For office use only	Team Control Number	For office use only
T1 ________________	**25844**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**A**	F4 ________________

2014 Mathematical Contest in Modeling (MCM) Summary Sheet

There are various traffic rules in the world, and the most basic one is the Right-Most Rule(basic rule). However, the efficiency of the left lane is always low and there exists potential risk of blocking. To improve the performance of freeways, we propose another two modified rules—Functionally Symmetrical Two Lane Rule (rule 2) and Three Lane Rule(rule 3). Then we simulate based on *NS Model* and develop earlier model to accommodate left rule and intelligent system.

To assess the performance of the rules, two aspects have to be taken into account—traffic flow and safety. Firstly, we consider the factors influencing them, such as uncertain factors like road conditions and habits of drivers, traffic volume, speed limits and so on.

By analyzing all possible parameters, we choose five most important ones that can best reflect the quality of safety and flow: flow index, speed variance, spacing index, ratio of overtaking and ratio of deceleration. We use the *Analytical Hierarchy Process (AHP)* to assign weights to these evaluation variables. Then, we use the parameters and weights correspondingly to propose two methods. Firstly, we use six levels which are similar to the level of service (LOS) for the performance. Each level is given a reference value for the five parameters. With the help of existing data of Florida's traffic system, we can validate the effectiveness of the method. Secondly, we analyze the results quantitatively, where we define an overall evaluation index based on the five weights of *AHP*.

Examining and comparing the three rules with the overall evaluation index, we find which performs better in a certain condition. In the case of fastigium, the overall evaluation indexes of rule 2 and rule 3 are much higher than that of the basic rule, and the gap is more apparent when congestion is growing serious. What is more, the flow of rule 2 is the largest of all rules. As for safety, rule 2 is the safest in light traffic while rule 3 is the safest in heavy traffic. In addition, the basic rule and rule 3 are more stable in spite of stochastic factors.

Furthermore, when comparing right and left rules, we find that the there are many similarities between them. However, the influence of the Coriolis force, field of sight and the driving habits of people makes left-driving less reasonable and more dangerous than right-driving to some extent.

Finally, under the control of an assumed intelligent system, we find that all rules perform better than the earlier analysis as expected. But the flows of all the three models tend to be almost identical. What is more, whether in the earlier analysis or in the intelligent system, rule 3 is the best in our evaluation system.

Faster Or Safer? Math Analysis On Freeway Rules

Contents

1 Introduction

There are several different traffic rules of freeways that are employed across the world. Some countries require driving on the right while others prefer on the left. Some freeways have lanes of different functions while others are approximately symmetrical in function.

To decide whether a rule is suitable, we need to consider different kinds of factors such as the road conditions, weather, the traffic volume, speed limits requirements, the psychology of the driver, and so on. What's more, we need to examine traffic flow, safety and the trade-off between them. We take all these things into consideration and try to evaluate the rules with simulated traffic model

This essay aims at developing the best traffic rule to promote traffic flow as well as safety under different circumstances. And we use the standards stipulated by us and that is based on different emphasis to determine the performance of the rule and give the best choice.Furthermore, we develop the model to analyze the left rule and the rode controlled by intelligent system.

2 Basic Models and Concepts

2.1 Parameters Of Models

Table 1: Variable Declaration

Variable	Meaning
L	The total number of cells in a line
d	The length of a cell
$v_{\max}$	The upper limit of speed
$v_{\min}$	The lower limit of speed
$v_i(t)$	The speed of the present car at time t
$v_{i+1}(t)$	The speed of the front next car at time t
$x_i(t)$	The potition of the present car at time t
$x_{i+1}(t)$	The position of the front next car at time t
r	The possiblity of incoming car
p	The possibility of acceleration
q	The posibility of random deceleration
δ	The gap index
v_d	The flow index
${\sigma_v}^2$	The speed variance
η_d	The times of deceleration
η_c	The times of overtaking
λ	The safe gap index

g	The gap between cars
g_{safe}	The safe gap
f	The evaluation index
$\bar{v}$	The average speed
$N_{density}$	The density of cars on lanes
N_{total}	The total number of cars on lanes

2.2 Basic Model–Cellular Automata

Cellular automata is a kind of dynamic system discrete in both time and space. Since we use it to solve the problem, we firstly introduce the basic model–cellular automata. Cellular automata describes a cellular space composed of discrete cells with finite states. These cells can change their states by some rules which makes the system a dynamics system. The construction of the cellular automata is showed in *Figure 1*.

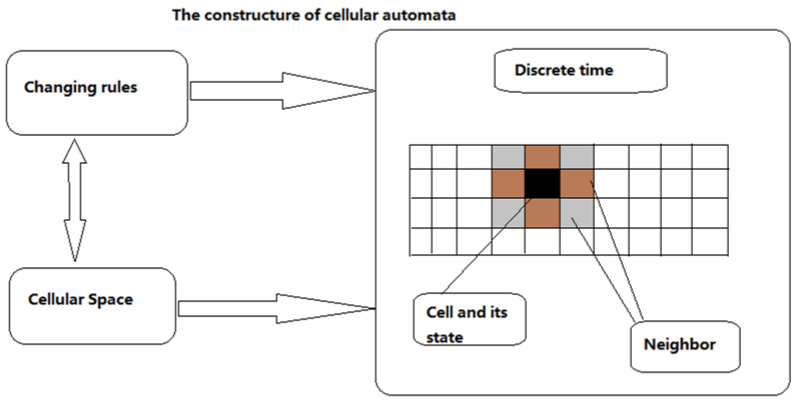


Figure 1: The Construction Of Cellular Automata

2.2.1 The concepts of the cellular automata

1. State of the cell

The variables of the state of the cell can be binary form like (0,1) and can also be a number in a finite integer collection S. For example, while it was used in traffic area, it can be any number in the data set $[v_{\max}, v_{\min}]$.

2. Cellular space

Cellular space tells us how the cells are distributed in space. It illustrates two things including the geometrical division and the boundary condition.

a. The geometrical division

The geometrical division for the one-dimensional space is sole while there are three kinds of division including triangle, square, and hexagon for the two dimensional space.

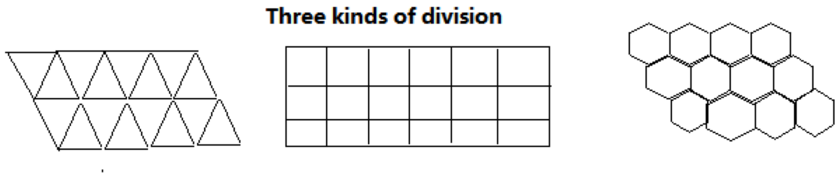


Figure 2: Three Kinds of Division

The square division is easy and visualized. Therefore, we use the square division for our model.

b. The boundary conditions

The boundary conditions that are usually used in daily life are periodic boundary, constant boundary, adiabatic boundary and constant boundary.

3. Neighbor

There are mainly four kinds of neighbor in the cellular automata. They are named as Von. Neumann, Moore, extended Moore and Margolus.

4. Rule

To know the prospective state of the cell, we need to have a function of the transition which is the rule.

5. Discrete time

The time used in the cellular automata is discrete which makes the time a series of integer.

2.2.2 184 Model

The cellular automata is widely used in different kinds of areas such as biology, computer science and so on. When it is used to the traffic, it is changed to the NS model and the BML model. However, the two models are both based on the 184 model.

In 184 model it assumes that:

- The road is divided to the L equidistant cells.

- At a moment, each cell can be empty or occupied by a car.
- All cars are moving to the same direction.
- At a moment, if there is no car being front of $(n+1)_{th}$ car, it can move forward. But if there is a car $(n+1)_{th}$, then even if the $(n+1)_{th}$ car leave away from here, the n_{th} car would not move.
- The model uses the periodic boundary to ensure that the total number of the cars will not change.

These assumptions can help us establish an attitude to the problem. And since the NS model is used for the freeway and BML model for the city, we solve the problem with the help of NS model.

2.3 NS Model

The NS Model(Nagel-schreekenberg Model)[3] is a one-dimensional one-lane cellular automata model, proposed in 1992. Because the model accounts for many phenomenon in the real word and is easy to simulate, it is widely used in the study of traffic flow.

In the Basic NS Model, at time t the i_{th} vehicle is fully described by its position $x_i(t)$ and its speed $v_i(t)$. If there exists an $(i+1)_{th}$ car in front of it, the gap is defined as g. Then during the time interval t, the state of the car is updated in the following order.

- ***Change of speed:*** $v_i(t+1) = \min\{g, v_i(t)+1, v_{\max}\}$

  In this process there exists random acceleration. When spacing in front of the car is far enough for acceleration, the driver would accelerate to a higher speed of $\min\{g, v_i(t)+1, v_{\max}\}$ at the probability $p_{acceleration}$. If the spacing is too close and the car could not change lanes, the driver would change the speed to g.

- ***Random deceleration:*** $v_i(t+1) = \max\{v_i(t)-1, 0\}$ at the probability p. With the random deceleration probability p, the NS Model takes into accounts the environmental conditions like weather and road conditions, the reaction time of the driver, and so on. The random deceleration contributes a lot to the traffic congestion.

- ***Update of position:*** $x_i(t+1) = x_i(t) + v_i(t+1)$

At each moment, the speed and position of the car can be updated based on the above NS Model.

3 Basic Model 1: Right-Most Rule and Evaluation

Our model is also based on the existing NS model but more factors and rules are taken into account. The first model analyzes the traffic of a two-lane highway which conforms strictly to the Keep-Right-Except-To-Pass Rule. Evaluation parameters and standards are also given to assess the rule according to the modeling results. First of all, we list the assumptions below.

3.1 Basic Hypothesis

- Initially there is no car on the freeway.
- New cars enter the freeway at a certain rate every time unit and a random speed v that is uniformly distributed on $[v_{\min}, v_{\max}]$.
- No more than one car enters each lane every time, with the possibility r, which means cars are not blocked at the entrance.
- We only study the cars traveling in a single direction without stopping or turning. Besides, no crossing or sideway exists.
- The lanes are evenly divided into "small cells", while time is also divided into slots. In each time slot, every car can only move the multiple number of cells.
- At a moment, each cell can be empty or occupied by a car.
- The right-most lane is for ordinary advancing while the left one is specially for overtaking. After passing another vehicle, returning is required if possible.
- Cars can only enter the road via the right-most road.
- Random deceleration exists, because of the condition of road, whether, and other possible factors.
- Every car on the road can only know the states, for example, speed, of its neighbors, including the ones on the adjacent lane.

3.2 The Rules of Model 1

The basic rule for the multi-lane freeways is: Drivers are required to drive on the right-most lane unless they are passing another vehicle, in which case they move one lane to the left, pass, and return to their previous lane. The freeway we study has a length of L cells, and vehicles out of the boundary will not be studied. Also for simplicity, we will apply the front-to-end method to analyze the movements of the traffic. During each time slot, all the cars will change their position and maybe speed. Some cars move out of the give area of lanes, some in a different position, while some new cars come into the area. In order to determine the behavior of the cars(ie, going straight, overtaking, returning whether or not to accelerate or decelerate), the movements of the front cars are studied first, and then the next-to-front cars, and so on. Finally the new comers will be put in. The process is illustrated in *Figure 3*.

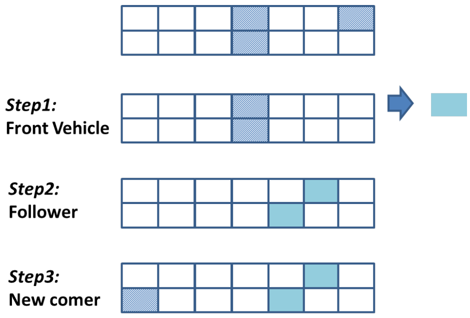


Figure 3: The Front-To-End Process

Because cars on the two lanes have different strategies, ie. cars on the traveling lane try to go ahead while cars on the overtaking lane tries to return to the traveling lane. However, they both have to follow the two principles[5] below.

- *Overtaking Principle:*

  If the speed of the vehicle $v_i > g + g_{safe}$, the overtaking principle is satisfied, where g is the distance from the car in front of it, and g_{safe} is the safe gap that should be maintained. The safe gap can be modeled as $g_{safe} = \lambda v_{i+1}$[5] with λ a coefficient and v_{i+1} to be the speed of the car in front of it.

- *Security Principle:*

  The car also has to check whether it is safe to transfer to the overtaking lane, as is shown in *Figure 4*. If $g_{front} - g_{safe} > v_i$ and $v_i - v_{back} > g_{back} + g_{safe}$, the security principle is satisfied.

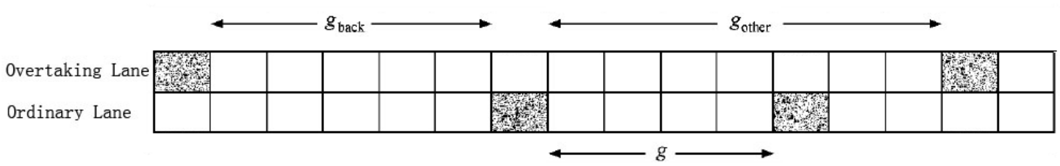


Figure 4: Two-lane System

With these principles, the behavior of a car can be uniquely defined as below. We will discuss this separately for the two lines.

Traveling Lane Procedure

(1)*Random Deceleration.* The car decelerates with the possibility p according to *NS model*.

(2)*Overtaking Principle Check.* The car will check the *Overtaking Principle*. If satisfied, go to **Step**(3). Otherwise, go to **Step**(4).

(3)*Security Principle Check.* The car will check the *Security Principle*. If satisfied, it will perform overtaking to the lane beside. Or it will go to **Step**(5).

(4)*Acceleration* The car accelerates by one unit with the possibility q, ie. the speed will be updated to be $\min\{v_{\max}, v+1, g-g_{safe}\}$.

(5)*Deceleration* The car decelerates to the same speed as the speed of the first car in front of it. Meanwhile the new gap between it and the front car is just g_{safe}.

Overtaking Lane Procedure

(1)*Random Deceleration.* The car decelerates with the possibility p according to *NS model*.

(2)*Security Principle Check.* The car will check the *Security Principle*. If satisfied, it will perform overtaking to the lane beside, ie. the traveling lane. Or it will go to **Step**(3).

(1)*Overtaking Principle Check.* The car will check the *Overtaking Principle*. If satisfied, go to **Step**(4). Otherwise, go to **Step**(5).

(4)*Deceleration* The car decelerates to the same speed as the speed of the first car in front of it. Meanwhile the new gap between it and the front car is just g_{safe}.

(5)*Acceleration* The car accelerates by one unit with the possibility q, ie. the speed will be updated to be $\min\{v_{\max}, v+1, g-g_{safe}\}$.

3.3 Evaluation Parameters and Methods

The model above gives the approach to simulate the traffic on the freeway that complies to the right-most rule. Then we will propose methods to tell whether this rule is good or not. Especially, we will pay attention to two aspects:

traffic flow and safety. Single parameters will be combined to obtain trade-off between these aspects.

3.3.1 Evaluation Parameters

Among those factors, there are five parameters that are especially in determining flow and safety:

1. Flow Index

The flow of the traffic is defined as the multiplication of the average speed and the traffic density $flow = \bar{v}N_{density}$. Furthermore, for the convenience of analysis, we will normalise the parameter to define the Flow Index as

$$v_q = \frac{\bar{v}N_{total}}{100v_{max}}$$

2. Gap Index

For safety consideration, we know that, the higher the speed of the back car and the smaller the gap, the more dangerous the road is. Thus we define the Gap Index as

$$\delta = \frac{g}{3.6v_i}$$

where the normalisation coefficient 3.6 is an empirical value. With the index growing, the road is becoming more and more unsafe.

3. Speed Variance

Speed Variance ${\sigma_v}^2$ is directly defined as the speed variation of the cars in the designated area. The higher the variance, the more overtaking occurs, and consequently, more dangerous.

4. The Times of Deceleration

The Times of Deceleration η_d is the total times of deceleration that happens in a single time step. More deceleration means less flow.

5. The Times of Overtaking

The Times of Overtaking η_c is the total times of overtaking that happens in a single time step. More deceleration implies more danger.

3.3.2 Evaluation Standards

1. Grading System Similar to LOS

Since the best case for traffic flow and safety can not be achieved simultaneously, we have to consider the trade off between these two aspects. We will first give a set of standards similar to LOS. The effectiveness of the standard will then be verified according to the existing LOS cases.

Level of service (LOS) is a measure used in the management of civil infrastructure to measure its fitness for purpose. LOS can also be used to analyze highways by categorizing traffic flow with corresponding safe driving conditions.In this case, LOS serves as a quality measure describing operational conditions within a traffic stream, generally in terms of such service measures as speed, freedom to maneuver, traffic interruptions and convenience. According to the Highway Capacity Manual (HCM), there are five levels of LOS in North American, from A to F, where A being the best and F being the worst. In this sense, LOS can represent the level of performance.

Similar to LOS, we will also give a set of evaluation standards using the five parameters above. The reference value of the parameters for the levels are given below.

Table 2: Influencing Factors

level	A	B	C	D	E	F
η_d	0.0160	0.2057	0.4378	0.6518	0.8702	1.1060
δ	3.5110	3.1327	2.6953	2.2536	1.8454	1.5129
η_c	0.0205	0.1475	0.2902	0.4538	0.6253	0.7460
v_q	0.3281	0.2759	0.2247	0.1796	0.1284	0.0875
σ_v^2	0.4372	0.5581	0.6807	0.7935	0.9052	1.0755

Then, by computing the weighed *Mean Square Error(MSE)*, defined as Index Deviation(ID), of each factor with the reference value in *Table 2*, we can determine which level a certain rule belongs to, as illustrated in (1).

$$\begin{aligned} p_{freeway} &= a_1 \cdot p_{flow} + a_2 \cdot p_{safety} \\ &= \omega_1(\Delta\eta_d)^2 + \omega_2(\Delta v_q)^2 + \omega_3(\Delta{\sigma_v}^2)^2 + \omega_4\delta^2 + \omega_5(\Delta\eta_c)^2 \end{aligned} \tag{1}$$

where $a_i,\ i = 1,2$ and $\omega_j,\ j = 1,2,...,5$ are the weights for each parameter. The values of those coefficients can be changed accordingly to satisfy the standard with different trade-off.

2. Weights Determination By APH

To determine the weights in (1), we use the Analytical Hierarchy Process (AHP)[Saaty 1982]. The structure for this AHP Model is shown as *Figure 5*. We

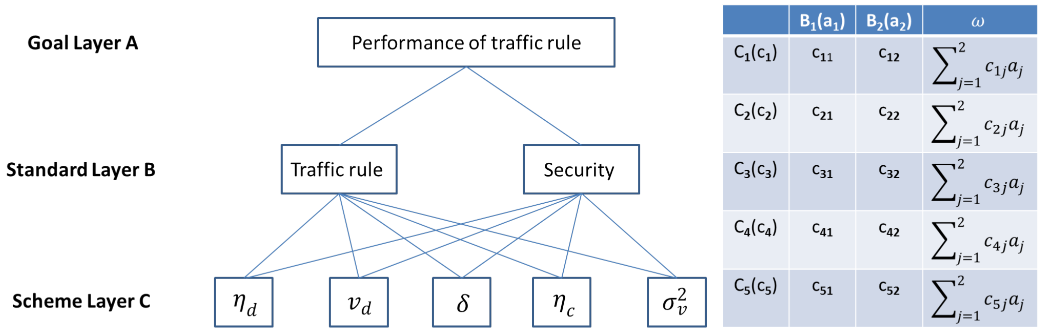


Figure 5: The structure of AHP Model

have to use the AHP preference scale in *Table 3* to form the comparison matrices for Standard Layer and Scheme Layer. The elements of those three matrices can be changed accordingly to satisfy tradeoffs between traffic flow and security, as well as tradeoffs between B_i *and* C_j. The matrices are chosen accordingly as blow. The values in it, however, are chosen by our own judgement of the importance of difference parameters.

Table 3: Preferences Made On 1-9 Scale

Intensity of value	Interpretation
1	Requirements i and j have equal value.
3	Requirement i has a slightly higher value than j.
5	Requirement i has a strongly higher value than j.
7	Requirement i has a very strongly higher value than j.
9	Requirement i has an absolutely higher value than j.
2,4,6,8	Intermediate scales between two adjacent judgments above.
reciprocal	Requirement i has a lower value than j.

Then we are able to calculate the numerical solutions as shown below.

A	B_1	B_2
B_1	1	1
B_2	1	1

B_1	C_1	C_2	C_3	C_4	C_5
C_1	1	3	2	9	9
C_2	1/3	1	1/2	3	3
C_3	1/2	2	1	5	5
C_4	1/9	1/3	1/5	1	1
C_5	1/9	1/3	1/5	1	1

B_2	C_1	C_2	C_3	$C4$	C_5
C_1	1	1/3	1	1/7	1/9
C_2	3	1	1/2	3	1/3
C_3	7	2	1	7	1
C_4	1	1/3	1/7	1	1/9
C_5	9	3	1	9	1

Table 4: APH-derided Weights

Parameter	η_d	δ	η_c	v_q	σ_v
Weight	0.2637	0.05	0.2319	0.1498	0.3045

We then test the consistency of the preferences for this instance of the AH-P. Good consistency should satisfy The consistency ratio $CR = CI/RI < 0.1$, where $CI = (\lambda_{\max} - n)/(n-1)$ and $RI = 1.12$ when $n = 5$. Good consistency requires $CR < 0.1$. By calculation we can deride $CR = 0.02 < 0.1$, which verifies that this standard is reasonable.

Standards test

Now we have to justify the validity of the evaluation standards. With the data from a certain freeway in Hendry county in Florida in 2011[12] , together with its official LOS, we can test whether our model will work.

Table 5: Data of Road R23

State Road Number	One-Way Volume	Lane	Posted Speed $mile/h$	LOS
RS23	2400	2	60	B

The posted speed is approximately 5units in the model. Thus, we will set $v_{\max}$ to be 5. Considering that the freeway is not blocked, the volume also equals to the input speed of cars. $1h$ is 7200 time slots in the model. As a result, we can set the possibility of new-coming cars to be $\frac{volume}{7200} = 0.33$. After simulation and calculation, he the results of comparison of the different levels are listed below.

Table 6: Index Deviation

Level	A	B	C	D	E	F
ID	0.1337	0.0744	0.1000	0.2271	0.4525	0.7643

From the table it can be figured out that the road should be rated into level B, the same as the official LOS. Therefore our assignment of weights are proved to be reasonable.

3. Evaluation Function

The method above can rate the freeways in different levels with good trade-off between flow and safety. However, it is not convenient for comparing different freeway models, ie. different rules. So based on the previous evaluation, we will illustrate another method, by defining the evaluation function below, where the negative sign means that the factors affects our evaluation in the opposite direction. The higher the value, the better the rule works in the given circumstance.

$$\begin{aligned} f &= -\omega_1\eta_d + \omega_2(v_q) - \omega_3{\sigma_v}^2 + \omega_4\delta + \omega_5\eta_c \\ &= -0.2637\eta_d + 0.1498v_q - 0.3045{\sigma_v}^2 + 0.05\delta + 0.2319\eta_c \end{aligned} \tag{2}$$

3.3.3 Model 1 simulation and result analysis

We simulate the model according to the steps in **3.1**. Meanwhile, we study how the input parameters will influence the performance of the rule. With the evaluation factors and standards, the characteristics of the systems can be analyzed and discussed.

1. Traffic Flow

a. The influence of p

p reflects the probability of acceleration. A higher p means that the driver is more adventurous and tends to accelerate when it is possible. When p is higher, the average speed of cars on the freeway may increase, while too many cars with high speed on lanes can leads to more possibilities of overpassing, thus resulting in potential block. Considering the above analysis comprehensively, the trend of $flow - p$ is figured out in *Figure 6*, with the increase of p, the performance of flow decreases on $[0, 0.5]$ and grows sharply on $[0.5, 1]$. Therefore, the traffic flow improves when there are more adventurous drivers on the road. On the other hand, higher speed may lead to more accidents. So considering the aspect of safety, although we want to improves the traffic flow, p should be controlled not too high.

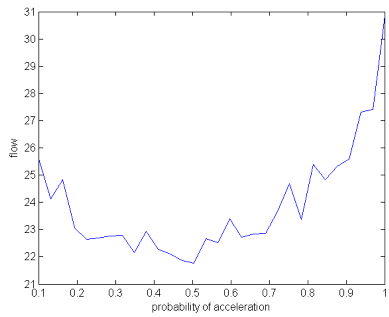


Figure 6: Flow With Different Random Acceleration Probability

b. The influence of q

q reflects the probability of acceleration. As $flow = density \cdot \bar{v}$, higher q could lead to lower average speed but more cars on lanes. Therefore, total level of flow can fluctuate within a small interval with the change of q, as is shown in *Figure 7*.

2. Number of deceleration

From *Figure 8*, we can find that to the basic model, when the rate of the entering car r increases, the times of deceleration,η_d increases, too. So when there are more cars entering the road, if they drive by the right rule, they will slow down their cars' speed more easily. It indicates that in case of heavy traffic it shows the disadvantage of the basic rule.

3. The variance of speed

Figure 9 shows that to the basic model, with the increasing of the entering car (r), the variance of speed (v_a) increases, too. This shows that when the possibili-

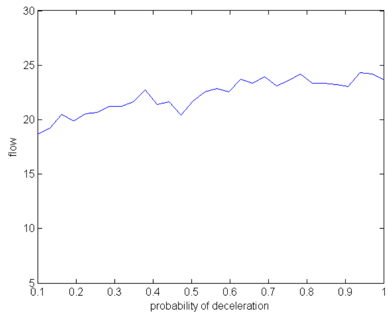


Figure 7: Flow With Different Random Deceleration Probability

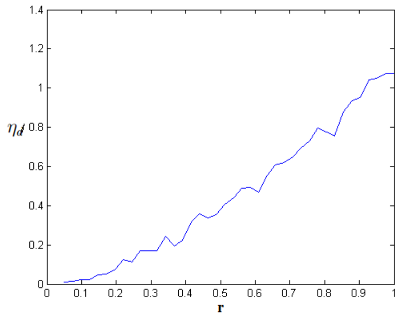


Figure 8: Image of η_d-r

ties of the cars' entering are larger, the cars' speed will have more difference than before. However, we can find that the va varies in a small area which shows the stability of the speed .

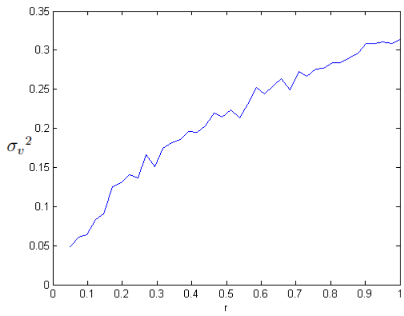


Figure 9: Image of $\sigma_v{}^2$-v

4. The number of overtaking

This figure shows that there is a positive correlation between the rate of entering car (r) and the times of overtaking η_c for the basic model. In basic model, we can know if there are more automobiles on the road, then the times of overtaking is larger. Under this circumstance, the safety of the model may not be so high because of the large amount of overtaking cars. In the normal life, when facing some festivals , the entering of the car will increase and the amount of overcoming others increases, too. The result is similar to our real life.

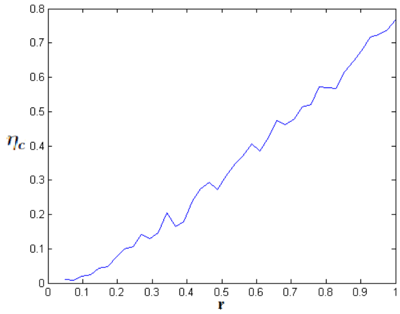


Figure 10: Image of η_c-r

4 Improved Freeway Model

4.1 Model 2 : The Improved Rule With Two Lanes

To improve the efficiency of both lanes, we develop a new rule based on two functionally symmetrical lanes. The new car can enter either lane. In addition, on each lane it is allowed to either follow-up or overpass, and the car does not necessarily have to return to the former lane after overtaking. That means both of the lanes have only to follow the **Traveling Lane Procedure** in model 1.

4.2 Model 3 : The Improved Rule With Three Lanes

We can perceive that both **Model 1** and **Model 2** are not efficient enough. In **model 1**, the usage of the lanes are pretty uneven, for the overpassing cars are supposed to returned to the previous lane. In **model 2** the two lanes are more symmetric. However, overtaking can make the lanes block each other.

So we propose a third model with *three* different lanes also based on *NS Model*, as illustrated below. The new car can enter either lane on the right, where the right-most lane has the higher possibility. The speed limits are also different, with Lane 1 the lower and Lane 2 the higher. Lane 1 is an ordinary lane while both Lane 2 and Lane 3 can be used for overtaking. Lane 1 and Lane 2 follow the **Traveling Lane Procedure** while lane 3 follows the **Overtaking Lane Procedure**.

4.3 Comparison of the Three Models

Then we will compare the three models in detail with the evaluation methods introduced in **3.3**. We will illustrate how the modified rules will improve the

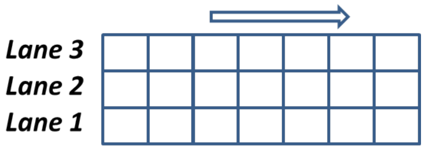


Figure 11: The Three-lane Model

performance of the free ways.

4.3.1 The evolution of flow

As is introduced in *Table 2*, the parameter $flow$ can reflect how traffic flow performs in a large scale. With the same input parameters, the trends of $flow$ of three models are shown in *Figure 12*.

We can figure out that apparently the total level of traffic flow is highest in Model 2, while the flow of Model 1 and Model 3 is much lower and similar in quantity. Therefore it is reasonable to come to the conclusion that compared with the basic rule, the Three-Lane Model improves the performance of traffic flow considerably.

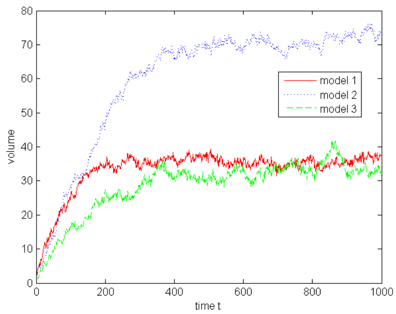


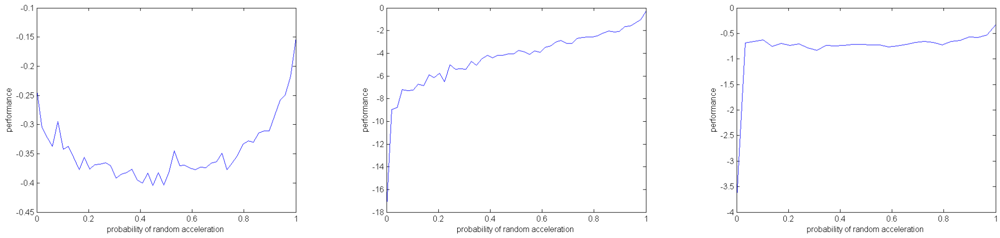
Figure 12: The Evolution of Flow

4.3.2 Overall evaluation

1. The influence of uncertain factors

p and q denotes random factors such as road conditions and weather. Good traffic should be stable and should not be affected too much by those uncertain factors. As we can see from *Figure 13.(a)*, with the increase of p, $Model\ 1$ and $Model\ 3$ remains almost stable while $Model\ 2$ changes a lot. *Figure 13.(b)* shows that with the increase of q, all the three models remain stable while the value of that of $Model\ 2$ is much lower.

Therefore, $Model\ 1\ and\ 3$ are more stable with the random acceleration probability. And all the three models are quite stable with the random deceleration probability.



(a) Model 1 (b) Model 2 (c) Model 3

Figure 13: The Influence of p on Evaluation

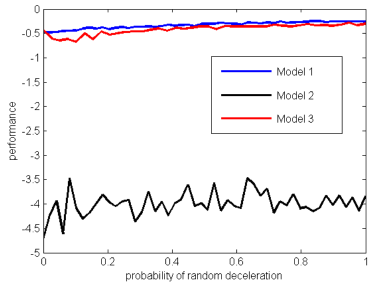


Figure 14: The Influence of q on Evaluation

2. The influence of rate of coming cars

Figure 15.(a) describes the relationship between the standard of the evaluation (f) and the rate of entering car (r) when the v_{min} is 4 for three models.

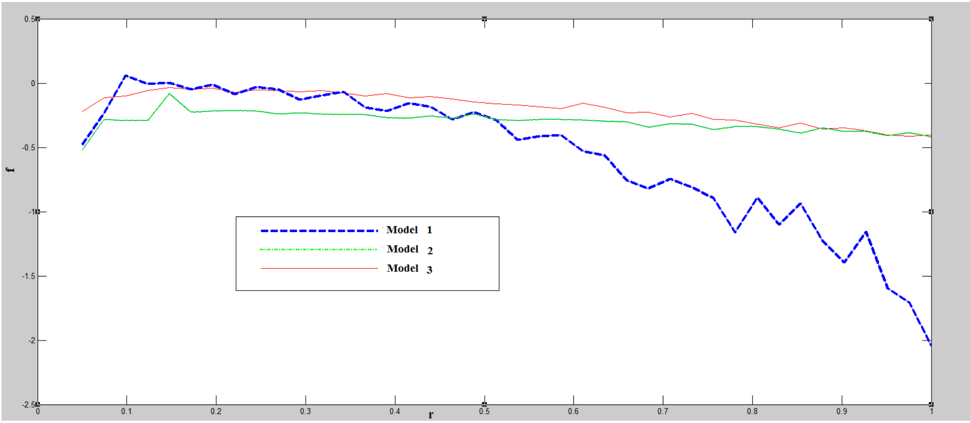
From the graph, we can find that when the rate is low, the three models evaluation standard is similar. However, when the rate is larger than 0.45, the standard of the basic model which employs the right rule decreases very fast. This means that our modified models are better than the basic one. The effect of our two models is better than that of the former one especially when the ratio is high.

Then we see the red line and the green line, which can tell us that when comparing the two models for the two-lane way and three-lane way, the three-line way model is better. When the ratio is too high, the two models are similar because the cars are very much making the road full of automobiles.

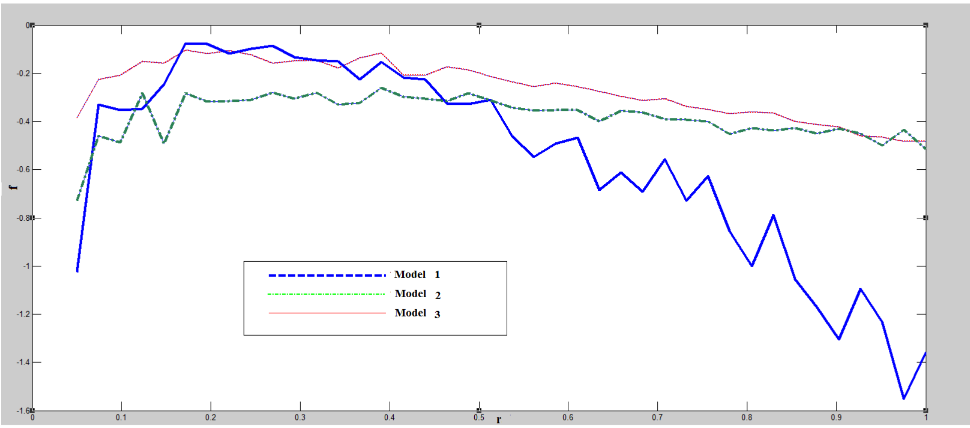
Figure 15.(b)describing the relationship between the standard of the evaluation f and the rate of entering car r when the $v_{min} = 3$ for three models.

From the figure, we can find the basic model may be a little bit better when r is low for the simplicity of the right rule, but it always performs worse than the other two models especially when the road is heavy. This shows the right traffic rule is worse for the road that is so busy.

What is more, the red line which presents the three-lane road and the green line which presents the green-lane are similar when the traffic is busy. And the three-lane roads standard is a little higher than that of the two-lane road.



(a) $v_{min} = 4$



(b) $v_{min} = 3$

Figure 15: The Influence of Rate of Coming Cars, $v_{\min} = 3$

If we compare the graph with that of the $v_{min} = 4$, you will find that the standard in the $v_{min} = 4$ is higher than this condition. This shows the speed limit should be a little strict to let the road unobstructed.

Figure 16, the comparison diagram, describes the relationship between the standards of the evaluation and the rate of entering car r when the v_{min} is 2 for three models.

From the comparison diagram, we can see that the evaluation of the three models is the highest when the rate is around 0.25, and when the rate is too high or too low the evaluation will be disadvantageous. Comparing the three lines, we can find a regular pattern that the red line and the green line are similar to each other. They are both higher than the third one which is the basic model.

What is more, we compare this graph with the ones when the $v_{\min}$ is 3 and 4. We can find this one is similar to the graph with $v_{\min}$ of 3. Meanwhile, we can also figure out that while $v_{\min}$ is 4, the evaluation is highest of the three low speed limits, which means we may choose $v_{\min}$ around 4 to build an excellent

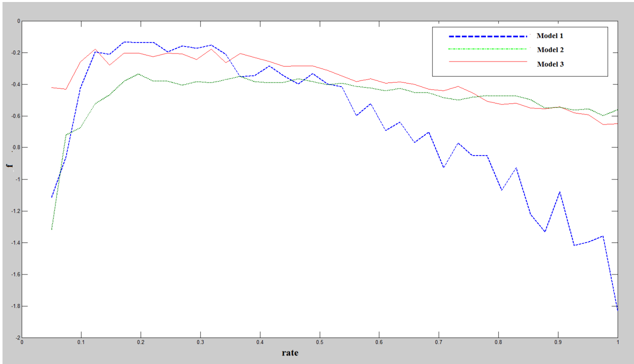


Figure 16: The Influence of Rate of Coming Cars, $v_{\text{min}} = 2$

rule.

4.3.3 The gap index

Figure 17 describes the relationship between the gap index δ and the rate of entering car r for the three models.

From the graph, we can find that the ratio of the three models are decreasing when the rate is increasing which shows that the time for the car to crash its front car is smaller when there are more cars coming in. For the three-lane road, the ratio is higher than the other two models. And when the rate is high, the ratio of three-lane model is similar to that of two-lane model.

However, the basic model which employs the right rule is the lowest of the three models when the road is light in traffic. Therefore, we can come to the conclusion that it is safer to choose the other two rules while the traffic is not busy. When the traffic is too busy, the three models are similar.

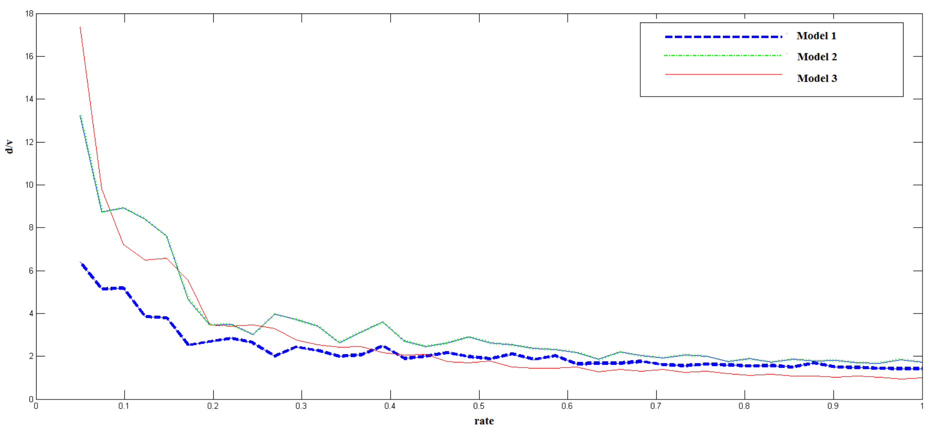


Figure 17: Image of δ-r

4.3.4 THe Number of overtaking

Figure 18 describes the relationship between the times of overtaking η_c and the rate of entering car r for three models.

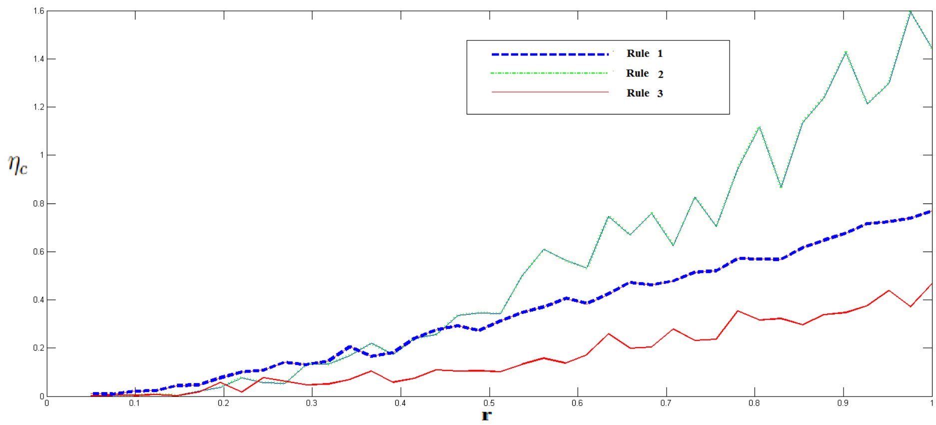


Figure 18: Image of η_c-r

In the diagram, we can find that the times of overcoming of the three models have positive relationship with rate r. The improved two-lane model is similar to the others when the rate is low. However, it can be very high when the traffic is busy.

Therefore, the two-lane improved traffic may have more space between the automobiles. The red line, we can see, is so low when the traffic is not busy, which can be interpreted that there is not many cars for us to pass. Most of the time, car owners can go straight ahead without blocking.

5 Models of Driving on the Left

To know the condition of the driving automobiles on the left side, we should know the difference and similarity between the left and the right rule. After that, we can utilize the models used in the right rule and change them.

5.1 Similarity

There are many similarities between the two rules according to our research. You can know by the general knowledge that if the cars run by the right rules, the automobiles on the right side will be much more than those on the left side. Similarly, when they run by the left rules, the cars on the left side will be more. Then the distribution of the automobiles on the road will be as the following in *Figure 19*.

So if we consider the two direction of the road, the distribution of the cars will be as the *Figure 20*.

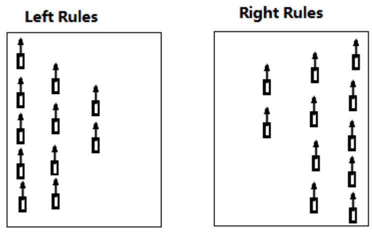


Figure 19: The Distribution of the Automobiles on the Road

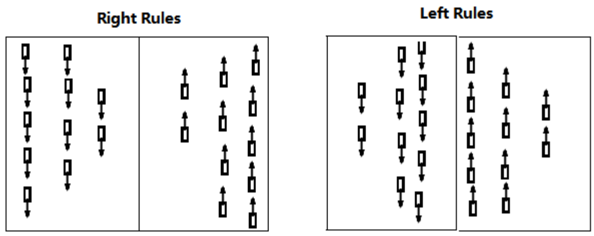


Figure 20: The Distribution Of The Automobiles On The Road

However, by our research, the real distribution is not like the above figure. Because on the left side of the road which employs the left roles the cars drive upward while the cars on the right side drive downward, the figure is wrong. The real distribution of the road is as *Figure 21*.

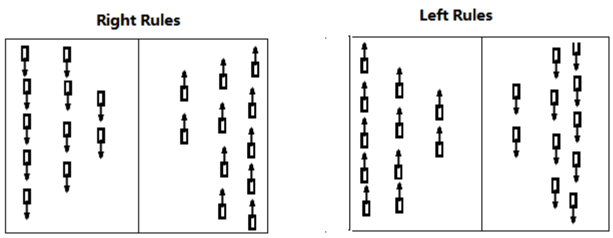


Figure 21: The Distribution of the Automobiles on the Road

This figure shows that the model set for right rule can be absolutely used on the left rule. The theoretical result of the two models are the same when we do not consider about other conditions like the different field of sight to different rules, the habit of the left-handers, and the influence of the Coriolis force.

5.2 Difference

1. Different field of view

a. Static field of view

According to the theory about the ergonomics, when a person stay quiet, his field of view can be divided into single-eye field and double-eyes field. The single-eye field is the field can be observed by only one eye which is usually about 150 degree. However, the double-eyes sight is usually about 120 degree horizontal angle, 50˜55 degree up angle and 60˜70 degree down angle[10].

b. Athletic field of view

When a person is moving, he focus on the way in front of him which will make the field of view different. When the person is speeding up, the longest distance of the driver is directly proportional to the speed while the field of view decreases with the increasing of the speed.

c. Distribution of the drivers field of view

According to the theory of Static field of view and Athletic field of view, we can know the distribution of the drivers field of view.

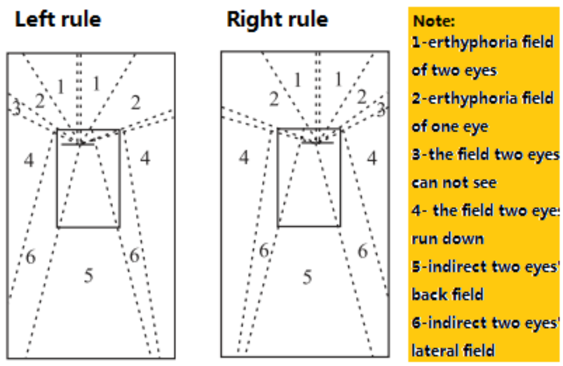


Figure 22: The Distribution of the Driver's Field of View

Since we know that a persons eye is sensitive to the objects in the erthyphoria field. And when the viewing angle is more than 30 degree[10], the drivers will turn to the object.

To the erthyphoria field of two eyes, the drivers eyes are the most accurate. But in the erthyphoria of one eye, drivers can observe objects directly with low accuracy and longer time. When it comes to the field that two eyes run down, drivers cannot see the objects directly, they always turn to the object which is very random. In the indirect field, drivers observe objects through rearview mirror whose effect is worse.

Therefore, we can get a conclusion that the erthyphoria field is better than the indirect field. In the erthyphoria field, it is better for observing by two eyes than by a single eye. In the indirect field, it is better for observing when objects are near to us.

d. Comparison between the two rules

To the right rule, we can know that the area in right front of the automobiles are mainly covered by orthophoria field of two eyes. The erthyphoria field of one eye and the field two eyes cannot see are smaller than the left side. On the two sides of the cars, since the right rearview mirror is near to the driver and the response time is shorter than that of the left side. Therefore, we can know that the left rule is better when considering about the field of view.

2. The influence of the Coriolis force

a. Coriolis accelerate

To know about the Coriolis force, we should firstly know the Coriolis accelerate.

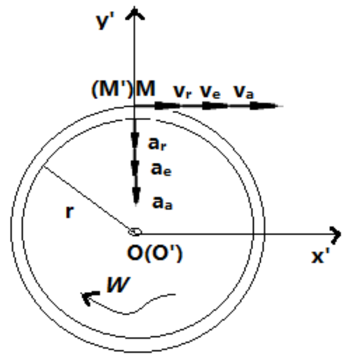


Figure 23: The Distribution of the Driver's Field of View

In *Figure 23*, the disk is rolling with angular speed w using constant speed vr . We calculate the absolute accelerate of point M:

We set athletic coordinate O X Y on the disk. Since the point M is doing the uniform circular motion, we suppose the relative speed is v_r and relative accelerate is a_r. We know the following formula[11]:

$$a_r = {a_r}^n = {v_r}^2/r \tag{3}$$

The transformational speed of point M is ve = rw whose direction is vertical to OM. And the transformational accelerate is as (4):

$$a_e = {a_e}^n = {\omega_r}^2 \tag{4}$$

Since the direction of v_r and v_e are the same, so the absolute speed v_a of point M is vertical to OM, as is shown in (5)[11].

$$v_a = v_r + v_e = v_r + \omega r \tag{5}$$

So the absolute movement of point M is also uniform circular motion. The absolute accelerate a_a is calculated by (6):

$$a_a = {a_a}^n = {v_a}^2/r = (v_r + \omega r)^2/r = \omega^2 r + {v_r}^2/r + 2\omega v_r \tag{6}$$

From the above formula, we know a_a contains not only a_e , ar but also contains an accelerate which is equal to $2\omega v_r$ which is always called as Coriolis accelerate.

b. Coriolis accelerate

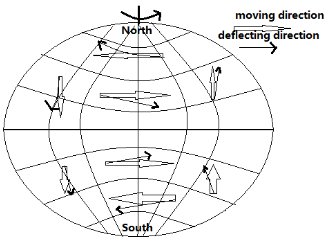
Since the earth moves from west to east, to the objects on the surface, the motion of the earth is translational motion. From the above analysis we know that:

$$a_a = a_e + a_r + a_c$$

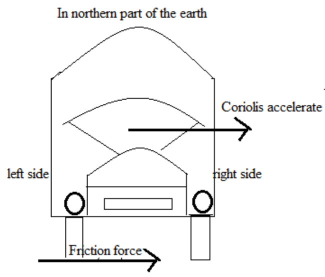
The a_e is equal to $2\omega v_r$. According to the Right-Hand Rule:

$$a_e = 2\omega v_r \sin\theta$$

Because of the existence of the Coriolis accelerate, the excursion of the automobiles cannot be ignored. What is more, in different place of the earth, the direction of the excursion is not the same. Therefore, the motion on the earth is as shown in *Figure 5.2*.



(a) Motion On the Earth



(b) Automobiles Moving On The Northern Part Of The Earth

Figure 24: Effect of Coriolis Accelerate

From the figure, we can find that to the cars in the northern part of the earth, when they are moving, they always like to move towards the right. However, to the automobiles in the southern part of the earth, while they are moving, they always like to move to the left.

Therefore, the automobiles moving in the northern part of the earth is like *Figure 5.2*.

The car need a Coriolis accelerate which can only be provided by the friction force. If the friction is small because of the weather, then the car will move to the right. And since the countries in the northern part like America allow the cars to move on the right side of the road, the car will not crash other cars on its left side. This helps to protect the other automobiles. In contrast with it, we can know the condition in the southern part of the earth which means that the left rule may work better.

3. The driving habits of people

By statistical result, we find that there are 10 12 percentage of people on the earth are left-handers, which means that most people like to use their right hands to do something.

According to the data, let us have a look at the structure of the cars in different countries with different driving rules–left-side driving with right driver seat and right-side driving with left driver seat.In the former pattern, if a person uses the car, he need to do the gear shifting using his left hand. Only if he is a left-hander, will he feel comfortable to drive the left car by the left rule. According to our research, most people are right-handler and will feel uncomfortable and tend to make less accurate judgements.

5.3 Conclusion

Through our analysis of the left rule, we can find that: due to the similarity between the two rules, the model used in the right rule can be used here for the left rule symmetrically. Theoretically speaking, the two rules have similar function.

However, there are still at least three things including the different field of sight to different rules, the influence of the Coriolis force, and the driving habits of people, that make the two rules different.

- The different field of sight:

  We find that the field of sight for the right rule is larger than that for the left rule. So driving cars by the right rule can be safer than by the left rule.

- The influence of the Coriolis force:

  According to the analysis to Coriolis force, we find the direction of the force in the northern part of earth is right while it is left for the southern part of world. Therefore, it can be better to utilize the right rule in the northern part and use the left rule in the southern part.

- The driving habits of people:

  By our research, we can find that most people are right-handers and they can easily use right hands to do the gear shifting which makes them accurate. So driving by the right rule can make people drive the car fast and safely which makes the float and safety for the road higher.

6 Ideal Model

Finally, regardless of human judgment for compliance, we consider the freeway system under the control of an intelligent system. In this case, we idealize the stochastic factors like the initial speed and coming rate of cars, the random deceleration probability q, the random acceleration probability p when it is safe for higher speed, the reaction level and driving habits of drivers, the conditions of road and weather, and so on.

6.1 New Hypothesis in the Ideal Condition

As weather, road conditions and driving habit considerably influence the rate of random acceleration or deceleration. So in this ideal case, we have several new assumptions.

- The rate of random acceleration $p = 1$. It means that the driver tends to accelerate whenever it is possible.
- The rate of random deceleration $q = 0$. Here we assume that factors like weather and road condition are so perfect that drivers will not slow down when there is no blocking in front.
- The factor that represents how drivers estimate the spacing and speed of the car in front: $\lambda = 0$. In this case driving is so adventurous that the safety distance is neglected.

6.2 Results and Analysis

Based on the above assumptions, we testify the 3 models again and the results are shown as follow.

6.2.1 Traffic flow

The evolution of flow in ideal conditions is as *Figure 25*. Compared with *Figure 12*, we can conclude that traffic flow evolves to a stable state more quickly in the ideal conditions. Also from the figure we can see that the trend of flow in all three models almost identical and $Model2$ is not the best in traffic flow this time.

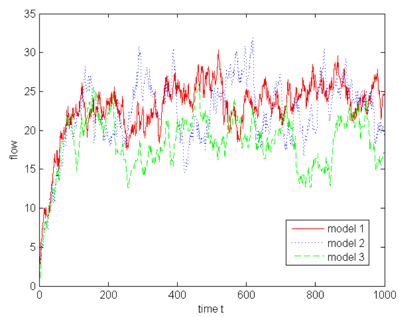
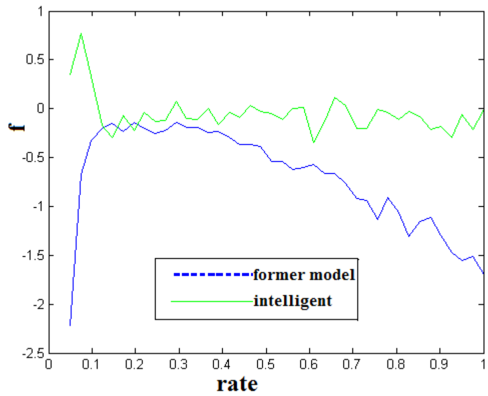
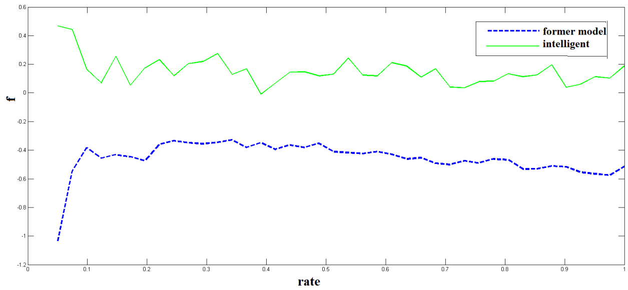


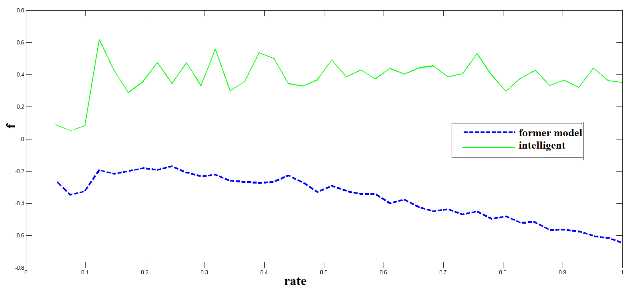
Figure 25: The Evolution Of Flow



(a) Model 1



(b) Model 2



(c) Model 3

Figure 26: Comparison of Evaluation Index

6.2.2 Evaluation Index

In ideal situations, the value of f will also differ from the no-ideal situations, as illustrated in Figure 26.

From the figures above, it is self-evident that the evaluation index is much better if ideal. This phenomenon can be interpreted in two aspects. As far as flow is concerned, it should increase since car can accelerate whenever possible, and no safety gap reduces the chance to decelerate. On the other hand, when it comes to safety, with our previous definition, the freeway should be a more dangerous place, for higher speed and narrower gap. However, this is no longer appropriate for the system is controlled by intelligence. As a result, if we correct the effect of safety, the evaluation of the rule shall be even better.

7 Strength and Weakness

Our model is based on the existing NS model to test the performance of different freeway traffic rules. But like all other systems, it has its own advantages and shortcomings.

7.1 Strength

- The model considered various factors including p, q, r, λ, ${}_{\max}$, $v_{\min}$, so that the model is a good estimation of the practical situation.
- We used the front-to-end method to update the state of the cars, which is simple and effective.
- Two modified rules are proposed to improve the right-most rule. The trade-off of the new rules perform better than the basic rule.
- We give good evaluation systems in both level and value. The two methods implement with each other to assess the road condition.

7.2 Weakness

- Our model is only available when the volume of the road is not large. If too many vehicles flood into the freeway, the model can no longer be used.
- The parameters of the evaluation relies on the system coefficients, such as the road length L. If the coefficients are changed, the evaluation system should be altered correspondingly.
- Because the model is a possibility model, the results of the model is not stable when the simulation times are not sufficient. This makes testing pretty hard, if the system model is complicated, resulting in large variance in the output values.

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