Minimal set of inference rules

This set of inference rules provides a type system that is ?decidable?, with a condition that all op definitions must have an annotation.

Notation

- All latin letters represent a single element type.
- All greek letters represent a monoid of stack element types.
- "τ" is a special case and represents a singleton.
- "$\cdot$" denotes a monoid concatenation, e.g., `mappend` in Haskell.
- "[*pre*][*post*]" represents an operator type. Where *pre* and *post* represent the argument and the return stack element types respectively.
- An operator(s) between the stack descriptions is a shorthand, e.g., writing $[a, b]$`foo bar`$[c, d]$ is equivalent to `foo bar` : $[a, b][c, d]$
- The leftmost element in the stack type description is the most recently pushed, e.g., $[]$`foo bar baz`$[Baz, Bar, Foo]$.

Specialization rule

Specialization rule for stack element types

$$\frac{\tau' = \{a \mapsto a'\}\tau}{\tau \sqsubseteq \tau'} \quad \text{(Spec)}$$

LHS is considered an $\sqsubseteq$ of RHS when there exists a substitution that turns the LHS into the RHS.

Specialization rule for operator types

$$\frac{[\alpha'][\beta'] = \{\tau_i \mapsto \tau_i'\}[\alpha][\beta]}{[\alpha][\beta] \sqsubseteq [\alpha'][\beta']} \quad \text{(Spec)}$$

Relaxed specialization with nop postfix

$$\frac{[\alpha][\beta] \sqsubseteq [\alpha'][\beta']}{[\alpha][\beta] \sqsubseteq [\alpha' \cdot \gamma][\beta' \cdot \gamma]} \quad \text{(Nop postfix)}$$

This rule is heavily used in the pattern matching to check if the case arms have the same type signature.

Operator name

$$\frac{[\alpha]\texttt{op}[\beta] \in \Gamma}{\Gamma \vdash [\alpha]\texttt{op}[\beta]} \tag{Op}$$

As a consequence, all operators should have an annotation. This allows the type system to get rid of the generalization rule from the HM completely

Instantiation

$$\frac{\Gamma \vdash [\alpha']\texttt{op}[\beta'] \quad [\alpha'][\beta'] \sqsubseteq [\alpha][\beta]}{\Gamma \vdash [\alpha]\texttt{op}[\beta]} \tag{Inst}$$

Instantiation is the same as in the HM.

Pattern matching

$$\frac{\Gamma \vdash [\alpha_i]\texttt{destr1 body1}[\beta_i] \sqsubseteq [\alpha][\beta] \quad \forall(\texttt{constr}, \texttt{body})}{\Gamma \vdash [\alpha]\texttt{case}\{\texttt{constr1}\{\texttt{body1}\},\texttt{...}\}[\beta]} \tag{Case}$$

Where destructor is a constructor with its pre and post flipped, i.e.

$$\mathrm{destr}([\alpha]\texttt{constr}[\beta]) = [\beta][\alpha]$$

An important thing to note is that this inference rule states that all match arms must have an op type that is $\sqsubseteq$ of the whole `case` expression op type.

Chaining

Operator chaining is split into $2 + 1$ possible variants: overflow, underflow, and exact, with the last being a consequence of any of the first two.

Overflow

$$\frac{\Gamma \vdash [\alpha]\texttt{x}[\beta \cdot \gamma] \qquad [\psi]\texttt{y}[\omega] \qquad \beta_i \sqsubseteq \psi_i}{\Gamma \vdash [\alpha]\texttt{x y}[\omega \cdot \gamma]} \tag{Overflow}$$

Underflow

$$\frac{\Gamma \vdash [\alpha]\texttt{x}[\beta] \qquad [\chi \cdot \psi]\texttt{y}[\omega] \qquad \beta_i \sqsubseteq \chi_i}{\Gamma \vdash [\alpha \cdot \psi]\texttt{x y}[\omega]} \tag{Underflow}$$

Exact

$$\frac{\Gamma \vdash [\alpha]\texttt{x}[\beta] \qquad [\psi]\texttt{y}[\omega] \qquad \beta_i \sqsubseteq \psi_i}{\Gamma \vdash [\alpha]\texttt{x y}[\omega]} \tag{Underflow}$$

Standard operations: dup, del, bury, dig

$$\frac{}{\Gamma \vdash [\tau]\texttt{dup}[\tau, \tau]} \tag{Dup}$$

$$\frac{}{\Gamma \vdash [\tau]\texttt{del}[]} \tag{Del}$$

$$\frac{||\alpha|| = n}{\Gamma \vdash [\tau \cdot \alpha]\texttt{br-n}[\alpha \cdot \tau]} \tag{Bury}$$

$$\frac{||\alpha|| = n}{\Gamma \vdash [\alpha \cdot \tau]\texttt{dg-n}[\tau \cdot \alpha]} \tag{Dig}$$

All four primitive operators are self explanatory, and are related to the stack elements manipulation.