 0; i < NALPHA; i++)
20         ret->nxt[i] = NULL;
21     ret->fail = NULL;
22     ret->end = 0;
23     return ret;
24 }
25 void myins(vtx *root, char const *s) {
26     for( ; *s ; s++ ) {
27         int of = (*s) - FIRSTA;
28         if ( NULL == root->nxt[of] ) {
29             root->nxt[of] = new_vtx();
30         }
31         root = root->nxt[of];
32     }
33     root->end++;
34 }
35 void build_ac(vtx *root) {
36     queue<vtx *> q;
37     q.push(root);
38     while(!q.empty()) {
39         vtx *p = q.front();
40         q.pop();
41         for(int i = 0; i < NALPHA; i++) {
42             if( NULL == p->nxt[i] ) continue;
43             if( root == p ) p->nxt[i]->fail = root;
44             else {
45                 vtx *t = p->fail;
46                 while ( t && NULL == t->nxt[i] ) {
47                     t = t->fail;
48                 }

```

```

49     if (t) p->nxt[i]->fail = t->nxt[i];
50     else p->nxt[i]->fail = root;
51 }
52 /* version[1] { */
53 p->nxt[i]->end += p->nxt[i]->fail->end; /* update this sum, add its existing
    prefix to this. */
54 /* } */
55 q.push(p->nxt[i]);
56 }
57 }
58 }
59 int qry(vtx *root, char const *s) {
60     vtx *p = root;
61     int cnt = 0;
62     for( ; *s ; s++ ) {
63         int of = (*s) - FIRSTA;
64         while( p != root && NULL == p->nxt[of] ) {
65             p = p->fail;
66         }
67         if (p->nxt[of]) p = p->nxt[of];
68         cnt += p->end; // correct when version[1] exists.
69         // if version[1] not exists, you need to add all ends from this vertex up.
70         //for( vtx *t = p ; t ; t = t->fail )
71         // cnt += t->end;
72     }
73     return cnt;
74 }
75 /* } */
76
77 /* — usage (of pointer version) — */
78 #include <bits/stdc++.h>
79 using namespace std;
80
81 char S[1000100];
82 char pat[64];
83
84 int main(void) {
85     int T, N;
86     vtx *root = NULL;
87     scanf("%d", &T);
88     while(T--) {
89         myclr();
90         root = new_vtx();
91         scanf("%s", S);
92         scanf("%d", &N);
93         while(N--) {
94             scanf("%s", pat);
95             myins(root, pat);
96         }
97         build_ac(root);
98         printf("%d\n", qry(root, S));
99     }
100     return 0;
101 }

```

## 6.6 Manacher

```

1 char t[MAXL<<1];
2 int p[MAXL<<1];
3 int manacher(char *s) {
4     int i;
5     int sl = strlen(s);

```

```

6   int pos = 0, mxr = 0;
7   int ret = 0;
8   t[0] = '^';
9   for(i = 0; i < sl; ++i) {
10      t[i*2+1] = '#';
11      t[i*2+2] = s[i];
12  }
13  t[sl*2+1] = '#';
14  t[sl*2+2] = '$';
15  sl = sl*2+2;
16  for(i = 1; i < sl; ++i) {
17      if(i <= mxr) {
18          p[i] = min(p[2*pos - i], mxr-i+1);
19      } else {
20          p[i] = 1;
21      }
22      while( t[i-p[i]] == t[i+p[i]] ) p[i]++;
23      if( i + p[i] - 1 > mxr ) {
24          mxr = i+p[i]-1;
25          pos = i;
26      }
27      ret = max(ret, p[i]-1);
28  }
29  return ret;
30 }

```

## 7 Utility

### 7.1 IO plug-in

```

1  /* I/O Plug-in, by Abreto <m@abreto.net>. */
2  #include <stdio.h>
3
4  #if ( _WIN32 || __WIN32__ || _WIN64 || __WIN64__ )
5  #define INT64 "%I64d"
6  #else
7  #define INT64 "%lld"
8  #endif
9
10 #if ( _WIN32 || __WIN32__ || _WIN64 || __WIN64__ )
11 #define UNS64 "%I64u"
12 #else
13 #define UNS64 "%llu"
14 #endif
15
16 #define ISDIGIT(x) ((x>='0')&&(x<='9'))
17 int readn(int *n) {
18     int c=0;
19     *n=0;
20     for(; !ISDIGIT(c); c=getchar());
21     for(; ISDIGIT(c); c=getchar()) *n=(*n)*10+c-'0';
22     return (*n);
23 }
24 void putn(int n) {
25     int ns[16]= {0,n%10},nd=1;
26     while(n/=10)ns[++nd]=n%10;
27     while(nd)putchar(ns[nd--]+'0');
28 }

```

  

```

1  #include <cstdio>
2
3  class abio {

```

```

4 static const unsigned BUF_SZ = 65536;
5 FILE *istream, *ostream;
6 char ibuf[BUF_SZ], obuf[BUF_SZ];
7 bool reached_eof;
8 size_t ip, isz;
9 size_t op, osz;
10 inline void clear_ibuf(void) {
11     ip = isz = 0u;
12 }
13 inline void clear_obuf(void) {
14     op = osz = 0u;
15 }
16 inline void clear_buffer(void) {
17     reached_eof = false;
18     clear_ibuf();
19     clear_obuf();
20 }
21 inline size_t read_buffer(void) {
22     isz = std::fread(ibuf, sizeof(char), BUF_SZ, istream);
23     ip = 0;
24     return isz;
25 }
26 inline size_t write_buffer(void) {
27     if(osz) {
28         size_t ret = std::fwrite(obuf+op, sizeof(char), osz-op, ostream);
29         op += ret;
30         if(op == osz) clear_obuf();
31         return ret;
32     }
33     return 0;
34 }
35 inline abio &reach_eof(void) {
36     reached_eof = true;
37     return (*this);
38 }
39 public:
40 static const char endl = '\n';
41 abio(FILE *input = stdin, FILE *output = stdout) {
42     this->istream = input;
43     this->ostream = output;
44     clear_buffer();
45 }
46 abio(const char *input, const char *output) {
47     this->istream = std::fopen(input, "r");
48     this->ostream = std::fopen(output, "w+");
49     clear_buffer();
50 }
51 ~abio(void) {
52     write_buffer();
53     std::fclose(istream);
54     std::fclose(ostream);
55 }
56 operator bool() const {
57     return (!reached_eof);
58 }
59 inline int getchar(void) {
60     if(isz == ip) read_buffer();
61     if(isz == ip) return EOF;
62     return ibuf[ip++];
63 }
64 inline int putchar(int ch) {
65     if(osz == BUF_SZ) write_buffer();
66     if(osz == BUF_SZ) return EOF;
67     return (obuf[osz++] = ch);

```

```

68 }
69 abio &read_int(int &x) {
70     int flag = 0, ch = getchar();
71     for (; (EOF!=ch)&&((ch<'0')||ch>'9')); ch=getchar(); if ('-' == ch) flag = 1;
72     if (EOF == ch) return (this->reach_eof());
73     x = 0;
74     for (; (ch>='0')&&(ch<='9'); ch=getchar()) x = x * 10 + (ch - '0');
75     if ( flag ) x *= (-1);
76     return (*this);
77 }
78 abio &read_ll(long long int &x) {
79     int flag = 0, ch = getchar();
80     for (; (EOF!=ch)&&((ch<'0')||ch>'9')); ch=getchar(); if ('-' == ch) flag = 1;
81     if (EOF == ch) return (this->reach_eof());
82     x = 0ll;
83     for (; (ch>='0')&&(ch<='9'); ch=getchar()) x = x * 10ll + (ch - '0');
84     if ( flag ) x *= (-1ll);
85     return (*this);
86 }
87 abio &read_unsigned(unsigned &x) {
88     int ch = getchar();
89     for(; (EOF != ch) && ((ch < '0') || (ch > '9'))); ch = getchar();
90     if (EOF == ch) return (this->reach_eof());
91     x = 0u;
92     for(; (ch >= '0') && (ch <= '9'); ch = getchar()) x = x * 10u + (ch - '0');
93     return (*this);
94 }
95 abio &read_ull(unsigned long long int &x) {
96     int ch = getchar();
97     for(; (EOF != ch) && ((ch < '0') || (ch > '9'))); ch = getchar();
98     if (EOF == ch) return (this->reach_eof());
99     x = 0ull;
100     for(; (ch >= '0') && (ch <= '9'); ch = getchar()) x = x * 10ull + (ch - '0');
101     return (*this);
102 }
103 /* set interrupt as '\n' to read a whole line. */
104 abio &read_s(char *s, const char interrupt = '\n') {
105     int ch = getchar();
106     while((EOF!=ch)&&(ch<'!'||ch>'~'))ch=getchar();
107     if(EOF==ch) return (this->reach_eof());
108     for(; (EOF!=ch)&&(interrupt!=ch)&&(ch>=' ' && ch<='~')); ch=getchar();(*s++)=ch;
109     (*s)=0;
110     return (*this);
111 }
112 abio &write_int(int x, char append = 0) {
113     int d[20],nd=0;
114     if(0==x) putchar('0');
115     if(x<0) {
116         putchar('-');
117         x=-x;
118     }
119     while(x) {
120         d[nd++]=x%10;
121         x/=10;
122     }
123     while(nd--)putchar('0'+d[nd]);
124     if(append)putchar(append);
125     return (*this);
126 }
127 abio &write_ll(long long int x, char append = 0) {
128     int d[20],nd=0;
129     if(0==x) putchar('0');
130     if(x<0) {
131         putchar('-');

```

```

132     x=-x;
133 }
134 while(x) {
135     d[nd++]=x%10;
136     x/=10;
137 }
138 while(nd--) putchar('0'+d[nd]);
139 if(append) putchar(append);
140 return (*this);
141 }
142 abio &write_unsigned(unsigned x, char append = 0) {
143     int d[20], nd=0;
144     if(0==x) putchar('0');
145     while(x) {
146         d[nd++]=x%10;
147         x/=10;
148     }
149     while(nd--) putchar('0'+d[nd]);
150     if(append) putchar(append);
151     return (*this);
152 }
153 abio &write_ull(unsigned long long int x, char append = 0) {
154     int d[20], nd=0;
155     if(0==x) putchar('0');
156     while(x) {
157         d[nd++]=x%10;
158         x/=10;
159     }
160     while(nd--) putchar('0'+d[nd]);
161     if(append) putchar(append);
162     return (*this);
163 }
164 abio &write_s(const char *s, char append = 0) {
165     while(*s) putchar(*s++);
166     if(append) putchar(append);
167     return (*this);
168 }
169 abio &operator>>(char &ch) {
170     ch = getchar();
171     if(EOF==ch) return (this->reach_eof());
172     return (*this);
173 }
174 abio &operator>>(int &x) {
175     return read_int(x);
176 }
177 abio &operator>>(long long int &x) {
178     return read_ll(x);
179 }
180 abio &operator>>(unsigned &x) {
181     return read_unsigned(x);
182 }
183 abio &operator>>(unsigned long long int &x) {
184     return read_ull(x);
185 }
186 abio &operator>>(char *s) {
187     return read_s(s);
188 }
189 abio &operator<<(const char ch) {
190     putchar(ch);
191     return (*this);
192 }
193 abio &operator<<(const int x) {
194     return write_int(x);
195 }

```

```

196     abio &operator<<(const long long int x) {
197         return write_ll(x);
198     }
199     abio &operator<<(const unsigned x) {
200         return write_unsigned(x);
201     }
202     abio &operator<<(const unsigned long long int x) {
203         return write_ull(x);
204     }
205     abio &operator<<(const char *s) {
206         return write_s(s);
207     }
208 } io;

```

## 7.2 Random Numbers

```

1  #include <algorithm>
2  #include <chrono>
3  #include <iostream>
4  #include <random>
5  #include <vector>
6  using namespace std;
7
8  const int N = 3000000;
9
10 double average_distance(const vector<int> &permutation) {
11     double distance_sum = 0;
12
13     for (int i = 0; i < N; i++)
14         distance_sum += abs(permutation[i] - i);
15
16     return distance_sum / N;
17 }
18
19 int main() {
20     /* use mt19937_64 if you want 64-bit random numbers */
21     mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
22     vector<int> permutation(N);
23
24     for (int i = 0; i < N; i++)
25         permutation[i] = i;
26
27     shuffle(permutation.begin(), permutation.end(), rng);
28     cout << average_distance(permutation) << '\n';
29
30     for (int i = 0; i < N; i++)
31         permutation[i] = i;
32
33     for (int i = 1; i < N; i++)
34         swap(permutation[i], permutation[uniform_int_distribution<int>(0, i)(rng)]);
35
36     cout << average_distance(permutation) << '\n';
37 }
38
39 // -----
40 // rand() * rand() % M
41 // or
42 // rand() << 16 | rand()
43 // maybe also work in some scene;

```

## 8 Appendix

### 8.1 C++ Reference

#### 8.1.1 STL

### 8.2 Java Reference

#### 8.2.1 Basic Structure

```
1 import java.io.*;
2 import java.util.*;
3 import java.math.*;
4
5 public class Main {
6     public static int[] int_array = new int[50050]; /** Array */
7     public static void main(String[] args) throws Exception {
8         Scanner cin = new Scanner(System.in);
9         int a = cin.nextInt(), b = cin.nextInt();
10        System.out.println(a + b);
11    }
12 }
```

### 8.3 Environment test items

- 一秒运算次数，带模一秒跑多少
- 行末空格
- `assert(0)` 是 WA 还是 RE
- `RAND_MAX` 大小