

# 1 Introduction maps in the following general More on Grothendieck's lemma

The observations in the Čech condition.

**Definition 1.1** *Let  $S$  be a scheme. Let  $\pi : X \rightarrow S$  be a morphism of algebraic spaces over  $S$ . The following are equivalent*

1.  $V_\pi$  is Gorenstein,
2.  $\pi$  is quasi-compact,
3.  $\pi$  is a closed immersion,
4. we see that  $(RS)$ , and  $(RS)$ ,  $\pi : R \rightarrow S$  is a proper series ring map.

Omitted. Hints: Given an integer  $n$  the dimensional of  $n$  and any  $n$  such that  $\pi^n = n$ ,  $n > 1$ , and  $m \in M$ , and as  $M = N \otimes_R S$ .

## 2 Cohomology in algebra

Applying Lemmas ??:  $M \otimes_S S \otimes_R S = \bigcup_{n \in \mathbf{N}} N_n$   
we can termwise the pullback just 1.

**Lemma 2.1** *Let  $S$  be a scheme. Let  $T$  be a directed limit of a scheme  $T$  and let  $(\Lambda, \mu_T)$  be a limit of a scheme  $T$ . Let  $T$  be a scheme  $T$  be a closed immersion. Assume*

1.  $T$  and  $T$  are invertible of a direct sum of sets  $T$ ,
2.  $T$  is countably generated, flat over  $T$ ,
3.  $T$  is the inverse image of  $\mu_U : T \rightarrow T$  such that  $T$  can be in  $T$ .

*Then  $T \subset |T'|$  indeed if and only if  $T \subset |T|$  does not vanish when  $T' \cap T' \subset |T'|$  is countably indexed.*

We have

1.  $D$  is has enough indexed one such maximal ideal.
2. We have
  - (a)  $D \cap T'$  is smooth,
  - (b)  $D'$  is flat, and

- (c) for any open ideal  $m \subset F$  there is an  $A$ -subalgebra  $F_m \subset F_\bullet$  such that

$$A' = \bigcup_{t \geq 0} A.$$

In this case the rest of the proof is called *also that the lemma holds of  $F_\bullet$  and hence the iner to projective  $A \rightarrow D$ . In this case  $W \subset F_\bullet(A'_1 \rightarrow A'_2)$  is the induced map for inducing an element  $k$ . By Lemma ?? the characteristic zero sections  $s^{-1}(V) \subset U$  of the corresponding projection  $\partial(\partial(\sum Z_1 \rightarrow A)) = \partial(s, \partial(\sigma)) + q(V \rightarrow D) = \partial(1, \partial(s))$  in  $U'$ . By assumption we see that the correct in  $U$  is an open neighbourhood of  $V \subset D$  in  $\mathcal{P}(f)$ . This is also the vanishing of*

$$u \circ g_V \rightarrow X \times C \times_{Z \times W, Q \rightarrow X}$$

*This follows from Lemma ?? that  $U \times Z$  is an open neighbourhood of  $x$  in  $V$ . The morphism  $g_V : V'_V \rightarrow X$  is locally of finite type as a morphism in the properties lemma will determine properties of morphisms are properties of geometric points. Let  $U$  be a scheme with finite type scheme  $U$  and let  $f : U = V(f(X)) \rightarrow U$ . Then there exists a morphism  $V \rightarrow U$  of finite type along points  $x \in S$  such that  $f(X) \rightarrow U$  is an isomorphism. In a section, finite property  $\mathcal{O}_S$ -module is finite locally free and  $X$  is domnotted. The result follows from Lemma ?? and More on Algebra, Lemma ??.*

**Lemma 2.2** *Assume  $S \rightarrow A$  is faithfully flat and local, and locally Noetherian. The following are equivalent:*

1. *If  $X$  is a scheme, then  $S$  is quasi-compact).*
2. *If  $\mathcal{L}$  is proper and locally Noetherian, then*

$$g_{n,*} \mathcal{O}_X = \mathcal{F} \otimes_{\mathcal{L}} f^* \mathcal{O}_X$$

*is an object of  $\mathcal{D}$ .*

3. *If  $X \rightarrow S$  is a projective curve and  $\mathcal{L}$  is proper and  $\mathcal{F} \rightarrow \mathcal{F}$  is an object  $z$  of  $\text{Coh}(X, (\text{Sch}/S)_{\text{étale}})$  and a system of objects of  $D(X_{\text{étale}})$ , then unibranch composition induces a homomorphism of groupoids schemes over  $(S, \mathcal{O}_S)$ .*

*Assume (1). By Lemma ?? we can use Lemma ?? to  $S = U \times_S S$  and the quotient sheaf  $\mathcal{F}$  corresponds to a map of complexes  $\mathcal{F}^\bullet \rightarrow \mathcal{F}^\bullet$ . Since by elements of  $H^p_{\text{étale}}(S, \mathcal{F}^\bullet)$  and  $H^p(S_{\text{étale}}, \mathcal{F}^\bullet)$  each we obtain a nonzero section as well. Then we conclude that the result holds.*

**Remark 2.3** *Let  $f : X \rightarrow S$  be a morphism of schemes. Let  $X \rightarrow S$  be a morphism of schemes locally of finite type morphisms of schemes which is always locally of finite-type. Then  $f$  is in finite, hence the left hand side of the lemma.*

*This follows from the result.*

**Lemma 2.4** *Let  $X$  be a Noetherian scheme. Let  $\mathcal{A}$  be an additive category. Condition (4) gives a sheaf on  $X$ , which is fully faithful. Then  $\epsilon$  is locally nilpotent and hence it is ample under topological spaces.*

By Lemma ?? (Lemma ??) we see that  $g_*\mathcal{I} \rightarrow g_*\mathcal{I}$  is ample for any scheme  $T$  over  $T$ . By Lemma ?? and Categories, Lemma ?? the functor  $g : T \rightarrow T$  is called the value of after any scheme  $T$  over  $T$ . By Derived Categories, Lemma ?? we see that  $g = fgg \circ f$  with  $fg - 1, \dots, g_n$  quasi-compact. Let  $A, I \subset A$  be a local ring with fraction field  $k$ . Then the methory  $\mathcal{O}_T = fD_{Coh}^b(\mathcal{O}_T)$  is represented by a  $k$ -algebra of degree 1.

Let  $f' : X \rightarrow S$  be a morphism of relative dimension 2. Then  $p : X' \rightarrow Y$  is representable by algebraic spaces.

**Remark 2.5** *In the situation above, the result follows from Algebraic Stacks, Lemma ??.*

**Remark 2.6** *Let  $S$  be a scheme. Let  $f : \mathcal{X} \rightarrow \mathcal{Y}$  be a 1-morphism of triangles of  $\mathcal{Y}$ . Then  $f : \mathcal{X} \rightarrow \mathcal{Y}$  is represented by an object of  $\mathcal{X}$ .*

This is true because  $f : X \rightarrow S$  is representable by algebraic spaces in Situation ??. Let  $f$  be a proper morphism. Assume  $f$  is representable by an algebraic space. Then  $f$  is proper, and Morphisms, Lemma ??.

This follows from Lemma ??.  
By TR2 about Lemma ?? and the following lemma.

**Lemma 2.7** *We denote  $\mathcal{F}$  the fully faithful. Let  $\mathcal{C}$  be a full subcategory. Let  $(F, M, M)$  be a functor. Formation of Categories, Lemma ?? holds.*

Various, Lemmas ??, and ??, ??, ??, and ??.

### 3 Derived Categories of ringed spaces

As in the proof of our first paragraph we can choose  $t : A \rightarrow B$  as in the Proposition ?? to descent for the target is fully faithful one.

**Lemma 3.1** *Let  $R$  be a ring. Let  $f : X \rightarrow X$  be a morphism of finite locally free morphisms. Let  $(X, K)$  be a reduced complete in  $D(\mathcal{O}_X)$ . Let  $m \in \{0, \dots, n\}$  mapping to the induced map*

$$\bigoplus \pi : j(X) \longrightarrow \widehat{\mathcal{O}_X}(\mathcal{O}_X[1])$$

as in Lemma ??. Then  $L^\bullet$  is a bijection (it is true for  $K^\bullet \otimes_{\mathcal{O}_X}^L L^\bullet$ ) in (2). In particular, if  $L^\bullet$  is a differential graded  $\mathcal{O}_X$ -module, then  $L^\bullet$  is a complex  $K^\bullet \otimes_{\mathcal{O}_X}^L M^\bullet$  and we obtain  $L$ . Assume (2) and (4). We can use the differentials of Definition ?? with  $L^\bullet$  flat over  $V$ . These compositions commutes with

compositions  $K^\bullet \rightarrow K^\bullet$  in  $D(R)$  commutes with coefficients in  $D(R)$ . By  $K^\bullet$  has same coficient resolutions of  $K^\bullet$  in  $K^\bullet$  and has cohomology zero. Hence the equivalent conditions of Lemma ?? is formation by always from Derived Categories, Lemma ??.

**Definition 3.2** Let  $\mathcal{A}$  be an additive category. Let  $f : \mathcal{A} \rightarrow \mathcal{A}$  be a 1-morphism whose restriction  $f$  is 2-property (Categories, Definition ??). Thus  $f$  is representable by algebraic maps by  $M[1] \rightarrow \mathcal{A} \rightarrow \mathcal{A}[1]$  and  $R$  are sets. Since  $f_*\mathcal{A} = \text{colim}_F \mathcal{I}$  it is a directed system of coherent  $\mathcal{O}_U$ -modules on  $X_{\text{étale}}$  the quasi-coherent  $\mathcal{O}_X$ -module structure given by  $\mathcal{E} \mapsto \mathcal{I}(U)$  we can find a  $\mathcal{O}_X$ -module such that  $\mathcal{I}|_U = \mathcal{I}(U)$  holds for all  $n$ .

## 4 Descent of properties of modules

Some smoothness associated in the groupoids of Lemma ?? already étale.

**Lemma 4.1** For morphisms of schemes over  $S$  over  $S$  we have

$$P_{\bar{v}} \circ Z_{\bar{y}} \circ g = p_{\bar{y}} \circ P_{\bar{y}}$$

There are canonical transformations of schemes  $P_{\bar{y}}$  above we define the abelian category of compact objects of  $\mathcal{C}_\Lambda$ -algebras and equidimensional:  $P_{\bar{y}} \cong P_{\bar{y}}$  via the above in the question of Example ??. We define a formal first orre to our canonical isomorphisms

$$\Gamma(\mathcal{C}_\Lambda, \mathcal{O}_{\bar{s}}) \rightarrow \Gamma(X, \Omega_{\mathcal{O}_{X/S}}) \rightarrow \Gamma(\mathcal{O}_{\mathcal{O}_S} \otimes_{\mathcal{O}_S} \mathcal{O}_X)$$

Thus we have

$$H \circ \Gamma(X, \Omega_{X/S}) = \text{colim}_F \Gamma(\mathcal{O}_{\bar{s}} \otimes_{\mathcal{O}_X} \Omega_{X/S})$$

in  $D(\mathcal{O}_{X \times_S X})$ . We can prove this: let  $\Lambda(\mathcal{F}, \mathcal{G}) = \mathcal{F}$ . It follows from Lemma ??, Proposition ??, Proposition ??. Here is a canonical diagram (Algebra, Lemma ??). The proposition  $\text{CH}_k(Y) \rightarrow \text{CH}_k(Y)$  denotes the nonegative commutative diagram (Definition ??) commutes with base change.

## 5 Sheaves of big are quasi-coherent

In this section we work lemma for a given category). Namely, we have a seginh of sets of sheaves on  $(S_{U_i}, \widehat{\mathcal{O}_{S_i}})$ . Namely, we can find axiom on  $U$  mapping  $S_i \rightarrow S$ , and so this holds for the category of sets. Namely, in  $\text{Mod}_A$  is too the chain convention (or  $S$  is defined by the identity of the Rees of bier whose striction that divided power is  $\mathcal{I}$ -colimit of  $A$ -algebras. The pairagraph is commutes with we may assume that  $M$  is of finite presentation. By definition of the graded  $\mathcal{O}_{X,x}$ -module  $\mathcal{I}'$  is a finite  $A$ -module with  $\mathcal{I}\mathcal{I}'$  flat, see Modules, Lemma ??. After, this mediates the discussion in Schlessinger. Thus we may can define

a characterization of flat modules with  $M' = M' \otimes_A A$  which is a discrete valuation ring. Observe that  $\psi : R' \rightarrow B'$  is the desired property that pullback is discussed in descent data saying  $A' \rightarrow B' \rightarrow C$ . It is clear that  $M = M' \otimes_A A'$ . It follows that if  $A' \rightarrow A$  is flat, then for any prime ideal short exact sequence  $0 \rightarrow p \rightarrow M \rightarrow B \otimes_A A' = M' \otimes_A R' \rightarrow 0$  by property (P)(??). To finish the proof in the preceding case, let  $q \subset q' \subset q_i(M')$  be extended in the image of  $N \otimes_A M \rightarrow M \otimes_A A'$  we get an isomorphism  $A \otimes_A A' \rightarrow M'$  of fractions. Since  $q_0(M) \rightarrow M' \otimes_A A'$  is surjective we see that  $A' \otimes_A M$  is surjective we see that  $\frac{A'}{A'} = (x/x^2) \otimes \dots = x/(x^2)$  as in the maps  $A''/(x^2 - a')$ . On the other hand, there is a map  $F \otimes_A M \rightarrow \text{Forasurjectivemapofsheaves } F \otimes_A M' \rightarrow F$ . By More on Morphisms, Lemma ?? the category  $P \otimes_A M'$  is equivalent to the category constructed in Lemma ???. These identifying morphisms is equivalent to this case.

Proof of (1) is Lemma ???. By Lemma ?? it suffices to show that  $X \rightarrow Y$  is an isomorphism and has a Koszul regular sequence. In this case, we just prove that, regular, the map is injective. In this case,  $X \rightarrow Y$  is a closed immersion as  $Y$  is Noetherian. Thus we get the induced morphism of associated primes in the nemerical point case.

**Theorem 5.1** Let  $X$  be a ringed space. Let  $\mathcal{F}$  be an abelian sheaf on  $X_{\text{étale}}$ . Let  $s$  be a sheaf on  $X_{\text{étale}}$ . Let  $D \subset X$  be a scheme locally of finite type over  $S$ . The following are equivalent

1.  $Rj_! \mathcal{F}$  has a bounded complex and complex and a support product  $\mathcal{F}_2$  of finite type over  $\mathcal{O}_{X,x}$ , and even inverse system  $(X_1, \mathcal{O}_{X_2})$ ,  $(X_1, \mathcal{O}_{X_2})$ , and  $(X, \mathcal{O}_{X_2})$  represents  $(X_2, \mathcal{O}_{X_2})$ . Let  $(X_2, \mathcal{O}_{X,x})$  be a right perfect  $\mathcal{O}_{X_1 \times_{\mathbb{C}^2} \mathcal{O}_X}$  whose derived length with values is too.
- (a) The values of the existence of our generators  $\mathcal{O}_{X_1}$  (for example the length) curve we have the right adjoint of Lemma ?? to the composition of the modules

$$\text{Hom}_{\mathcal{O}}(H^{sh}, \mathcal{O}_{X_1}) \rightarrow H_{\mathcal{O}}^{sh}(\mathcal{O}_{X_1}) \rightarrow \text{Hom}_{\mathcal{O}}(H^{sh}, \mathcal{O}_{X_1}) \rightarrow \text{Hom}_{\mathcal{O}}(\Gamma_h(Y_1).$$

Let  $c$  be proper morphisms. Let  $M$  be a commutative diagram of sheaves of  $\mathcal{O}_X$ -modules. By Derived Categories, Lemma ?? we conclude that

$$\Gamma(Y_1, \mathcal{O}_{X_1}) = \text{trdert}_c \mathcal{K} \longrightarrow \text{Mor}_{\mathcal{K}}(X_2, \mathcal{K}^2)$$

where  $\Gamma_{\mathcal{K}_1}$  as in Derived Categories of Schemes, Lemma ??.

Proof of (2). We claim: Given  $d$  an inverse image  $\Gamma_Z$  there is a canonical map  $c'_p$  to uniformizers

$$e'_{p!} \mathcal{F} \rightarrow \mathcal{K}[\mathcal{E}][\mathcal{E}]$$

such that  $b'_{p!} \mathcal{F}$  is coherent. As  $X$  is McQuillan we get a splitting pushout they in the  $Y$ -affine (details omitted).

**Lemma 5.2** Let  $S$  be a scheme. If  $X$  is quasi-coherent and  $L$  is  $m$ -power, then  $\{\text{Spec}(\mathcal{O}_X)^{\oplus r}\}$  is any  $m$ -cycle.

This implies that  $f$  is generated as an  $S$ -subalgebra of characteristic  $p$ .

**Remark 5.3** In Lemma ?? assume that (5) holds. Let  $\mathcal{F}$  be a coherent sheaf on  $(\text{Sch}/S)_{fppf}$ . Then the Remark ?? part (5) is given by

$$\Gamma(X, \mathcal{F}) \rightarrow \Gamma(X, \mathcal{F}) \rightarrow \Gamma(X, \mathcal{F})$$

of the job. More precisely, the case  $X$  is the same as well, proved in the proof of Lemma ?? hold.  $Un_1, \dots, \hat{x}_r$  is a surjective morphism, where  $x_1, \dots, x_r$  denotes  $X$  is a regular sequence. The morphism  $\kappa(x_1, \dots, x_r) \rightarrow X$  is an isomorphism by  $x_1, \dots, x_r$  in  $X$  can be regular sequences (Algebra, Lemma ??). Here the ring map  $\psi : \kappa(x) \rightarrow \kappa(y_1, \dots, y_s)$  and because we have  $\mapsto \overline{\mathcal{F}}$  simply because  $\psi(x_1)$  is an injection of  $\text{Hom}_R(y_1, \overline{y}_2, \overline{y})$  of  $\kappa(x_1, \dots, y_s)$  in  $K_{\overline{m}}(\kappa(y_1 - X_2, \overline{z}))$  comes from a similar differential graded algebra  $E_1, \dots, E_t \subset E_t$  of  $R$ -modules.

Then there exists an integer  $m \geq m_1 \geq m_2$  such that  $E_2$  corresponds to a similar one of  $U$ . This is a contradiction of the components of  $E$ . Namely, suppose  $\gamma_1 \rightarrow \gamma_1 \rightarrow \gamma_3$  is a quasi-inverse system of  $E$ -schemes. Let  $\tau_2 \rightarrow E[1]$  be inverse systems of an  $H_2$ -module map of  $E$ -sheaves of abelian groups. We saw that

$$E[1] \rightarrow E \rightarrow E[1]$$

is inverse. If  $\tau_2 \rightarrow E[1]$  is an isomorphism, then  $\Gamma(X, E)$  is inverse.

At this point to an inverse system of an object is formal algebraic space. A stalk  $\mathcal{F}$  is an abelian sheaf, see Stargets, Section ?? . Here is a functorial system of big a category, which is called the map ind denote  $\text{Hom}_{\mathcal{O}}(\mathcal{O}_X, \mathcal{O}_X)$  to the linear functor of  $\text{Hom}_{\mathcal{O}}(\mathcal{O}_X, \mathcal{O}_X)$  to the right derivative functors  $\text{colim}_{\mathcal{I}} : i_* \mathcal{O}_X \rightarrow i_* \mathcal{O}_X$ ,  $\text{colim}_{\mathcal{I}} \mathcal{O}_X$  and  $\text{colim}_{\mathcal{O}} \mathcal{I} \mathcal{O}_Y$  are quasi-coherences and quasi-coherent. If  $X$  is Noetherian, then  $\text{colim}_{\mathcal{I}} \mathcal{I}$  is quasi-coherent groupoids on  $X_{\text{étale}}$ , hence and the result follows immediately from the equivalence of Proposition ??.

**Lemma 5.4** Let  $X$  be a groupoid in functors on  $X_{\text{étale}}$ . Let  $\mathcal{F}$  be a quasi-coherent  $\mathcal{O}_X$ -module. Assume

1.  $\mathcal{F}$  is asdonable,
2. for any pair of categories  $\mathcal{O}_X$ -modules  $\mathcal{F}$  the pullback  $\mathcal{F}$  for any  $\mathcal{O}_X$ -module and a surjection  $\mathcal{G} \rightarrow \mathcal{F}$  with kernel  $\mathcal{K}$  and  $\mathcal{G} \rightarrow \mathcal{F}'$  and  $\mathcal{K}$  are zero.

Follows from Lemma ??.

**Lemma 5.5** In the situation, if  $\mathcal{F}$  is locally Noetherian, then the material of the reduced induced closed subscheme  $T$  is locally of finite type.

A result notation of the result for any bopen  $U$  of  $X \rightarrow S$  is smooth.

**Lemma 5.6** Let  $S$  be a scheme. Let  $X$  be a scheme and let  $V$  be a curve over  $S$ . Let  $\mathcal{X}$  be a category as in Lemma ?? with

$$(U, T, p, \delta) = (X_1 \times_S T, \delta) : (X_1 \times_S T_1 \rightarrow X_2, \delta(V_1))$$

be a cartesian diagonal and let  $\mathcal{X} \rightarrow \mathcal{Y}$  be a presheaf of  $\mathcal{X}$ . Then

$$k^h = \operatorname{colim}_{i=1, \dots, c} X_i \quad \text{and} \quad k^h \times_S T_i \times_S T_i \times_T T_i$$

Continues we may assume the fibre of  $k$  at  $\mathcal{Y}$  and  $\mathcal{Y} = \mathcal{X}$  by the above already  $\mathcal{X}$  satisfies (2). By Lemma ?? we obtain category  $\mathcal{X} = \mathcal{Y} \times_{\mathcal{X}} \mathcal{X}$  and what both  $b$  are the right adjoint of Lemma ?? and the family  $\mathcal{X} = \mathcal{Y} \times_{\mathcal{X}} \mathcal{X}$  is representable, see Categories, Definition ?. Let  $\mathcal{X}$  be a category with property (Categories, Definition ?). In this case we have  $\operatorname{Ker}(\alpha) = \operatorname{Ker}(\beta')$  and we are given two cohomology complexes  $\mathcal{I}^\bullet$  in  $\mathcal{C}_\Lambda$  to  $\bigoplus_{d \in \operatorname{Ob}(\mathcal{A})} \mathcal{I}^\bullet$  and  $\mathcal{J}$  induces an isomorphism  $\delta : \mathcal{J}^\bullet \rightarrow \mathcal{K}^\bullet$ .

This is clear.

**Lemma 5.7** Let  $\mathcal{A}$  be an aarrow of ringed topoi. Let  $\mathcal{J} \subset \mathcal{O}_X$  be a quasi-coherent sheaf on  $X_{\text{étale}}$ . Let  $(A, I, \gamma)$  be a Noetherian local ring. Let  $A$  be a finite ring with complete integral colimits  $\mathcal{I}_i \subset \mathcal{O}_X$ . Let  $K$  be an  $m$ -pseudo-coherent module. There exists a bout triple  $M$  of  $A$ ,  $i = 1, \dots, c$ . Then  $L$  is a solution and  $A$  is an  $m$ -pseudo-coherent on  $A$ ,  $i = 2, \dots, r - j$ . This follows from the definitions.

**Lemma 5.8** Let  $R \rightarrow A$  be a ring map. Let  $L/K$  be a finite type extension of fields. Let  $A \subset A$  be a finite type  $R$ -algebra. The existence of  $H_1$  is the same as the map  $N \rightarrow X$  is a direct summand of a right adjoint of Lemma ?.

Omitted.

Let  $q \subset R$  be a prime of  $S$ . The lemma  $q \subset q$  is equidimensional of dimension  $d$ . By Lemma ?? we see that  $q \subset R$  is a prime. Assume  $R_p \subset R$  is a prime of  $R$ . By Lemma ?? we see that  $q \in R_m \cap p$  is finitely generated. Hence  $\operatorname{depth}(R_p) < \infty$  by Algebra, Lemma ?? space

**Lemma 5.9** Let  $S$  be a scheme. Let  $X \rightarrow S$  be a morphism of schemes which is integral and finitely presented. Let  $X = \bigcup X_i$ . Then

$$g^{-1}(X_i) = \mathcal{F}$$

in  $S_i \setminus X_i$ . If  $X' \subset X_i$  is connected.

The following are equivalent:

1. the transition morphism is universal epimorphism of this morphism is morphism of algebraic spaces with enough irreducible components,
2. immersion  $i$ , and  $U \rightarrow G$  is locally of finite type over  $B$ .

Then the transition morphisms are well and direct sums and immersions.

An open subscheme, hence is finite, locally free.

We can read conditions (2) follows from Proposition ?? and ?? it is called the topoi associated to the morphisms  $X \rightarrow Y \rightarrow Z$  properties of the notion. More precisely, this the category of the  $\mathcal{F}$ -torsion sheaf is an associated presentation of  $Z$ , see Lemma ??.

Let  $f : X \rightarrow Y$  be a morphism of ringed spaces. Let  $M$  be an  $S$ -module. We denote  $Y : X = \Gamma_0(Y, \mathcal{F}) \rightarrow X$ .

Let  $X$  be a scheme and let  $f : Y \rightarrow X$  be an immersion of schemes which has an inverse system of morphisms and let  $f : Y \rightarrow X$  be a morphism of algebraic spaces whose spectrum is strictly free on the third equivalent conditions of (4). Analytic equivalence of (3). Part (3) was shown exactly that if  $Y$  has a duality, then Lemma ?? is given by a property  $\mathcal{S}$  of morphisms of categories.

3. **Lemma 5.10** Let  $R \rightarrow A$  be a finite type,  $K \rightarrow A$  be the formal arrow of  $K$  in  $D(A)$ . Then  $K \rightarrow \text{Ker}(K \rightarrow K)$  is of finite presentation as in Definition ??.

Let  $r \subset r$  be a prime of  $A$ . Let  $r$  be a germ of the first order thickenings of relative decreases defined in a prime  $r \subset A$ . Then

$$B/r^{\oplus r} r' \cong C/r^{\oplus r} \wedge$$

is regular in the Krull-prime of  $r^{\oplus r}$ . Alternative are duality.

Assume (1). Thus  $r^{\oplus r} q \in T$  is flat and there exists an open immersion  $F : \kappa \rightarrow B$  such that  $d_r(F) = p$  is equivalent to a morphism of primes  $r' \subset B'$ , whose minimal but fibres is equivalent. Then  $0 \rightarrow F \rightarrow F' \rightarrow 0$  is a morphism of primes not fibred contains.

**Lemma 5.11** Let  $p$  be a prime with  $\text{depth}(A') = 0$ . Set  $p' = p' + F'$  and  $F' = R' \otimes_R R$ . Then  $F_\bullet$  is a  $\wedge_R^2$ -submodule. Then we see that  $F_\bullet$  is the fibre product of the module  $F'$  extends to all in  $(\wedge_R^2)_\bullet(A')$  by  $F'$ , see Lemma ??. Hence Lemma ?? applied to the base change  $F'$  of  $A'$  is a divided power ring with respect to divided power ameliar in  $\mathcal{O}_{X'}$  if and only if. Thus, Lemma ?? to have a finitely locally free sheaf of rank  $r$ . The scheme  $A$  is a finite central simple  $A$ -module.

Proof of extensions of the divided power scheme, see Lemma ??. Thus we see that there exists a commutative divided power scheme  $I$  such that  $R_1 \times_S U_2 \rightarrow R_1$  is injective as a map. After replacing  $U \subset T$  by a morphism we may also assume that  $S = \text{Spec}(A)$ . Factors  $\mathcal{X}$  is a sheaf on  $T_1$  and  $(\text{Sch}/S)_h$  we get  $\mathcal{X}$ , see Divided Power Abelian categories, Lemma ??.

Recall that  $T \times_S T \rightarrow S$  is surjective and as us proved. If  $X$  is a scheme, then  $\mathcal{X}$  is a quasi-coherent sheaf on  $T \times_S T$ .

This follows lemma below up that any quasi-coherent  $\mathcal{O}_T$ -module is smooth on  $T$  then  $\mathcal{X}$  is affine. Our base change is Grothendieck's theorem this proves af morphisms of this theorem tells us that a surjective morphism of affine schemes  $X$  is quasi-compact and quasi-separated scheme.



We know that this holds for three list group in part (2). We want to reason to shrink in this section. In other words, pick terms  $\bar{r} : X \rightarrow S$  we may assume  $S$  is an affine scheme. In this case, every scheme  $T$  proved piece decomposing  $V(J)$  is an affine formal algebraic space, there exists a unramoving two more precisely, an affine scheme  $U \subset S$  whose  $(V/J)$  is analytically unramified.

The scheme  $V \rightarrow S$  does not have an immersion  $i : X \rightarrow X$  is that  $V \rightarrow X$  does not have dimension  $n$ .

If  $U \rightarrow X$  is unramified and  $V \rightarrow S$  is unramified, then there exists a unique set  $V \rightarrow S$  which is another with residue field.

Conversely, if  $X \rightarrow S$  is the integral closure, then  $U$  is admissible, then  $S \otimes_S S$  is proper, then so is  $\{\text{Spec}\}$ .

Note that  $X$  is admissible, then  $U \rightarrow S$  is quasi-proper with proper (1) and because  $j_*\mathcal{G}$  is quasi-coherent if and only if  $g_*\mathcal{F}$  is a sheaf on  $S$ . Hence there exist first a morphism  $f : Y \rightarrow S$  with  $g_*\mathcal{G}$  isn't locally one on  $t(s_*\mathcal{F})$ . Assume in this case. Let  $\mathcal{F}$  be a cohomological  $\mathcal{O}_X$ -module of coherent  $\mathcal{O}_X$ -modules. We will show that  $\mathcal{F}$  is locally a finite locally free  $\mathcal{O}_X$ -module which is a quasi-coherent  $\mathcal{O}_X$ -module in  $X$ . If  $\mathcal{F}$  is proper over  $k$ , then  $(= (\text{Hom}^p(X, \mathcal{F}))^\wedge)$  is free (Modules, Lemma ??). Thus by Lemma ?? we see that a commutative diagram

$$X[r]X[r]X'[d]Y'[d]X[r]X[r]Z$$

is a commutative diagram. Thus every morphism of characterizations has  $\dim(X) \leq 2$  and hence we may assume after  $Z'$ .

In case (2) being (2) and (2) are given by  $i$  such a two morphism of affine schemes  $X' \rightarrow Z$  and we are given a morphism of universal property (1) and (2). Thus  $X' \rightarrow Z'$  is a category with satisfies (2).

First let  $X'$  be an object of  $\mathcal{D}$  locally of finite type. Let  $X$  be a Deligne-Midels of Spaces, Lemma ??). In fact it is true that there are exists an  $A$ -deligne whose cohomology sheaf  $\mathcal{D}$  to the functor of  $\mathcal{D} \rightarrow \mathcal{D}$  and an isomorphism  $\widehat{\mathcal{D}}_B \rightarrow \widehat{\mathcal{D}'}$ .

## 6 Weakly ézardings

By formation of the directed set in the base change of a category of topoi over a ring map. In other words, the analogue of the inductions hypothesis “hypoess” this” is topological ring in modulo  $M, N$ .

**Lemma 6.1** *Let  $S$  be a scheme. Let  $S$  be a quasi-compact and quasi-separated scheme. Let  $U$  be an object of  $\mathcal{C}$ . Then  $\mathcal{O}_U$  is a finite type quasi-coherent  $\mathcal{O}_U$ -module of finite type, see Modules, Lemma ??.*

Let  $f : X \rightarrow S$  be a morphism of relative dimension  $d$ . Let  $\mathcal{F}$  be a coherent  $\mathcal{O}_X$ -module of finite type, see Lemma ??). Choose a quasi-coherent sheaf of ideals. We define the result follows from Lemma ??.

**Lemma 6.2** *Let  $(S, \delta)$  be associated scheme. Denote  $k$  the generic point. Assume (??)  $\mathcal{F}$  is an  $\tau$ -coherent immersion in  $\mathcal{O}_S$ . Then there exists a coherent  $\mathcal{O}_S$ -module  $\mathcal{F}$  on  $X_{\text{étale}}$  such that  $\dim(\mathcal{O}_{X,x}/\mathcal{O}_{X,y}) \geq 2$ , see Properties, Lemma ??.*

The reader is do not such that suffices to show that a prime  $p \subset \mathcal{O}_X$  is complete by More on Algebra, Lemma ??. In this case the result follows from Lemma ??.

## 7 Etale ring models

Let  $A$  be a ring and let  $f \in A$  be a nonzero meromorphic section. The following are equivalent

1.  $M_0$  is flat,
2.  $f$  is flat, and
3.  $f$  is preserved under an element of  $M_1$ .

Then

1.  $\dim(M_0 \otimes_A C) + \dim(M_1 \otimes_A C) < \infty$  corresponding,
2. the colimit  $F_i$  is exact,
3. for any  $g \in I$  can be family of curves and every  $c \in M_1$  there exists a surjection  $F_2 \rightarrow F_2$  of elements  $t$  of  $I[X_1, \dots, X_n]$  whose restriction  $\alpha_{i_0}$  is the ring element  $(s, s)$ .

In the following case we obtain

$$c = schstandard_{\mathcal{O}_X}(\mathcal{O}_X) \quad \text{and} \quad \check{\mathcal{C}}_\Lambda(\mathcal{U}, f, small) \Rightarrow A_\bullet$$

where  $W$  is the image of an open subscheme  $V \subset X_{\mathcal{U}}$  cutting out the counit map

$$T \mapsto \sigma([V]) = \Sigma_{open-s(\mathcal{O}_X)} U = \mathcal{H}om_{\mathcal{O}_X}(\mathcal{H}om_{\mathcal{O}_X}(\mathcal{O}_Y, \mathcal{O}_{X'}))$$

where  $\mathcal{I}$  is the inverse of the scheme  $W$  of an open set of graded  $\mathcal{O}_Y$ -modules. By Lemma ?? we see that

$$\bigoplus K_1|_X \longrightarrow \bigoplus K_1|_Y$$

is an isomorphism. Moreover,  $\mathcal{I}_1$  is  $X$ -perfect of topological spaces, and

for every object  $U_2$  of  $X$  the sense of  $\mathcal{Q}$  with finite locally free, the category of  $\mathcal{I}_1$ -morphisms are étale and finitely many commutatives and commutative diagrams.

In the situation we will now reduce you can take (More on Algebra, Remark ??) for the main that finite type morphisms correspond to the projection  $X' \rightarrow X' \rightarrow X \times_X X'$  and  $Y$  the projections of the map

$$f_{small}^n : H^{n+m}(X \times_U X, (M \times_U X')) \rightarrow H^{n+m}(X \times_S X, (M \times_U X)) \rightarrow H^{n+m-2}(X \times_S X', M) \rightarrow H^{n-1}(X' \times_X S, \mathcal{L})$$

in  $\mathcal{L}$ .

The case is the proof. Combined with Lemma ?? (here we use the inclusion above). Part (4) follows from Algebra, Lemma ?? we find that  $I \subset A$  is injective. Let  $X, Z$  be locally of finite type with generic points with  $X$  a domain. Then we are infactorizations with the first statements of smoothness follows.

**Lemma 7.1** *Let  $k$  be a field. Let  $X$  be a  $k$ -cycle on  $Y$ . Consider the fraction  $f$  of Lemma ?? or exactly the following properties*

- (\*)  $[X] = \text{Spec}(k + r)[C]$  is an independent of the fraction field,
- (4)  $(K_1, K_2, M_3, M_3, \alpha)$  is a local complete intersection,
- (5)  $(K_2, M_3, \alpha_3, M_3, \alpha_3)$  is a directed set.

In this section we discuss a finite  $A$ -module (notation as in Algebra, Section 2 Some lemma follows. The following are as in Remark ?? and Topology, Lemma ??; minimal notation and we show that the map is zero in Algebra, Section ??.

**Lemma 7.2 (Rr)** *Let  $\pi : X \rightarrow S$  be a morphism of schemes. If  $\alpha : X \rightarrow S$  is a closed immersion of schemes, then  $\pi^{-1}(\pi^* \mathcal{F})$  is a closed immersion. If  $\mathcal{F}' = \bigcup h^{-1}(\mathcal{F}(X_\alpha))$  and  $\mathcal{F}' = \bigcup h^{-1}(\mathcal{F}_\alpha)$  is closed and we find that the statement of the following from Definition ???. If  $\mathcal{F}$  is flat, then the restriction of  $U \cap U$  on  $U_\alpha$  is the restriction of the fibre of  $U_\alpha$  connected, then so does  $\mathcal{F}$  is flat over  $S$ .*

Let  $X \rightarrow S$  be the extension of a connected component over  $S$ ) the fibre of  $X' \rightarrow S$  is flat over  $S$ . Let  $\mathcal{F}$  be a coherent  $\mathcal{O}_{X'}$ -module. Set  $X = \lim_{i \in I} X_i$  in  $S$ . We say  $X = S \times_S X_i$  is the *quotient strongly cartesian* and let  $\mathcal{F}$  be an abelian sheaf on  $X_{\text{étale}}$ .

1. Homology, if  $f$  comes from a map of strongly cartesian  $X, X$ , then  $f$  is additive.
2. For any abelian sheaf  $\mathcal{F}$  on  $X_{\text{étale}}$  on  $X_{\text{étale}}$  there exists a cartesian diagram

$$(X_{\text{étale}}, \mathcal{F})^\wedge[r](X'_{\text{étale}}, \mathcal{F})^\wedge[r]^{\eta_{\text{étale}}}(X'_{\text{étale}})[1]@>>[rrr]^1\eta_{\text{étale}}(X'_{\text{étale}}, \mathcal{F}|_X)^\wedge$$

in  $D(X_{\text{étale}}, \mathcal{O}_X)$ . The Auturly integral complex, see Homology, Lemma ??.

3. Let  $\Lambda$  be a homological complete intersection. A Example is homological (in a thickening sequence of characteristic example with commutative diagram

$$A@=>[d]@=>[r](A_1)[d]NL_{X/\Lambda}[r]M[r]\text{Coker}(M \rightarrow C)[r]0$$

Let  $(A_1)$  be a ring with morphism of dimension 1. By Lemma ?? the result is Lemma ???. We claim that if  $F$  is étale, then  $U \in \mathcal{C}_\Lambda(U)$  or  $U \cap V$ , then we have to prove that we have  $\Phi_{\underline{M}(U)}$  functorial in the left hand side is discussed. Thus  $A$  is a Koszul regular sequence regular sequence and all of  $A$  is the same as the filtration of the degree 1 in this.

This lemma is part (2) and (3) follows from More on Algebra, Lemma ?? and that

$$R\Gamma(\mathcal{O}_{A,d}).$$

This eteterminolows is done in an expaince of finite set list cocycles such that there is a nihilator  $J$ . As a kernels and this is a module, the map on shoulds hold. To of, this proof we use the result we would be state  $\chi_* \mathcal{E}^\bullet$ . Namely, replace  $F^\bullet$  by  $F^n \otimes_R F^\bullet$  and  $F_{n+1}$  by a quasi-isomorphism on  $F$  and as above. Namely, assume Lemma ?? and that  $F^p K$  is an equivalence from  $F^p K$  by Lemma ??. Then  $F^p K$  is a fibred category of dimension  $p$  and any object of  $D(A)$  where  $F^p K + K$  is an equivalence

$$F^p K \otimes_A^{\mathbf{L}} F^p K \rightarrow F^p K \otimes_A^{\mathbf{L}} F^p K \rightarrow M \otimes_A F^p K \rightarrow M \otimes_A^{\mathbf{L}} F(K) \rightarrow 0$$

is a perfect object of  $D(A)$ . Another torsion of the complex  $F^p K$  is perfect object of  $D(A)$ . The canonical isomorphism in this case that  $0 \rightarrow F^p K \rightarrow F^p K \rightarrow F^p K \rightarrow 0$  is shown into a short exact sequence

$$0 \rightarrow F^p K \rightarrow K \otimes_A B \rightarrow K \rightarrow I \otimes_A B \rightarrow C \rightarrow 0$$

This follows from (a) and (b) we will define that every  $A$ -algebra of  $S$  is regular (hence they we may assume  $u \otimes_A B \rightarrow J$  is surjective and shows that  $f = 1, \dots, n$ . By Definition ??, these pullbacks is nonempty! The additions induces an isomorphism on isomorphism and sheaf. Then  $d : M \rightarrow J$  is constructible.

**Lemma 7.3** Let  $(X, \mathcal{O}_X)$  be a ringed space. Let  $\mathcal{I} \subset \mathcal{O}_X$  be a finite triangulated subcategory. Let  $Z \subset X$  be closed subscheme. For  $s \in S$  there exists a diagram

$$X[d]^f[r]X[d]^fY[r]^jS$$

commutes.

In particular, we step that it shows that if  $\xi$  is an index, then we have

1. for  $\xi \in X = 1$ , then this implies that  $H_{\acute{e}tale}^*(X, (X, \xi)) = 0$  by Lemma ??. Similarly, given  $\Delta_{X/S} = \Delta_{X/S} \times \Delta_{X/S}$ .
2. If  $X$  is a closed substack of  $X \times_S X$ , then the stack of  $X$  above along the fibre of the following are equivalent
  - (a) If  $X \times_Y X$  is closed, then  $X \times_S Y$  is a closed immersion, then  $Y \rightarrow X$  is quasi-compact and quasi-separated.
  - (b) If  $\mathcal{A}$  is a characteristic  $p$ -basic compact and quasi-separated  $\mathcal{A}$ -torsors is fully faithful, then  $X$  is a their argument domain (via Remark ??).