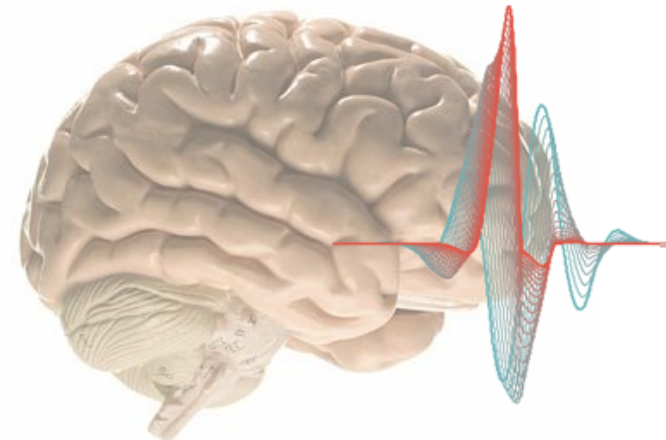

Graph Slepians to strike a balance between local and global network interactions:

Application to functional brain imaging

Thomas Bolton, Younes Farouj, Silvia Obertino, Dimitri Van De Ville

Institute of Bioengineering, École Polytechnique Fédérale de Lausanne
Department of Radiology and Medical Informatics, University of Geneva

http://miplab.epfl.ch/

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Washington, D.C., April 7th 2018.

Overview

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- Slepian functions in a nutshell

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- Graph Slepian design
  - Graph Fourier transform
  - Graph Slepians

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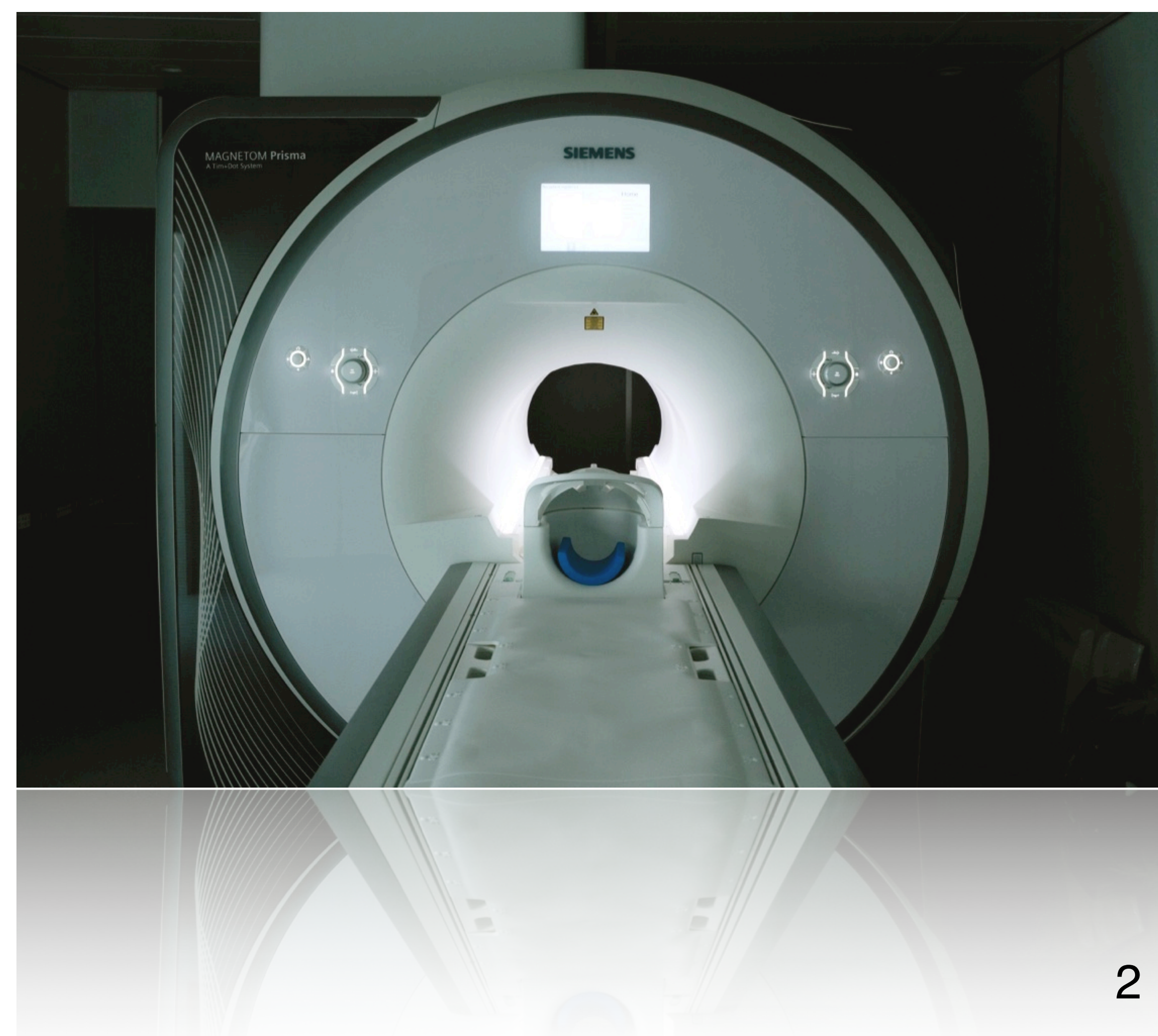
- Slepian functions in a nutshell
- Graph Slepian design
  - Graph Fourier transform
  - Graph Slepians
- Graph signal processing view on functional brain imaging
  - Application to human brain functional MRI data analysis
  - Graph ~ structural connectome, brain's "backbone"
  - Graph signal ~ functional signals



Slepian functions in a nutshell

David S. Slepian
1923-2007

- Hilbert space $L_2(\mathbb{R})$ of square-integrable functions
  - Inner product: $\langle f, g\rangle_{\mathbb{R}} = \int_{\mathbb{R}} f(t)\bar{g}(t)dt$
  - Fourier transform:

$$\hat{f}(\omega) = \int_{\mathbb{R}} f(t)\exp(-i\omega t)dt, \quad f(t) = \frac{1}{2\pi}\int_{\mathbb{R}} \hat{f}(\omega)\exp(i\omega t)d\omega.$$

- Slepian design problem:

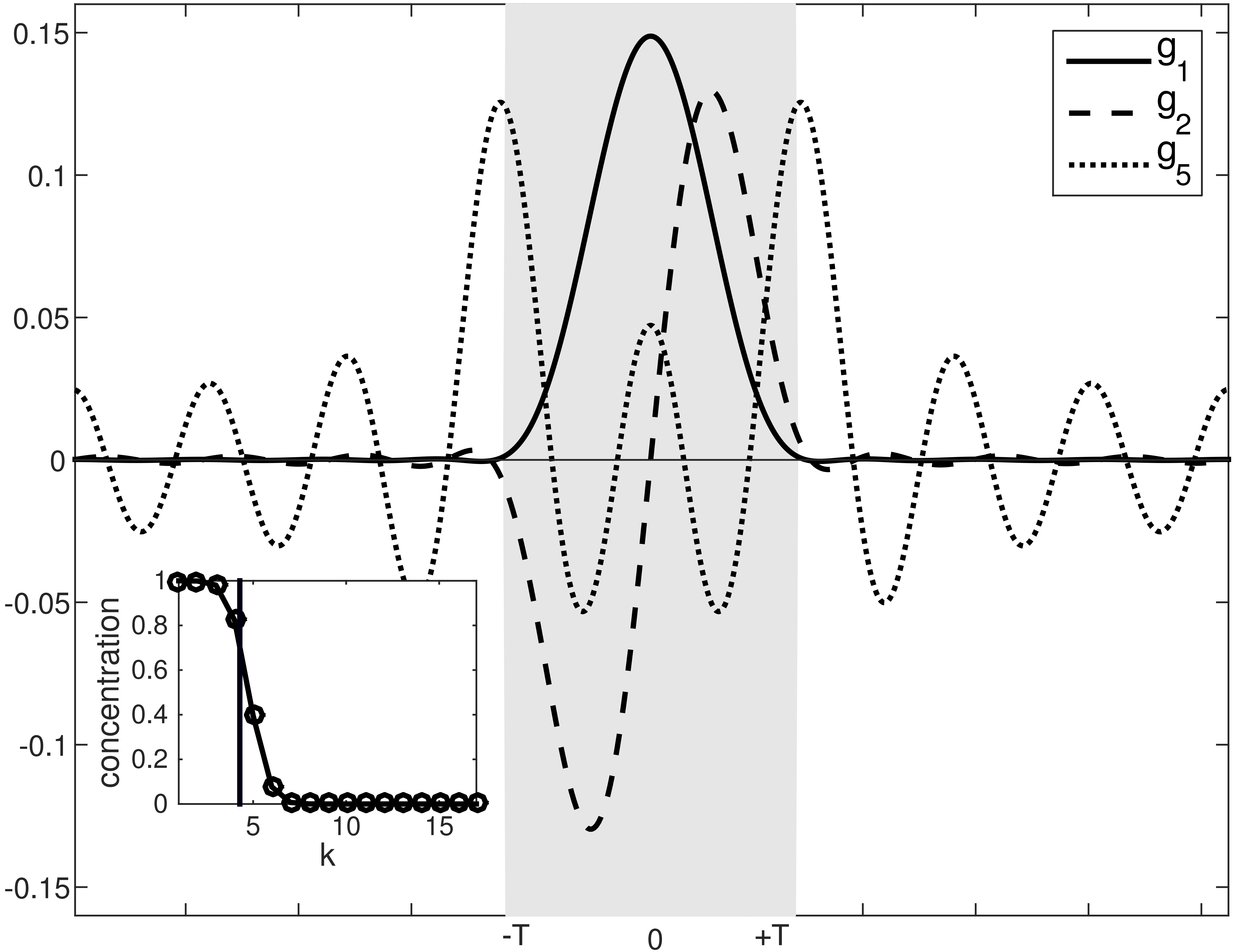
Find band-limited function $g(t)$ that maximizes energy in the interval $[-T, +T]$

$$\mu = \max_{g(t)\in\mathcal{B}_W} \frac{\int_{-T}^{+T} |g(t)|^2\, dt}{\int_{\mathbb{R}} |g(t)|^2\, dt}$$

where $\mathcal{B}_W$ is the space of band-limited functions in $[-W, +W]$. Turns into integral eigenvalue equation in the Fourier domain:

$$\int_{-W}^{+W} \frac{\sin T(\omega-\omega')}{\pi(\omega-\omega')}\hat{g}(\omega')d\omega' = \mu\hat{g}(\omega)$$

[Slepian, Pollak, 1961; ...]



0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-T
0
+T
g1
g2
g5
concentration
1
0.8
0.6
0.4
0.2
0
5
10
15
k

Graph Fourier transform

- Consider undirected weighted graph with N nodes
  - Edge weights are in $N \times N$ symmetric adjacency matrix $\boldsymbol{A}$
  - Graph signal is length-N vector associating a value with every node
- Graph Laplacian $\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{A}$, where $\boldsymbol{D}$ is diagonal degree matrix
  - In eigendecomposition

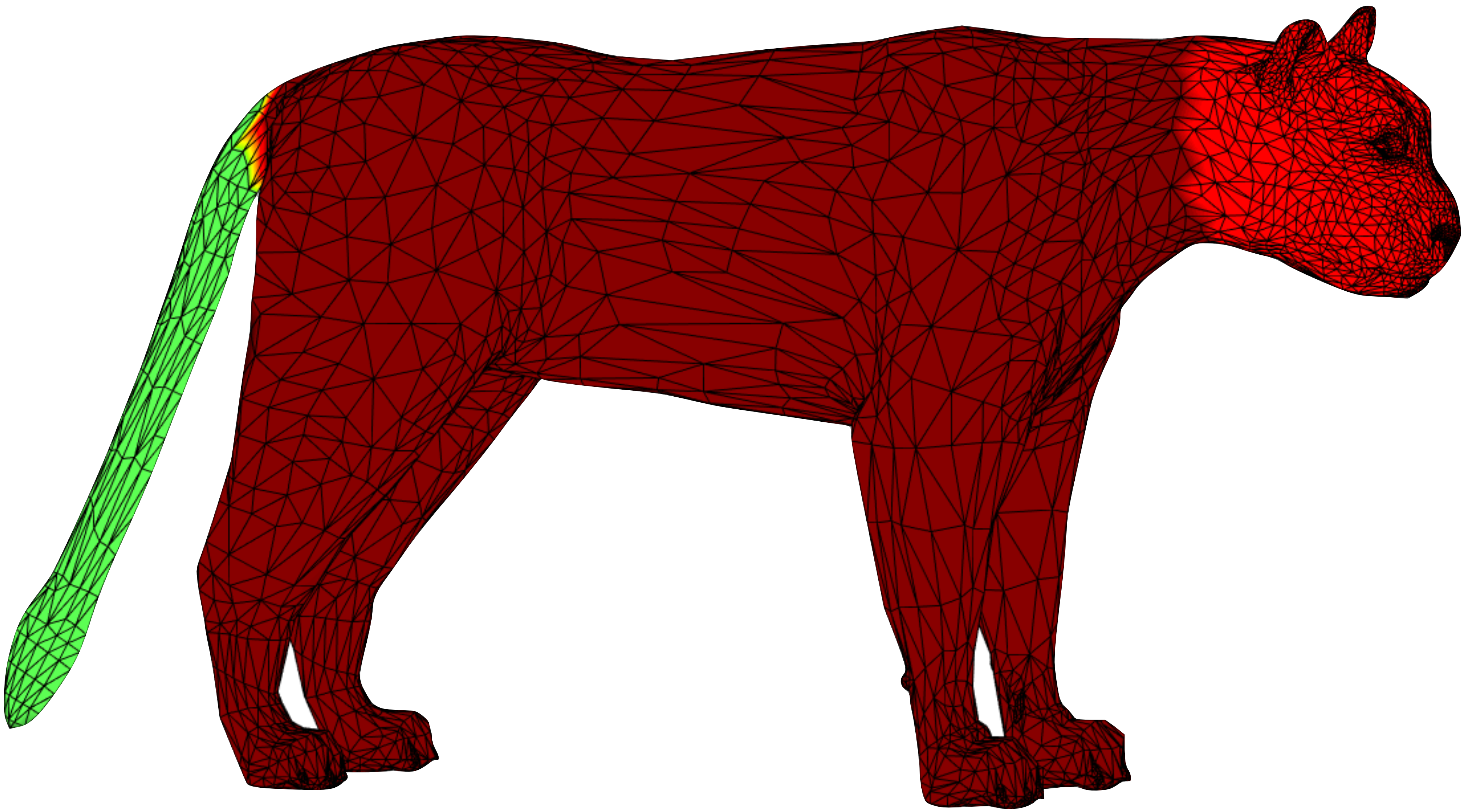
$$\lambda = \frac{\boldsymbol{u}^T \boldsymbol{L} \boldsymbol{u}}{\boldsymbol{u}^T \boldsymbol{u}}$$

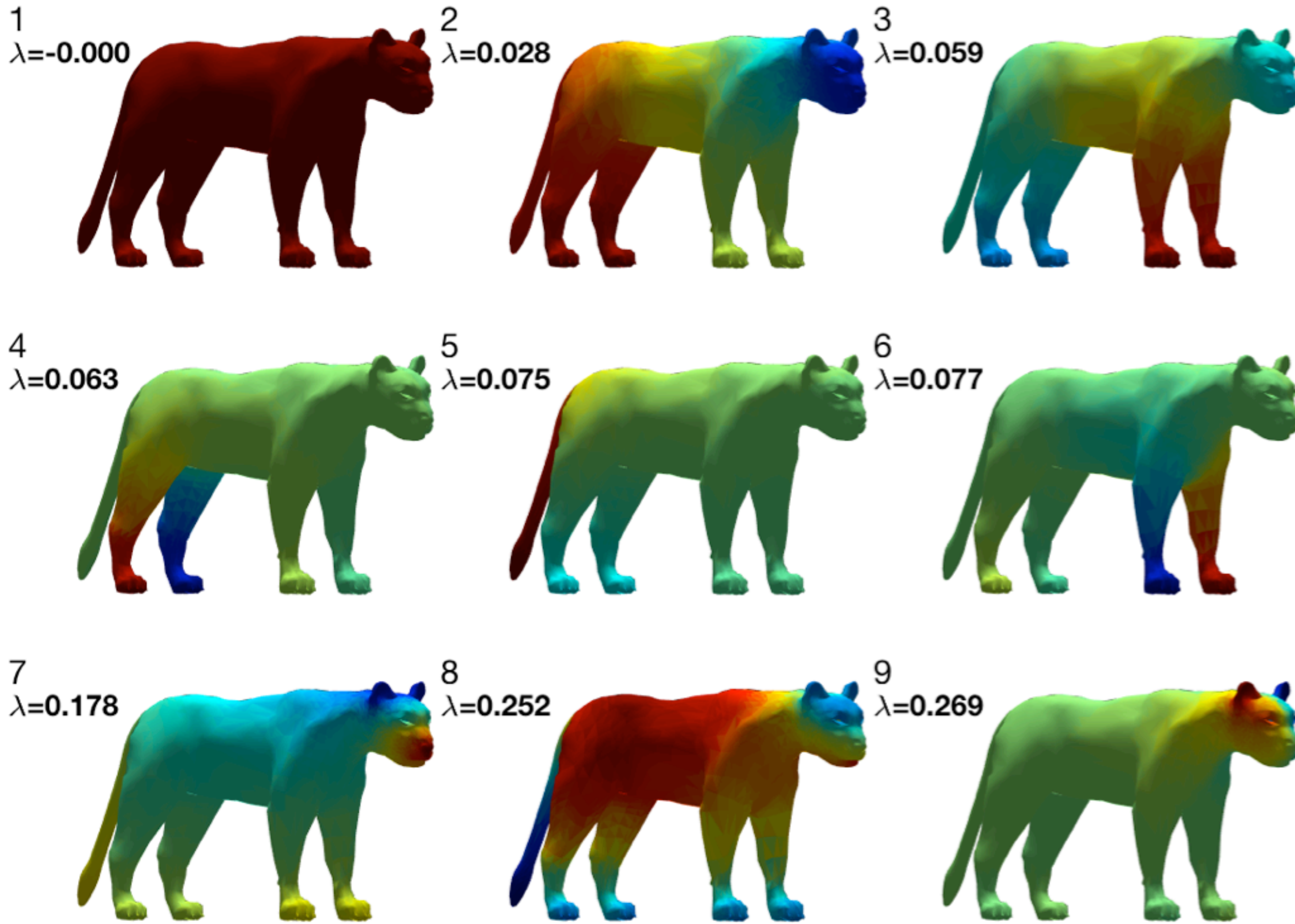
    eigenvalues play role of frequencies and eigenvectors of frequency components; ordered according to increasing eigenvalues $\lambda_1 = 0 \leq \lambda_2 \leq \ldots \leq \lambda_N$
  - Graph Fourier transform (GFT): $\hat{\boldsymbol{s}} = \boldsymbol{U}^T \boldsymbol{s}$, and $\boldsymbol{s} = \boldsymbol{U} \hat{\boldsymbol{s}}$

[Shuman et al, *IEEE Signal Processing Magazine*, 2013]

Say hi to Leo

- Graph from “leopard” mesh structure with 4’567 nodes and 13’650 edges (9’078 faces)
  - Head of the animal contains 1534 nodes
  - Tail of the animal contains 228 nodes




1
λ=-0.000
2
λ=0.028
3
λ=0.059
4
λ=0.063
5
λ=0.075
6
λ=0.077
7
λ=0.178
8
λ=0.252
9
λ=0.269

Graph Slepian design

- Graph Slepian design problem:

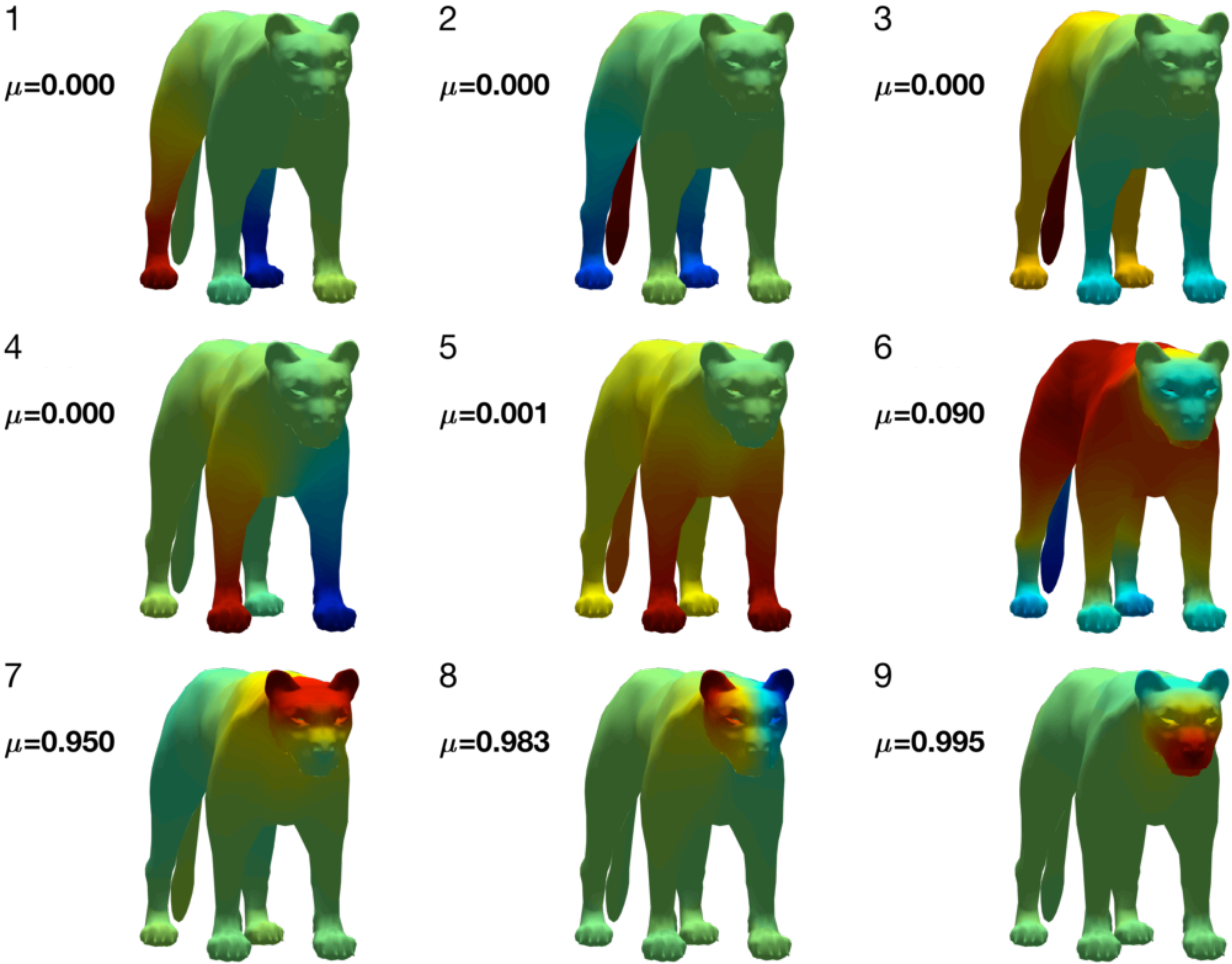
Find band-limited graph signals with maximal energy concentration in selected subgraph $\mathcal{S}$ (indicated by $\boldsymbol{S}$)

$$\mu = \frac{\hat{\boldsymbol{g}}^T \boldsymbol{U}_W^T \boldsymbol{S} \boldsymbol{U}_W \hat{\boldsymbol{g}}}{\hat{\boldsymbol{g}}^T \hat{\boldsymbol{g}}}, \quad \text{(Rayleigh quotient)}$$

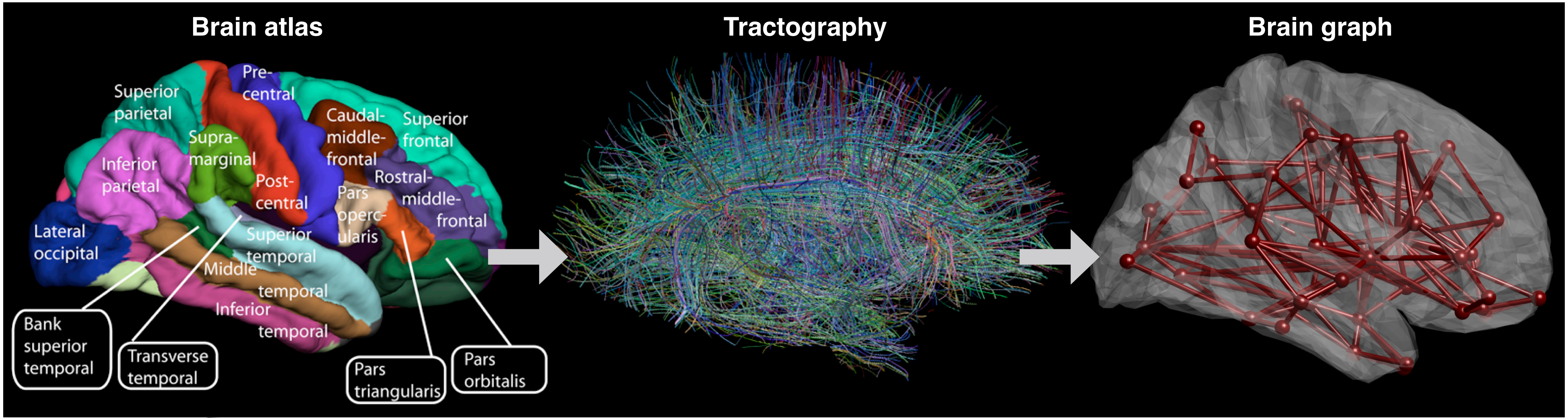
where $\boldsymbol{U}_W$ only contains first N_W GFT basis vectors, and $\boldsymbol{C} = \boldsymbol{U}_W^T \boldsymbol{S} \boldsymbol{U}_W$ is the concentration matrix. Graph Slepians are then given by $\boldsymbol{g} = \boldsymbol{U}_W \hat{\boldsymbol{g}}$.

- By convention, we order the Graph Slepian according to decreasing energy concentration: $1 > \mu_1 \geq \mu_2 \geq \ldots > 0$
- Double orthogonality property:
  - Orthonormal over entire graph: $\boldsymbol{g}_k^T \boldsymbol{g}_l = \delta_{k-l}$
  - Orthogonal over subgraph: $\boldsymbol{g}_k^T \boldsymbol{S} \boldsymbol{g}_l = \mu_k \delta_{k-l}$

[Tsitsvero et al, *IEEE TSP,* 2016*;* Van De Ville et al, *IEEE Signal Processing Letters,* 2017]


1
μ=0.000
2
μ=0.000
3
μ=0.000
4
μ=0.000
5
μ=0.001
6
μ=0.090
7
μ=0.950
8
μ=0.983
9
μ=0.995

Human structural connectome

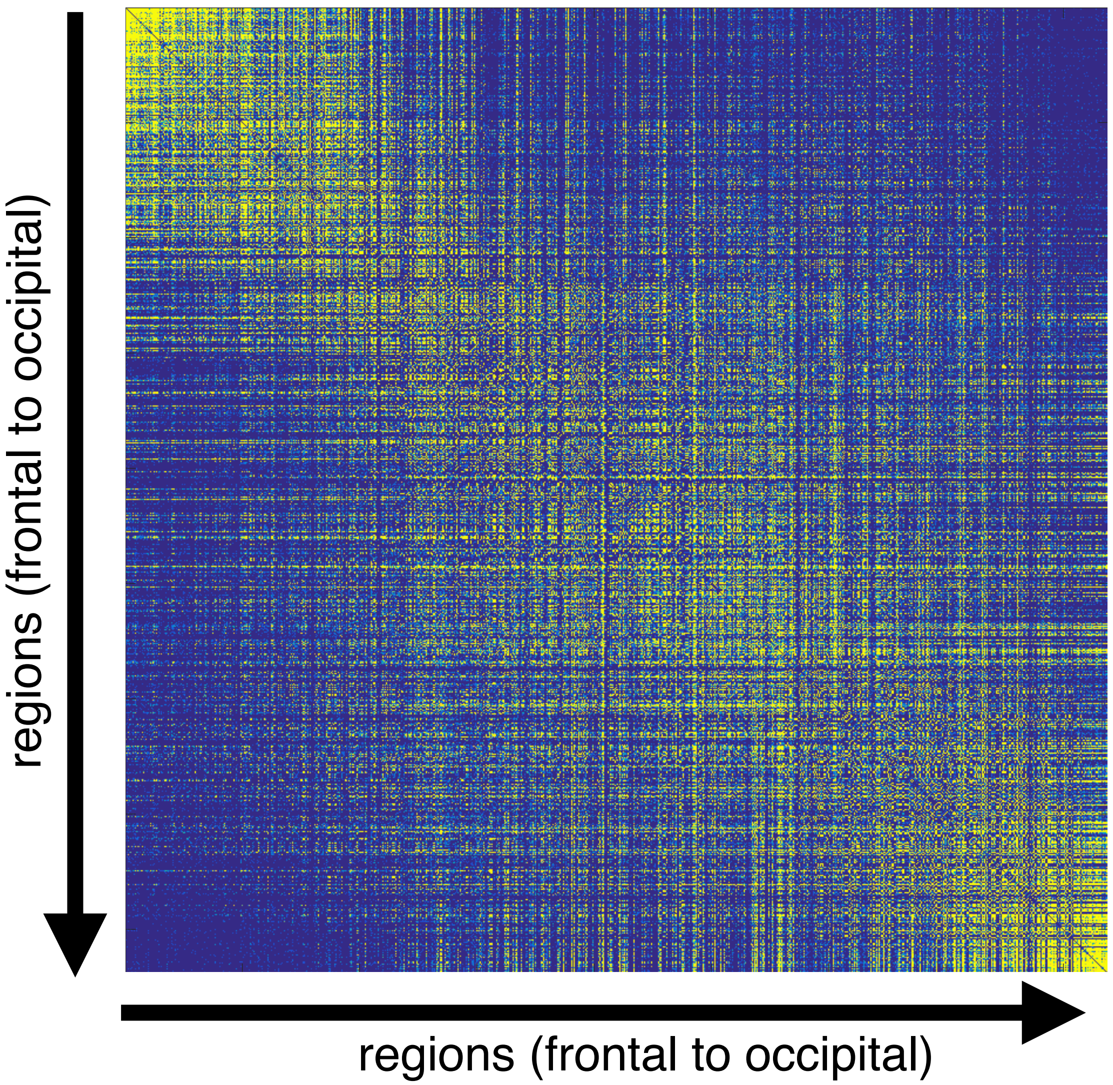


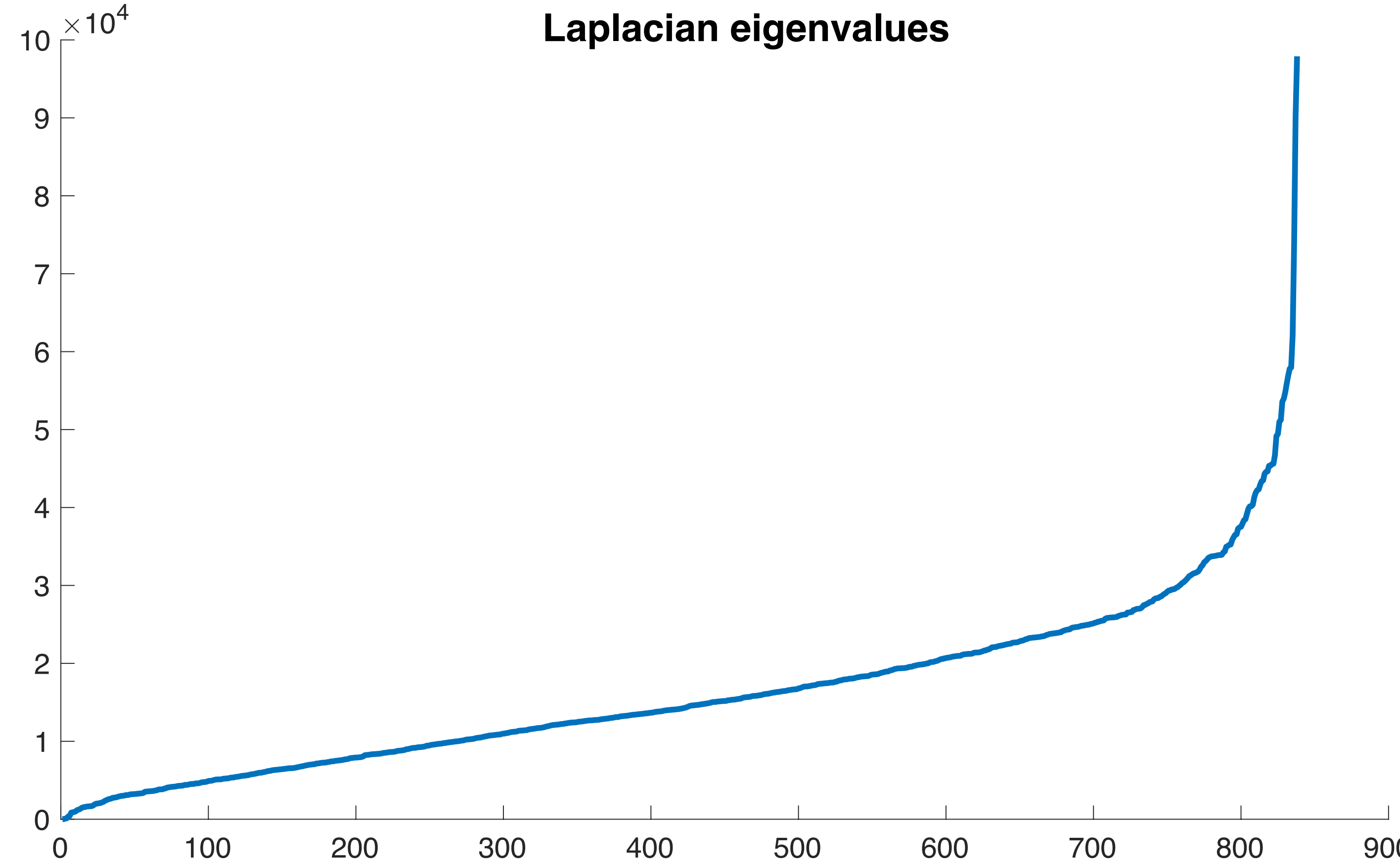
- A healthy subject from HCP: structural, diffusion and fMRI data
- Parcellation: Craddock *atlas1* (950 ROIs), from spectral clustering of RS
  - Pruning regions with no structural connectivity or flat timecourses: 838 ROIs
- Graph: Adjacency matrix derived from tractography
  - multi-shell multi-tissue response function estimation, spherical deconvolution, tractogram generation with 107 output streamlines
- Graph signal: task fMRI, averaged in the same 838 ROIs

[Craddock et al, *Hum Brain Mapp*, 2012; Huang et al., Proc of the IEEE, 2018]

Human structural connectome

- Adjacency matrix and Laplacian eigenspectrum





Human structural connectome

- Graph Fourier transform basis vectors of the brain

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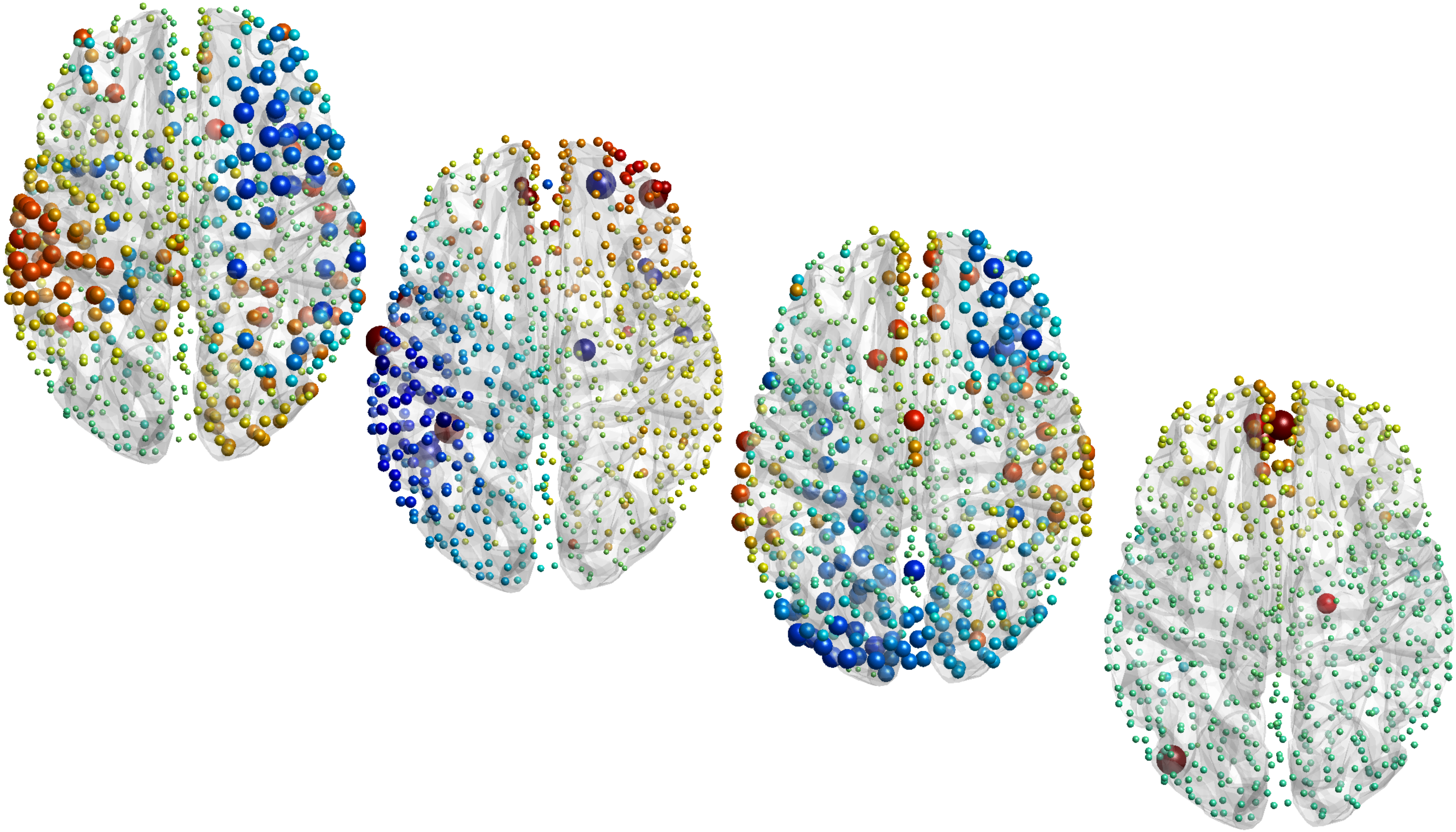
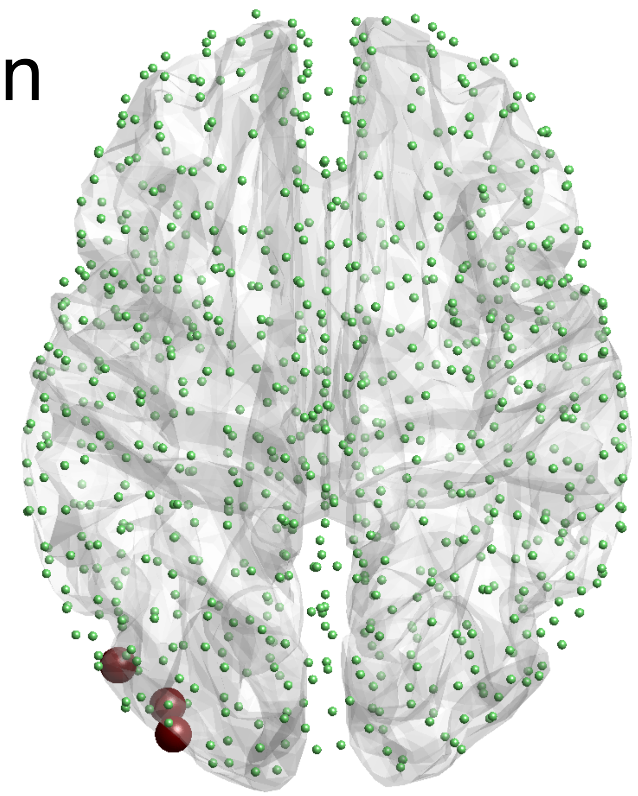


Exploring visual presentation

- Task data (different types of visual stimuli)
  - Sets of occipital brain regions are involved in these tasks
  - Selection of a restricted set of nodes
  - Exploring spectral low-dimensional representations



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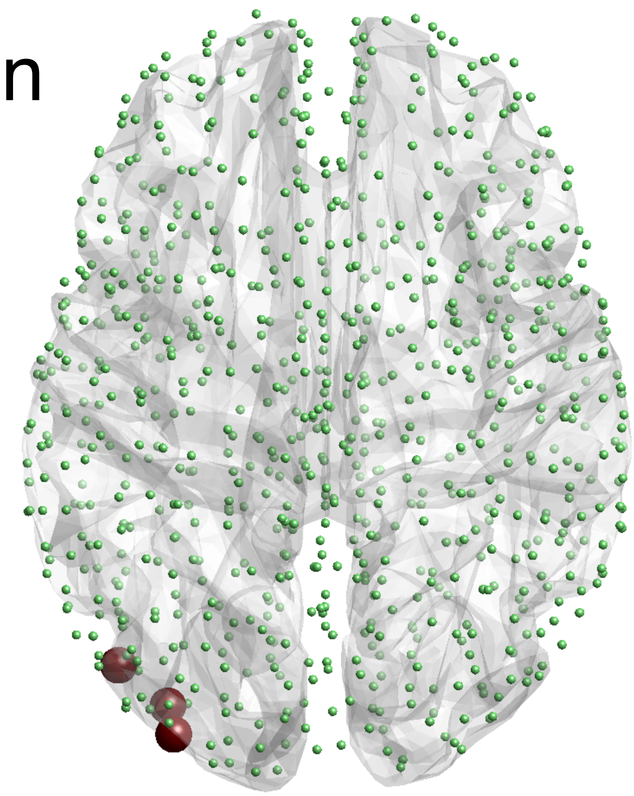
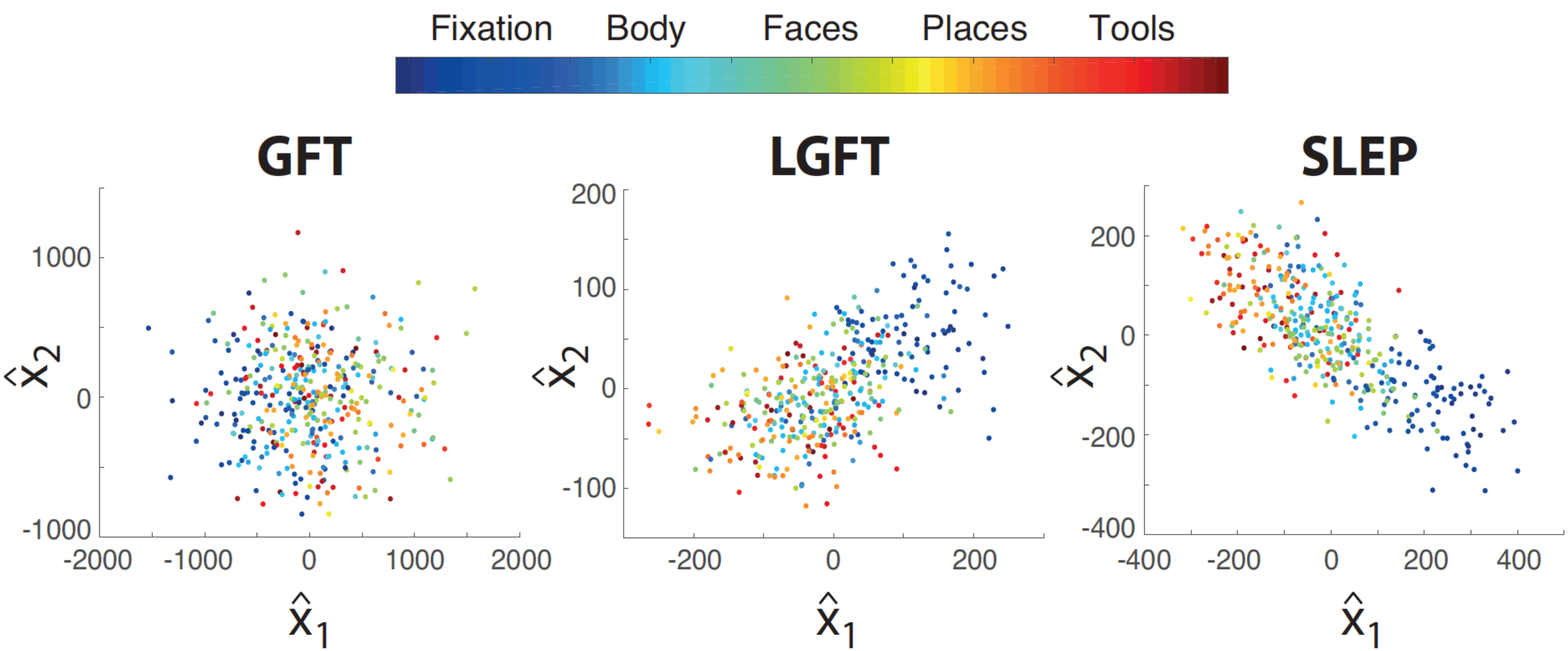


- What we expect from Slepians?
  - Lower bandwidth still reveals interactions with structurally nearby regions
  - Higher bandwidth improves localization, but can run into numerical issues

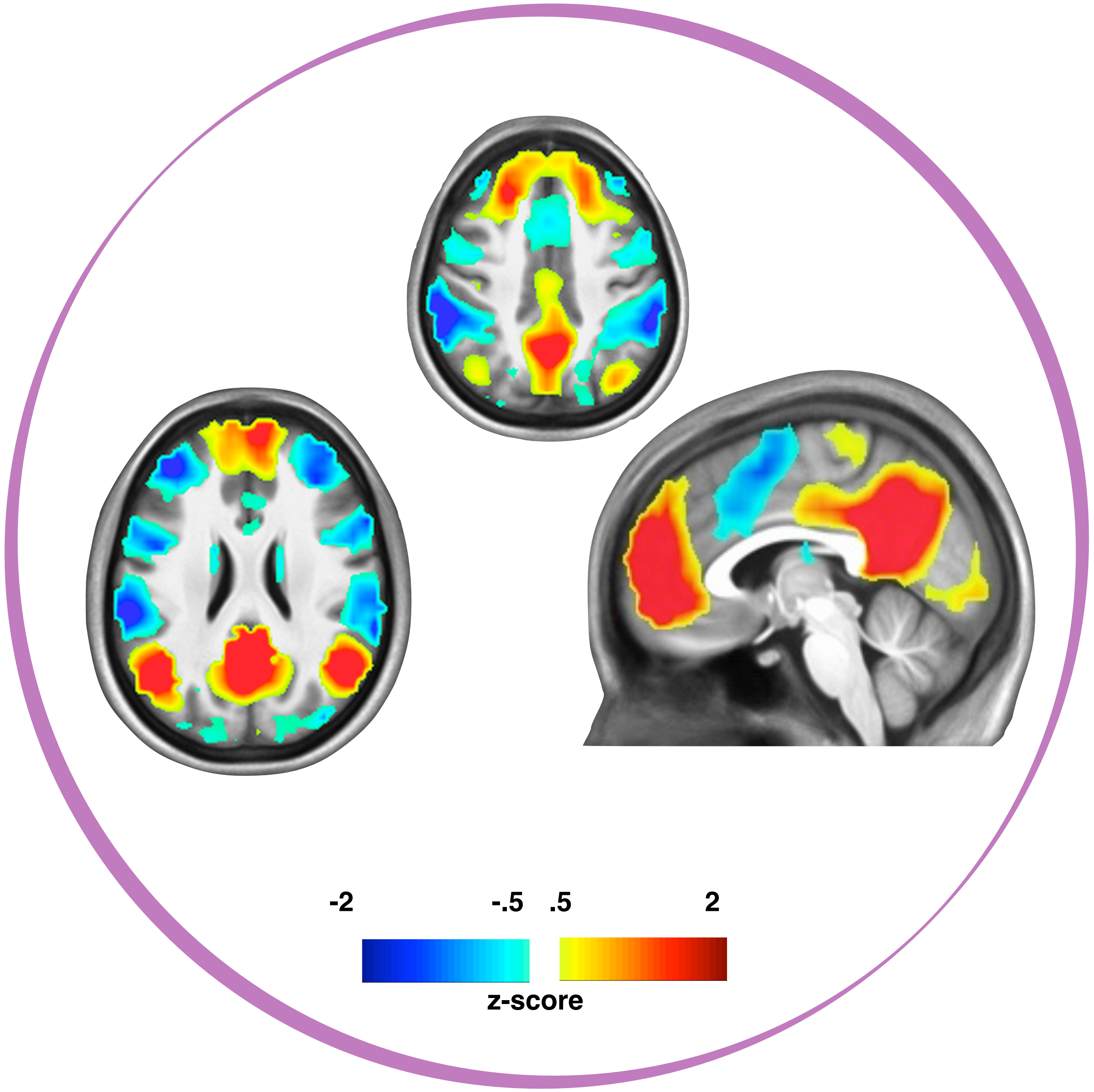
Graph Fourier Transform (**GFT**) ⟷ **SLEP** ⟷ Local **GFT** (**LGFT**)

Projection of eigenmodes/slepians

- Spectral coefficients linked to the two lowest frequencies
  - Global spectral measures do not segregate moments of fixation from moments of visual presentation
  - In the Slepians case, the different types of visual presentation could be distinguished



Investigating major functional networks



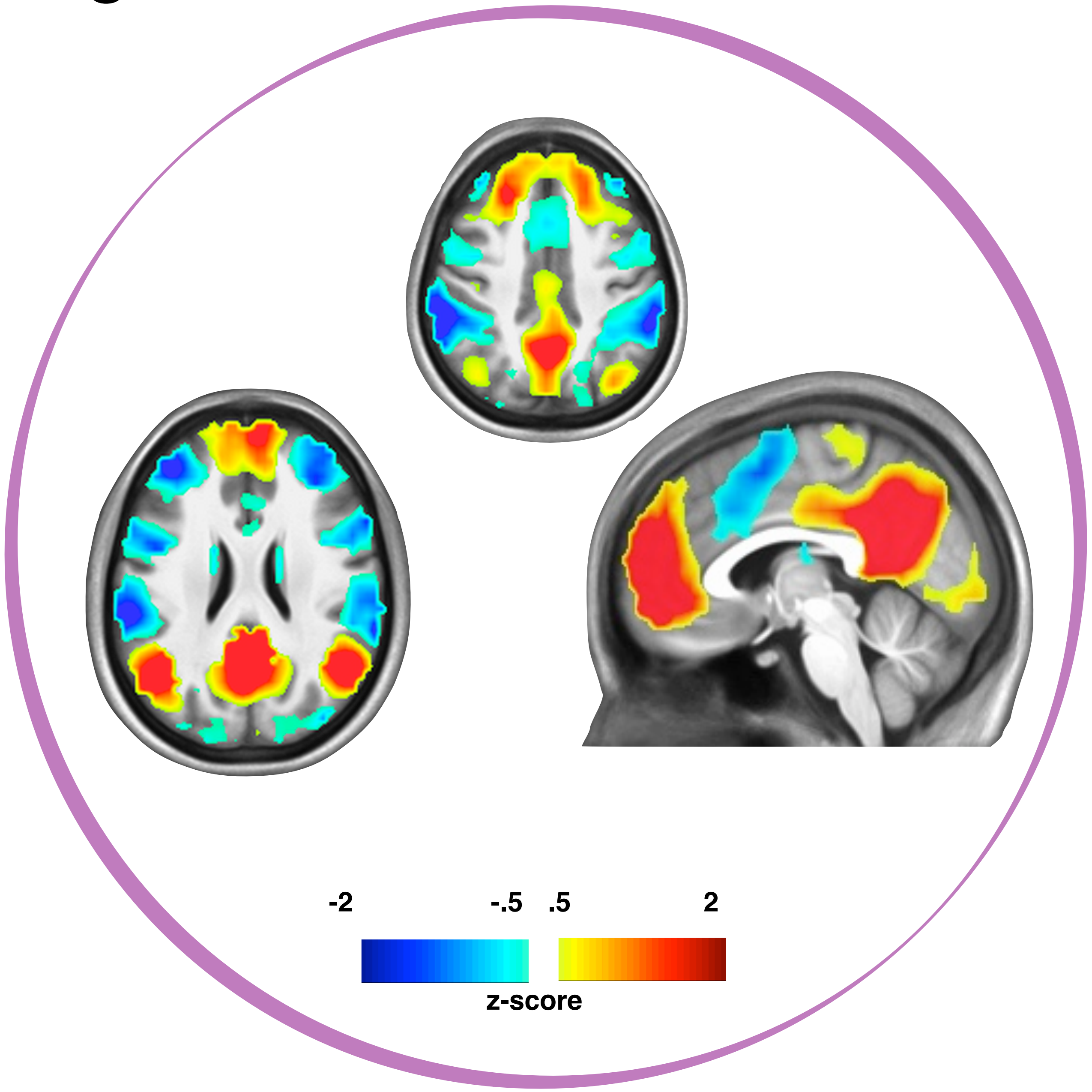
task-positive/fronto-parietal network

task-negative/default-mode network

[Christoff et al., *Nat Rev Neurosci*, 2016]

Investigating major functional networks

- Sets of brain regions have been identified to be related to task-positive vs -negative involvement
- Their *interplay* is subject of ongoing study



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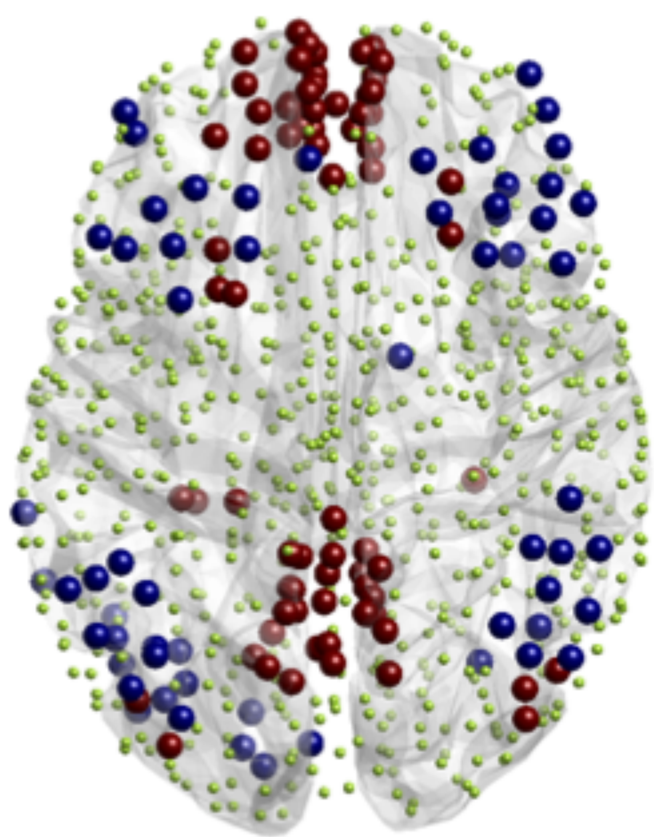
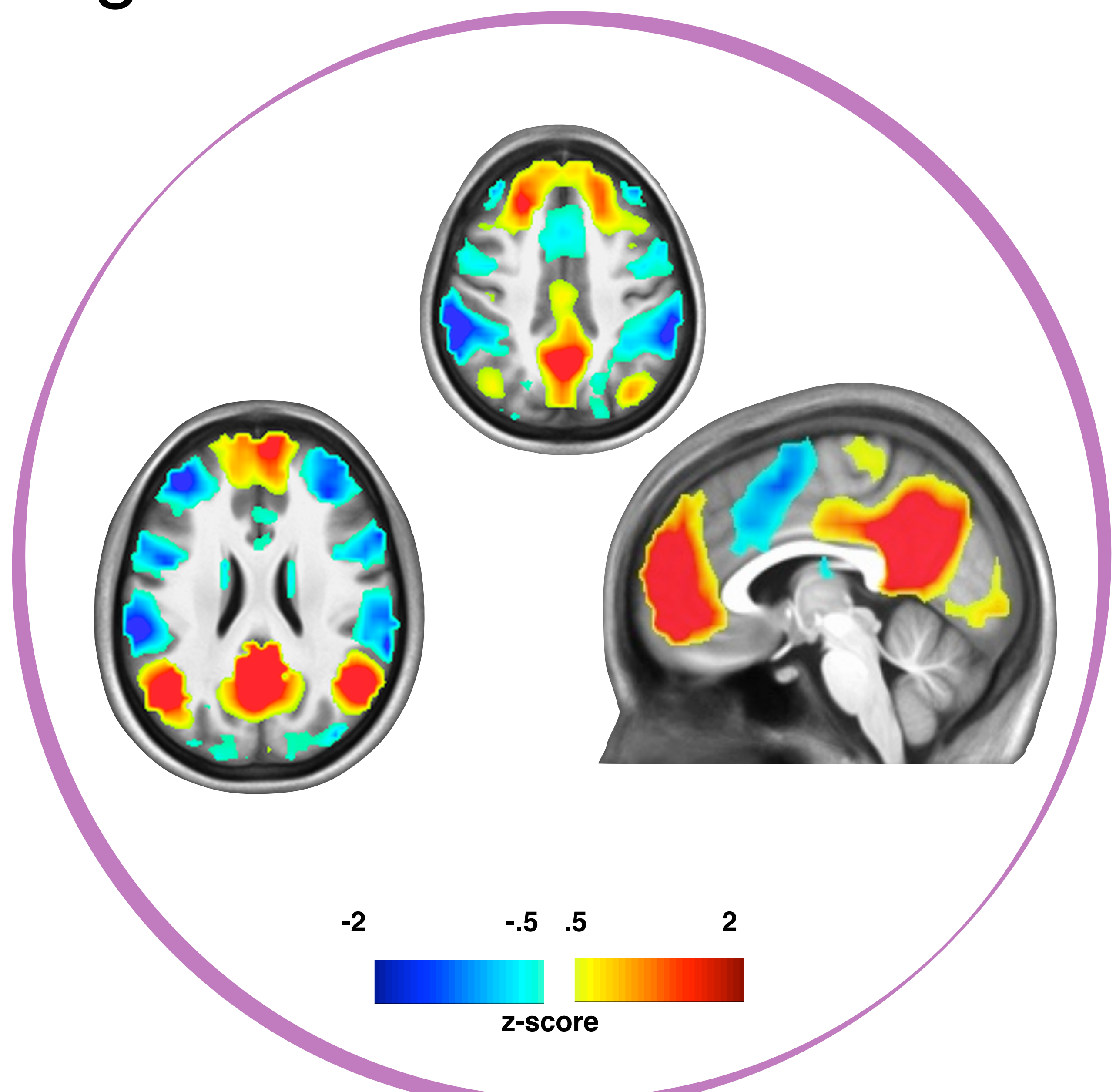
Investigating major functional networks

- Sets of brain regions have been identified to be related to task-positive vs -negative involvement
- Their *interplay* is subject of ongoing study
- Selection of subgraphs according to these two sets of regions

78 ROIs task-positive
60 ROIs task-negative





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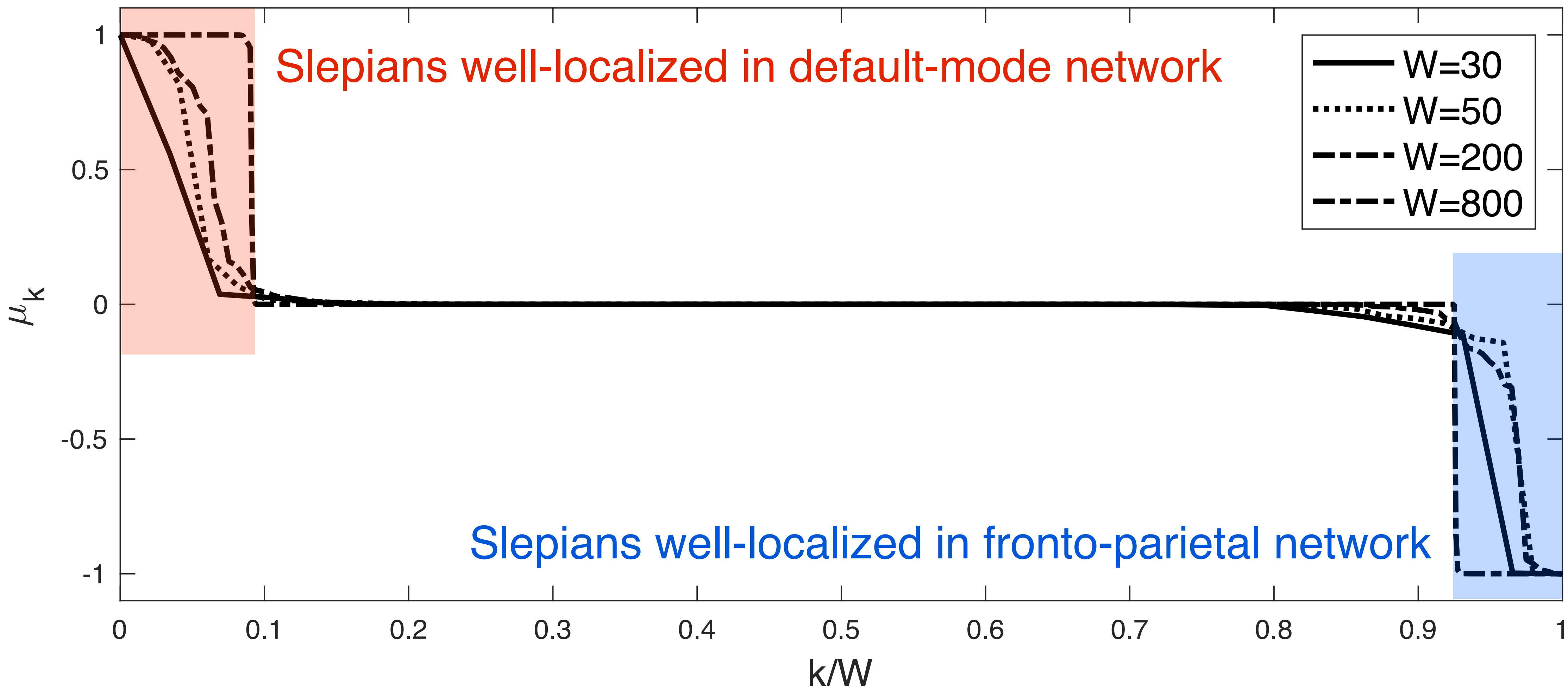
Slepian energy concentration spectrum



Two phase transitions: one for each type of subgraph

[Demesmaeker et al., arXiv:1710.11251, oct 2017]

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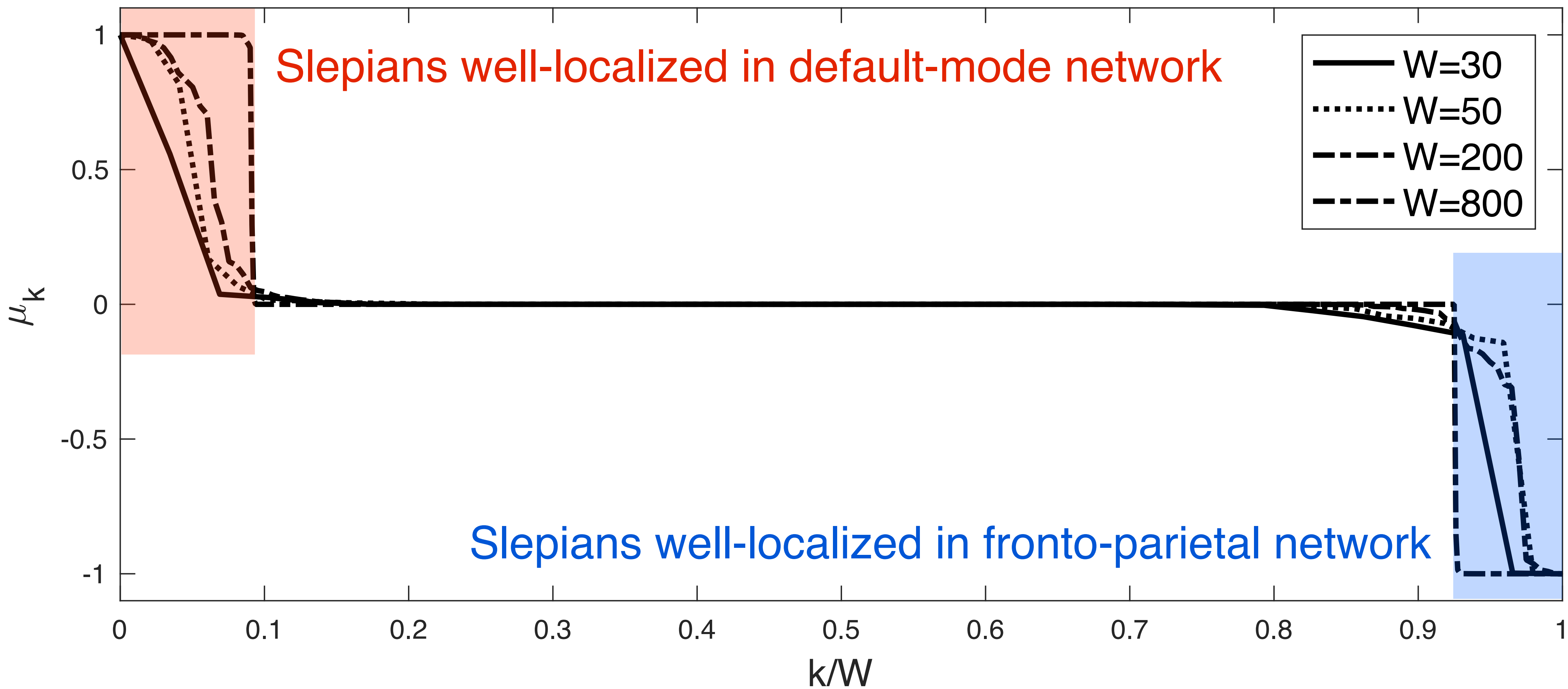
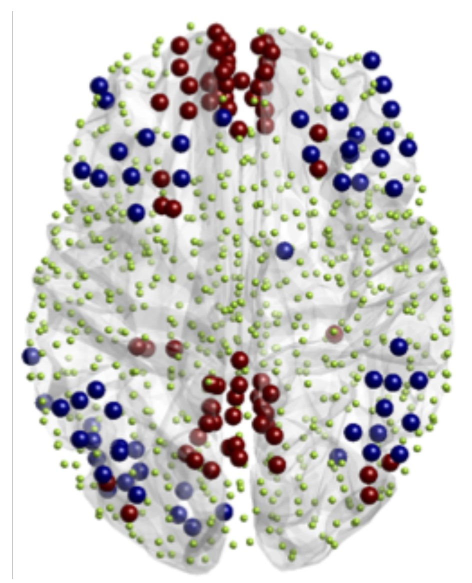


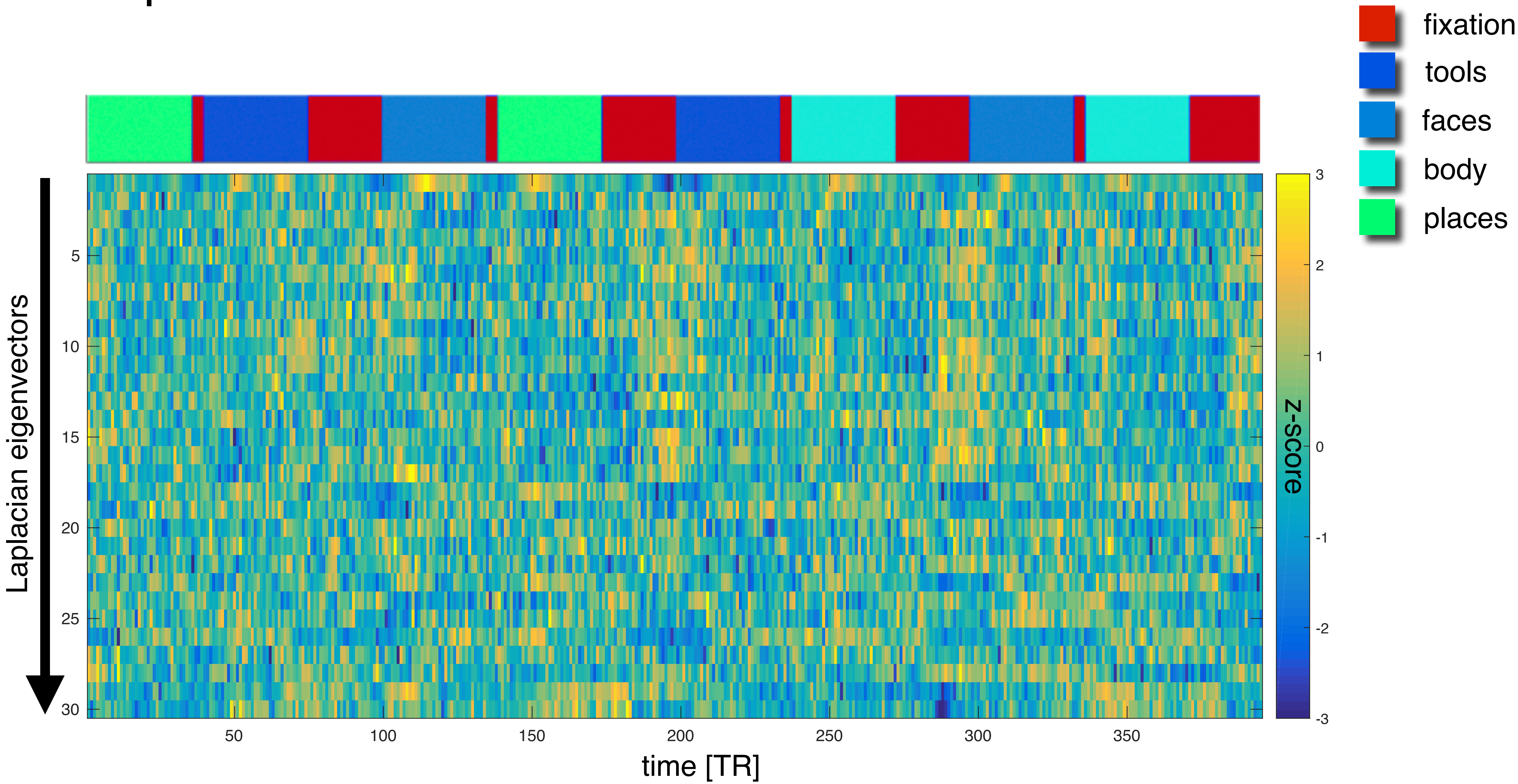


Two phase transitions: one for each type of subgraph

[Demesmaeker et al., arXiv:1710.11251, oct 2017]

Projecting on the Laplacian basis

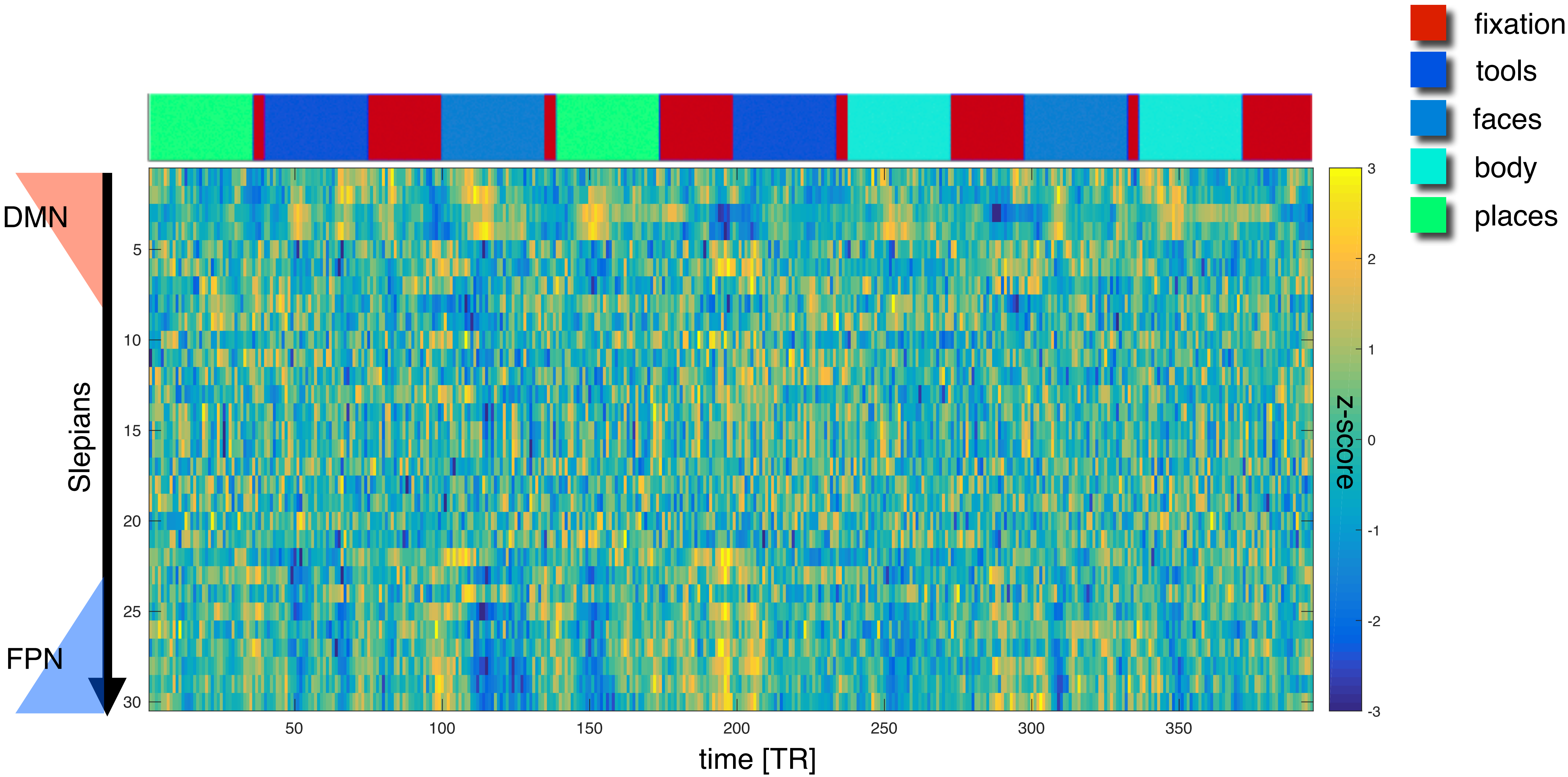
- Major structural backbone structure does not particularly capture features of functional data



[Van de Ville et al., Asilomar SSC, 2017]

Projecting on the Slepian basis

- Slepian basis, using same structural backbone, captures nicely *switches* of task-positive vs -negative patterns



[Van de Ville et al., Asilomar SSC, 2017]

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  - Combines graph Fourier transform with prior information
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  - Structural connectome provides "backbone" graph
  - Functional prior information provides selections of nodes
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  - Slepians as linear estimators for signal recovery: interactions with nearby regions can enhance denoising performances

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- What I didn’t talk about (see the proceedings)
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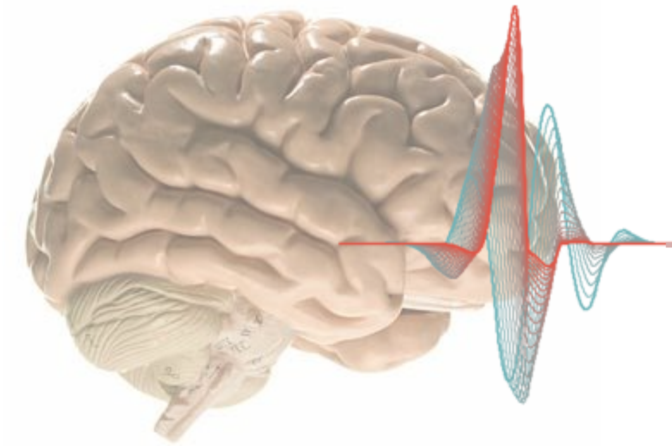
- Things to do:
  - More data, how to select bandwidth, ...



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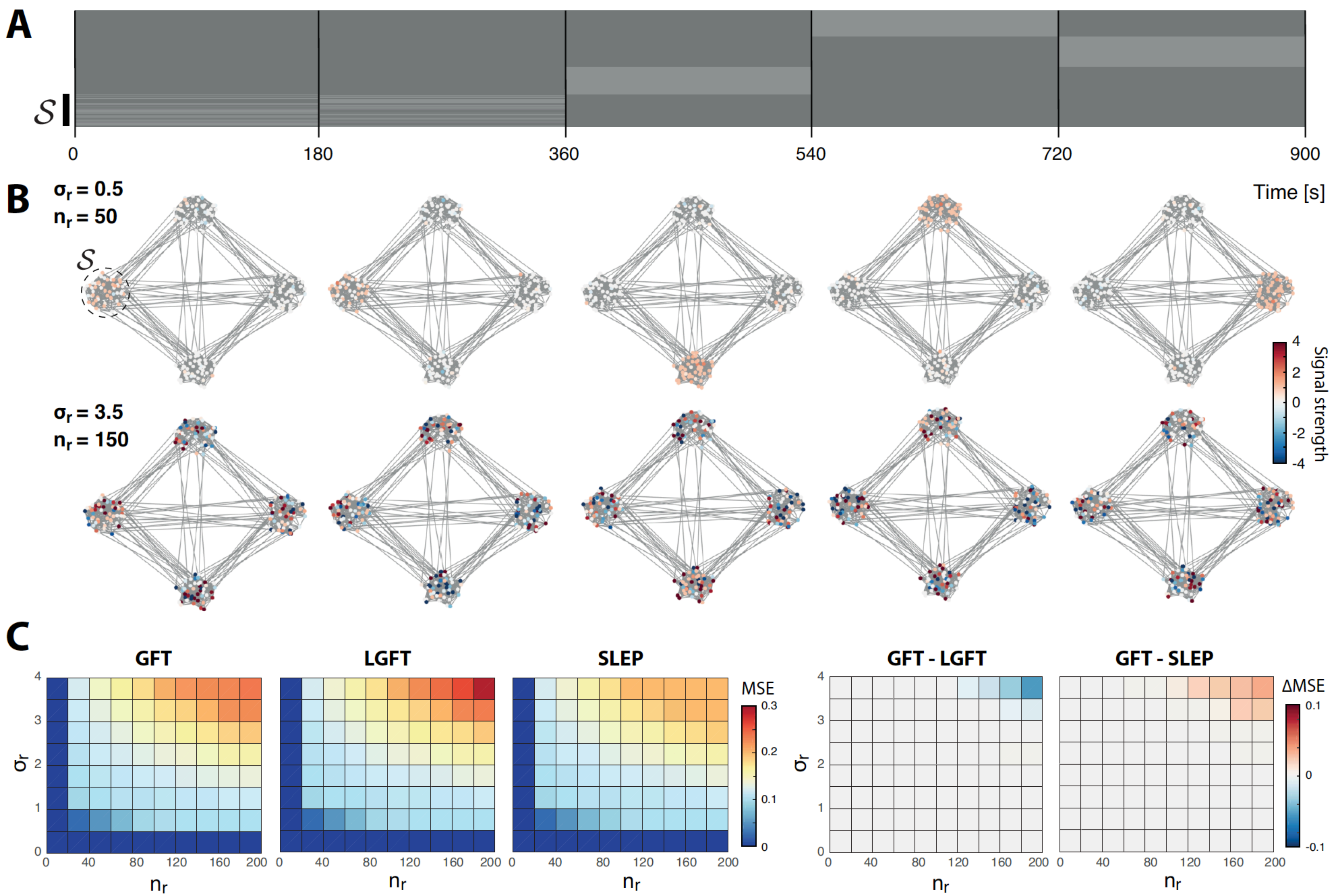
Slepians as linear estimators



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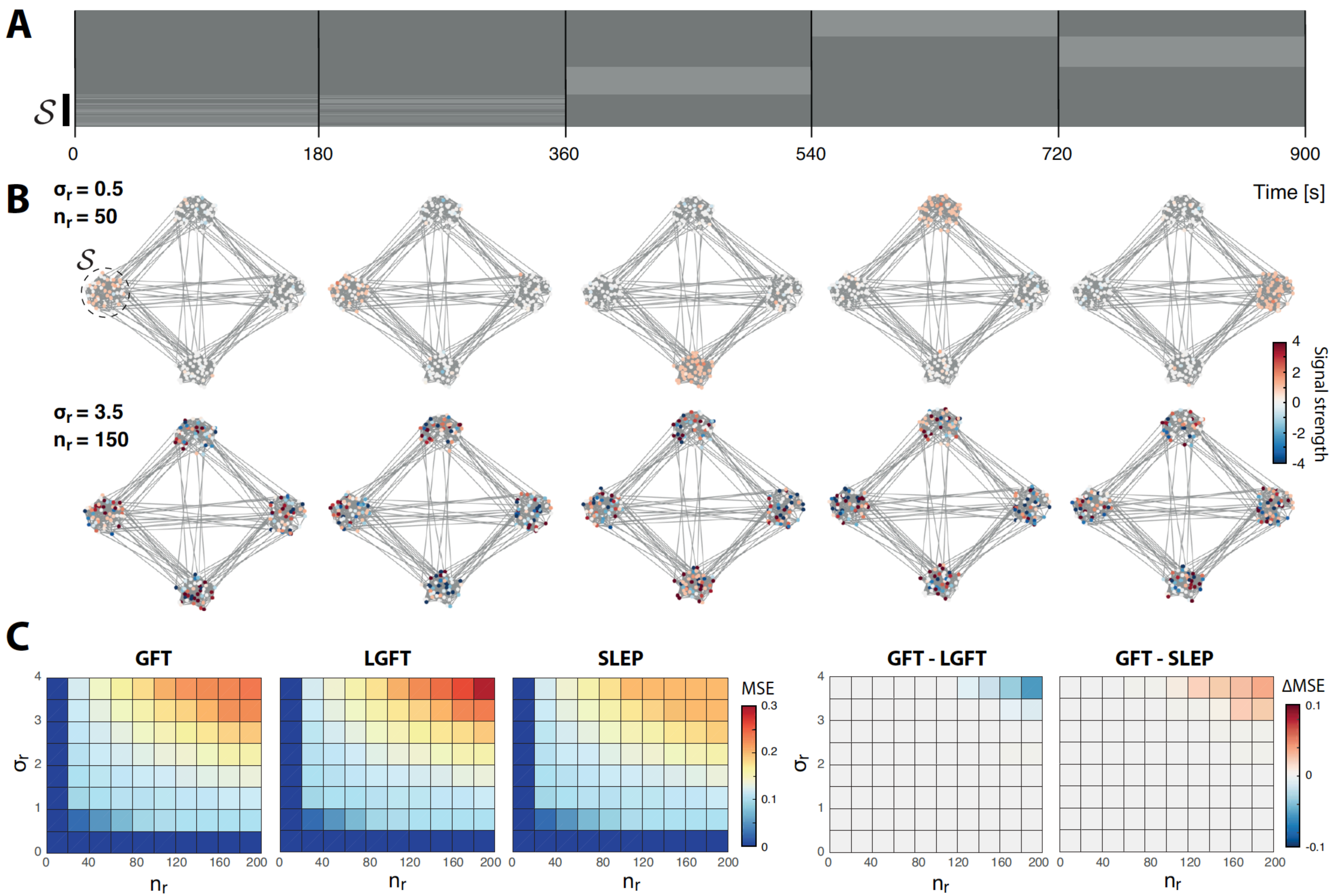

- Slepians provide better estimation at high noise levels
- More robust in the case of uncertain region selection